

Index

• *Symbols & Numerics* •

ϕ spherical coordinate, 318
 \int symbol, 17
 θ spherical coordinate, 318
 Σ symbol, 24, 51–52
! symbol, 38
3-D coordinate, 314–319
3-D problems
 meat-slicer method
 overview, 220
 pyramids, 222–224
 rotating solids, 225–226
 solids between two surfaces, 230–234
 solids of revolution, 227–228
 solids with congruent cross sections, 220–221
 solids with similar cross sections, 221–222
 weird solids, 224–225
 overview, 219–220
 shell method
 overview, 234
 peeling and measuring can of soup, 235–236
 use of, 236–238
 surface of revolution, 229–230
 tips for solving, 238–239

• *A* •

absolute convergence, 280–281
adding vector, 313–314
advanced math
 differential equations, 34
 Fourier analysis, 34
 multivariable calculus, 33
 numerical analysis, 34–35
 real analysis, 35
algebraic function, Diagonal Method, 145–148
algebraic times cosine function, 139

algebraic times exponential function, 139
algebraic times sine function, 139
algorithm, 35
alternating series
 absolute convergence, 280–281
 based on convergent positive series, 277
 conditional convergence, 280–281
 defined, 257
 divergence, 348
 making new series from old, 276
 overview, 261, 275
 sequence of partial sums, 248
 testing, 277–279, 281–282
 two forms of basic, 276
alternative 3-D coordinate system, 316–319
alternative indeterminate forms of limits, 68–72
altitude, 318
analytic geometry, 12–14
angle, measuring, 3
angular distance, 50
anti-derivative, 28, 73
anti-differentiation
 defined, 4
 indefinite integrals, 96–97
 in integration, 94, 346
approximate integration
 overview, 74
 with rectangles, 74–77
 Simpson's Rule, 80–83
 slack factor, 78
 Trapezoid Rule, 79–80
approximating
 area problem, 25–27
 definite integrals, 23
 functions in Taylor series, 300–301
 in numerical analysis, 35
arc notation, 58
arccos x function, 138, 143
arccot x function, 138, 143
Archimedes, 13
arc-length formula, 5, 215–217, 229
arcsin x function, 138, 143

arctan x function, 138, 143

area function, 92–94

area problems. *See also* approximate integration; definite integral; indefinite integral

advanced math

- differential equations, 34
- Fourier analysis, 34
- multivariable calculus, 33
- numerical analysis, 34–35
- real analysis, 35

building formula

- height, 25
- limiting margin of error, 23–24
- other ways of approximating, 25–27
- overview, 22–23
- sigma notation, 24–25
- width, 24

calculating arc length, 215–217

classical versus analytic geometry, 12–14

generalizing, 15–16

improper integral

- horizontally infinite, 199–201
- overview, 199
- vertically infinite, 201–204

infinite series

- convergent versus divergent, 32–33
- distinguishing sequences from, 31
- evaluating, 32

Mean Value Theorem for Integrals, 213–215

overview, 11–12, 197

rule for, 198–199

slicing space into rectangles, 19–22

solving with integration

- finding area between curves, 29
- measuring curve lengths, 29–30
- overview, 28–29, 343
- solid of revolution, 30–31

solving with more than one function

- finding area between two functions, 206–209
- finding area under functions, 205–206
- measuring unsigned areas, 211–213
- overview, 204–205
- signed areas, 209–211

solving without Riemann sum formula, 97–99

asymptote, 47, 202

autonomous equation, 333

axe

- in Cartesian coordinate system, 314
- in cylindrical coordinate systems, 316

x

- area problems, 15
- Cartesian coordinates, 314
- definite integrals, 12
- signed areas, 209

• B •

Barrow, Isaac, 91

basic anti-derivative, 106–107

basic integral

- basic anti-derivatives, 106–107
- integration rules, 107–110
- overview, 106

benchmark series, 264, 267

boldfaced text, 3

braces in notations, 244

breathing exercise, 349

• C •

Calculus I

- derivatives
 - limit formula for, 56
 - notation for, 56–57
 - overview, 55–56
- differentiation
 - Chain Rule, 62–64
 - Constant Multiple Rule, 59
 - memorizing key derivatives, 57–59
 - overview, 57
 - Power Rule, 60
 - Product Rule, 61
 - Quotient Rule, 61–62
 - Sum Rule, 59
- limits, 53–55
- overview, 37, 53

Calculus II

- definite integrals, 345
- indefinite integrals, 346
- infinite series
 - convergence or divergence, 348
 - related sequences, 347

