



Effective duration of callable corporate bonds: Theory and evidence

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Abstract

This paper computes the effective duration of callable corporate bonds, using a contingent-claims model that incorporates both default risk and call risk. The model generates empirical implications regarding the cross-sectional variation and the firm-specific determinants of duration, and demonstrates that the effect of the call feature is to shorten duration (except for low-grade bonds). The effective duration is also estimated empirically for a large sample of long-term corporate bonds, using monthly bond price and interest rate data. Cross-sectional regression analysis is used to test the empirical implications of the model regarding the determinants of effective duration, and the empirical results are quite supportive of the model's predictions.

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1. Introduction

The analysis of duration is an important component of the evaluation and control of risk in fixed-income securities. Duration is a widely used measure in bond investment and portfolio management and, not surprisingly, there is a large literature on

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this topic (see Bierwag and Gordon, 1990, or Fooladi et al., 1997, for a brief review). In this paper, we compute the theoretical effective duration of a callable corporate bond, using a contingent-claims model that also allows us to identify the firm-specific determinants of duration. Further, we test empirically the implications of the model, using a sample of long-term callable corporate bonds. There is very little research in the current literature on the duration of callable corporate bonds, and none (to our knowledge) on the cross-sectional determinants of duration. But this is surely an important issue in investing and hedging decisions, since most bond investors and portfolio managers take positions in individual bonds rather than entire bond indices.

Traditionally, analytical work on bond duration has focused on default-free non-callable bonds (see Bierwag, 1987, for an overview of this literature). Such analysis is not very useful for corporate bonds that are subject to default risk (always) and call risk (very often), since default or call will clearly alter the timing and amount of cash flows and thereby alter the sensitivity of bond value to interest rate changes (the effective duration). Recently, there have been a few papers that examine the duration of default-risky corporate bonds, using either reduced-form or structural models for bond valuation. Reduced-form models use default probabilities and recovery rates to capture default risk, e.g., Bierwag and Kaufman (1988), Fooladi et al. (1997), Skinner (1998), and Jacoby (2002). Structural models use the contingent-claims approach pioneered by Merton (1974), where the unlevered firm value (or the value of the firm's assets) is usually the underlying state variable. An advantage of such a contingent-claims model is that both default and call can be explicitly incorporated in the model, along with other option-like features embedded in the bond. Another advantage of a contingent-claims model is that it can examine firm-specific behavior, which is important because default risk and call risk are, after all, determined mostly by firm-specific factors such as firm risk (volatility), leverage ratio, etc. Using the contingent-claims approach, Chance (1990) derives the duration of a zero-coupon corporate bond, Longstaff and Schwartz (1995) and Babbel et al. (1997) derive duration when both firm value and interest rates are stochastic (and possibly correlated), and Leland and Toft (1996) derive the corporate bond duration when the interest rate is constant but the bankruptcy trigger is endogenous. The general conclusion of the theoretical research is that the effective duration of a default-risky bond is shorter than that of an otherwise identical default-free bond, and the difference increases with bond risk.²

There are also some empirical papers on duration of corporate bonds subject to default risk. Fons (1990) estimates the effective duration for a sample of corporate bonds, and finds it to be significantly shorter than the Macaulay duration, with the spread between the two being a function of the bond rating. Ilmanen et al. (1994) conclude that the traditional duration measure is a reasonable proxy only

² There are two exceptions: (i) Jacoby (2002) argues that default risk has no significant impact on duration of investment-grade bonds but increases duration for high-yield bonds, especially for short-maturity bonds; and (ii) Nawalkha (1996) shows that the duration of a (default-risky) corporate bond may be shorter or longer than the Macaulay (default-free) duration, depending on the correlation between the return on the firm's assets and changes in the short risk-free interest rate.

for high-grade corporate bonds. Landes et al. (1985) also conclude that the traditional Macaulay duration works reasonably well for high-grade (AAA or AA rated) bonds, but not for lower ratings. Finally, Ogden (1987) reports that the effective duration of a corporate bond is significantly shorter than the Macaulay duration, and shows (using a dummy variable) that certain embedded options (such as the call feature) have a significant impact on duration.

1.1. Callable bonds

A large number of corporate bonds are callable (Kish and Livingston, 1992), hence the effect of callability on bond duration should be of interest to fixed-income security researchers and portfolio managers. Since the call feature is likely to have a significant effect on bond duration, the traditional duration (either Macaulay or default-adjusted) is unlikely to be a good measure of interest rate risk for callable bonds. Nevertheless, there are only two theoretical papers, to our knowledge, on callable bond duration. Dunetz and Mahoney (1988) use option theory to incorporate the call option, but they express the callable bond duration in terms of non-callable bond duration and the *delta* of the call option. This formulation is not useful because the option delta cannot be computed without a valuation model; clearly the first step in computing callable bond duration is to derive the value of the bond. A forthcoming paper by Acharya and Carpenter (2002) has a callable bond valuation model with both default risk and interest rate risk. They derive some general duration results, e.g., both default and call feature shorten duration in isolation but each may lengthen duration in the presence of the other, and callable bond duration is an inverse-U shaped function of the asset value. However, their main focus is on valuation and optimal call/default policies, and they do not examine the determinants of duration; moreover, they ignore potentially relevant factors such as corporate taxes.

Further, neither of the two papers mentioned above carry out any empirical tests of callable bond duration. Indeed, there is very little empirical research on the duration of callable bonds. Ogden's (1987) empirical examination was limited to merely recognizing that the call feature had a significant effect on duration (using a dichotomous dummy variable to represent callability). Jacoby and Roberts (2002) compare default- and call-adjusted durations with Macaulay durations, and conclude that adjusted durations are significantly shorter; moreover, call adjustment is more important than default adjustment in their sample. However, they were not able to look at cross-sectional differences and firm-specific determinants of duration, since their study used a sample of bond indices. Since default and call risk are mostly firm-specific in nature, a study of effective duration of corporate bonds would be more useful, we feel, if the analysis included individual firm characteristics. Gebhardt et al. (2001) show that these firm-specific characteristics play an important role in the corporate bond market, hence it is reasonable to expect that they will also be important in determining the effect of interest rate risk on bond values.

This paper focuses on the effective duration of a callable corporate (default-risky) bond. Our objectives are to

- derive theoretically the effective duration of a callable default-risky corporate bond, using a contingent-claims model to incorporate both default risk and call risk
- compare the (theoretical) duration of callable and non-callable bonds
- identify theoretically the determinants of callable bond duration (comparative static analysis)
- estimate empirically the effective duration of corporate bonds from monthly price data, and empirically identify their determinants, and thereby test the implications of the model.

We use a structural model of corporate bond valuation, which allows us to incorporate rational default and call policies. The key findings of this paper are as follows. The effect of the call feature is to shorten duration, except for low-grade bonds. The important determinants of effective duration of callable corporate bonds are firm-specific characteristics such as leverage ratio, volatility, tax rate, and payout ratio. Empirical tests with callable corporate bond data support the model's predictions.

