



Banks as delegated risk managers

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Received 22 April 2002; accepted 17 September 2003

Available online 22 January 2004

Abstract

Risk management, although of major importance in the banking industry in practice, plays only a minor role in the theory of banking. We reduce this gap by putting forward a model in which risk managers – specialists that can find out correlations between risky assets – endogenously take over typical functions of banks. They *grant loans*, they consult on financial questions with firms that are threatened by bankruptcy, and they sign tailor-made hedge transactions with these firms. Delegation costs are innately low if banks assume the function of risk managers in an economy. Risk management can be seen as a core competence of banks.

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JEL classification: G21; D80

Keywords: Risk management; Delegation; Financial intermediation

1. Introduction

Risk management is often seen as a core function of the banking business. Allen and Santomero (1998, p. 1462) suggest that it becomes more and more a central activity of banks, but that current theories have noticeably little to say about why risk management should play such an important role in the activities of intermediaries at all. ¹ On p. 1465, they explicate that little is offered as a cogent argument as to why intermediaries should be the ones offering the services of risk management, and

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¹ For the present, the terms *bank* and *financial intermediary* are used synonymously.

what value they bring to the activity. Hellwig (1998) sees the need for a systematic theoretical assessment of the role of financial intermediation in allocating risks in the economy.

The notion of *risk management* contains basically two essential aspects, (i) *risk analysis* as the cumulation and aggregation of information about distributions and correlations of risky assets, and (ii) *risk controlling* as the active structuring and shaping of a risky portfolio. Especially the knowledge of correlation seems to be important for risk management in reality, since it is the basis of portfolio management. A firm can only hedge its risk with financial contracts if it is aware of the correlation between its own business and traded financial contracts (feasible hedging tools).

Actually, there is a variety of approaches to explaining the existence of financial intermediaries.² All these have one characteristic in common. Risk allocation is often modeled, the acquisition of information about the distribution of random variables is sometimes modeled, whereas the acquisition of information about the correlation of random variables typically remains unmodeled. As a result, an important aspect of risk management is ignored.

One might think that risk management should be an essential ingredient in models about risk sharing – such as those presented by Allen and Gale (1994). However, risk analysis rarely plays an explicit role.³ As an example, take the model of Allen and Gale (1999), which describes a risk sharing problem between an entrepreneur and a specialist with superior information. The *specialist* knows the state dependence of the yield of any asset. Therewith, he knows their distribution and correlations. The *entrepreneur* is affected by incomplete information regarding the random variables of traded securities' outcomes. He wants to hedge the risk of his random income, but refrains from engaging on the financial markets because he fears unpleasant surprises. The specialist possesses the information the agent needs but cannot credibly convey it. In a one-period setting, although specialist and entrepreneur can conclude risk sharing contracts that are favorable for either party, the entrepreneur refuses to do so. He fears being cheated by the specialist. In a multi-period setting, though, the specialist would have to expect the termination of the profitable relationship by the entrepreneur if he cheats. Therefore, he refrains from doing so. He offers a risk sharing contract that fits the entrepreneur's needs in the sense that it reduces the entrepreneur's risk. The anticipating entrepreneur dares to conclude the contract, which he needs not necessarily understand – he thus saves information costs.

However, the model of Allen and Gale (1999) contains only aspects of *risk controlling* (the entrepreneur reduces risk by signing the risk sharing contract) but not of *risk analysis* (the aggregation of information about correlation is not costly for the specialist). Hence, the focus of the model is risk sharing, not *risk management*.

² Certainly, Diamond (1984) must be mentioned as the seminal article. See also Baltensperger (1980), Santomero (1984), Bhattacharya and Thakor (1993), Thakor (1995) or Swank (1996) for overviews. Boot (2000) concentrates on the relationship aspect of banking.

³ One reason is often that correlations of risky assets do not appear, because random variables are assumed to be independent. Another possible reason is that information about correlation is publicly accessible, so that no meaningful decision about the acquisition of correlation data needs to be made.

Furthermore, most theoretical approaches view banks as competitors with financial markets.⁴ By contrast, empirical evidence suggests that the relationship between banks and financial markets may not be characterized by competition alone, but may also comprise cooperative aspects. According to Allen and Santomero (2001), the importance of banks has decreased only in relation to the importance of the financial industry. Yet in comparison with GDP, the importance of banks and financial firms increases. This hypothesis is supported by empirical research carried out by Scholtens and Wensveen (2000) for US-American markets and by Schmidt et al. (1999) for French, German and British data. Allen and Gale (1999) conclude that a change of paradigm lies ahead. The “traditional paradigm” in which banks and financial markets are seen as competitors needs to be replaced by an “emerging paradigm” in which banks act as a link between entrepreneurs and financial markets.

In the following, we put forward a *model* in which risk management is the only exogenous function of banks: An indebted entrepreneur has access to a risky project. In the case of low outcomes, he faces costly bankruptcy. Hence he has an incentive to hedge his risk. However, as he is not a specialist regarding financial markets, he does not know exactly which contracts to hedge with. This is where the specialist – in the following called *risk analyst* (or in short *analyst*) – plays a role. He can gather the information the entrepreneur needs: He can acquire information about correlations of assets, he has the ability of *risk analysis*. However, because information gathering is costly and unobservable to the entrepreneur, a principal–agent problem between entrepreneur and risk analyst emerges. We show that if the risk analyst himself acts as counterparty for the contracts, and if he grants a loan to the entrepreneur, the problem is reduced. He thus

1. carries out the *risk analysis* on behalf of the entrepreneur by gathering information about the correlation between the entrepreneur’s business and states of nature.
2. carries out the *risk controlling* on behalf of the entrepreneur by selling him a hedging tool that reduces the entrepreneur’s risk.
3. grants a *loan* to the entrepreneur.

Items 1 and 2 imply that the analyst acts as *delegated risk manager*. Item 3 is more subtle. Through a loan, a financial relation between risk analyst and entrepreneur is created, which in turn alleviates the problem of delegating the risk analysis. The loan harmonizes the interests of the entrepreneur and of the risk analyst. The entrepreneur is genuinely interested in avoiding his own bankruptcy. After granting the loan, the risk analyst bears a counterparty risk and is therefore (more) willing to help the entrepreneur. Because risk analysis (gathering information about risk), risk controlling (selling tailor-made contracts to customers) and granting loans are typical

⁴ As an example, Hellwig (1998) discusses whether the risk of an economy should be borne by banks or directly by markets.

functions of banks, we may call the risk analyst a *bank*.⁵ In our model, banks act as agents who manage risk for the principals, their borrowers (the entrepreneurs). Because of this financial relation, delegation costs are reduced. Hence banks have an innate cost advantage when managing and redistributing risk in an economy.

The remainder of the paper is organized as follows. Section 2 deals with a basic model that does not consider loans between entrepreneur and risk analyst. Section 3 extends the basic model by allowing the risk analyst to grant loans to the customer. The discussion and conclusion of Section 4 complete the paper.

2. Basic model – hedge contracts only

2.1. Model structure and assumptions

Consider a two date ($t = 0, 1$) economy with two agents, an entrepreneur and a risk analyst.⁶ Both agents are risk neutral. The *entrepreneur* has debts with face value K . If he fails to repay in $t = 1$, he undergoes bankruptcy costs ϕ_U . His only asset is a project with uncertain nonnegative return $\tilde{Y} : \Omega \rightarrow \mathbb{R}_+$, where Ω is the set of states of nature in $t = 1$. The entrepreneur is not liable with private means. He knows the distribution function of all random variables, e.g. those of financial contracts and especially that of his own project, $F : \mathbb{R} \rightarrow [0, 1]$. He does *not* know the state dependencies and correlations of random variables, especially not the correlations between financial contracts and his project.⁷ In this situation, the entrepreneur may want to hedge away some of his bankruptcy risk by trading on financial markets, say by concluding a financial transaction that leads to state contingent payments $\tilde{Z} : \Omega \rightarrow \mathbb{R}$ which reduces the probability of bankruptcy. However, he cannot assess which one is the right transaction to conclude.

The *risk analyst* also knows the distribution functions of random variables, especially that of the entrepreneur's project, F . Furthermore, he knows the correlations between all random variables apart from that of the entrepreneur's project. By exerting an effort c_B , he can even learn these last correlations. Formally, he knows the state dependencies of the returns of any conceivable financial contract $\tilde{Z} : \Omega \rightarrow \mathbb{R}$, but not that of the project $\tilde{Y} : \Omega \rightarrow \mathbb{R}_+$. In short, the risk analyst lacks only one bit of information – the state dependency of the project \tilde{Y} . From this information, he can derive the correlations between the project and all other financial contracts.

⁵ However, the approach follows the functional approach proposed by Merton and Bodie (1995). The question answered is whether there are economies of scope between the functions of (delegated) risk management and lending.

⁶ In the following, the index U stands for the entrepreneur. B stands for the risk analyst because we want to show that B behaves like a Bank.

⁷ Note that throughout this paper, the notion of *correlation* contains all information about the dependency on two random variables, not only the correlation coefficient ρ . Formally, it is the copula that contains this information about the correlation of two random variables whose distributions are known. The copula is the two-dimensional distribution function modulo the marginals. For a more detailed discussion, cf. Bickel et al. (1993, pp. 155–157) or Embrechts et al. (2001).

The missing bit of information is accessible to him at costs c_B , but not to the entrepreneur. After gathering the information, the risk analyst may advise the entrepreneur on which contract to conclude. He may even sell a contract himself. The generated incentive problem – whether to spend c_B and gather information or not – is analyzed in the following sections. The risk analyst is assumed to dispose of unlimited capital.

We impose three assumptions on *hedge contracts*, the financial contracts between entrepreneur and risk analyst.