- integration
 - as fancy addition, 344
 - finding area, 343
 - as inverse differentiation, 346–347
 - signed areas, 344
 - slack factor, 345
 - slices, 344–345
 - overview, 1–8, 343
- Calculus III. *See also* vector
- dimension
 - 3-D Cartesian coordinates, 314–315
 - alternative 3-D coordinate systems, 316–319
 - functions of several variables, 319–321
 - multiple integrals
 - evaluating, 324–326
 - measuring volume under surface, 323–324
 - overview, 323
 - overview, 33, 307
 - partial derivatives
 - evaluating, 322–323
 - measuring slope in three dimensions, 321–322
 - overview, 321
- Cartesian coordinate
 - 3-D, 314–315
 - analytic geometry, 13
 - versus polar coordinates, 50–51
 - vector basics, 308
- case, trig substitution
 - distinguishing, 162–163
 - secant, 169–171
 - sine, 164–166
 - tangent, 166–169
- Chain Rule
 - differentiating functions, 62–64, 106
 - finding derivative of functions, 115
 - finding integrals
 - of nested functions, 120
 - of products, 121
- checking test answer, 351–352
- circle
 - area problems, 12
 - as cross sections, 227, 239
- circumference of circle formula, 229
- classical geometry, 12–14
- coefficient, 310
- comma, 252
- comparison test
 - direct, 265–267
 - limit, 267–270
 - overview, 264–265
- composition of function
 - Chain Rule, 62–63
 - finding integrals of, 118–120
 - integrating function multiplied by set of, 121–122
 - variable substitution
 - integrating with, 123–125
 - overview, 117
 - shortcut for, 125–128
- computing integrals, 114
- conceptual understanding, 352
- conditional convergence, 280–281
- congruent cross section, volume of solids
 - with, 220–221
- conic section, 11
- Constant Multiple Rule
 - differentiation, 59
 - finding integrals of nested functions, 119
 - finding values using roots, 182
 - integration
 - overview, 108
 - polynomials, 110
 - power series, 287
 - moving denominator, 154
 - to separate integrals, 153
 - for series, 250–251
- constant of integration C , 97, 99
- constant, 310
- continuity, 115
- continuous function, 19, 47–48
- convergence
 - absolute, 280–281
 - conditional, 280–281
 - intervals of in power series, 288–290
 - sequences, 245–246
 - series, 32–33, 52, 277
 - Taylor series, 298–300
- tests of
 - integral, 270–272
 - n th-term, 263
 - one-way, 263–264
 - overview, 261
 - ratio, 273–274

convergence, tests of (*continued*)

root, 274–275

starting, 262

two-way, 264

coordinate

alternative 3-D, 316–319

Cartesian

3-D, 314–315

analytic geometry, 13

vector basics, 308

cylindrical, 316–317

polar, 50–51, 314

spherical, 317–319

cos x function, 293

cosecant, 159–161

cosine

double-angle identities for, 50

integrating powers of, 152–155

cosine function, 46, 148

cosine times exponential function, 139

cotangent, 159–161

counting numbers, summation formula for,
83–84

cross section

circular, 227, 239

horizontal, 238

meat-slicer method

area between curves, 231–233

congruent, 220–221

rotating solids, 225–226

similar, 221–222

solids of revolution, 228

volume of pyramids, 223

weird solids, 224–225

vertical, 238

cubic number, summation formula for,
84–85

curve

finding area between, 29

measuring lengths, 29–30

measuring unsigned area between,
211–213

solid of revolution, 30

cylinder, 220

cylindrical coordinate, 316–317

• D •

DE (differential equation)

