1 Number System Basics

Digital technology has become widespread and encompasses virtually all aspects of our everyday lives. We could see it being used in computers and related gadgets, entertainment, automation (robotics), medical etc. Though physical quantities measured in the real world are analogue, most of these are processed by digital means. In order to do this, we have to convert the measured analogue quantity into digital, process the digital quantity using digital circuitry and then reconvert to analogue.

The contents of this book concentrate on the digital circuit design to enable the processing of the digital quantity. But before we look into the principles of such designs, we need to understand the basics of number systems.

1.1 Decimal Numbers

Decimal number system is the commonly used number system that has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. It is also known as base (or radix) ten system since it has ten digits that can be used to represent any number. Figure 1.1 shows the positional values or weights of the decimal number system for an integer.

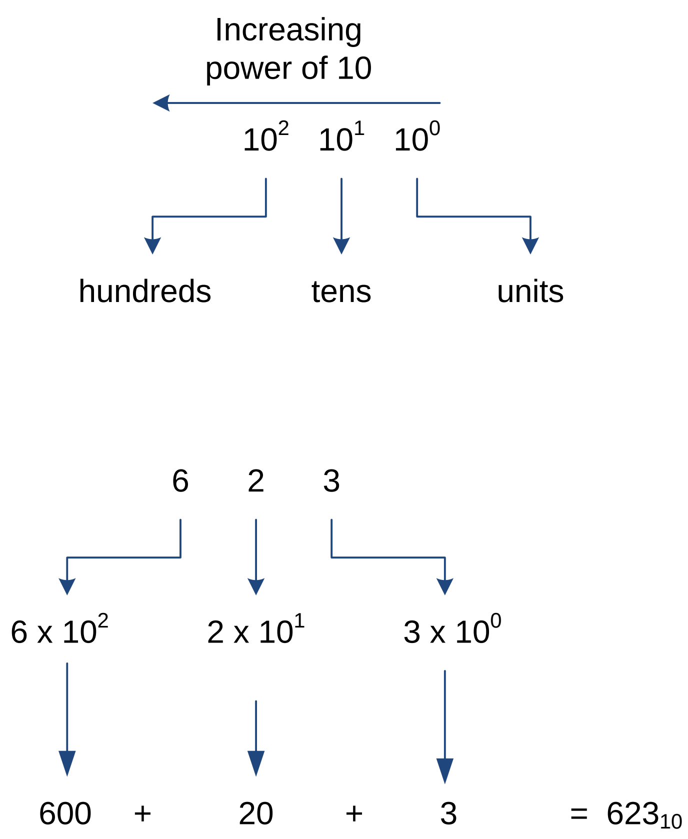
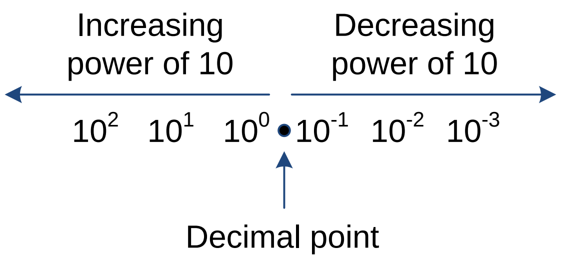


Figure 1.1: Decimal number system for integers.

The digit with least weight (i.e. the one on the foremost right) is known as the least significant digit (LSD) while the highest weight digit is known as the most significant digit (MSD). In the example shown in Figure 1.1, the MSD is digit 6 while the LSD is digit 3. Figure 1.2 shows the case for fractional decimal number.