Our starting point is the contingent-claims valuation model for callable debt in Sarkar (2001). The model, incorporating both default risk and call risk, provides the bond price as an output. By computing bond prices at different interest rate levels, we are able to derive (numerically) the effective duration of the bond. Section 2 describes the valuation model for callable and non-callable bonds, and clarifies the important role played by default and call policies. Section 3 shows how to compute bond duration from the valuation model, and presents the main theoretical results including the determinants of duration under default and call risk. Section 4 empirically estimates the effective duration of a sample of long-term corporate bonds, and tests the cross-sectional implications of the model. Section 5 summarizes the main results and concludes.

The introduction of endogenous default and call triggers makes the model quite complicated, hence we make two simplifying assumptions in order to keep the model tractable. Although they are discussed in detail in the next section, we mention them here since they will limit somewhat the applicability of our model. First, we assume perpetual bonds; while this is not a good approximation for short- or intermediate-term bonds, it is quite reasonable for long-term bonds. Accordingly, we limit our empirical tests to long-term bonds. Second, we assume a constant interest rate, as in Bierwag (1987); Leland and Toft (1996), etc. Introducing stochastic interest rates would certainly make the model more realistic, but would significantly increase the complexity because there would be two state variables. Acharya and Carpenter (2002) incorporate stochastic interest rates in their model, although they abstract from frictions such as corporate taxation that might be relevant in the determination of duration. They focus mainly on valuation and optimal default/call policies and only derive some general results on duration. Their model does not investigate, for instance, the determinants of duration, and thus does not generate the set of testable hypotheses that our paper derives and tests.

2. The valuation model

The starting point is the contingent-claims model of Sarkar (2001). There exists a firm whose unlevered value (i.e., value of the firm's assets) V follows the standard continuous diffusion process with constant proportional volatility:

$$\frac{dV}{V} = (\mu - \delta) dt + \sigma dZ, \quad (1)$$

where μ is the expected return, δ the payout rate (constant fraction of firm value paid out to all security holders), σ the volatility, and dZ the increment of a standard Brownian Motion Process.

There is an outstanding issue of callable debt of infinite maturity³ with a continuous coupon rate of c (as a fraction of the face value $\$F$; that is, the coupon payment⁴ is $\$cF$ per unit time). The call premium p is also expressed as a fraction of the face value; thus the call price is $\$(1 + p)F$, which is what the bondholders receive when the bond is called by the firm. There exists a risk-free asset that pays a continuous constant⁵ interest rate r . Finally, the effective tax rate for the firm is given by a constant τ . If there are no personal taxes, then τ is simply the corporate tax rate. In the presence of personal taxes, however, τ will depend on the corporate tax rate as well as the personal tax rates on income from debt and equity, as discussed in Leland (1994, footnote 27, page 1231).

When V falls to a default-triggering level Θ , equity holders declare bankruptcy and bondholders take over the assets of the firm after incurring proportional bankruptcy costs of α , where $0 \leq \alpha \leq 1$. Thus, the bankruptcy cost is $\alpha\Theta$, the payoff to bondholders is $(1 - \alpha)\Theta$, and equity holders are left with nothing. In our model, the default-triggering level Θ is determined endogenously as an optimal decision by shareholders. That is, in Leland's (1994) terminology, this callable bond is not protected debt (as would be the case with an exogenously specified default-triggering level). This specification is more realistic since most long-term corporate debt is unprotected in practice (Leland, 1994).

The callable bond ceases to exist when it is called, at which time the bondholder receives the call price in exchange for the bond. Suppose the firm's policy is to call the

³ This is a common assumption in the callable bond literature, e.g., Mauer (1993); Fischer et al. (1989), etc. With long-maturity bonds, the return of principle has negligible value and can be ignored (Leland, 1994). Moreover, infinite maturity permits time-independent valuation formulas and keeps the analysis tractable.

⁴ Note that the total payout rate from operations is given by δV (from Eq. (1)). If this is insufficient to make coupon payments, the shortfall is financed by issuing additional equity. Of course, shareholders will do this only if it is optimal to keep the firm going; otherwise they will default on the debt, as discussed below. See Leland (1994, footnote 12) for a further discussion of this point.

⁵ There are two factors driving the call decision – default risk and interest rate risk. Interest rate risk has been examined by Brennan and Schwartz (1977), Kraus (1983), Mauer (1993), Sarkar (1997), etc., and default risk by Fischer et al. (1989), Longstaff and Tuckman (1994), Leland and Toft (1996), etc. We have decided to follow the approach of Leland and Toft (1996) and focus on default risk. In any event, the effect of default risk is of interest because of the cross-sectional implications from such a model.

bond when the state variable V reaches an upper trigger level V^* . Thus, at the first instant that $V = V^*$, the bond is called and replaced by a perpetual non-callable⁶ bond with the same dollar coupon amount ($\$cF$ per unit time). In the process, the firm incurs a refunding cost, given by β times the value of the replacement (non-callable) bond, where $0 \leq \beta \leq 1$. Hence, β is just the fractional refunding (or flotation) cost; we have assumed it is constant, but it can be made variable (see Mauer, 1993). Like the default trigger, the call trigger V^* is also determined optimally by the firm.

2.1. Non-callable bond valuation

In the model described above, if the existing bond is non-callable, its value is given by

$$NCP(V) = \frac{cF}{r} - \left[\frac{cF}{r} - (1 - \alpha)V_B \right] \left(\frac{V}{V_B} \right)^{\gamma_2}, \tag{2}$$

where γ_2 is defined in Eq. (5) below, and V_B is the default trigger with non-callable debt (i.e., with non-callable debt, the firm will default when the value V falls to V_B). For the derivation of the non-callable bond value and the analytical expression for V_B , please refer to Sarkar (2001).

2.2. Callable bond valuation

Also as shown in Sarkar (2001), the value of a callable bond in the model described above is given by

$$P(V) = \frac{cF}{r} + P_1 V^{\gamma_1} + P_2 V^{\gamma_2}, \tag{3}$$

where

$$\gamma_1 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2}, \tag{4}$$

$$\gamma_2 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\frac{2r}{\sigma^2} + \left(\frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2}, \tag{5}$$

and

$$P_1 = \frac{F \left(1 + p - \frac{c}{r} \right) \Theta^{\gamma_2} - \left[(1 - \alpha)\Theta - \frac{cF}{r} \right] (V^*)^{\gamma_2}}{(V^*)^{\gamma_1} \Theta^{\gamma_2} - (V^*)^{\gamma_2} \Theta^{\gamma_1}}, \tag{6}$$

⁶ The assumption of replacement by a non-callable bond is a standard one in the refunding literature; the arguments justifying this assumption are given by Mauer (1993) and others.

$$P_2 = \frac{\left[(1 - \alpha)\Theta - \frac{cF}{r} \right] (V^*)^{\gamma_1} - F \left(1 + p - \frac{c}{r} \right) \Theta^{\gamma_1}}{(V^*)^{\gamma_1} \Theta^{\gamma_2} - (V^*)^{\gamma_2} \Theta^{\gamma_1}}. \quad (7)$$

In Eqs. (6) and (7), Θ is the default trigger and V^* the call trigger.