building versus solving, 331–332

checking solutions, 332–333

defined, 6

integrals, 330–331

linear, 329–330

order of, 329

ordinary and partial, 328–329

overview, 34, 327–328

solving

initial-value problems, 334–336

separable equations, 333–334

using integrating factor, 336–339

defining sequence, 252–253, 347

definite integral

approximate integration

overview, 74

with rectangles, 74–77

Simpson's Rule, 80–83

slack factor, 78

Trapezoid Rule, 79–80

approximating, 23

area problem, 12, 16–19

in Calculus II, 345

defined, 4

Fundamental Theorem of Calculus

additional part of, 95

area function, 92–94

connecting slope and area, 94

overview, 89–91

slope, 92

indefinite integral

anti-differentiation, 96–97

versus definite, 101–102

overview, 95–96

signed areas, 99–101

solving without Riemann sum formula,
97–99

Mean Value Theorem for Integrals, 214

overview, 73

Riemann sum formula

defined, 4

evaluating limit, 89

expressing function as sum, 86–87

- limits of integration, 86
 - overview, 23–27, 85–86
 - solving problem, 88
 - signed areas, 344
 - summation formulas, 83–85
 - unsigned areas, 210
 - variable substitution to evaluate, 132–133
 - degree, 42, 187
 - denominators in partial fraction, 181
 - derivative. *See also* differentiation
 - defined, 33
 - limit formula for, 56
 - memorizing, 57–59, 106
 - notation for, 56–57
 - overview, 55–56
 - partial
 - evaluating, 322–323
 - measuring slope in three dimensions, 321–322
 - in multivariable calculus, 33
 - overview, 321
 - of trig functions, 58
 - determinate form of limit, 65–66
 - DI-agonal Method
 - algebraic functions, 145–148
 - inverse trig functions, 143–145
 - logarithmic functions, 141–143
 - overview, 140–141
 - trig functions, 148–150
 - Difference Rule, 59, 108
 - differentiability of polynomial, 285
 - differential equation (DE)
 - building versus solving, 331–332
 - checking solutions, 332–333
 - defined, 6
 - integrals, 330–331
 - linear, 329–330
 - order of, 329
 - ordinary and partial, 328–329
 - overview, 34, 327–328
 - solving
 - initial-value problems, 334–336
 - separable equations, 333–334
 - using integrating factor, 336–339
 - differentiation
 - Chain Rule, 62–64
 - Constant Multiple Rule, 59
 - formulas for inverse trig functions, 163
 - memorizing key derivatives, 57–59
 - overview, 57
 - Power Rule, 60
 - Product Rule, 61
 - Quotient Rule, 61–62
 - Sum Rule, 59
 - dimension in multivariable calculus
 - 3-D Cartesian coordinates, 314–315
 - alternative 3-D coordinate systems, 316–319
 - discontinuous function, 115
 - discontinuous integrand, 203–204
 - distinct linear factor, 177–178
 - distinct quadratic factor, 178
 - divergence
 - sequences, 245–246
 - series, 32–33, 52
 - Taylor series, 298–300
 - tests of
 - integral, 270–272
 - nth-term, 263
 - one-way, 263–264
 - overview, 261
 - ratio, 273–274
 - root, 274–275
 - starting, 262
 - two-way, 264
 - division, polynomial, 188–191
 - does not exist (DNE), limit
 - common functions, 65
 - defined, 54
 - improper integral, 201
 - sequence, 246
 - double integral 323–325
 - double-angle identities, 50
 - dx constant, 350
- **E** ●
- elementary functions
 - advantages of polynomials, 285
 - drawbacks of, 284–285
 - overview, 284
 - representing
 - integrals as, 114–115
 - as polynomials, 285
 - as series, 285–286
 - ellipse, 13–14

- equation. *See also* differential equation (DE)
- autonomous, 333
 - heat, 34
 - Laplace, 34
 - separable, 333–334
 - systems of, 182–183
- error bound for Taylor series, 301–303
- even power integration
- cosines, 154–155
 - secants
 - with tangents, 155
 - without tangents, 157
 - sines, 154–155
 - tangents
 - with odd powers of secants, 158–159
 - without secants, 156–157
- exam-taking tips, 349–352
- exercise, breathing, 349
- expanded notation, 247, 249–250
- exponent
- integrating
 - cotangents and cosecants, 159–160
 - sines and cosines, 152–155
 - tangents and secants, 155–159
 - negative, 40, 109, 160
 - Power Rule, 60
 - in Pre-Calculus, 39–41
- exponential curve, 14, 224
- exponential function, 44–45
- expressing functions, 300–301
- expression
- of form $f(x) \cdot g(x)$, 129–130
 - of form $f(x) \cdot h(g(x))$, 130–132
- **F** ●
- factorial, 38–39, 273
- first-degree polynomial, 268
- formula. *See also* Riemann sum formula
- arc-length, 5, 215–217, 229
 - building for area problems
 - approximating definite integral, 23
 - height, 25
 - limiting margin of error, 23–24
 - other ways of approximating, 25–27
 - overview, 22–23
 - sigma notation, 24–25
 - width, 24
 - circumference of circle, 229
 - for finding surface of revolution, 229–230
 - half-angle, 228
 - for inverse trig functions, 163
 - limit, for derivatives, 56
 - summation, 83–85
- Fourier analysis, 34
- fourth-order ODE, 329
- fractional coefficient, 190
- fractional exponent, 60
- fraction, 38–39. *See also* partial fraction
- FTC (Fundamental Theorem of Calculus)
- additional part of, 95
 - anti-derivatives, 106–107
 - area function, 92–94
 - connecting slope and area, 94
 - indefinite integrals, 73
 - overview, 28, 89–91
 - slope, 92
- functions. *See also individual functions by type*; nested function
- area, in Fundamental Theorem of Calculus, 92–94
 - DI-agonal Method
 - algebraic, 145–148
 - inverse trig, 143–145
 - logarithmic, 141–143
 - differentiating, 63–64
 - elementary
 - advantage of polynomials, 285
 - drawbacks of, 284–285
 - overview, 284
 - representing as polynomials, 285
 - representing as series, 285–286
 - expressing as series
 - cos x , 293
 - overview, 291
 - sin x , 291–292
 - graphing common
 - exponential, 44–45
 - linear and polynomial, 43–44
 - logarithmic, 45
 - trigonometric, 46–47