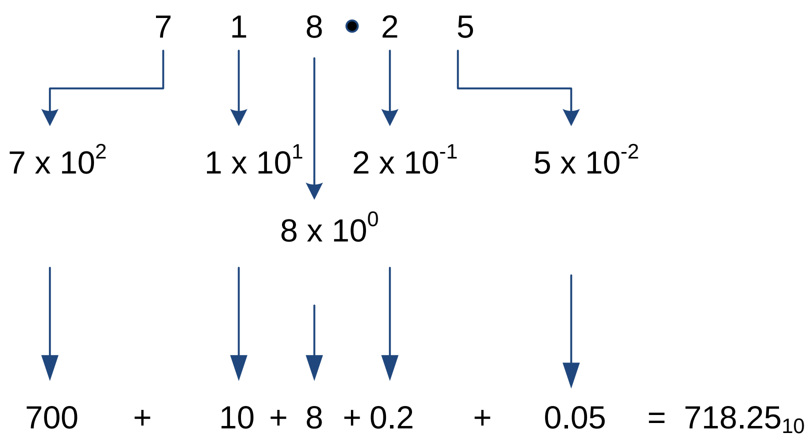
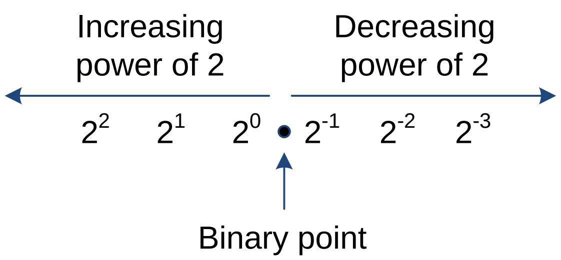


Figure 1.2: Decimal number system for fractional numbers.

1.2 Other Number Systems – Binary, Octal and Hexadecimal

While decimal number system is the commonly used number system in everyday lives, digital devices uses only binary number system that consists of 0 and 1. The base is two for this system and Figure 1.3 show an example of binary number for decimal equivalent of 6.25_{10}



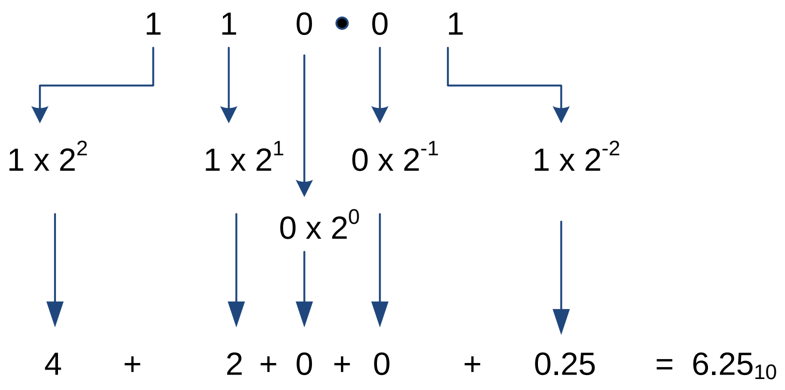
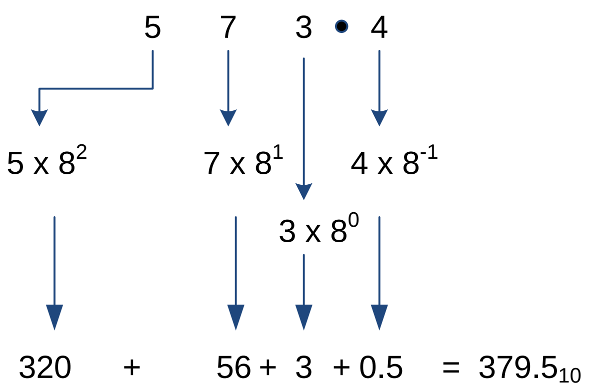
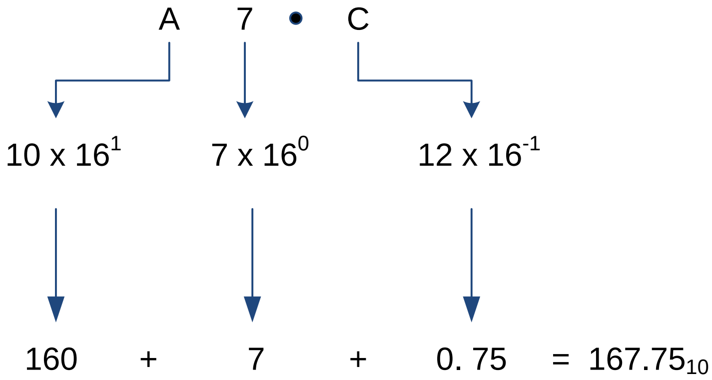