2.3. Optimal default and call triggers

The default and call triggers are optimally chosen in our model, by maximizing the ex-post value of equity, as in Leland (1994) and Sarkar (2001). To compute the optimal triggers, Eqs. (8) and (9) below must be solved simultaneously:

$$\Theta^* + \gamma_1 (\Theta^*)^{\gamma_1} [T_1 - B_1 - R_1 - P_1] + \gamma_2 (\Theta^*)^{\gamma_2} [T_2 - B_2 - R_2 - P_2] = 0, \quad (8)$$

$$\begin{aligned} & \gamma_1 (V^*)^{\gamma_1} [T_1 - B_1 - R_1 - P_1] + \gamma_2 (V^*)^{\gamma_2} [T_2 - B_2 - R_2 - P_2] \\ & + \gamma_2 \left(\frac{V^*}{V_B} \right)^{\gamma_2} [\tau cF/r + \alpha V_B + (1 - \tau)\beta(V_B(1 - \alpha) - cF/r)] = 0, \end{aligned} \quad (9)$$

where the terms T_1 , T_2 , B_1 , B_2 , R_1 , R_2 are all defined in Sarkar (2001). Since Eqs. (8) and (9) have no analytical solutions, they are solved numerically.

The determination of the optimal call and default triggers is an important step in our model, because they can have a significant impact on debt value, as discussed below. A number of parameters affect the bond value (and thereby the duration) indirectly via their effect on the two triggers. Incidentally, this aspect of default and call risk has generally been ignored in the existing duration studies.

2.4. A Numerical illustration

Since there are no closed-form expressions for the default and call triggers, we illustrate the results numerically. For base case, we use the parameter values:⁷

$$\begin{aligned} \sigma &= 0.3, \quad c = 7\%, \quad F = 100, \quad p = 10\%, \quad r = 6\%, \\ \delta &= 3\%, \quad \alpha = 0.5, \quad \beta = 0.01, \quad \tau = 0.15. \end{aligned}$$

Solving Eqs. (8) and (9) simultaneously, we get the following optimal default and call triggers: $\Theta^* = 47.22$ and $V^* = 1329.87$. The corresponding non-callable default trigger is $V_B = 47.24$.

The default and call triggers will be affected by the various parameters, as discussed in Sarkar (2001). What is of particular interest here is the effect of interest rate, since we are studying the interest rate sensitivity of the bonds. Repeating the computation with different interest rates, we observe that the call trigger is an increasing function of r , rising from 307.01 to 1329.87 as r is increased from 2% to

⁷ As in Leland (1994, footnote 27), we arrive at an effective tax rate of 15% as follows: if the effective personal tax rate is 20% (reflecting tax deferment), the tax rate on bond income is 40%, and the corporate tax rate is 35%, then the effective tax advantage of debt (or the effective tax rate) τ is $[1 - (1 - 0.35) \times (1 - 0.2)/(1 - 0.4)] = 13.3\%$. Hence we use 15% for the base case.

6%. The default trigger is a decreasing function of r , falling from 58.58 to 47.22 as r is increased from 2% to 6%. (Incidentally, the non-callable default trigger V_B is also a decreasing function of interest rate).

Another result of interest is that the optimal default trigger with callable debt is always lower than that with non-callable debt (or $V_B > \Theta^*$). The economic intuition behind this result is quite simple: equity holders will default on the debt payments when equity value falls to zero, i.e., conditions are so bad that there is no likelihood of equity value rising again. With callable debt, however, the upside potential for equity value is greater because equity holders can effectively place an upper limit on debt value by calling the debt. That is, when the company is doing well, equity holders will be better off if the debt is callable. Therefore, equity holders are more willing to accept short-term adversity and keep the firm alive (i.e., less willing to default), when there is callable (rather than non-callable) debt in the capital structure.

This is a relevant point when studying corporate bond durations. What it implies is that the call feature reduces default risk by pushing the default trigger further down. Thus, the call feature indirectly delays the bankruptcy decision, and thereby lengthens the duration of the bond. When the company is close to default (bond rating is very low, or leverage ratio is very high), therefore, a callable bond should have a longer duration than a non-callable bond. Thus, although a call feature generally shortens the duration of a bond by reducing its effective maturity, it can also lengthen bond duration when the firm is close to defaulting. This point is discussed further in Sections 3 and 4.

2.4.1. Bond value and duration

The bond value $P(V)$ is a function of the firm value V . The behavior of bond value with respect to interest rate is illustrated in Figs. 1–3 for three different values of V .

In Fig. 1, the firm value $V = 60$ is close to the default trigger. Note that there is no difference between callable and non-callable bond values for high interest rates. This is because, at high interest rates, the call trigger reaches very high levels which implies that a call is virtually ruled out; thus the callable bond behaves just like a non-callable bond. (This is true for all cases, Figs. 1–3).

Also, it is well known that the value of a default-free bond is a monotonically decreasing function of interest rate. However, in Fig. 1, as the interest rate is reduced, both bond values first rise and then fall. This is because of default risk and the behavior of the default trigger with respect to the interest rate. We saw above that the default trigger is a decreasing function of r . Thus, as r is reduced, there are two effects: (i) bond value increases because of the lower discount rate, and (ii) the default trigger gets closer, which reduces the bond value. For high r , the first effect dominates because the default trigger is still somewhat far away. But for lower r , the default trigger is closer to the current value of V ; hence the second effect dominates. The same argument holds for non-callable debt, since the non-callable default trigger is also a decreasing function of r . Note, however, that this behavior should not be observed for higher values of V ; this is exactly what we find in Figs. 2 and 3.

Finally, we note in Fig. 1 that, for low interest rates, the callable bond is worth more than the non-callable bond. This is a counter-intuitive result, and seems to sug-

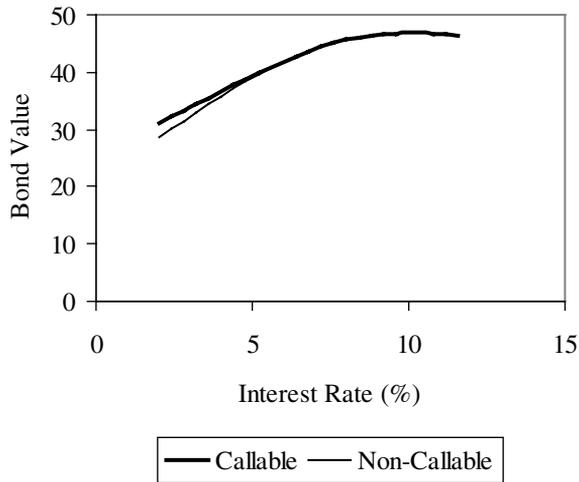


Fig. 1. Shows callable and non-callable bond value as a function of the interest rate, for $V = 60$. The following “base case” parameter values are used: $\sigma = 0.3$, $\delta = 3\%$, $\alpha = 0.5$, $\beta = 0.01$, $\tau = 0.15$, $c = 7\%$, $F = 100$, and $p = 10\%$.