horizontal transformations, 48
 indefinite integrals, 28
 integrating, multiplied by set of nested,
 121–122
 limits, 53–54, 65
 Maclaurin series, 293–296
 multiplied by functions, 123
 overview, 283–284
 power series
 integrating, 287–288
 interval of convergence, 288–290
 overview, 286–287
 related area functions, 94
 representing integrals as, 114–115
 Riemann sum formula, 86–87
 of several variables, 319–321
 solving area problems with more
 than one
 finding area between two, 206–209
 finding area under, 205–206
 measuring unsigned area, 211–213
 overview, 204–205
 signed areas, 209–211
 substitution when one part differentiates
 to another, 129–132
 Taylor series
 calculating error bounds for, 301–303
 computing with, 297–298
 constructing, 303–304
 convergent and divergent, 298–300
 expressing versus approximating,
 300–301
 overview, 296–297
 transforming continuous, 47–48
 trigonometric
 derivatives of, 58–59
 DI-agonal Method, 148–150
 integrating combinations of, 160–161
 vertical transformations, 48
 Fundamental Theorem of Calculus (FTC)
 additional part of, 95
 anti-derivatives, 106–107
 area function, 92–94
 connecting slope and area, 94
 indefinite integrals, 73
 overview, 28, 89–91
 slope, 92

• G •

Gauss, Karl Friedrich, 84
 general expression, 87
 general form of power series, 295
 general solution, 334
 generalizing area problem, 15–16
 geometric series, 258, 286
 geometry, 12–14
 graphing common function
 exponential and logarithmic, 44–45
 linear and polynomial, 43–44
 logarithmic, 45
 trigonometric, 46–47

• H •

half-angle identities, 50, 154, 228
 harmonic series
 defined, 32
 divergence of, 258–259
 making new from old, 276
 sequence of partial sums, 254
 heat equation, 34
 height
 area problem, 25
 of rectangles, 22
 horizontal axes in polar coordinate
 system, 316
 horizontal cross section, 238
 horizontal transformations of function, 48
 horizontally infinite improper integral,
 199–201
 hyperbola, 14

• I •

identities
 integration of trig functions using, 112–113
 trig
 even powers of sines and cosines, 156
 half-angle, 154
 important, 48–50
 using to integrate trig functions, 112–113
 using to tweak functions, 160–161

- improper integral
 - defined, 5
 - horizontally infinite, 199–201
 - overview, 199
 - vertically infinite, 201–204
- improper polynomial fraction, 191
- improper rational function
 - integrating
 - distinguishing from proper, 187
 - overview, 187
 - polynomial division, 188–191
 - overview, 173
- incorrect test answer, 352
- indefinite integral. *See also* integration by
 - parts; partial fraction; variable substitution
 - anti-differentiation, 96–97
 - area problem, 27–28
 - in Calculus II, 346
 - versus definite integrals, 101–102
 - limits of integration, 17
 - overview, 4–5, 95–96
 - signed areas, 99–101
 - solving without Riemann sum formula, 97–99
- indeterminate forms of limit
 - alternative, 68–72
 - L'Hospital's Rule, 66–68
 - overview, 55, 65–66
- infinite improper integral
 - horizontally, 199–201
 - vertically, 201–204
- infinite sequence
 - convergent, 245–246
 - converting into infinite series, 31
 - divergent, 245–246
 - notation for, 244–245
 - overview, 244
- infinite series. *See also* functions; test
 - alternating
 - absolute convergence, 280–281
 - based on convergent positive series, 277
 - conditional convergence, 280–281
 - defined, 257
 - divergence, 348
 - making new series from old, 276
 - overview, 275
 - sequence of partial sums, 248
 - testing, 277–279, 281–282
 - two forms of basic, 276
 - basics, 247–249
 - in Calculus II
 - convergence or divergence, 348
 - related sequences, 347
 - connecting with related sequences, 252–254
 - convergent versus divergent, 32–33
 - defined, 2
 - distinguishing from sequences, 31
 - evaluating, 32
 - expressing functions as
 - versus approximating, 300
 - cos x , 293
 - overview, 291
 - sin x , 291–292
 - geometric, 255–257
 - harmonic, 258
 - infinite sequences
 - convergent, 245–246
 - divergent, 245–246
 - notation for, 244–245
 - overview, 244
 - overview, 5–6, 243
 - power series
 - differentiating from other series, 295
 - integrating, 287–288
 - interval of convergence, 288–290
 - overview, 286–287
 - p-series, 257–259
 - representing elementary functions as, 285–286
 - sigma notation
 - Constant Multiple Rule, 250–251
 - overview, 249
 - Sum Rule, 251–252
 - ways to use, 250
 - writing in expanded form, 249–250
 - Sum Rule, 286
- initial-value problem, 334–336
- inner function, 63, 125–128
- input values to indefinite integral, 346
- integrability, 113–116