1. No payments are made in $t = 0$. Only in $t = 1$ payments are swapped.
2. The stipulated payments \tilde{Z} in $t = 1$ are state contingent. There are no return contingencies.⁸ \tilde{Z} can assume positive values – when the entrepreneur is due to pay – and negative values – when the entrepreneur receives money and the risk analyst pays.

The following third property of a hedge contract is slightly more complicated. The entrepreneur knows what he wants – a contract \tilde{Z} that eliminates his entire bankruptcy risk, hence he would like $\tilde{Y}(\omega) - \tilde{Z}(\omega) > K$ for all $\omega \in \Omega$. In addition, assume that entrepreneur and risk analyst agree that in $t = 1$, the hedge contract should not lead to more cash payment flows than necessary. Optimally, it then has the form

$$\tilde{Z}(\omega) = \min\{S, \tilde{Y}(\omega) - K\} \quad (1)$$

for some $S \in \mathbb{R}$. If the project performs poorly, the entrepreneur wants to be paid $\tilde{Y} - K$ in order to avert bankruptcy, whereas in case of good performance, he is willing to pay a “premium” S . However, because the entrepreneur does not know the state dependencies \tilde{Y} , he cannot check whether a contract \tilde{Z} follows Eq. (1). At least, he can observe the contract’s distribution function, and he knows that a contract \tilde{Z} as in (1) must have the distribution function

$$F_{\tilde{Z}}(z) = \begin{cases} F_{\tilde{Y}}(z + K) & : z < S, \\ 1 & : \text{else.} \end{cases} \quad (2)$$

Therefore, if he notices that the risk analyst proposes a contract \tilde{Z} which does not have the distribution as in (2), he infers he does not get what he wants and rejects the contract. We consequently assume

3. $F_{\tilde{Z}}$ has the form of Eq. (2).

Item 3 has one important implication. It involves that from the entrepreneur’s point of view, any contract can be completely described by one variable S , the “terms” of the contract. $S = 0$ means that the contract never stipulates payments

⁸ Consequently, the contract can be written as a random variable $\tilde{Z} : \Omega \rightarrow \mathbb{R}$: For every state of nature $\omega \in \Omega$, a payment $\tilde{Z}(\omega)$ is stipulated.

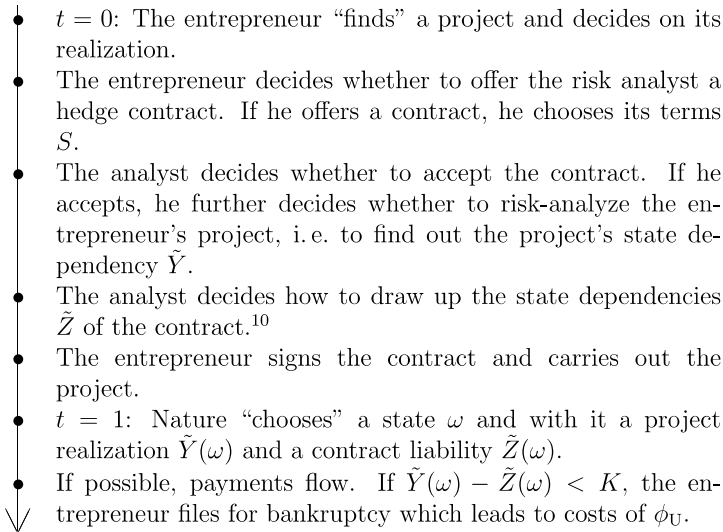


Fig. 1. Time structure of Game 1 (without loan).

from the entrepreneur to the risk manager. $S \rightarrow \infty$ implies that the entrepreneur must forward all project returns to the risk analyst. The illustration for a hedge contract \tilde{Z} is a bundle of financial instruments like swaps, futures, forwards and options depending on exogenous variables such as interest rates, exchange rates, commodity prices, macroeconomic variables and the like. Item 3 then implies that not more instruments than necessary are “bought”.

For simplicity, the future is not discounted, the risk free rate is standardized to zero. Just as well, assume that $0 < F(Y) < 1$ everywhere, and that F is sufficiently smooth so that the density function $f(Y) = F'(Y)$ exists. The game described so far is called *Game 1* in the following; its time structure can taken from Fig. 1.

In Game 1, the entrepreneur is the first mover, his strategy space is denoted by $\sigma_U \in \{S, \dagger\}$, where $S \in \mathbb{R}$ stands for the entrepreneur’s proposal of the hedge contract’s terms, \dagger for not offering any contract. The analyst is the second mover, his strategy space is $\sigma_B \in \{\oplus, \ominus, \dagger\}$, where \oplus denotes accepting the proposal S and investing c_B (risk-analyzing), \ominus for accepting S but not exerting c_B (not risk-analyzing), \dagger for not accepting the entrepreneur’s proposal.⁹ At his decision node,

⁹ For a precise definition of game theoretical notions, cf. e.g. Fudenberg and Tirole (1991). Note that the chosen way of notation contains already some simplification and is not formally correct. Because the analyst’s move is already a response to the entrepreneur’s move, one should correctly write $\sigma_B \in \{(\oplus|S), (\ominus|S), (\dagger|S), (\dagger|\dagger)\}$. In order to avoid clutter, we quote only the moves instead of the whole strategies. Moreover, B’s strategy space is actually richer, for he might risk-analyze the project but choose to draw up a contract that does not *fit* the project.

the risk analyst is in a position where he can only reject or accept a take-it-or-leave-it offer. The entrepreneur is implicitly assumed to possess all the bargaining power.

Let us now discuss the assumptions. The assumption that the entrepreneur is threatened by bankruptcy is needed to induce some motivation for risk sharing. Bankruptcy risk is assumed away for the risk analyst to keep the risk sharing problem as simple as possible.¹⁰ The assumption that distributions of random variables are publicly known directs the scope of the model on correlations, as being the principal item of risk management. The assumption that the risk analyst already knows the correlations between random variables, especially between financial securities, reflects the idea that there may be someone who has an information advantage on the financial markets. The assumption that the entrepreneur is completely ignorant of correlations is, of course, strong. However, it keeps calculations simple. Alternatively, one may assume that the entrepreneur has *some* information on correlations, but that this information is already incorporated in the project's return \tilde{Y} . Property 2 of the contract, the assumption that payments are state contingent only, is crucial for the results. The idea behind is that in reality, return contingencies would entail moral hazard problems that are not to be modelled here.¹¹ Finally, the time structure of the model reflects the bargaining power of the agents. The entrepreneur is unique with his project, whereas there may be several specialists on the financial markets. In the model, the entrepreneur has the power to confront the risk analyst with a take-it-or-leave-it offer.

2.2. Sequential equilibrium

We first calculate the expected profits, depending on the moves chosen by entrepreneur and risk analyst.¹² That way, the strategic form of Game 1 becomes known, which simplifies the analysis of equilibrium. The following remark studies the case that the risk analyst invests in information gathering about the project's correlations.¹³

¹⁰ If the risk analyst were affected by bankruptcy risk, he would in turn want to hedge away any risk generated from the contract with the entrepreneur. By engaging on financial markets, he would become an actual *financial intermediary*.

¹¹ A return contingent contract may provide that in case the project performs poorly, the risk analyst always pays the missing amount to prevent the entrepreneur's bankruptcy. In this case, if we assume that the entrepreneur can to a certain degree influence the return of his project by exerting effort, these effort may be reduced if poor performance becomes foreseeable.

¹² In order to keep track of the calculations, we have to define indices for different moves. In the brackets in the subscript, the first symbol denotes the entrepreneur's move, the second denotes the risk analyst's response.

¹³ For proofs, cf. Appendix A.

Remark 1 (Hedge contract after risk analysis). *If the risk analyst analyzes the project ($\sigma_U = S, \sigma_B = \oplus$), then he draws up a hedge contract, and this contract fits the project. The entrepreneur’s bankruptcy risk is eliminated.*

As derived in the proof, the risk analyst’s expected profits are

$$E[\Pi_B]_{(S,\oplus)} = S - \mathcal{F}(S + K) - c_B, \tag{3}$$

where F is the distribution function of \tilde{Y} , and \mathcal{F} is its primitive function. The entrepreneur’s expected profits can be calculated as a residual,

$$E[\Pi_U]_{(S,\oplus)} = E[\tilde{Y}]_{(S,\oplus)} - E[\tilde{Z}]_{(S,\oplus)} = E[\tilde{Y}] - S + \mathcal{F}(S + K). \tag{4}$$

If the risk analyst does not risk-analyze the entrepreneur’s project, he cannot build the contract on the project’s state dependencies \tilde{Y} . There are then several possibilities. By chance, the risk analyst may draw up a contract \tilde{Z} with a high correlation with the fitting (desired) contract. On the other hand, he may also happen to choose an unfortunate contract \tilde{Z} that stipulates payments from the entrepreneur when the project’s returns are low, possibly even causing his bankruptcy. The following remark shows that, taking into account all possibilities with their according probabilities, the assumption that there is a random variable \tilde{X} , i.i.d. with \tilde{Y} , on which the contract is based, leads to correct expected profits.¹⁴ As an illustration, the entrepreneur does not need to take all possible contracts written by the risk analyst into account: If anticipates the analyst will not analyze the project, he may assume that the payments of the contracts he gets are stochastically independent of the contract he wants. Also the contract \tilde{Z} is then stochastically independent from the project \tilde{Y} .