Figure 1.3: Binary number system with an example.

Similarly, octal and hexadecimal (hex in short) number systems have number bases of 8 and 16. For octal number system, the eight digits are 0, 1, 2, 3, 4, 5, 6, and 7 while hexadecimal number system has 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. Figure 1.4 gives examples on these number systems.



(a)



(b)

Figure 1.4: Number system examples (a) octal (b) hex.

1.3 Conversion between different number systems

It is often necessary to convert a number from one base system to another. Converting a number to decimal is rather straightforward as we have seen in the previous examples. The weights or positional values (for the appropriate base) are multiplied with the digit and summed to give the decimal value. In this section, we will look at methods to convert numbers from decimal to binary, octal and hex. Other conversions such as octal to binary (and vice versa), binary to hex, hex to binary, octal to hex and hex to octal are also possible.

1.3.1 Decimal to binary, octal and hex conversions

There are two methods that can be used to achieve decimal to binary conversion. The first method is by presenting the decimal value in units, tens, hundreds etc. For example:

$$26 = 16 + 8 + 2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 11010_{10}$$

The problem with this method is that certain positional values (such as 2^2 and 2^0 in the example above) can easily be forgotten. There is another method called repeated division that is more frequently employed. Figure 1.5 illustrates this method. It works by repeated division with a value of 2 (until the quotient is 0) and the remainder digits from each step represent the binary number (in reverse order).

				remainder	
$\frac{34}{2}$	=	17	→	0	LSD
$\frac{17}{2}$	=	8	→	1	
$\frac{8}{2}$	=	4	→	0	
$\frac{4}{2}$	=	2	→	0	
$\frac{2}{2}$	=	1	→	0	
$\frac{1}{2}$	=	0	→	1	MSD

Figure 1.5: Decimal to binary conversion example, $34_{10} = 100010_2$.

Similarly, we can convert a decimal number to octal and hex. Figures 1.6 and 1.7 illustrate the steps for these conversions. Do remember that the final answer is in the reverse order!

				remainder	
$\frac{149}{8}$	=	18	→	5	LSD
$\frac{18}{8}$	=	2	→	2	
$\frac{2}{8}$	=	0	→	2	MSD

Figure 1.6: Decimal to octal conversion example, $149_{10} = 225_8$.

				remainder	
$\frac{564}{16}$	=	35	→	4	LSD
$\frac{35}{16}$	=	2	→	3	
$\frac{2}{16}$	=	0	→	2	MSD

Figure 1.7: Decimal to hex conversion example, $564_{10} = 234_{16}$.

1.3.2 Binary to Octal and vice versa

Any binary number can be converted to octal simply by grouping them in groups of three digits. For example, 100101110_8 can be converted to 456_8 as shown in Figure 1.8 (a). The reverse procedure of converting an octal number to binary can be done by writing three binary digit equivalent for each octal digit. This is shown in Figure 1.8 (b).