- integral. *See also* definite integral;
 indefinite integral; partial fraction
 computing, 114
 Constant Multiple Rule, 119, 153
 differential equations, 330–331
 evaluating basic
 anti-derivatives, 106–107
 integration rules, 107–110
 overview, 106
 improper
 horizontally infinite, 199–201
 overview, 199
 vertically infinite, 201–204
 Mean Value Theorem for Integrals,
 197, 213–215
 multiple
 evaluating, 324–326
 measuring volume under surface,
 323–324
 in multivariable calculus, 33
 overview, 323
 Power Rule, 153
 representing as functions, 114–115
 Sum Rule, 136, 153
 variable substitution
 to evaluate definite, 132–133
 of nested functions, 118–120
 of product, 120–121
 well defined, 115
 integral test, 270–272
 integrands, discontinuous, 203–204
 integrating factor, 336–339
 integration. *See also* 3-D problems; area
 problems; definite integral; partial
 fraction; trig substitution; variable
 substitution
 approximate
 overview, 74
 with rectangles, 74–77
 Simpson's Rule, 80–83
 slack factor, 78
 Trapezoid Rule, 79–80
 asymptotic limits of, 202
 in Calculus II
 as fancy addition, 344
 finding area, 343
 as inverse differentiation, 346–347
 signed areas, 344
 slack factor, 345
 slices, 344–345
 defined, 2
 evaluating basic integrals
 17 basic anti-derivatives, 106–107
 integration rules, 107–110
 overview, 106
 integrability, 113–116
 overview, 4–5, 11, 105
 polynomials, 110–111
 power series, 287–288
 rational expressions, 111
 solving problems with
 finding area between curves, 29
 measuring curve lengths, 29–30
 overview, 28–29
 solid of revolution, 30–31
 of trig functions using identities, 112–113
 integration by parts
 DI-agonal Method
 algebraic functions, 145–148
 inverse trig functions, 143–145
 logarithmic functions, 141–143
 overview, 140–141
 trig functions, 148–150
 overview, 135
 reversing Product Rule, 136–137
 use of, 137–139
 intervals of convergence, 288–290
 inverse identities, 49
 inverse trig function
 derivatives of, 58–59
 DI-agonal Method, 143–145
 integration by parts, 139
 inverses of function, 225
 italicized text, 3
- **L** •
- Laplace equation, 34
 latitude, 318
 left rectangle, 25, 74–75
 left-hand limits of integration, 74
 Leibniz, Gottfried, 57, 91
 Leibniz notation, 56–57

- length
 - calculating arc, 215–217
 - measuring curve, 29–30
 - L'Hospital's Rule
 - alternative indeterminate forms, 68–72
 - determinate form of limits, 65–66
 - indeterminate form of limits, 65–66
 - limit comparison tests, 269
 - overview, 64–65
 - use of, 66–68
 - limit
 - alternative indeterminate, 68–72
 - asymptotic, of integration, 202
 - in Calculus I, 53–55
 - determinate form of, 65–66
 - does not exist
 - common functions, 65
 - defined, 54
 - improper integral, 201
 - sequence, 246
 - formulas for derivatives, 56
 - indeterminate form of, 65–66
 - of integration, 12, 15
 - Riemann sum formula, 86, 89
 - linear differential equation, 329–330
 - linear factor
 - distinct, 177–178
 - integrating partial fractions, 184
 - repeated, 178–179
 - linear function, 43–44
 - log composed with algebraic function, 139
 - log function, 139
 - log rolling, 71
 - log times algebraic function, 139
 - logarithmic curve, 14
 - logarithmic function
 - DI-agonal Method, 141–143
 - integration by parts, 138
 - overview, 45
 - longitude, 318
- *M* •
- Maclaurin, Colin, 295
 - Maclaurin series, 291, 293–297
 - magnitude, vector, 310–311
 - margin of error, 23–24
 - Mean Value Theorem for Integrals, 197, 213–215
 - meat-slicer method
 - overview, 220
 - pyramids, 222–224
 - solids
 - with congruent cross sections, 220–221
 - of revolution, 227–228
 - rotating, 225–226
 - with similar cross sections, 221–222
 - between two different surfaces, 230–234
 - weird, 224–225
 - memorizing derivatives, 57–59, 106
 - method of exhaustion, 13
 - midpoint rectangle, 26
 - Midpoint Rule, 74, 76–77
 - minus sign, 56
 - monofont text, 3
 - multiple integral
 - evaluating, 324–326
 - measuring volume under surface, 323–324
 - in multivariable calculus, 33
 - overview, 323
 - multiplication, scalar, 311–312
 - multivariable calculus. *See also* vector
 - dimension
 - 3-D Cartesian coordinates, 314–315
 - alternative 3-D coordinate systems, 316–319
 - functions of several variables, 319–321
 - multiple integrals
 - evaluating, 324–326
 - measuring volume under surface, 323–324
 - overview, 323
 - overview, 33, 307
 - partial derivatives
 - evaluating, 322–323
 - measuring slope in three dimensions, 321–322
 - overview, 321
- *N* •
- natural log function
 - DI-agonal Method, 141–143
 - integration by parts, 138
 - overview, 45