Remark 2 (The representative hedge contract). *Assume that the analyst draws up a hedge contract without having risk-analyzed the project ($\sigma_U = S, \sigma_B = \ominus$). Then the assumption that the contract is based on a random variable \tilde{X} which is i.i.d. with the project returns \tilde{Y} leads to the correct expected profits.*

It is now possible to calculate the expected profits of a contract which is based on an \tilde{X} which is i.i.d. from \tilde{Y} . The entrepreneur’s limited liability must be taken into account – the risk analyst can never obtain more than the minimum of the returns of the project Y and the contract’s claims, $\min\{X - K, S\}$. We obtain¹⁵

$$\begin{aligned} E[\Pi_B]_{(S,\ominus)} &= \int_0^\infty \int_0^\infty \min\{Y, X - K, S\} f(Y) dY f(X) dX \\ &= S - \mathcal{F}(S + K) - \int_0^S F(X)(1 - F(X + K)) dX. \end{aligned} \tag{5}$$

¹⁴ Note that we do not claim that the contract is always independent of the project \tilde{Y} . As described, good fits are possible as well as bad fits. \tilde{X} is a mere representative with “average fit” leading to right expectations.

¹⁵ For an extensive derivation, cf. Appendix A.

Table 1
Survey of expected profits in dependence on selected moves

	Risk analysis $\sigma = (S, \oplus)$	No risk analysis $\sigma = (S, \ominus)$	No contract $\sigma = (\dagger, \dagger)$
$E[\Pi_B]_\sigma$	$S - \mathcal{F}(S + K) - c_B$	$S - \mathcal{F}(S + K) - \int_0^S F(X)(1 - F(X + K)) dX$	0
$E[\Pi_U]_\sigma$	$E[\tilde{Y}] - S + \mathcal{F}(S + K)$	$E[\tilde{Y}] - S + \mathcal{F}(S + K) + \int_0^S F(X)(1 - F(X + K)) dX - \phi_U F(S + K) \frac{2 - F(S + K)}{2}$	$E[\tilde{Y}] - \phi_U F(K)$
Σ	$E[\tilde{Y}] - c_B$	$E[\tilde{Y}] - \phi_U F(S + K) \frac{2 - F(S + K)}{2}$	$E[\tilde{Y}] - \phi_U F(K)$

The table contains the strategic form of Game 1: Expected profits are listed for all possible moves.

Again, the entrepreneur’s expected profits can be calculated as a residual, additionally taking into account his positive probability of bankruptcy.

$$E[\Pi_U]_{(S,\ominus)} = E[\tilde{Y}] - E[\Pi_B]_{(S,\ominus)} - E[\phi_U]_{(S,\ominus)} \tag{6}$$

with

$$\begin{aligned} E[\phi_U]_{(S,\ominus)} &= \phi_U \Pr\{Y - Z < K\} = \phi_U \Pr\{Y < S + K \text{ and } Y < X\} \\ &= \phi_U F(S + K) \left(1 - \frac{F(S + K)}{2}\right) \end{aligned}$$

and $E[\Pi_B]_{(S,\ominus)}$ as calculated in (5). Finally, we have to determine the expected profits of both players for the case that no contract is written.

$$E[\Pi_U]_{(S,\dagger)} = E[\Pi_U]_{(\dagger,\dagger)} = E[\tilde{Y}] - \phi_U \Pr\{Y < K\} = E[\tilde{Y}] - \phi_U F(K). \tag{7}$$

The risk analyst does not participate, which implies $E[\Pi_B]_{(S,\dagger)} = E[\Pi_B]_{(\dagger,\dagger)} = 0$. Now the expected profits given the moves of the players are known (cf. Table 1).

When choosing between nonperformance and performance of the risk analysis, the analyst is confronted with a tradeoff. If he analyzes the project and draws up the fitting contract, he has to spend c_B , but faces no counterparty risk – the entrepreneur’s possible bankruptcy is prevented by the contract. If he draws up a contract without risk-analyzing, he saves c_B , but runs the risk that the entrepreneur cannot pay although intended by contract. As one can see from Table 1, the expected financial loss due to counterparty risk amounts to $\kappa(S) := \int_0^S F(X)(1 - F(X + K)) dX$. The analyst is indifferent between moves \oplus and \ominus (and assumed to opt for \oplus) iff $c_B = \kappa(S)$.

In Fig. 2, an example for profit functions is pictured. Here, all three indifference points exist. The profit function of the entrepreneur jumps at indifference points of the risk analyst. In sequential equilibrium, the entrepreneur chooses $\sigma_U = S_{\oplus \sim B \ominus}$, the

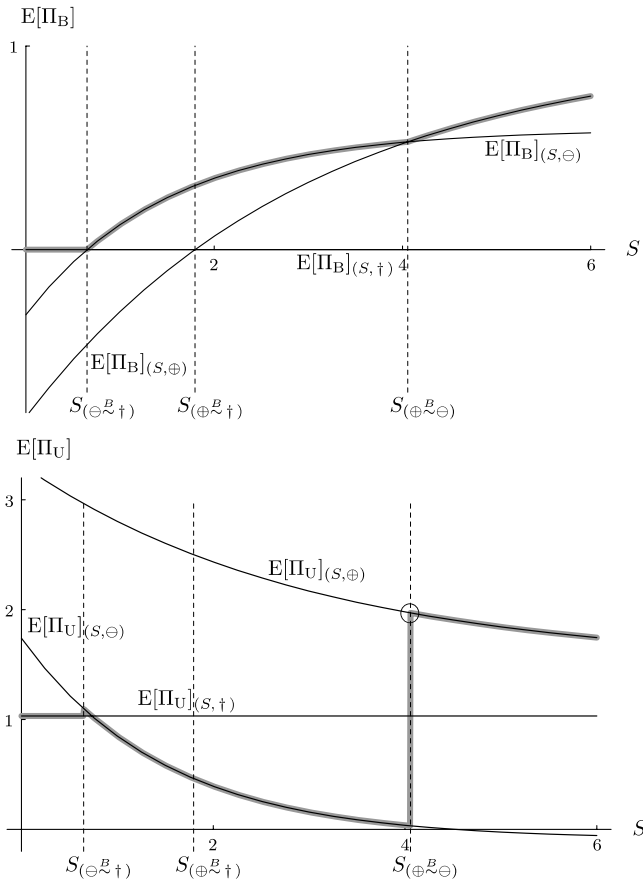


Fig. 2. Expected profits of risk analyst and entrepreneur as a function of S .
 Note: The figure is based on exponential distribution of \tilde{Y} , thus $F(Y) = 1 - e^{-Y/\mu}$ with $\mu = 3$. Furthermore, $K = 1.5$, $c_B = 0.5$, and $\phi_U = 5$. The entrepreneur's expected profits are maximized when the analyst is indifferent between risk-analysis and no risk-analysis (at 'O'); equilibrium has the form σ_2^* . Here, B's expected profits are positive.

analyst answers with $\sigma_B = \oplus$, he analyzes the project. The following Proposition 1 states that this is one of four possible types of sequential equilibria.¹⁶

Proposition 1 (Categorization of sequential equilibria). *Depending on the exogenous parameters F , K , ϕ_U and c_B , sequential equilibrium $\sigma^* = (\sigma_U^*, \sigma_B^*)$ is in one of the four categories, one of which provides positive expected profits for the risk analyst,*

¹⁶ In Appendix A, we prove that $E[\Pi_U]$ is locally non-increasing with S , which implies that the entrepreneur always proposes an S that leaves the analyst indifferent between two strategies (Proposition 4). If indifference points exist, they are unique (Remark 5).

$$\begin{aligned}
 \sigma_0^* & \text{ with } \sigma_U^* = \dagger, & \sigma_B^* & = \dagger \text{ and } E[\Pi_B] = 0, \\
 \sigma_1^* & \text{ with } \sigma_U^* = S_{(\oplus \rightsquigarrow \dagger)}, & \sigma_B^* & = \oplus \text{ and } E[\Pi_B] = 0, \\
 \sigma_2^* & \text{ with } \sigma_U^* = S_{(\oplus \rightsquigarrow \ominus)}, & \sigma_B^* & = \oplus \text{ and } E[\Pi_B] > 0, \text{ or} \\
 \sigma_3^* & \text{ with } \sigma_U^* = S_{(\ominus \rightsquigarrow \dagger)}, & \sigma_B^* & = \ominus \text{ and } E[\Pi_B] = 0.
 \end{aligned}$$

Proof. See Appendix A.

Here, the category of σ_2^* contains the sequential equilibrium of Fig. 2. There, a delegation of risk analysis is advantageous for the entrepreneur. However, if he offered the risk analyst a contract setting the terms S sufficiently low that $E[\Pi_B] = 0$, then the analyst would rather choose not to risk-analyze the project. In order to create sufficient incentives, the entrepreneur must “raise” S and thus the analyst’s counterparty risk $\kappa(S)$ until $\kappa(S) = c_B$. Under this condition, the analyst examines

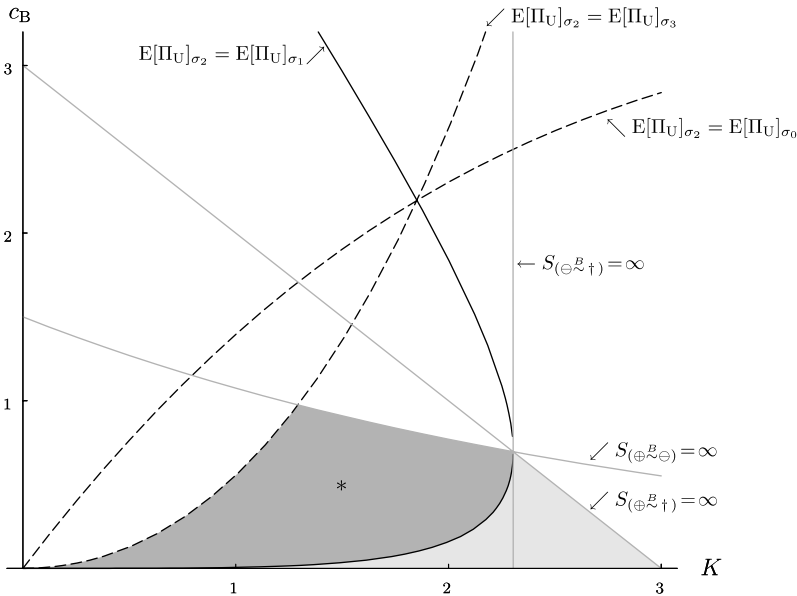


Fig. 3. Scope of sequential equilibria of Game 1.