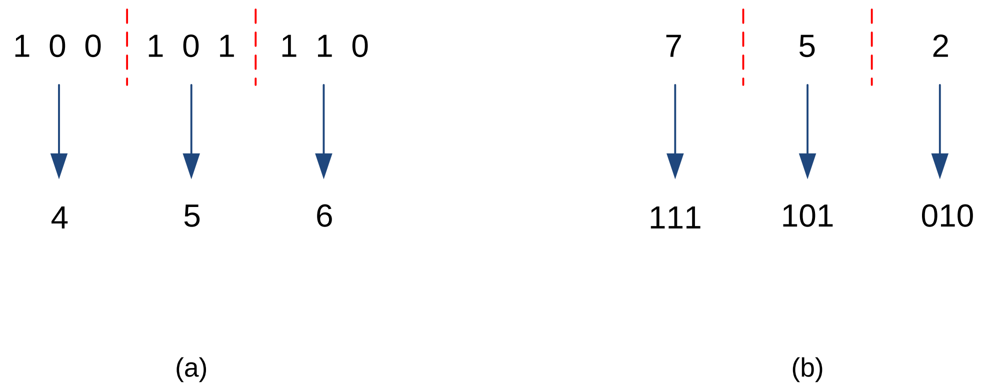


Figure 1.8: Octal to binary conversion example and vice versa: (a) $100101110_2 = 456_8$ (b) $752_8 = 111101010_2$.

1.3.3 Binary to Hex and vice versa

Similar to octal number, binary number can be converted to hex simply by grouping them in groups of four digits. For example, 10010111_2 can be converted to 97_{16} as shown in Figure 1.9 (a). A hex number can be converted to binary by writing four binary digit equivalent for each hex digit. This is shown in Figure 1.9 (b).

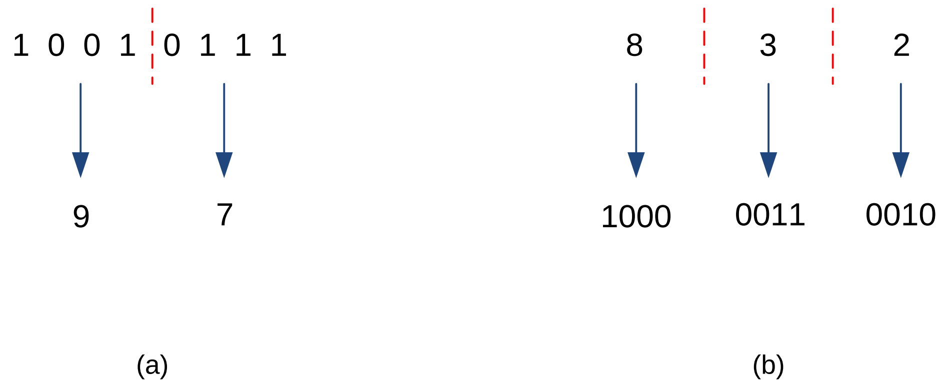


Figure 1.9: Hex to binary conversion example and vice versa: (a) $10010111_2 = 97_{16}$ (b) $832_{16} = 100000110010_2$.

1.4 Other number codes

In this section, several other commonly used codes will be discussed.

1.4.1 ASCII code

ASCII stands for American Standard Code for Information Interchange. Characters such as 'a', 'A', '@', '$' each have a code that is recognised by the computer. Standard ASCII has 128 characters (represented by 7 binary digits; 2^7=128), though the first 32 is no longer used. Extended ASCII has another 128 characters, mostly to represent special characters and mathematical symbols such as 'ÿ', 'ë', 'Σ', and 'σ'. Table 1.1 shows the standard ASCII code.