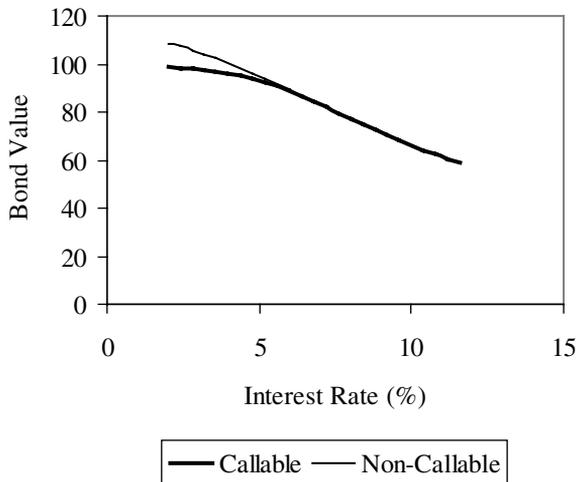


Fig. 2. Shows callable and non-callable bond value as a function of the interest rate, for $V = 180$. The following base case parameter values are used: $\sigma = 0.3$, $\delta = 3\%$, $\alpha = 0.5$, $\beta = 0.01$, $\tau = 0.15$, $c = 7\%$, $F = 100$, and $p = 10\%$.

gest a negative option value (option to call the bond). However, this is a result of the difference between callable and non-callable default triggers. For high r , there is virtually no difference between the two triggers, but for low r the non-callable default trigger is higher. For low V , therefore, the non-callable bond will be close to the

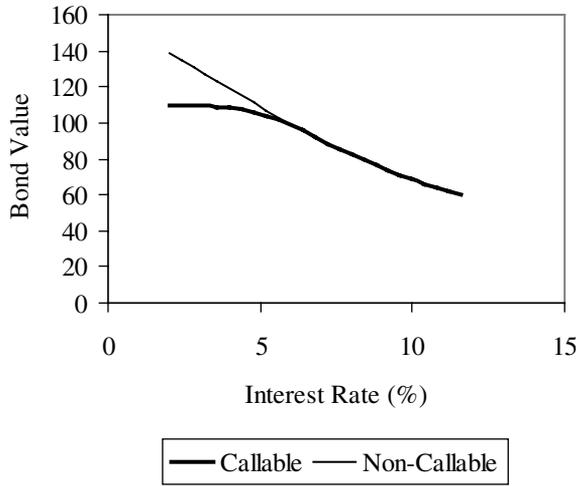


Fig. 3. Shows callable and non-callable bond values as a function of the interest rate, for $V = 300$. The following base case parameter values are used: $\sigma = 0.3$, $\delta = 3\%$, $\alpha = 0.5$, $\beta = 0.01$, $\tau = 0.15$, $c = 7\%$, $F = 100$, and $p = 10\%$.

default trigger. Since default involves significant bankruptcy costs, the non-callable bond will be worth less than the callable bond, explaining the apparent anomaly. But this is true only for low values of V (or high leverage ratios), when the bond is close to the default trigger. The result is reversed for higher V , as we see in Figs. 2 and 3.

Fig. 1 has two important implications for duration. The first implication is that duration is negative for both callable and non-callable bonds over most of the range (since the bond value curve is upward-sloping). The second implication is that callable bond has higher duration than non-callable (since the callable bond curve is flatter and both durations are negative). Therefore, when the firm is in financial distress (i.e., V is very close to the default trigger), both callable and non-callable debt should have negative durations and the callable bond should have the higher duration (more on this later).

Fig. 2 shows bond value as a function of interest rate, for $V = 180$. Here we note that both curves are downward sloping, implying positive durations. The firm value V is much larger than the default trigger, hence the first effect dominates, giving a positive duration. Unlike in Fig. 1, the callable bond value never exceeds the non-callable bond value. Since the callable bond price curve is flatter, callable duration should be smaller than non-callable duration. At low interest rates, the call trigger is lower, hence call risk is greater, and the difference between callable and non-callable widens. Thus, callable durations are shorter for low interest rates. As V gets closer to the call trigger (Fig. 3, with $V = 300$), the difference between callable and non-callable widens (the callable value curve flattens further). Thus, callable bond duration should be lower for large values of V (or low leverage ratios).

3. Theoretical results and empirical implications

The duration (D) of a bond is defined as follows:

$$D = -\frac{1}{P} \frac{\partial P}{\partial r}, \quad (10)$$

where P is the bond price and r the interest rate. Unfortunately, because of the complexities resulting from the two endogenous boundaries for V , an analytical expression could not be found for the duration. We therefore compute it numerically, by computing the bond price at three different interest rates. Suppose that, at the three interest rate levels ($r - \Delta r$), r and ($r + \Delta r$), the bond prices are P_1 , P_2 and P_3 respectively. Then the derivative is approximated by the following expression:

$$\frac{\partial P}{\partial r} \approx \frac{\Delta P}{\Delta r} = \frac{1}{2} \left(\frac{P_2 - P_1}{\Delta r} + \frac{P_3 - P_2}{\Delta r} \right). \quad (11)$$

By making Δr small enough, we can approximate duration fairly accurately.

3.1. Numerical illustrations

For the base case parameter values of Section 2, we computed bond values for $r = 0.059, 0.06$ and 0.061 . Using these bond values, we computed the duration using Eq. (11). The durations of callable and non-callable bonds for different values of V are shown in Table 1.

The figures in Table 1 are consistent with our expectations from the corporate bond price behavior illustrated in Figs. 1–3. The duration of a non-callable corporate bond is a monotonically increasing function of V (or a decreasing function of

Table 1
Computed callable and non-callable durations from the model

V	Duration	
	Non-callable	Callable
50	-12.78	-12.19
100	1.93	1.71
150	5.43	4.90
200	7.32	6.49
400	10.66	8.44
600	12.05	8.12
800	12.85	6.93
1000	13.38	5.21
1050	13.49	4.72
1100	13.59	4.21

The table shows duration of callable and non-callable corporate bonds computed from the model, for various levels of asset value V . The durations were computed using the base case parameter values:

$$\sigma = 0.3, c = 7\%, F = 100, p = 10\%, r = 6\%, \delta = 3\%, \alpha = 0.5, \beta = 0.01, \tau = 0.15.$$

leverage), and rises from -12.78 to 13.59 as V is increased from 50 to 1100. This is not surprising because, for a non-callable corporate bond, default risk is the most important factor. For very high values of V (or low leverage ratios), default risk becomes negligible and the duration approaches that of a risk-free bond (which is $1/r$ or 16.67 in the base case). As V is reduced (or leverage ratio increased), default risk becomes more serious and this reduces the duration of the bond accordingly.