Note: Like in Fig. 2, \tilde{Y} is exponentially distributed with $\mu = 3$, and $\phi_U = 5$. Depending on c_B and K , different sequential equilibria are played. Points where σ_2^* (with $\sigma_B^* = \oplus$ and $E[\Pi_B] > 0$) is played are backed dark gray, σ_1^* (with $\sigma_B^* = \oplus$ and $E[\Pi_B] = 0$) is backed light gray and σ_3^* is backed white. Under exponential distribution, σ_0^* is never played. Therefore, demarcation lines between σ_0^* and other equilibria are not noted. As demonstrated in Fig. 2, σ_2^* is the equilibrium when $c_B = 0.5$ and $K = 1.5$ (*’).

the project, draws up the fitting contract, and his expected profits $E[\Pi_B]$ are positive. Hence the entrepreneur must “overpay” the analyst for his service.¹⁷

By contrast, in σ_1^* the delegation of risk analysis is also favorable for the entrepreneur, but an S with $E[\Pi_B] = 0$ is sufficient to implement incentives for the risk analyst to inspect the project.¹⁸ In σ_3^* , the analyst writes the contract blindfold – the entrepreneur’s project is never risk-analyzed. Because $E[\Pi_B] = 0$ in this case, the contract leads to a mean preserving spread for the entrepreneur, which in this case reduces his bankruptcy risk. If the mean preserving spread generated by the contract cannot reduce the bankruptcy risk, then no contract can be agreed upon (the case of σ_0^*). The scopes of these four classes of equilibria, depending on K and c_B , are represented in Fig. 3.

3. A model with lending

3.1. Model structure and assumptions

We have seen that in some cases, expected profits of the risk analyst are positive, although he only receives a take-it-or-leave-it offer. Therefore, the entrepreneur may seek ways to appropriate some of the analyst’s profits. In this section, we analyze how a loan – as an example for a deeper financial intertwining between entrepreneur and risk analyst – can be used as an instrument for appropriation. First, we take the level of debt as given exogenously.

Assume the entrepreneur has already received a loan of the amount $l \geq 0$ with repayment $L \geq 0$. L contains redemption and interest. The maturity period is identical with the project’s life span. The entrepreneur is liable only with the project’s results. If he does not need l to finance the project, one may imagine that he consumes l directly.¹⁹ Because of the additional liabilities of the entrepreneur due to the loan, the bankruptcy threshold rises from K to $K + L$. Taking this into account, a hedge contract must shed the whole, increased risk, it must be based on $K + L$.

As in Section 2.2, we can now calculate expected profits for the different strategies. First, assume that the risk analyst has inspected the project and written the fitting contract ($\sigma_B = \oplus$). Then he receives S (plus L because of the entrepreneur’s liabilities towards him) only if the project delivers more than $S + K + L$. Otherwise he receives the whole output Y less K which he transfers to the entrepreneur in order to avoid bankruptcy. Thus²⁰

¹⁷ The mechanism is similar to Stiglitz and Weiss (1981), where the lender chooses undesirable projects if the bank sets interest rates too high.

¹⁸ Here, profits are identical to those under symmetric information.

¹⁹ Note that the reason for the funding is irrelevant – there need not be an explicit financing problem. If there is a financing problem, the loan has two effects. It solves the problem *and* it has implications on the problem of delegating risk management.

²⁰ Unfortunately, the expected profits must now be indexed with two more variables, L and l . Note that $E[\Pi_B]_{(S,\oplus),(L=0,l=0)} = E[\Pi_B]_{(S,\oplus)}$ and $E[\Pi_U]_{(S,\oplus),(L=0,l=0)} = E[\Pi_U]_{(S,\oplus)}$, for $L = l = 0$ means that no loan exists. The same applies for $E[\Pi_B]_{(S,\ominus)}$, $E[\Pi_U]_{(S,\ominus)}$, and so forth.

$$\begin{aligned} E[\Pi_B]_{(S,\oplus),(L,l)} &= \int_0^{S+K+L} (Y - K)f(Y) dY + \int_{S+K+L}^{\infty} (S + L)f(Y) dY - c_B - l \\ &= S + L - \mathcal{F}(S + K + L) - c_B - l \end{aligned} \tag{8}$$

and

$$E[\Pi_U]_{(S,\oplus),(L,l)} = E[\tilde{Y}] - S - L + \mathcal{F}(S + K + L) + l. \tag{9}$$

The same train of thought can be applied to calculate the expected profits for the other strategies,

$$\begin{aligned} E[\Pi_B]_{(S,\ominus),(L,l)} &= \int_0^{\infty} \int_0^{\infty} \min\{Y, X - K, S + L\}f(Y) dY f(X) dX - l \\ &= S + L - \mathcal{F}(S + L) - \int_0^{S+L} F(X)(1 - F(X + K)) dX - l, \end{aligned} \tag{10}$$

$$E[\Pi_U]_{(S,\ominus),(L,l)} = E[\tilde{Y}] - E[\Pi_B]_{(S,\ominus),(L,l)} - E[\phi_U]_{(S,\ominus),(L,l)} \tag{11}$$

with

$$\begin{aligned} E[\phi_U]_{(S,\ominus),(L,l)} &= \phi_U \Pr\{Y - Z < K + L\} = \phi_U \Pr\{Y < S + L + K \text{ and } Y < X\} \\ &= \phi_U F(S + L + K) \left(1 - \frac{F(S + L + K)}{2}\right) \end{aligned}$$

and $E[\Pi_B]_{S,\ominus}$ as calculated in (10). Furthermore,

$$\begin{aligned} E[\Pi_B]_{(\dagger,\dagger),(L,l)} &= L - \mathcal{F}(L) - l, \\ E[\Pi_U]_{(\dagger,\dagger),(L,l)} &= E[\tilde{Y}] + l - L + \mathcal{F}(L) - \phi_U F(K + L). \end{aligned}$$

One can now determine how the loan parameters l and L influence the expected profits of each strategy. Interestingly, in the two cases $\sigma_B = \oplus$ and $\sigma_B = \ominus$, S and L influence the analyst's profits only through their sum $S + L$.

Remark 3 (Relocation of indifference point). *Lending changes the expected profit curves of the risk analyst so that the indifference point S between analysis and no analysis is shifted downwards,*

$$E[\Pi_B]_{(S,\oplus),(L,l)} = E[\Pi_B]_{(S+L,\oplus),(0,l)} \tag{12}$$

and

$$E[\Pi_B]_{(S,\ominus),(L,l)} = E[\Pi_B]_{(S+L,\ominus),(0,l)}, \tag{13}$$

thus

$$S_{(\oplus \rightsquigarrow \ominus),(L,l)} = S_{(\oplus \rightsquigarrow \oplus)} - L. \tag{14}$$

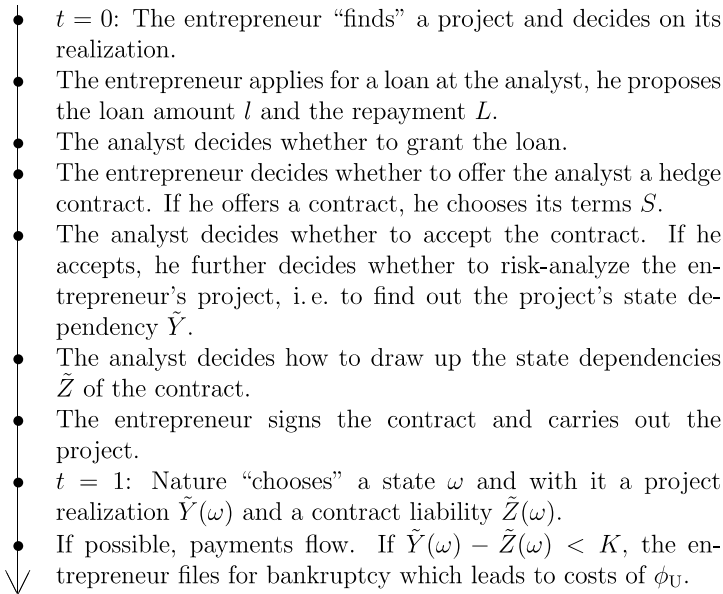


Fig. 4. Time structure of the Game 2 (with loan).

Proof. See Appendix A.

Remark 3 has far-reaching consequences on equilibrium. As (14) states, the higher the entrepreneur’s level of debt at the analyst, the less *further* incentives (by raising S) he must offer to make the analyst indifferent between risk-analysis and no risk-analysis. As a result, one can summarize that debt mitigates the delegation problem of risk analysis between entrepreneur and risk analyst.

Let us now extend the above game by assuming that L and l are chosen endogenously: First, the entrepreneur may apply for a loan (determined by L and l) from the risk analyst. Then, he may mandate the analyst to analyze his project and write an appropriate contract (hence he enters the subgame of risk analysis delegation as described in Fig. 1). The analyst may accept or reject each proposal. Furthermore, assume that the entrepreneur disposes of all market power. If the analyst rejects him, he can apply for a loan at another risk analyst.²¹ Because the entrepreneur can now choose L and l , his space of moves is enlarged: $\sigma_U \in \{L, l\} \times \{S, \dagger\}$ with $L, l, S \in \mathbb{R}$. The analyst can reject the loan *and/or* the hedge contract, $\sigma_B \in \{*, \dagger\} \times \{\oplus, \ominus, \dagger\}$, where $*$ stands for accepting the loan. In the following, the game described is called Game 2 (cf. Fig. 4).