negative area, 99, 344
 negative power
 cotangents and cosecants, 160
 overview, 40
 Power Rule, 60, 109
 nested function
 Chain Rule, 62–63
 finding integrals of, 118–120
 integrating function multiplied by set of,
 121–122
 variable substitution
 integrating with, 123–125
 overview, 117
 shortcut for, 125–128
 Newton, Isaac, 57, 91
 nonnegative integer exponent, 40
 notation. *See also* sigma notation
 arc, 58
 braces in, 244
 defined, 4
 for derivatives, 56–57
 expanded, 247, 249–250
 for infinite sequences, 244–245
 Leibniz, 56–57
 trig, 41–42
 with and without braces, 244
 nth-term test, 256, 263, 279
 numerators in partial fraction, 181
 numerical analysis, 34–35

• 0 •

octant, 314
 odd power integration
 secants
 with even powers of tangents, 158–159
 without tangents, 157–158
 sines and cosines, 152–153
 tangents, 156
 ODE (ordinary differential equation),
 328–329
 one-way test, 261, 263–264
 ordinary differential equation (ODE),
 328–329
 outer function, 63, 125–128

• p •

pairing trig function, 160–161
 parabola, 14
 partial derivative
 evaluating, 322–323
 measuring slope in three dimensions,
 321–322
 in multivariable calculus, 33
 overview, 321
 partial differential equation (PDE),
 34, 328–329
 partial fraction
 example, 191–193
 integrating improper rationals
 overview, 187
 polynomial division, 188–191
 versus proper rational expressions, 187
 overview, 173–174
 with rational expressions, 175–176
 solving integrals by using
 distinct linear factors, 177–178
 distinct quadratic factors, 178
 finding unknowns, 181–183
 integrating, 184–186
 overview, 176
 repeated linear factors, 178–179
 repeated quadratic factors, 179–180
 setting up, 180–181
 partial sum, sequences of, 253–254
 past material. *See* Calculus I; L'Hospital's
 Rule; Pre-Calculus
 PDE (partial differential equation),
 34, 328–329
 phi, 317
 plotting cylindrical coordinate, 316
 plus sign, 252
 polar coordinate, 50–51, 314
 polynomial
 advantage of, 285
 benchmark series, 268
 converting from functions, 153
 division, 188–191
 elementary functions, 285
 graphing common functions, 43–44
 integration, 110–111

- polynomial (*continued*)
 - overview, 39
 - representing elementary functions as, 285
 - Taylor, 301–303
 - positive integer exponent, 40
 - positive series, 275, 277
 - power
 - integrating
 - cotangents and cosecants, 159–160
 - sines and cosines, 152–155
 - tangents and secants, 155–159
 - negative, 40, 109, 160
 - Power Rule, 60
 - in Pre-Calculus, 39–41
 - Power Rule
 - differentiation, 60
 - evaluating integrals, 153
 - integrating
 - overview, 109
 - polynomials, 110–111
 - power series, 287–288
 - power series
 - differentiating from other series, 295
 - integrating, 287–288
 - interval of convergence, 288–290
 - overview, 286–287
 - practice problem, 2
 - Pre-Calculus
 - asymptotes, 47
 - exponents, 39–41
 - factorials, 38–39
 - graphing common functions
 - exponential, 44–45
 - linear and polynomial, 43–44
 - logarithmic, 45
 - trigonometric, 46–47
 - important trig identities, 48–50
 - overview, 37–38
 - polar coordinates, 50–51
 - polynomials, 39
 - radians, 42–43
 - sigma notation, 51–52
 - transforming continuous functions, 47–48
 - trig notation, 41–42
 - precision, 35
 - prism, 220
 - product, integral of, 120–121
 - Product Rule
 - differentiation, 61, 106, 114
 - integration by parts, 135
 - linear first-order DEs, 338
 - reversing, 136–137, 339
 - proper rational expression, 173, 187
 - pyramid, volume of, 222–224
- *Q* •
- quadratic factor
 - distinct, 178
 - of form $(ax^2 + bx + c)$, 185–186
 - of form $(ax^2 + c)$, 184–185
 - repeated, 179–180
 - quadrature method, 13
 - Quotient Rule, 61–62, 106
- *R* •
- r spherical coordinate, 318
 - radian, 3, 42–43
 - ratio test, 273–274
 - rational expression
 - integration, 111
 - limit comparison tests, 268
 - partial fractions with, 174–176
 - rational power, 109
 - reading through exam, 350
 - real analysis, 7, 35
 - rectangle
 - approximate integration with, 74–77
 - approximating area with
 - left, 25
 - midpoint, 26
 - right, 26
 - in classical geometry, 12
 - finding height of, 22
 - slicing space into to calculate area, 19–22
 - rectangular coordinate, 3-D, 314–315
 - remainder
 - polynomial division with, 189–191
 - polynomial division without, 188–189
 - remainder term, Taylor, 283–284, 301–303