The duration of a callable bond is, however, not monotonic in V (or leverage ratio). This happens because there are two sources of risk in a callable bond, default risk and call risk. For very low V (high leverage ratio) default risk is significant and this results in a shorter duration. On the other hand, for very high values of V (low leverage ratio), default risk becomes negligible but call risk becomes significant. The risk of an early call also shortens the duration of the callable bond. Therefore, as V is increased (or leverage reduced), the duration of the callable bond first rises and then falls. In the base case, the callable duration rises from -12.19 for $V = 50$ to 8.44 for $V = 400$, and then falls to 4.21 for $V = 1100$. Callable duration rises to only about half the Macaulay duration before it starts falling. The non-monotonicity of callable bond duration with respect to the leverage ratio is a major difference between callable and non-callable bonds. This result is consistent with Acharya and Carpenter (2002); see their Fig. 4, for instance.

Another feature worth noting in Table 1 is that default risk adjustment is more important for high leverage ratios (or for low-grade bonds) and call risk adjustment is more important for low leverage ratios (or for high-grade bonds). For example, with $V = 150$ (high leverage ratio), non-callable duration is 5.43 years (significantly smaller than the default-free or Macaulay duration of 16.67 years), whereas the callable duration is 4.90 years (only slightly smaller than the non-callable duration). Thus, for high leverage ratios, the adjustment for default risk is much more important than the adjustment for call risk. For $V = 1000$ (low leverage ratio), non-callable duration is 13.38 years (somewhat smaller than the default-free bond duration), but the callable duration is only 5.21 years (significantly smaller than the non-callable duration). For low leverage ratios (or high-grade bonds), therefore, call risk adjustment is much more important than default risk adjustment. This finding is consistent with Jacoby and Roberts's (2002) observation: "... we can comfortably conclude that callability causes a greater distortion for Macaulay duration when compared to the distortion caused by default risk for AAA bonds" (page 19).

We also note that callable duration is generally shorter than non-callable duration, which is not surprising since the call option gives the firm the right to terminate the bond prior to maturity. However, for very low firm value V (or very high leverage ratio), i.e., when default risk is high, callable duration exceeds non-callable duration. This is because the call feature reduces default risk (by lowering the default trigger) and thereby lengthens the effective maturity of the bond, as discussed in Section 2. This result implies that, for low-grade bonds only, callable bond durations can be longer than non-callable bond durations. Our empirical results (Section 4) confirm this intuition.

3.2. Determinants of callable bond duration

Here we report and discuss the comparative static results, which are summarized in Table 2. The results are displayed for three scenarios: $V = 100, 170$ and 320 . These

Table 2
Comparative static results for callable bond durations

V	Callable duration for		
	$\sigma = 0.275$	$\sigma = 0.300$	$\sigma = 0.325$
100	1.2472	1.7051	2.0496
170	5.6708	5.6485	5.5775
320	7.8900	8.1097	8.1143
	$p = 5\%$	$p = 10\%$	$p = 15\%$
100	1.4172	1.7051	1.9259
170	4.8818	5.6485	6.2370
320	6.2112	8.1097	9.5528
	$\tau = 0.05$	$\tau = 0.15$	$\tau = 0.25$
100	1.0244	1.7051	2.4861
170	5.2456	5.6485	6.1143
320	7.9507	8.1097	8.2804
	$c = 6\%$	$c = 7\%$	$c = 8\%$
100	3.4100	1.7051	-0.1322
170	7.2732	5.6485	3.1124
320	10.3882	8.1097	3.1961
	$\delta = 3\%$	$\delta = 4\%$	$\delta = 5\%$
100	1.5184	1.7051	1.8239
170	5.6791	5.6485	5.5139
320	7.8195	8.1097	8.1797
	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
100	3.4818	1.7051	-0.4454
170	6.6503	5.6485	4.4998
320	8.4730	8.1097	7.6084
	$\beta = 0$	$\beta = 1\%$	$\beta = 2\%$
100	1.7347	1.7051	1.6789
170	5.6667	5.6485	5.6354
320	8.1271	8.1097	8.1029
	$r = 5\%$	$r = 6\%$	$r = 7\%$
100	0.2223	1.7051	2.9296
170	3.4605	5.6485	6.8703
320	3.9730	8.1097	9.7784

This table illustrates the predicted effects of the various parameters on the duration of a callable bond, for three levels of asset value V . The same base case parameter values were used:

$$\sigma = 0.3, c = 7\%, F = 100, p = 10\%, r = 6\%, \delta = 3\%, \alpha = 0.5, \beta = 0.01, \tau = 0.15.$$

values of V were chosen because, under the base case parameters, they represent a broad range of leverage ratios and yield spreads, i.e., leverage ratio of 59%, 47% and 30% and yield spread of 406, 200 and 100 BP (basis points) respectively for $V = 100, 170$ and 320 . We use these three cases to represent (somewhat arbitrarily) high, moderate and low leverage ratios respectively. The effects of various parameters on callable bond duration are described below.

Leverage ratio. As discussed above (and illustrated in Table 1), the duration of a callable bond first rises and then falls as leverage ratio is increased (as opposed to a non-callable bond, whose duration decreases monotonically with leverage).

Volatility. The effect of volatility (σ) depends on the leverage ratio. For low V (high leverage), duration is an increasing function of volatility; for intermediate values of V , duration is a decreasing function of volatility; and for high V (low leverage ratio) duration is a generally increasing function of volatility. The intuition is fairly straightforward: call risk is important for low leverage ratios, and a higher volatility pushes the call trigger farther (from basic option theory); this reduces call risk and thereby makes the duration longer. Thus the duration is an increasing function of σ for low leverage ratios. For high leverage ratios, default risk is important and a higher σ lowers the default trigger; thus the duration is increasing in volatility for high leverage ratios as well.

Call premium. Duration is an increasing function of the call premium (p). This is quite obvious, since a larger call premium will make the issuing firm more hesitant to call the bond, thus increasing its duration.

Coupon rate. Duration is a decreasing function of the coupon rate (c). A high-coupon bond is more likely to be called early, as a result of which it has a shorter duration.

Payout. The effect of the payout ratio (δ) on duration depends on the leverage ratio. For high leverage (low V) duration is an increasing function of δ , for intermediate leverage duration is slightly decreasing in δ , and for low leverage duration is again an increasing function of δ . The intuition is similar to that for volatility, since the payout affects the call and default triggers in a similar manner.

Bankruptcy cost. Duration is a decreasing function of the bankruptcy cost (α).

Refunding cost. Duration is a (slightly) decreasing function of the refunding cost (β).

Tax rate. Duration is an increasing function of the firm's effective corporate tax rate (τ).

Interest rate. Duration is an increasing function of the interest rate (r).