²¹ Note that the reason for rejection cannot be a low credit standing, as the entrepreneur’s probability of default is public information. If his credit standing is low, he knows that no risk analyst would grant the loan, so he does not need to apply.

3.2. Sequential equilibrium

Proposition 2 (Relocation of expected profits). *Assume that F, c_B, ϕ_U and K are such that in Game 1 (without debt) expected profits of the risk analyst are positive ($E[\Pi_B]_{l=0,L=0} > 0$, equilibrium is of type σ_2^*). Then with the same exogenous parameters in Game 2 (allowing for debt), if the entrepreneur chooses $\bar{l} \leq E[\Pi_B]_{l=0,L=0}$ and S and \bar{L} such that $\bar{l} = \bar{L} - \mathcal{F}(\bar{L})$, $S = S_{(\oplus \sim \ominus), (\bar{l}, \bar{L})} - \bar{L}$, then*

$$E[\Pi_U]_{l=\bar{l}} = E[\Pi_U]_{l=0} + \bar{l} \text{ and } E[\Pi_B]_{l=\bar{l}} = E[\Pi_B]_{l=0} - \bar{l}.$$

If the entrepreneur chooses $\bar{l} \geq E[\Pi_B]_{l=0,L=0}$, then

$$E[\Pi_U]_{l=\bar{l}} = E[\Pi_U]_{l=0} + E[\Pi_B]_{l=0} \text{ and } E[\Pi_B]_{l=\bar{l}} = 0.$$

Proof. See Appendix A.

According to Proposition 2, if the risk analyst’s expected profits are positive, an increasing level of debt l lets them shrink, whereas those of the entrepreneur grow. This is intuitive. Without the loan, the entrepreneur would have to oblige the analyst by increasing S , otherwise no analysis would take place ($\sigma_B = \ominus$). Now if the entrepreneur is indebted at the analyst, this already has a genuine incentive to help the entrepreneur to avoid his bankruptcy. This is due to the interest-harmonizing impact of debt. Because the entrepreneur understands this impact, he applies for a loan at the analyst. In other words, he holds out the prospect of the positive expected profits of the hedge contract for the analyst, but only after accepting the loan. If the analyst rejects the loan, then the entrepreneur turns to another risk analyst.

Proposition 3 (Categorization of sequential equilibria). *Depending on the exogenous parameters F, K, ϕ_U and c_B , sequential equilibrium $\sigma_i^* = (\sigma_{i,U}^*, \sigma_{i,B}^*)$ is in one of the four categories*

$$\begin{aligned} \sigma_{i,0}^* \text{ with } \sigma_{i,U}^* &= (l = 0, \dagger), & \sigma_{i,B}^* &= \dagger \text{ and } E[\Pi_B] = 0, \\ \sigma_{i,1}^* \text{ with } \sigma_{i,U}^* &= \left(l = 0, S_{(\oplus \sim \dagger)} \right), & \sigma_{i,B}^* &= \oplus \text{ and } E[\Pi_B] = 0, \\ \sigma_{i,2}^* \text{ with } \sigma_{i,U}^* &= \left(l > 0, S_{(\oplus \sim \ominus)} \right), & \sigma_{i,B}^* &= \oplus \text{ and } E[\Pi_B] = 0, \quad \text{or} \\ \sigma_{i,3}^* \text{ with } \sigma_{i,U}^* &= \left(l = 0, S_{(\ominus \sim \dagger)} \right), & \sigma_{i,B}^* &= \ominus \text{ and } E[\Pi_B] = 0. \end{aligned}$$

Proof. See Appendix A.

In $\sigma_{i,0}^*$ (as in σ_0^*), no contract between entrepreneur and analyst is reached, so there is no need for harmonization of interests. In $\sigma_{i,3}^*$ (as in σ_3^*), the entrepreneur chooses not to raise $S + L$ until incentive compatibility is given, hence again nothing speaks for $l > 0$. In $\sigma_{i,1}^*$ (as in σ_1^*), incentive compatibility is reached already with $l = L = 0$. Only in $\sigma_{i,2}^*$, the entrepreneur applies for a loan in order to lower delegation costs by harmonizing interests with the analyst. Proposition 3 is visualized in Fig. 5.

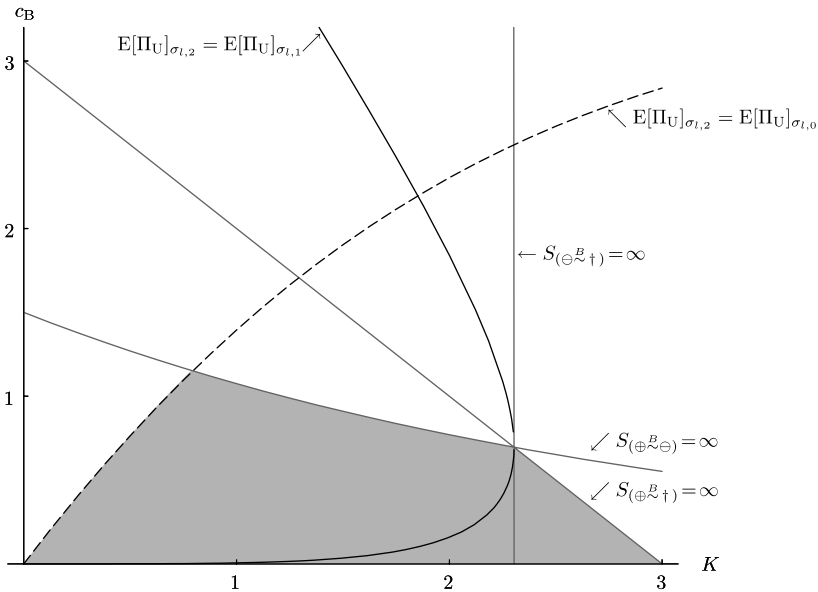


Fig. 5. Scope of sequential equilibria of Game 2.

Note: Like in Fig. 3, \tilde{Y} is exponentially distributed with $\mu = 3$, and $\phi_U = 5$. The area where the entrepreneur risk-analyzes ($\sigma_U^* = \oplus$) is backed gray. l is positive in the gray area between the two curved black lines. In comparison with Fig. 3, one can see that the gray area has expanded: The possibility of a loan makes the performance of risk analysis more likely.

Because loans are advantageous in only one of the four types of equilibria, a remark on the relevance of the results may be appropriate. From an economic point of view, it is clear that not in *all* equilibria there is a use for loans. The reason is that only one aspect of loans is considered – the harmonization of diverging interests. Other important aspects, such as the raising of funds for investment, are not taken into account.²² Of course, parameter constellations under which there is no scope for the delegation of risk analysis are easy to think of. Analysis costs may be prohibitively high ($c_B \rightarrow \infty$) or the initial bankruptcy probability of the entrepreneur may be too low ($K \rightarrow 0$). In these cases, the missing desirability of risk analysis inhibits possible advantages of a loan. However, which type of equilibrium is most realistic depends on the parameter assignment (of F , K , ϕ_U and c_B) that comes closest to reality.

Remark 4 (Comparison of equilibria). *If the entrepreneur is allowed to apply for a loan from the analyst, there are more parameter constellations that if he is not allowed to. The delegation of risk analysis gains scope.*

²² The lack of disadvantages of debt is also the reason that the l of Proposition 3 is only a lower bound for an optimal loan amount. A further increase of l leads neither to advantages nor to disadvantages for entrepreneur or analyst.

In other words, parameter constellations under which σ_0^* of σ_3^* are equilibria in Game 1 (with $\sigma_B^* = \ominus$, no analysis) may lead to equilibria of the type $\sigma_{l,2}^*$ in Game 2 (with $l > 0$ and $\sigma_B^* = \oplus$, analysis). Delegation of risk analysis becomes more likely. The reason is that delegation costs that may be prohibitively high in Game 1 are lowered in Game 2. The opposite direction of change of equilibrium is not possible. The parameter constellations under which $\sigma_{l,1}^*$ and σ_1^* form the equilibrium are identical. Therefore, the “area” in which risk analysis takes place expands.²³ Beyond these points, some further aspects are worth discussing.

(i) In our setting, the loan does not serve any funding purposes. This does not seem to match reality. However, if we alternatively assume that the entrepreneur needs additional funding in order to carry out the project, he has the choice between debt from the capital markets and debt from the risk analyst. In this environment, debt from the analyst has again the property of mitigating the delegation problem, in addition to the funding property. With the same reasoning, we can analyze the effect of collateral on the delegation problem. Because collateral curtails the potential losses for the risk analyst, it worsens incentives. Therefore, the costs of collateral for the entrepreneur may be higher and the optimal degree of collateral lower than they would be in the absence of the delegation problem.

(ii) In the model, only the effects of a loan on the delegation problem are considered. Alternatively, the problem of delegating risk management can also be mitigated if the analyst provides share finance for the entrepreneur. Also shares increase the counterparty risk to be considered by the analyst. One may ask whether share or debt finance generate more incentives for the analyst. Unfortunately, the question cannot be answered from within our setting: In Item 2 (see Section 2.1) we have assumed that financial contracts do not contain return contingencies, and share finance is, of course, highly return contingent. In a modified setting, one might expect that also share finance mitigates the delegation problem, albeit to a lesser degree.

(iii) Why does the analyst, paying c_B , have access to information that the entrepreneur does not have? One may imagine that two types of information are needed for risk management: general information about the capital markets that costs a fixed C to acquire, and firm-specific information that costs c_B . If C is large, there are strong economies of scale, and some firms may want to specialize in information-gathering in order to economize on these costs. Risk analysts emerge endogenously. One may ask *who* is going to become a risk specialist. The answer of our paper is that *banks* (lenders) will assemble this expertise, because they will subsequently have lower delegation costs in comparison with potential competitors.