- repeated linear factor, 178–179
 repeated quadratic factor, 179–180
 reviews of past material. *See* Calculus I;
 L'Hospital's Rule; Pre-Calculus
 revolution
 solids of
 meat-slicer method, 227–228
 overview, 30–31
 surfaces of, 229–230
 Riemann, Bernhard, 91
 Riemann sum, 12, 78
 Riemann sum formula
 calculating definite integral
 calculating sum, 88
 evaluating limit, 89
 expressing function as sum, 86–87
 limits of integration, 86
 overview, 85–86
 solving problem, 88
 defined, 4
 overview, 23–27
 right rectangle, 26, 75–76
 right-hand limit of integration, 75
 root, 181–182
 root test, 274–275
 rotating problems, meat-slicer method,
 225–226
 rule. *See also* Constant Multiple Rule;
 Power Rule; Sum Rule
 for area problems, 198–199
 Chain Rule
 differentiating functions, 106
 differentiation, 62–64
 finding derivative of functions, 115
 finding integral of nested functions, 120
 finding integral of products, 121
 Difference Rule, 59, 108
 integration, 107–110
 L'Hospital's Rule
 alternative indeterminate forms, 68–72
 determinate form of limits, 65–66
 indeterminate form of limits, 65–66
 limit comparison tests, 269
 overview, 64–65
 use of, 66–68
 Midpoint Rule, 74, 76–77
 Product Rule
 differentiation, 61, 106, 114
 integration by parts, 135
 linear first-order DEs, 338
 reversing, 136–137, 339
 Quotient Rule, 61–62, 106
 Simpson's Rule, 74, 80–83
 Trapezoid Rule, 74, 79–80
 Rumsey, Deborah, 37
 Ryan, Mark, 37, 53
- S ●
- scalar, 308, 310
 scalar multiplication, 311–312
 scribbling during exam, 351
 secant, integrating powers of, 155–159
 secant case, trig substitution, 163–164,
 169–171
 second-degree polynomial, 268
 second-order ODE, 329
 separable equation, 333–334
 sequence
 connecting series with related, 252–254
 infinite
 convergent and divergent, 245–246
 notations for, 244–245
 overview, 244
 overview, 31
 of partial sums, 32, 248, 347
 series. *See* infinite series
 shell method
 overview, 234
 peeling and measuring can of soup,
 235–236
 use of, 236–238
 shortcut
 for intergrating nested functions, 128
 for variable substitution of nested
 functions, 125–128
 sigma notation
 area problem, 24–25
 overview, 51–52, 247

sigma notation (*continued*)