The last four comparative static results are driven by the effects of the parameters on the call and default triggers. For these results, the intuition is not always clear because there are complex interactions between various parameter effects; for example, the direct effect on bond value, indirect effect on default and call triggers, etc.

3.3. Empirical implications

We have seen above that the effective duration initially rises and then falls with the leverage ratio. Therefore, in a cross-sectional regression with *effective duration* as the

dependent variable, the independent variables *leverage* and (*leverage*²) should have positive and negative coefficient respectively. *Call premium* should have a positive coefficient, along with *tax rate* and *interest rate*. *Bankruptcy cost*, *refunding cost*, and *coupon rate* should all have negative coefficients.

For *payout* and *volatility*, the predictions are a little more complicated since the effect depends on the leverage ratio, hence we need to use some joint terms (joint with leverage) in the regression. Since duration is positively related to volatility for high and low leverage ratios and negatively related to volatility for intermediate leverage ratios, the coefficients of *volatility*, (*volatility * leverage*) and (*volatility * leverage*²) should be positive, negative and positive respectively. Similarly for payout, we need to use the three terms *payout*, (*payout * leverage*) and (*payout * leverage*²), with the coefficients expected to be positive, negative and positive respectively.

4. Empirical estimation and tests

In our empirical study, we follow a two-stage procedure similar to Ogden (1987). In the first stage, we estimate empirically the effective duration of a sample of long-term corporate bonds, from monthly data on bond prices and long-term interest rates. In the second stage, we identify the determinants of effective duration of callable bonds, using cross-sectional regression analysis.

4.1. Estimation of effective duration

In the first stage, time series regression analysis is used to estimate the effective duration of a sample of long-term corporate bonds (both non-callable and callable), based on their price changes over time in response to interest rate changes. The inputs for such an estimation procedure consists of bond prices and risk-free interest rates, the former available from the *Lehman Brothers Fixed Income Database*⁸ and the latter from the *Federal Reserve Bank of St. Louis* web page. We used the monthly time series of long-term corporate bond prices and long-term interest rates (long-term government bond yield) for the 36-month period January 1994–December 1996. There are two reasons for limiting the empirical study to long-term bonds (bonds with *more than twenty years left to maturity*): (i) the model's theoretical results are based on infinite-maturity debt, hence the results are not applicable to short-term bonds; and (ii) we need a reasonably long time series to estimate bond duration, hence we used three years of data; but this also limits the analysis to long-term bonds, since only the duration of a long-term bond would remain approximately constant during this three-year period.

We use a time-series regression as in Ogden (1987), and regress the change in interest rate on the change in bond price. However, we also include a squared term to

⁸ For a detailed description of the database, see Hong and Warga (2000).

capture any second-order effects (related to convexity, for instance). Thus the following regression is run:

$$\frac{\Delta P}{P} = B_0 + B_1(\Delta y) + B_2(\Delta y)^2 + \tilde{\varepsilon}, \quad (12)$$

where P is the bond price, y is the long-term interest rate, $(\Delta P, \Delta y)$ refer to monthly increments (since we are using monthly data), $\tilde{\varepsilon}$ is a random error term, and (B_1, B_2) the regression coefficients. The bond's effective duration is then given by $-B_1$. The sample consists of 275 bonds, of which 110 are non-callable and the remaining 165 are callable.

Table 3 displays some interesting statistics for the sample. As expected, the average estimated (effective) duration is shorter for the callable bond sub-sample (5.62 years) than the non-callable sub-sample (7.49 years), and the difference is statistically significant. For this sample, the call feature shortens the effective duration of long-term corporate bonds by an average of 1.9 years. However, the lowest non-callable duration is -3.21 years while the lowest callable duration is larger at -2.57 years. This is consistent with our discussion in Section 3 that the call feature actually increases the duration for low-grade bonds (note that duration is negative only for low-grade bonds, because of the high default risk).

Consistent with our discussion in Section 2, we find that it is possible for both callable and non-callable bonds to have negative duration. As far as the Macaulay duration (also available from the Lehman Brothers Database) is concerned, the difference between callable and non-callable is statistically insignificant. Relative duration (that is, the ratio of effective to Macaulay duration) ranged from -0.2919 to 0.8916 , with an average of 0.6612 for non-callable bonds and 0.5285 for callable

Table 3
Empirical estimates of duration

	Non-callable	Callable
<i>Number of bonds</i>	110	165
<i>Average McCaulay duration</i>	11.3258	10.5627
<i>Effective duration</i>		
Minimum	-3.2086	-2.5678
Maximum	9.5290	9.2280
Average	7.4889	5.6233
<i>Relative duration (ratio of effective to Macaulay duration):</i>		
Minimum	-0.2919	-0.2360
Maximum	0.8714	0.8916
Average	0.6612	0.5285

This table summarizes the results from the estimation of effective durations for a sample of corporate bonds, both callable and non-callable. The following time-series regression was estimated for the 36-month period January 1994–December 1996: $\Delta P/P = B_0 + B_1(\Delta y) + B_2(\Delta y)^2 + \tilde{\varepsilon}$, where P is the bond price, y is the long-term risk-free interest rate, $(\Delta P, \Delta y)$ refer to monthly increments, $\tilde{\varepsilon}$ is a random error term, and (B_1, B_2) the regression coefficients. The bond's effective duration is given by $-B_1$.

bonds. Ogden (1987) empirical study found a range of 0.085–0.958 for the relative duration, while Jacoby and Roberts (2002) reported larger relative durations (which is not surprising, since their sample consisted of investment-grade bonds only).

4.2. Cross-sectional regression

The next step is to test the empirical implications of Section 3 and to identify empirically the major determinants of effective duration of callable corporate bonds. For explanatory variables, we use the parameters that emerge from our model in the previous section. The necessary data were collected for the sample period January 1994–December 1996 from the *Compustat* files and other sources such as the Lehman Brothers Bond Database and the *John Graham Corporate Tax Database* at Duke University. The following explanatory variables were used:

LEV (from Compustat) = average leverage ratio over the sample period, given by the book value of long-term debt divided by the market value of equity⁹

TAX (proxy for τ , from John Graham Corporate Tax Database) = average marginal corporate tax rate for the sample period

COUPON (from Lehman Brothers Bond Database) = coupon rate of the bond

PAYOUT (proxy for δ , from Compustat) = the average payout rate for the firm over the sample period, given by (dividend + interest payment) divided by total firm value (market equity + book debt)

FACE (proxy for refunding cost β , from Lehman Brothers Bond Database) = face value of the outstanding debt, in thousands of dollars. This is used as a (possibly coarse) proxy for β , since percentage transactions costs usually decline with issue size. Therefore, since the effective duration is a decreasing function of β , the coefficient of FACE should be positive.