(iv) One might ask whether large or rather small firms delegate risk analysis. For simplicity, assume that there is a “size” variable γ and that all monetary variables concerning the enterprise (\tilde{Y} , K and ϕ_U) are proportional to γ .²⁴ Then the answer can be derived easily. The only variable left is c_B , the costs of risk analysis. Choosing

²³ For an illustration, compare Fig. 3 with Fig. 5. The gray area grows.

²⁴ Of course, alternative assumptions, such as K growing more than proportionally with γ , or ϕ_U growing less than proportionally with γ , may be even more plausible.

γ as numéraire, c_B becomes inversely proportional to γ . A doubling of the firm's size γ has the same effect as a halving of the costs of risk analysis c_B . Now assume that c_B is low ($c_B \approx 0$). The terms S^* that make the analyst indifferent between \oplus and \ominus are small ($S^* \approx 0$). Therefore, the entrepreneur does not need to oblige the analyst by raising S^* above the “fair” level in order to make him risk-analyze. There are no delegation costs for the entrepreneur, and the first-best solution can be reached. Hence large firms delegate risk analysis. They need not raise a loan in order to reduce delegation costs. By contrast, $\gamma \approx 0$ is equivalent to $c_B \approx \infty$, implying that the entrepreneur does not delegate risk analysis. However, no monotonic relation between e.g. expected profits of the risk analyst and size of the firm exist.²⁵ In short, large firms have no costs when delegating risk management. The fixed costs of risk analysis imply economies of scale already in the first-best world without a delegation problem. The above considerations – even if simplistic – suggest that economies of scale are aggravated in the presence of the delegation problem.

(v) A property of Figs. 3 and 5 can be generalized: Entrepreneurs with either very high or very low credit standing do not delegate risk management (note that a high K implies a high probability of bankruptcy $F(K)$). For entrepreneurs with a low $F(K)$, insuring their risk is not worthwhile. If $F(K)$ is high, insurance of that much risk is prohibitively costly. As a result, the clientele of banks is entrepreneurs with a medium credit standing.²⁶

(vi) An important question is whether the risk analyst may be interpreted as a bank. To be sure, the answer to this question depends on how narrow one defines the word “bank”. If one regards the refinancing through illiquid deposits as the key property of banks, the question remains unanswered by the model. If one defines banks as firms that grant loans, the risk analyst surely is a bank. The question of how the risk analyst is financed is not addressed in the model – it is assumed that the analyst has sufficient funds to pay all obligations. It is legitimate to ask what changes if the analyst has only limited funds and is himself threatened by bankruptcy. Assume also that the analyst has access to credit markets in order to obtain additional funding, and to financial markets to reinsure the risk he incurs from the entrepreneur. This complicates the model fundamentally, augmenting the number of players from two to four. However, the basic properties of the model remain unchanged. The entrepreneur, borrowing from the risk analyst, creates incentives for the analyst to avert the entrepreneur's bankruptcy, which in turn mitigates the costs of delegating risk management.²⁷ Yet another approach would be to assume that there exist further market imperfections that typically entail the existence of banks, such as costly

²⁵ The reason is that an increase of the numéraire γ has a positive direct effect on expected profits, but lets c_B and therewith $E[I_B]$ shrink. There are therefore two opposite effects of which none strictly dominates the other.

²⁶ For a formal discussion, see Proposition 7.

²⁷ However, this mitigating effect may be stronger than in the benchmark case of this paper, as a bankruptcy of the entrepreneur can have contagious effects on the solvency of the analyst. On the other hand, it may also be weaker because of the analyst's limited liability. The relative size of these effects depends on the parameterization of the extended model.

state verification as in Diamond (1984), Williamson (1986) or Winton (1995).²⁸ In this case, it seems natural that it is the risk analyst who assumes the task of delegated monitoring. That way, the problem of delegating risk management is presumably mitigated again.²⁹ Additionally in the setting of Williamson (1986), the analyst can economize on monitoring costs.³⁰

(vii) Note that the preceding results can be reinterpreted in a different manner. In Game 1 (without loan), the constellations of parameters are such that in equilibrium, the expected profits of the analyst are positive. In Game 2 (with the loan), the entrepreneur has to cede part of his expected profits to the analyst in order to satisfy incentive compatibility. Now Remark 3 states that it is irrelevant whether incentive compatibility is achieved by higher terms S or by a loan repayment L (which is linked to the loan amount l). One can define a different game which leads to the same results. The entrepreneur auctions off the prospect of the contract. The bidders are risk analysts. A risk analyst then bids as high as the future expected payoffs of the hedge contract.³¹ In equilibrium, all expected cash flows are identical to the ones of Game 2.

4. Discussion and conclusion

As shown, an agent that is able to obtain information about the correlations of risky assets endogenously carries out risk analysis, risk controlling (thus *risk management*) and grants loans, on behalf of a principal, an entrepreneur. The model suggests that there are economies of scope between lending and risk management. It can therefore be seen as one possible answer to the assertion of Allen and Santomero (1998) that there is no cogent theory as to why banks should be the ones offering the service of risk management.

Several simplifying assumptions allowed us to derive analytical results. The following extensions seem desirable from an empirical point of view, but may render the explicit analysis of the model more difficult.

(i) Unlike the entrepreneur, the risk analyst is not threatened by bankruptcy in the model. This has two implications. First, the entrepreneur does not need to take the risk analyst's possible insolvency into consideration. As a result, the risk analyst can easily insure any conceivable risk. Second, unlike a real bank, the risk analyst has no intrinsic incentive to manage (own) risk. The incentive emerges extrinsically from the

²⁸ Note that the sources of information asymmetry are primarily different in our paper and Diamond (1984). There, ex post state verification is costly. Here, there is no ex post information asymmetry. Ex ante, correlations are unobservable for the entrepreneur.

²⁹ Still, one caveat may make delegated risk management for several entrepreneurs problematic. Under certain circumstances, the risk analyst may be tempted to exploit his limited liability and sell the entrepreneurs highly correlated hedging contracts, no matter what the correlation of their projects is.

³⁰ If he exerts c_B and learns \tilde{Y} , he can infer $Y = \tilde{Y}(\omega)$ from the state of nature ω . He needs to monitor only in the cases where the entrepreneur reports that the project return is less than the (inferred) Y .

³¹ The auction's mechanism (English, Dutch, Vickrey, First Price) is irrelevant if risk analysts have the same information about the value of the future contract.

entrepreneur's delegation. As a possible extension of the model, one can impose financial restrictions on the risk analyst. Because of potential bankruptcy, the risk analyst then acts no longer risk neutrally. When risk sharing, he weighs up his own against his customer's interests. In case of conflict, the customer's interests may be neglected. Still, the risk analyst may create *some* value by bearing some of the entrepreneur's risk.

(ii) The entrepreneur's initial probability of bankruptcy is exogenous. In reality, it seems obvious that this probability can be influenced, e.g. by the way the entrepreneur operates his business. For example, the entrepreneur may be able to agree upon different terms with his customers. The terms that his business partners grant may in turn depend on the entrepreneur's decision about whether to hedge against bankruptcy risk.³² A possible extension of the model might consider the reaction of the entrepreneur's business partners on his hedging decision.

(iii) In the model, the risk analyst can choose between carrying out the risk analysis or refrain from it.³³ Alternatively, one may assume the risk analyst's set of options to be larger and smoother. He may then e.g. choose between different levels of effort when carrying out the risk analysis.

However, the idea that loans mitigate the delegation problem and therefore lenders have comparative advantages in selling advice concerning risk management to their lenders seems to be very robust regarding different assumptions. This paper is the first to point out this possible function of banks.

Acknowledgements

The author thanks Elena Carletti, Juliane Godehardt, Martin Hellwig, Dirk Krüger, Peter Nippel, Andreas Pfingsten, the participants at the 2001 Doctoral Seminar for the Promotion of the Young Generation in the Banking Science in Bochum, at the Banking Workshop in Münster, at the Theory Workshop in Mannheim, at the European Meeting of the Econometric Society 2002 in Venice and at the annual meeting of German Economic Association 2002 in Innsbruck for helpful comments. Finally, valuable remarks and suggestions by editors and referees, which resulted in a major revision, are gratefully acknowledged. Still, the sole responsibility for remaining shortcomings rests with us.

³² The price a customer is willing to pay for a good may e.g. depend on the producer's probability of bankruptcy, especially if the good need future servicing. Even multiple equilibria might exist. If an entrepreneur's business partners – especially his lenders – believe that his credit worthiness is high, they will demand only a low risk premium, which reduces the entrepreneur's costs of funds and thus his probability of bankruptcy. When they believe that the credit worthiness is low, one can argue the other way round.

³³ This causes the jumps in the expected profits functions of Fig. 2.