series

Constant Multiple Rule, 250–251

overview, 249–252

Sum Rule, 251–252

use of, 250

writing in expanded form, 249–250

signed area, 99–101, 209–211

similar cross sections, volume of solid
with, 221–222

Simpson's Rule, 74, 80–83

sine

double-angle identities for, 50

expressing as series, 291–292

half-angle formulas for, 228

integrating powers of, 152–155

pairing with cosines, 161

sine case, trig substitution, 163–166

sine curve, 14

sine function, 46, 148

sine times exponential function, 139

slack factor, approximate integration, 78

slope

Fundamental Theorem of Calculus, 92, 94

measuring in three dimensions, 321–322

solid

meat-slicer method to find volume of

with congruent cross sections, 220–221

with similar cross sections, 221–222

between two different surfaces,
230–234

weird, 224–225

overview, 219

solids of revolution

meat-slicer method, 227–228

overview, 30–31

specific form of power series, 295

spherical coordinate, 317–319

square identities, 49

square number, summation formula for, 84

straight-line distance, 215

study tip, 2

substitution. *See* trig substitution; variable
substitution

subtracting vector, 313–314

sum formula. *See* Riemann sum formula

Sum Rule

differentiation, 59

even powers of tangents with secants, 159

finding values using roots, 182

infinite series, 251–252, 286

integrating

overview, 108

polynomials, 110

power series, 287

to separate integrals, 153

solving rational expressions, 175–176

split integrals, 136

splitting functions, 154

summation formula, 83–85

surface, measuring under volume, 323–324

surface of revolution, 229–230

system of equation, 182–183



tangent

double-angle identities for, 50

integrating powers of, 155–159

pairing with secants, 161

tangent case, trig substitution, 163–164,
166–169

Taylor polynomial, 301

Taylor series

calculating error bounds for, 301–303

computing with, 297–298

constructing, 303–304

convergent, 298–300

divergent, 298–300

expressing versus approximating
functions, 300–301

versus other series, 295

overview, 296–297

remainder term, 283–284, 301–303

term, 244

test

alternating series, 277–279, 281–282

comparison

direct, 265–267

limit, 267–270

overview, 264–265

- of convergence and divergence
 - integral, 270–272
 - n*th-term, 263
 - one-way, 263–264
 - overview, 261
 - ratio, 273–274
 - root, 274–275
 - starting, 262
 - two-way, 264
 - failing, 264
 - passing, 264
 - of *p*-series, 258–259
 - tips for taking math, 349–352
 - theta, 317
 - third-order ODE, 329
 - three-dimensional coordinate, 314–319
 - three-dimensional problem
 - meat-slicer method
 - overview, 220
 - pyramids, 222–224
 - rotating solids, 225–226
 - solids between two surfaces, 230–234
 - solids of revolution, 227–228
 - solids with congruent cross sections, 220–221
 - solids with similar cross sections, 221–222
 - weird solids, 224–225
 - overview, 219–220
 - shell method
 - overview, 234
 - peeling and measuring can of soup, 235–236
 - use of, 236–238
 - surface of revolution, 229–230
 - tips for solving, 238–239
 - tip
 - for studying, 2
 - for test-taking, 349–352
 - top-and-bottom trick, 212–213
 - tractability, 35
 - transforming continuous functions, 47–48
 - Trapezoid Rule, 74, 79–80
 - triangle
 - area problems, 12
 - as trapezoids, 80
 - trig substitution
 - calculating arc length, 217
 - distinguishing cases for, 162–163
 - integration
 - combinations of trig functions, 160–161
 - overview, 163–164
 - powers of cotangents and cosecants, 159–160
 - powers of sines and cosines, 152–155
 - powers of tangents and secants, 155–159
 - secant case, 169–171
 - sine case, 164–166
 - tangent case, 166–169
 - trig functions, 151–152
 - overview, 151
 - when to avoid, 171
 - trigonometry
 - functions
 - derivatives of, 58–59
 - DI-agonal Method, 148–150
 - graphing common, 46–47
 - integrating, 151–152, 160–161
 - identities
 - even powers of sines and cosines, 156
 - half-angle, 154
 - important, 48–50
 - using to integrate functions, 112–113
 - notation, 41–42
 - triple integral, 325–326
 - two-way test
 - defined, 261
 - integrals, 270–272
 - overview, 264
 - ratio, 273–274
 - root, 274–275
- *u* •
- unary operator, 56
 - understanding, conceptual, 352
 - unit vector, 312–313
 - unknowns, 176, 181–183
 - unsigned area, 209, 211–213, 344

• U •

variable

- functions of several, 319–321
- Fundamental Theorem of Calculus, 95

variable substitution

- anti-differentiation, 4
- integrating rational function, 193
- linear factor cases, 184–185
- overview, 117
- versus trig substitution, 161–162
- use of
 - to evaluate definite integrals, 132–133
 - finding integral of nested functions, 118–120
 - finding integral of product, 120–121
 - integrating function multiplied by set of nested functions, 121–122
 - overview, 118
- when to use
 - integrating nested functions, 123–125
 - overview, 123
 - shortcut for nested functions, 125–128
 - when one part of function differentiates to another, 129–132

vector

- basics, 308–309
- calculating with
 - adding and subtracting, 313–314
 - finding unit vector, 312–313
 - magnitude, 310–311
 - overview, 310
 - scalar multiplication, 311–312
- overview, 308
- versus scalars, 310
- vertical asymptote, 201
- vertical cross section, 238
- vertical transformations of function, 48
- vertical z-axes in cylindrical coordinate system, 316
- vertical-line test, 319–320
- vertically infinite improper integral, 199, 201–204

volume

- measuring under surface, 323–324
- meat-slicer method to find
 - overview, 220
 - pyramids, 222–224
 - rotating solids, 225–226
 - solids between different surfaces, 230–234
 - solids of revolution, 227–228
 - solids with congruent cross sections, 220–221
 - solids with similar cross sections, 221–222
 - weird solids, 224–225
- shell method
 - overview, 234
 - peeling and measuring can of soup, 235–236
 - use of, 236–238

• W •

- well defined integral, 115
- width, 24

• X •

x-axis

- area problems, 15
- Cartesian coordinates, 314
- definite integrals, 12
- signed areas, 209

• Y •

- y-axis in Cartesian coordinate system, 314

• Z •

- z-axis in Cartesian coordinate system, 314