VOLA (proxy for σ) = volatility of the firm value. Since the volatility is not directly observable, it is estimated as follows: we first generate (from the Compustat database) the quarterly time series of total firm value V (book value of long-term debt plus market value of equity) for each firm for the period ending Quarter 4, 1996. The database that was used had 12 years of quarterly data, so that a maximum of 48 points were available. From this, we create the discretized version of the time series dV/V (i.e., $\Delta V/V$), the annualized standard deviation of which gives the volatility σ or VOLA of the firm. (Firms with less than 20 continuous data points were excluded from the sample.)

MACDURA (from Lehman Brothers Bond Database) = the Macaulay duration of the bond. We decided to add this to the list of explanatory variables in order

⁹ Since our model is concerned with long-term debt (LTD), we use LTD/Equity as a measure of leverage. Debt was measured in terms of book value (rather than market value) because of data limitations. In any case, since the cross-sectional correlation between market and book values of debt is very large (see Bowman, 1980), the misspecification from using book values is probably negligible. It is, in fact, quite common to use book values for debt, e.g., Titman and Wessels (1988).

to capture any effects ignored by the other variables, and to minimize misspecification errors. In particular, we are using long-term but finite-maturity bonds in the empirical tests while the model considers infinite-maturity debt; any maturity effects will hopefully be captured in the Macaulay duration.

The summary statistics for these variables are reported in Table 4, for the entire sample and also split into callable and non-callable sub-samples.

From the empirical implications of the model, we saw that call premium (p) and bankruptcy cost (α) were determinants of effective duration. However, appropriate data on call premiums were not available since most of the bonds were call-protected at the time. For bankruptcy cost, we considered a number of proxies such as percentage of intangible assets and R&D expenses as a percentage of sales. However, there were so many missing data points for these proxies that our sample would have become too small to be informative. We therefore decided to carry out the regressions without these two variables, even though some information is lost by excluding them. We ran the following cross-sectional regression:

Table 4
Summary statistics for variables used in cross-sectional regressions

Variable	<i>N</i>	Mean	Std. Dev.	Minimum	Maximum
<i>A. All bonds</i>					
LEV	184	0.5700	0.6240	0.0140	3.7170
TAX	178	0.3460	0.0150	0.2130	0.3590
COUPON	275	8.47%	0.96%	6.63%	10.75%
PAYOUT	184	0.018	0.009	0.002	0.107
FACE	275	204917	120898	100000	1000000
VOLA	184	12.78%	23.79%	4.83%	325.05%
<i>B. Only callable bonds</i>					
LEV	92	0.5900	0.5270	0.0540	2.3100
TAX	91	0.3460	0.0130	0.2820	0.3590
COUPON	165	8.41%	0.97%	6.63%	10.75%
PAYOUT	92	0.017	0.006	0.004	0.031
FACE	165	189383	93638	100000	550000
VOLA	92	13.25%	33.15%	4.93%	325.05%
<i>C. Only non-callable bonds</i>					
LEV	92	0.5520	0.7100	0.0140	3.7170
TAX	87	0.3460	0.0170	0.2130	0.3540
COUPON	110	8.56%	0.93%	6.65%	10.25%
PAYOUT	92	0.018	0.012	0.002	0.107
FACE	110	228218	150455	100000	1000000
VOLA	92	12.31%	6.20%	4.83%	40.14%

The variables are defined as follows: LEV is the company's leverage ratio; TAX is the company's marginal corporate tax rate; COUPON is the bond's coupon rate; PAYOUT is the total payout rate of the firm; FACE is the face value of the bond; and VOLA is the standard deviation of the percentage change in firm value.

This table presents the summary descriptive statistics for the explanatory variables used to empirically identify the determinants of bond duration.

$$\begin{aligned}
\text{EFFDURA} = & \beta_0 + \beta_1 \text{MACDURA} + \beta_2 \text{LEV} + \beta_3 \text{LEV}^2 + \beta_4 \text{TAX} \\
& + \beta_5 \text{COUPON} + \beta_6 \text{PAYOUT} + \beta_7 (\text{PAYOUT} * \text{LEV}) \\
& + \beta_8 (\text{PAYOUT} * \text{LEV}^2) + \beta_9 \text{FACE} + \beta_{10} \text{VOLA} \\
& + \beta_{11} (\text{VOLA} * \text{LEV}) + \beta_{12} (\text{VOLA} * \text{LEV}^2) + \tilde{\varepsilon},
\end{aligned}$$

where EFFDURA is the (empirically) estimated or effective duration and $\tilde{\varepsilon}$ is an error term. The predicted coefficient signs from the model are as follows: β_1 , β_2 , β_4 , β_6 , β_8 , β_9 , β_{10} and β_{12} should be positive, and β_3 , β_5 , β_7 and β_{11} should be negative.

The results from the cross-sectional regression for callable bonds are reported in Table 5. Because of missing data, the number of usable observations drops substantially, to 91 callable bonds. The signs of the coefficients were generally as predicted by the model. The one exception was the coefficient of TAX, which was insignificant in any case, with a t -statistic of -0.36 and a p -value of 0.7179 . The coefficient of MACDURA was positive, as expected, and quite significant with a p -value of

Table 5
Empirical determinants of callable bond duration

Variable	Expected sign	Parameter estimate	t -Statistic	p -Value
INTERCEPT		8.6191	0.68	0.4990
MACDURA	+	0.3161	3.00	0.0036
LEV	+	22.7503	2.06	0.0424
(LEV ²)	-	-15.7212	-1.97	0.0526
TAX	+	-12.9855	-0.36	0.7179
COUPON	-	-0.9823	-3.95	0.0002
PAYOUT	+	283.1550	2.06	0.0428
(PAYOUT * LEV)	-	-1230.32	-2.29	0.0250
(PAYOUT * LEV ²)	+	864.009	2.08	0.0413
FACE	+	0.0001	0.97	0.3329
VOLA	+	0.1912	1.41	0.1628
(VOLA * LEV)	-	-0.4557	-1.48	0.1437
(VOLA * LEV ²)	+	0.2637	1.20	0.2346
		$R^2 = 57.01\%$	F -statistic = 8.62	
		Adjusted $R^2 = 50.39\%$	Prob(F) < 0.0001	

The explanatory variables are defined as follows: MACDURA is the Macaulay duration of the bond; LEV is the company's leverage ratio; TAX is the company's marginal corporate tax rate; COUPON is the bond's coupon rate; PAYOUT is the total payout rate of the firm; FACE is the face value of the bond; and VOLA is the standard deviation of the percentage change in firm value.

This table shows the results of the cross-sectional regression

$$\begin{aligned}
\text{EFFDURA} = & \beta_0 + \beta_1 \text{MACDURA} + \beta_2 \text{LEV} + \beta_3 \text{LEV}^2 + \beta_4 \text{TAX} + \beta_5 \text{COUPON} + \beta_6 \text{PAYOUT} \\
& + \beta_7 (\text{PAYOUT} * \text{LEV}) + \beta_8 (\text{PAYOUT} * \text{LEV}^2) + \beta_9 \text{FACE} + \beta_{10} \text{VOLA} \\
& + \beta_{11} (\text{VOLA} * \text{LEV}) + \beta_{12} (\text{VOLA} * \text{LEV}^2) + \tilde{\varepsilon},
\end{aligned}$$

where EFFDURA is the (empirically) estimated duration and $\tilde{\varepsilon}$ is an error term.