Appendix A

Proof of Remark 1. If the risk analyst draws up the fitting contract, he can expect ³⁴

$$E[II_B]_{(S,\oplus)} = E[\tilde{Z}] - c_B = \int_{\Omega} \tilde{Z}(\omega) dPr\{\omega\} - c_B \tag{A.1}$$

$$\begin{aligned} &= \int_0^{\infty} \min\{S, Y - K\} f(Y) dY - c_B \\ &= \int_0^{S+K} (Y - K) f(Y) dY + \int_{S+K}^{\infty} S f(Y) dY - c_B \\ &= SF(S + K) - \mathcal{F}(S + K) + (1 - F(S + K))S - c_B \\ &= S - \mathcal{F}(S + K) - c_B. \end{aligned} \tag{A.2}$$

These expected profits are an upper bound for the possible expected profits of other contracts with the same distribution,

$$E[II_B] = \int_{\Omega} \min\{\tilde{Z}(\omega), \tilde{Y}(\omega)\} dPr\{\omega\} - c_B \leq \int_{\Omega} \tilde{Z}(\omega) dPr\{\omega\} - c_B. \tag{A.3}$$

Thus if the analyst draws up a contract at all, he selects the *fitting* contract. However, if the analyst had chosen to enter into no contract at all, then he would have foreseen this before spending c_B – hence this cannot occur, if he acts rationally. \square

Proof of Remark 2. We lead the proof for the case of finite Ω with $P(\omega) = 1/|\Omega|$ for all $\omega \in \Omega$ only. We have thus an equal distribution on Ω . Arbitrary distributions can be approximated by taking the limit $|\Omega| \rightarrow \infty$. ³⁵

Now be a function $g : \mathbb{R} \rightarrow \mathbb{R}$ given such that the fitting (desirable) contract \tilde{Z} has the form $\tilde{Z}(\omega) = g(\tilde{Y}(\omega))$. In the case of the model of Section 2.1, $g(Y) = \min\{S, Y - K\}$. Be $\mathcal{G} := \{g(\tilde{Y}(\omega)) | \omega \in \Omega\}$ the image set of $g(\tilde{Y})$, and $\gamma : \mathcal{G} \rightarrow \mathbb{N}$, $\gamma(z) := \#\{\omega \in \Omega | g(\tilde{Y}(\omega)) = z\}$ the function that counts how often a $z \in \mathcal{G}$ is hit.

Now the risk analyst, without gathering information about the state dependency of the project \tilde{Y} , sets

$$\tilde{Z}_{\pi}(\omega) = g(\tilde{Y}(\pi(\omega)))$$

each \tilde{Z}_{π} chosen with equal likelihood for each $\pi \in \Pi_{\Omega}$, where Π_{Ω} is the set of permutations on Ω . Hence, $|\Pi_{\Omega}| = |\Omega|!$ Then

$$P(\tilde{Z} = z | \omega) = \frac{\gamma(z)(|\Omega| - 1)!}{|\Omega|!} = \gamma(z) / |\Omega|$$

and likewise

³⁴ Let $F(f)$ without index always stand for $F_{\tilde{Y}}(f_{\tilde{Y}})$. Be \mathcal{F} the primary function of F .
³⁵ An arbitrary distribution function F can be approximated by assuming that $\Omega = \{1, \dots, N\}$, $N \in \mathbb{N}$ and setting $\tilde{Y}(i) = F^{-1}\left(\frac{i-1/2}{N}\right)$.

$$P(\tilde{Z} = z | \tilde{Y}(\omega)) = \frac{\gamma(t)^2(|\Omega| - 1)!}{\gamma(t)|\Omega|} = \gamma(z)/|\Omega|.$$

The same result one would get when assuming that $\tilde{Z}(\omega) = g(\tilde{X}(\omega))$, with \tilde{X} i.i.d. of \tilde{Y} . \square

The derivation of Eq. (5) is based on partial derivation.

$$\begin{aligned} & E[\Pi_B]_{(S,\oplus)} \\ &= \int_0^\infty \int_0^\infty \min\{Y, X - K, S\} f(Y) dY f(X) dX \\ &= \int_{S+K}^\infty \int_S^\infty Sf(Y) dY f(X) dX + \int_{S+K}^\infty \int_0^S Yf(Y) dY f(X) dX \\ &+ \int_K^{S+K} \int_{X-K}^\infty (X - K)f(Y) dY f(X) dX + \int_K^{S+K} \int_0^{X-K} Yf(Y) dY f(X) dX \\ &+ \int_0^K \int_0^\infty (X - K)f(Y) dY f(X) dX = (1 - F(S + K))(1 - F(S))S \\ &+ (1 - F(S + K))(SF(S) - \mathcal{F}(S)) + \int_K^{S+K} (1 - F(X - K))(X - K)f(X) dX \\ &+ \int_K^{S+K} ((X - K)F(X - K) - \mathcal{F}(X - K))f(X) dX - \mathcal{F}(K) \\ &= (1 - F(S + K))(S - \mathcal{F}(S)) + \int_K^{S+K} ((X - K) - \mathcal{F}(X - K))f(X) dX - \mathcal{F}(K) \\ &= (1 - F(S + K))(S - \mathcal{F}(S)) + F(S + K)\mathcal{F}(S) - (\mathcal{F}(S + K) - \mathcal{F}(K)) \\ &- \int_K^{S+K} \mathcal{F}(X - K)f(X) dX - \mathcal{F}(K) \\ &= S - \mathcal{F}(S + K) - \int_0^S F(X)(1 - F(X + K)) dX, \end{aligned}$$

which was to be shown. \square

Proposition 4 (Indifference points of the risk analyst). *If the entrepreneur offers a contract to the risk analyst, he proposes terms S so low that the risk analyst is indifferent between two moves.*

Proposition 4 implies that only a finite set of terms S needs to be considered. The point $S_{(\oplus \sim \ominus)}^B$ where the analyst is indifferent between a contract with analysis and a contract without, $S_{(\oplus \sim \dagger)}^B$ where the analyst is indifferent between a contract with analysis and no contract at all, and $S_{(\ominus \sim \dagger)}^B$ where the analyst is indifferent between a contract without risk analysis and no contract.

Proof of Proposition 4. The assertion follows from the fact that the entrepreneur’s expected profits decrease monotonically with S , for

$$\begin{aligned} \partial E[\Pi_U]_{(S,\oplus)} / \partial S &= F(S + K) - 1 < 0 \quad \text{and} \\ \partial E[\Pi_U]_{(S,\ominus)} / \partial S &= (F(S) - 1 - \phi_U f(S + K))(1 - F(S + K)) < 0. \end{aligned}$$

Of course, $\partial E[\Pi_U]_{(S,\dagger)} / \partial S = 0$. Hence for every move of the risk analyst $\sigma_B \in \{\oplus, \ominus, \dagger\}$, the entrepreneur preferably chooses S low, only restricted by the risk analyst's indifference points. \square

Remark 5 (Existence and uniqueness of indifference points). *If an indifference point $S_{(\oplus \rightsquigarrow \ominus)}$ exists, it is unique, and for all $S < S_{(\oplus \rightsquigarrow \ominus)}$, the analyst prefers not to carry out the risk analysis. If $S_{(\oplus \rightsquigarrow \ominus)}$ does not exist, the analyst always prefers to carry out the risk analysis. Analogous statements apply for $S_{(\oplus \rightsquigarrow \dagger)}$ and $S_{(\ominus \rightsquigarrow \dagger)}$.*³⁶

Proof of Remark 5. The claim follows from the fact that

$$\begin{aligned} 0 < 1 - F(S + K) - F(S)(1 - F(S + K)) < 1 - F(S + K) &\Rightarrow \\ \frac{\partial E[\Pi_B]_{(\dagger, \dagger)}}{\partial S} < \frac{\partial E[\Pi_B]_{(S, \ominus)}}{\partial S} < \frac{\partial E[\Pi_B]_{(S, \oplus)}}{\partial S} \quad \text{and in } S = 0, & \\ 0 < -\mathcal{F}(K) < -\mathcal{F}(K) &\Rightarrow \\ E[\Pi_B]_{(\dagger, \dagger)} > E[\Pi_B]_{(S=0, \ominus)} > E[\Pi_B]_{(S=0, \oplus)}. & \end{aligned}$$

Hence, indifference points are unique, and if they fail to exist, the option preferred at $S = 0$ is preferred everywhere. \square

Proof of Proposition 1. The proof draws on Remark 5. If the entrepreneur's global optimum is (as illustrated in Fig. 2) reached with $\sigma_B^* = \oplus$, then he implements S as small as possible. That being so, either $\sigma_U^* = S_{(\oplus \rightsquigarrow \dagger)}$ with $E[\Pi_B] = 0$ or $\sigma_U^* = S_{(\oplus \rightsquigarrow \ominus)}$ with $E[\Pi_B] > 0$ holds. If $\sigma_B^* = \oplus$ is optimal for the entrepreneur, then the lower bound of potential S is $S_{(\ominus \rightsquigarrow \dagger)}$. If no contract is better than any contract, then the entrepreneur's strategy will be $\sigma_U^* = \dagger$.³⁷ \square

Having categorized the possible sequential equilibria, it is of interest to deduce how the endogenous variables react to changes of exogenous parameters (comparative statics). In the following Propositions 5 and 6, we concentrate on equilibria of the form σ_2^* .³⁸ The entrepreneur can solve the delegation problem and make the analyst inspect ($\sigma_B = \oplus$) only by obliging him through raising the terms S to $S_{\oplus \rightsquigarrow \ominus}^B$.

³⁶ To be precise, if $S_{(\oplus \rightsquigarrow \dagger)}$ ($S_{(\ominus \rightsquigarrow \dagger)}$ respectively) exists, it is unique, and for all $S < S_{(\oplus \rightsquigarrow \dagger)}$ ($S_{(\ominus \rightsquigarrow \dagger)}$ respectively), the analyst prefers not to participate. If $S_{(\oplus \rightsquigarrow \dagger)}$ ($S_{(\ominus \rightsquigarrow \dagger)}$) does not exist, the analyst prefers not to participate (compared to participating and analyzing/not analyzing).

³⁷ There are in fact more equilibria. However, the corresponding payments are equal those of one of the four equilibria listed above. For example, instead of playing $\sigma_U^* = \dagger$, the entrepreneur can set $S = 0$, which implies $E[\Pi_B] < 0$. As a result, the analyst cannot possibly agree. The equilibrium $\sigma^* = (\sigma_U^* = 0, \sigma_B^* = \dagger)$ is therefore considered as equivalent to $\sigma_0^* = (\sigma_U^* = \dagger, \sigma_B^* = \dagger)$.

³⁸ For the other types, comparative statics are trivial. $E[\Pi_B] = 0$ in any event, and $\partial E[\Pi_U] / \partial c_B$, $\partial E[\Pi_U] / \partial K$ and $\partial E[\Pi_U] / \partial \phi_U$ are negative or vanish.