Table 1.1: Standard ASCII code

D'mal	Hex	B'ary	Char	D'mal	Hex	B'ary	Char	D'mal	Hex	B'ary	Char
32	20	0100000	space	48	30	0110000	0	64	40	1000000	@
33	21	0100001	!	49	31	0110001	1	65	41	1000001	A
34	22	0100010	“	50	32	0110010	2	66	42	1000010	B
35	23	0100011	#	51	33	0110011	3	67	43	1000011	C
36	24	0100100	$	52	34	0110100	4	68	44	1000100	D
37	25	0100101	%	53	35	0110101	5	69	45	1000101	E
38	26	0100110	&	54	36	0110110	6	70	46	1000110	F
39	27	0100111	’	55	37	0110111	7	71	47	1000111	G
40	28	0101000	(	56	38	0111000	8	72	48	1001000	H
41	29	0101001	)	57	39	0111001	9	73	49	1001001	I
42	2A	0101010	*	58	3A	0111010	:	74	4A	1001010	J
43	2B	0101011	+	59	3B	0111011	;	75	4B	1001011	K
44	2C	0101100	,	60	3C	0111100	<	76	4C	1001100	L
45	2D	0101101	-	61	3D	0111101	=	77	4D	1001101	M
46	2E	0101110	.	62	3E	0111110	>	78	4E	1001110	N
47	2F	0101111	/	63	3F	0111111	?	79	4F	1001111	O

D'mal	Hex	B'ary	Char	D'mal	Hex	B'ary	Char	D'mal	Hex	B'ary	Char
80	50	1010000	P	96	60	1100000	`	112	70	1110000	p
81	51	1010001	Q	97	61	1100001	a	113	71	1110001	q
82	52	1010010	R	98	62	1100010	b	114	72	1110010	r
83	53	1010011	S	99	63	1100011	c	115	73	1110011	s
84	54	1010100	T	100	64	1100100	d	116	74	1110100	t
85	55	1010101	U	101	65	1100101	e	117	75	1110101	u
86	56	1010110	V	102	66	1100110	f	118	76	1110110	v
87	57	1010111	W	103	67	1100111	g	119	77	1110111	w
88	58	1011000	X	104	68	1101000	h	120	78	1111000	x
89	59	1011001	Y	105	69	1101001	i	121	79	1111001	y
90	5A	1011010	Z	106	6A	1101010	j	122	7A	1111010	z
91	5B	1011011	[	107	6B	1101011	k	123	7B	1111011	{
92	5C	1011100	\	108	6C	1101100	l	124	7C	1111100	|
93	5D	1011101	]	109	6D	1101101	m	125	7D	1111101	}
94	5E	1011110	^	110	6E	1101110	n	126	7E	1111110	~
95	5F	1011111	_	111	6F	1101111	o	127	7F	1111111	.

1.4.2 Binary coded decimal (BCD)

BCD is actually a set of binary numbers where a group of four binary numbers represent a decimal digit. As there are 10 basic digits in the decimal number system, four binary digits (bits) are required[1]. Figure 1.10 shows an example, while Table 1.2 gives the BCD code.

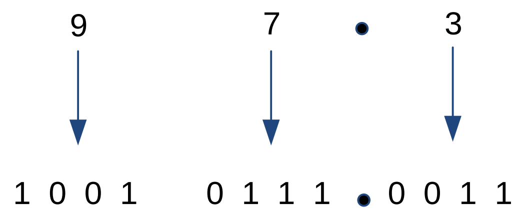


Figure 1.9: Hex to binary conversion example and vice versa: $973_{10} = 10010111.0011_{BCD.}$

1 Three bits will only give eight representations, which is not enough for a decimal system.

Table 1.2: BCD code

Decimal	BCD	Decimal	BCD
0	0000	5	0101
1	0001	6	0110
2	0010	7	0111
3	0011	8	1000
4	0100	9	1001

1.4.3 Gray code

Gray code is another commonly encountered code system. The main feature of this code is that only one bit changes between two successive values. This system is less prone to errors and is considered very useful for practical applications such as mechanical switches and error correction in digital communication as compared to the standard binary system. Table 1.3 gives the BCD code with 4 bits (i.e. up to decimal value of 15).

Table 1.3: Gray code

Decimal	Gray	Decimal	Gray
0	0000	8	1100
1	0001	9	1101
2	0011	10	1111
3	0010	11	1110
4	0110	12	1010
5	0111	13	1011
6	0101	14	1001
7	0100	15	1000