0.0036. The coefficients of both LEV and LEV² were also as expected, with *t*-statistics of 2.06 and -1.97 (*p*-values of 0.0424 and 0.0526) respectively.

The coefficient of COUPON had the expected sign and was also very significant, with a *t*-statistic of -3.95 and a *p*-value of 0.0002, while the coefficient of FACE had the correct sign but was insignificant. The coefficients of PAYOUT, (PAYOUT * LEV) and (PAYOUT * LEV²) all had the expected signs and were significant, with *t*-statistics of 2.06, -2.29 and 2.08 (*p*-values of 0.0428, 0.0250 and 0.0413) respectively. Finally, the coefficients of VOLA, (VOLA * LEV) and (VOLA * LEV²) had the predicted signs but were insignificant. The overall regression diagnostics were quite satisfactory, with *F*-statistic of 8.62 (Prob(*F*) < 0.0001) and an adjusted *R*-squared statistic of 50.39%. Thus, slightly over half the cross-sectional variation in callable bond duration is explained by the model.

Table 6
Empirical determinants of bond duration

Variable	Parameter estimate	<i>t</i> -Stat	<i>p</i> -Value	Parameter estimate	<i>t</i> -Stat	<i>p</i> -Value
	<i>(Non-callable bonds only)</i>			<i>(All bonds)</i>		
INTERCEPT	-3.5252	-0.57	0.5715	0.1535	0.03	0.9727
MACDURA	0.8039	2.83	0.0060	0.6930	8.15	<0.0001
LEV	10.2980	1.59	0.1163	4.7466	1.38	0.1709
(LEV ²)	-4.9671	-1.62	0.1100	-2.2817	-1.51	0.1327
TAX	-8.4132	-0.95	0.3475	-6.8956	-0.65	0.5142
COUPON	0.3294	1.45	0.1516	-0.0425	-0.25	0.8009
PAYOUT	74.0364	1.34	0.1848	46.3246	0.85	0.3940
(PAYOUT * LEV)	-477.215	-2.44	0.0170	-286.534	-1.77	0.0780
(PAYOUT * LEV ²)	252.767	2.52	0.0138	151.875	1.97	0.0510
FACE	0.0001	1.69	0.0945	0.0001	1.99	0.0485
VOLA	0.0541	0.49	0.6260	0.0707	0.81	0.4201
(VOLA * LEV)	-0.2220	-0.73	0.4698	-0.0997	-0.75	0.4526
(VOLA * LEV ²)	0.0444	0.50	0.6193	0.0080	0.24	0.8080
<i>R</i> -sq		24.69%			45.39%	
Adj <i>R</i> -sq		12.48%			41.42%	
<i>F</i> -stat		2.02			11.43	
Prob(<i>F</i>)		3.38%			<0.0001	

The dependent variable EFFDURA is the (empirically) estimated duration.

The explanatory variables are defined as follows: MACDURA is the Macaulay duration of the bond; LEV is the company's leverage ratio; TAX is the company's marginal corporate tax rate; COUPON is the bond's coupon rate; PAYOUT is the total payout rate of the firm; FACE is the face value of the bond; and VOLA is the standard deviation of the percentage change in firm value.

This table also shows the results of the cross-sectional regression

$$\begin{aligned}
 \text{EFFDURA} = & \beta_0 + \beta_1 \text{MACDURA} + \beta_2 \text{LEV} + \beta_3 \text{LEV}^2 + \beta_4 \text{TAX} \\
 & + \beta_5 \text{COUPON} + \beta_6 \text{PAYOUT} + \beta_7 (\text{PAYOUT} * \text{LEV}) \\
 & + \beta_8 (\text{PAYOUT} * \text{LEV}^2) + \beta_9 \text{FACE} + \beta_{10} \text{VOLA} \\
 & + \beta_{11} (\text{VOLA} * \text{LEV}) + \beta_{12} (\text{VOLA} * \text{LEV}^2) + \tilde{\epsilon}
 \end{aligned}$$

for the non-callable sample and the entire sample.

Most of the model's predictions for callable debt duration therefore seem to be supported by the data, in spite of the exclusion of the two explanatory variables discussed above and the substantial shrinkage in sample size due to missing data. In order to see whether non-callable bond duration behaves in the same manner, we repeated the regression with the entire sample (callable plus non-callable bonds) and with the non-callable sample. The results are reported in Table 6.

Not surprisingly, the model's predictions were not supported by the data for either the entire sample or the non-callable bond sample. For the non-callable sample, the only significant variables were MACDURA, (PAYOUT * LEV) and (PAYOUT * LEV²). None of the other coefficients were significant, although they generally had the predicted signs (except for COUPON and TAX). The regression diagnostics were also unsatisfactory, with an *F*-statistic of 2.02 (Prob(*F*) = 3.38%) and an adjusted *R*-squared statistic of only 12.48%. Thus the performance of the regression deteriorated sharply for non-callable bonds.

For the entire sample, all the coefficients (except that of TAX) had the predicted signs. The overall diagnostics were somewhere between those of the callable and non-callable bond samples, with an *F*-statistic of 11.43 (Prob(*F*) < 0.0001) and adjusted *R*-squared statistic of 41.42%.

5. Conclusion

This paper's contribution to the duration literature is three-fold. First, it computes theoretical measures of effective duration of callable corporate bonds, using a contingent-claim valuation model that allows the explicit consideration of default risk and call risk. Second, it estimates empirically the effective durations for a large sample of long-term US corporate bonds from actual price data, and uses these estimates to identify the factors affecting corporate bond duration. Thirdly, it focuses on the firm-specific nature of these measures in both the theoretical and empirical treatments, which is important because both default risk and call risk are largely determined by firm-specific variables such as leverage ratio and payout rate. By examining individual bonds, we get a better picture than would be possible with aggregate data in the form of bond indices. For investors and portfolio managers who take positions in individual bonds rather than bond indices, the cross-sectional determinants of duration will be important for hedging decisions.

So, in conclusion, what is the effect of the call feature on bond duration? It generally shortens duration (by an average of about 1.9 years in our sample). Also, call risk adjustment for duration is more important in the case of high-grade bonds, while default risk adjustment is more important in the case of low-grade bonds. The individual effects can be complicated; for instance, the call feature lengthens bond duration for low-grade bonds but shortens duration for high-grade bonds. Moreover, the extent to which the call feature affects duration depends on company-specific and bond-specific characteristics such as leverage ratio, firm risk, payout ratio, call premium, coupon rate, etc. We identify these determinants from the model, and empirically test the

model's predictions using the estimated durations for the sample. The results are quite supportive of the model's predictions.

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