Intuitively, if c_B is high, the entrepreneur must provide more incentives. If K is high, he must pay more for the “insurance”.

Proposition 5 (Comparative statics for S^*). *Let F, K, ϕ_U and c_B be such that in equilibrium, expected profits of the risk analyst are positive (equilibrium has the form σ_2^*). Then*³⁹

$$\frac{\partial S^*}{\partial c_B} > 0 \quad \text{and} \quad \frac{\partial S^*}{\partial K} > 0. \tag{A.4}$$

The results of Proposition 5 can be used to analyze the expected profits’ dependence on exogenous parameters.

Proof of Proposition 5. In σ_2^* , the analyst is indifferent between \oplus and \ominus , so

$$c_B = \int_0^{S^*} F(X)(1 - F(X + K)) dX. \tag{A.5}$$

Differentiation of (A.5) subject to c_B yields

$$1 = F(S^*)(1 - F(S^* + K)) \frac{\partial S^*}{\partial c_B}$$

and thus

$$\frac{\partial S^*}{\partial c_B} = \frac{1}{F(S^*)(1 - F(S^* + K))} > 0.$$

Analogously,

$$\frac{\partial S^*}{\partial K} = \frac{\int_0^{S^*} F(X)f(X + K) dX}{F(S^*)(1 - F(S^* + K))} > 0,$$

which was to be shown. \square

Proposition 6 (Comparative statics for $E[\Pi_B]$ and $E[\Pi_U]$). *Let F, K, ϕ_U and c_B be such that in equilibrium, expected profits of the risk analyst are positive (equilibrium has the form σ_2^*). Then*

$$\begin{aligned} \frac{\partial E[\Pi_B]}{\partial K} < 0, \quad \frac{\partial E[\Pi_U]}{\partial K} > 0, \\ \frac{\partial E[\Pi_B]}{\partial c_B} > 0, \quad \frac{\partial E[\Pi_U]}{\partial c_B} < 0. \end{aligned}$$

Furthermore, $\partial E[\Pi_B]/\partial \phi_U = \partial E[\Pi_U]/\partial \phi_U = 0$.

³⁹ Note that if one of the variables changes too much, equilibrium may switch from σ_2^* to a different type. Accordingly, the derived inequations apply only locally in the interior of σ_2^* .

Proof of Proposition 6

$$E[\Pi_U] = E[\tilde{Y}] - S^* + \mathcal{F}(S^* + K)$$

$$\text{s.t. } c_B = \int_0^{S^*} F(X)(1 - F(X + K)) dX,$$

thus

$$\frac{\partial E[\Pi_U]}{\partial K} = F(S^* + K) - (1 - F(S^* + K)) \frac{\partial S^*}{\partial K}$$

$$= F(S^* + K) - (1 - F(S^* + K)) \frac{\int_0^{S^*} F(X)f(X + K) dX}{F(S^*)(1 - F(S^* + K))}$$

which is negative if and only if

$$F(S^* + K)F(S^*) < \int_0^{S^*} F(X)f(X + K) dX$$

$$= F(S^*)F(S^* + K) - \int_0^{S^*} f(X)F(X + K) dX \iff 0$$

$$> \int_0^{S^*} f(X)F(X + K) dX,$$

which is a contradiction. Hence $\partial E[\Pi_U]/\partial K > 0$ holds. Furthermore,

$$\frac{\partial E[\Pi_U]}{\partial c_B} = -(1 - F(S^* + K)) \frac{\partial S^*}{\partial c_B} < 0,$$

$$\frac{\partial E[\Pi_B]}{\partial K} = -\frac{\partial E[\Pi_U]}{\partial K} < 0 \quad \text{and}$$

$$\frac{\partial E[\Pi_B]}{\partial c_B} = -\frac{\partial E[\Pi_U]}{\partial c_B} - 1$$

$$= (1 - F(S^* + K)) \frac{1}{F(S^*)(1 - F(S^* + K))} - 1 = \frac{1}{F(S^*)} - 1 > 0.$$

Obviously, $\partial E[\Pi_B]/\partial \phi_U = \partial E[\Pi_U]/\partial \phi_U = 0$. \square

Nothing can be said about how the expected profits of the players react to changes of F in general.⁴⁰ One of the results surprises at first sight. The risk analyst’s expected profits increase with the costs. Intuitively, if the costs of risk analysis rise, the entrepreneur must raise the analyst’s counterparty risk $\kappa(S)$ and as a result S in order to keep the analyst indifferent between risk analysis and randomizing. Proposition 6 shows that the effect of the rising S outweighs the effect of the higher c_B . Consequently, the analyst is overcompensated for his increasing costs.

⁴⁰ This is due to the multidimensionality of F . If one e.g. assumes that $\log \tilde{Y}$ is normally distributed, one can calculate $\partial E[\Pi_U]/\partial \mu$, $\partial E[\Pi_U]/\partial \sigma$ and so forth.

Proposition 7 (Credit standing of analyzed entrepreneurs). *Given F , c_B and ϕ_U , an entrepreneur with especially high or low credit standing does not offer the risk analyst a contract such that this examines the project in equilibrium (hence equilibrium has the form σ_1^* or σ_2^*).⁴¹*

Consequently, the customers of the analyst have a medium credit standing. For entrepreneurs with a high standing, the costs of risk analysis (inclusive of delegation costs) are too high, compared with potential gains when saving bankruptcy costs. Entrepreneurs with a very low standing cannot afford a hedge contract, because the costs of insurance exceed expected profits.

Proof of Proposition 7. First note that because of F 's monotonicity, a high (low) K is equivalent to a high (low) initial bankruptcy probability and thus to a low (high) credit standing. If K is low ($K \approx 0$), then $E[\Pi_U]_{\dagger} = E[\tilde{Y}] - \phi_U F(K) \approx E[\tilde{Y}]$. If analyst and entrepreneur sign a contract and the analyst risk-analyzes, then $E[\Pi_B] + E[\Pi_U] = E[\tilde{Y}] - c_B$, thus $E[\Pi_B] \geq 0$ implies $E[\Pi_U] \leq E[\tilde{Y}] - c_B$. Risk analysis is too expensive for $K \approx 0$. For the case of high K , note that

$$\kappa(S) = \int_0^S F(X)(1 - F(X + K)) dX \leq SF(S)(1 - F(K)).$$

If \tilde{Y} is integrable then $SF(S)$ is bounded, and for high K and thus small $1 - F(K)$, there is no S with $\kappa(S) = c_B$. Incentives are never sufficient to guarantee the analysis. \square

Proof of Remark 3. Clearly, (8) remains unchanged when substituting $L \rightarrow 0$ and $S \rightarrow S + L$, which proves (12). The same applies for (10), yielding (13). Hence concerning the indifference point,

$$\begin{aligned} E[\Pi_B]_{(S,\oplus),(L,l)} &= E[\Pi_B]_{(S,\ominus),(L,l)} \text{ iff} \\ E[\Pi_B]_{(S+L,\oplus),(0,l)} &= E[\Pi_B]_{(S+L,\ominus),(0,l)} \text{ iff} \\ E[\Pi_B]_{(S+L,\oplus),(0,0)} &= E[\Pi_B]_{(S+L,\ominus),(0,0)} \text{ iff} \\ S + L &= S_{(\oplus \sim \ominus)^B}, \end{aligned}$$

thus $S_{(\oplus \sim \ominus)^B, (L,l)} = S_{(\oplus \sim \ominus)^B} - L$, which was to be shown. \square

Proof of Proposition 2. (14) implies that with raising L , $S_{(\oplus \sim \ominus)^B, (L,l)}$ is lowered by the same amount. The sum $S_{(\oplus \sim \ominus)^B, (L,l)} + L$ remains constant, and as can be seen in (12) and (13), the analyst's expected profits stay constant. Because analysis takes place, also the entrepreneur's profits in (9) depend only on the sum $S + L$ and stay unchanged. Thus only the change of l affects expected profits, and as can be seen from

⁴¹ Fig. 3 furnishes an illustration for Proposition 7. For fixed c_B , the analyst risk-analyzes only for medium K .

(8) and (9), $\partial E[\Pi_B]/\partial l = -1$ and $\partial E[\Pi_U]/\partial l = 1$ if $\sigma_U = \oplus$. The condition $\bar{l} = \bar{L} - \mathcal{F}(\bar{L})$ guarantees that after granting the loan, analyst and entrepreneur are still interested in achieving a hedge contract. \square

Proof of Proposition 3. The proof can be led analogously to that of Proposition 1. The entrepreneur chooses $S + L$ just large enough to guarantee the desired response of the analyst. A positive $E[\Pi_B]$ is impossible, because of Proposition 2, the entrepreneur would rather raise l and pocket the expected profits himself.⁴²

We can now examine how the loan amount l reacts to changes in exogenous parameters (comparative statics). Only the case $\sigma_{l,2}^*$ is of interest, because in the other cases, we have $l = 0$.

Remark 6 (Comparative statics for l). Let F , K , ϕ_U and c_B be such that in equilibrium, the amount of debt is positive (equilibrium has the form $\sigma_{l,2}^*$). Then

$$\frac{\partial l}{\partial K} < 0 \quad \text{and} \quad \frac{\partial l}{\partial c_B} > 0.$$

Furthermore, $\partial l / \partial \phi_U = 0$.

Note that the analyst grants loans only to entrepreneurs with medium credit standing. This can be derived directly from Proposition 7.

Proof of Remark 6. By Proposition 2, each additional monetary unit of l increases $E[\Pi_U]$ and decreases $E[\Pi_B]$ by one unit. The rest can be derived by substituting $E[\Pi_B] \rightarrow l$. \square

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⁴² Analogously to Proposition 1 (as stated in Footnote 38), there are additional equilibria which are equivalent to the ones listed above concerning payments.

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