# Chapter 12

# Putting Geometry into Story Problems

### In This Chapter

- Putting Pythagoras to work
- Making area and perimeter formulas standard fare
- Volumizing with volume problems

You can't get away from it: square yards of carpeting, miles per gallon for the car, capacity of the new freezer. In this chapter, I reacquaint you with area, perimeter, and volume. You also see how to deal with those awkward, irregularly shaped objects. It isn't all that important that you memorize the formulas — the main emphasis is on how to use the formula and where to find it when you need it.

## Triangulating a Problem with the Pythagorean Theorem

A wonderful formula to use when working with lengths and triangular situations is the Pythagorean theorem. The Pythagorean theorem is a formula that shows the special relationship between the three sides of a right triangle.

ALGEBRA RULES

According to the Pythagorean theorem, if $a$, $b$, and $c$ are the lengths of the sides of a right triangle (like the ones shown in Figure 12-1), and $c$ is the longest side (the hypotenuse), then $a^2 + b^2 = c^2$.

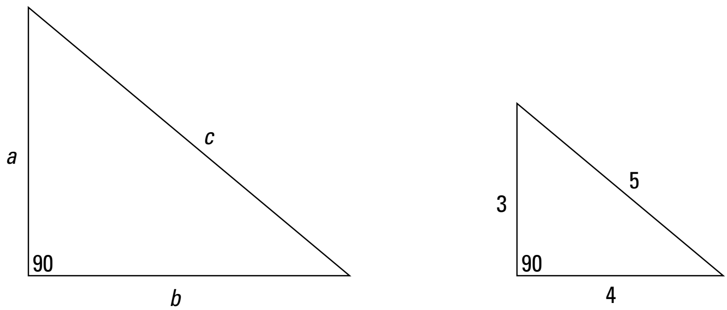



**Figure 12-1:** Triangulating the "right" way.

A carpenter wants to determine whether a garage doorway has square corners or if it's really leaning to one side. She measures 30 inches from one corner along the bottom of the doorway and makes a mark. She measures 40 inches up along the door frame from the same corner and makes a mark on the side. She then takes a tape measure and measures the distance between the marks; it comes out to be 49 inches.

Find the squares of the measures:

$$30^2 = 900 \qquad 40^2 = 1{,}600 \qquad 49^2 = 2{,}401$$

Then, $900 + 1{,}600 = 2{,}500 \neq 2{,}401$. The two smaller squares don't add up to the larger square, so the corner isn't square.

# Being Particular about Perimeter

How long is the running track around the field? What's the distance around the room? How many feet of fencing do you need to go around the pool? The *perimeter* is the distance around the outside of a given figure — the total length of the periphery that borders a region.

In general, the perimeter of a figure is the sum of the lengths of the sides.

## Triangulating triangles

The perimeter of a triangle is equal to the sum of the measures of the three sides: $P = s_1 + s_2 + s_3$.

Find the amount of fencing you'll need for a triangular area if the two sides that form a right triangle are 7 yards and 24 yards, and you can't measure the longest side, the hypotenuse, because it's too muddy right now.

Because you have a right triangle, the sum of the squares of 7 and 24 is equal to the square of the longest side:

$$7^2 + 24^2 = 49 + 576 = 625$$

Because 625 is the square of 25, the sides of the area are 7, 24, and 25 yards. Then $P = 7 + 24 + 25 = 56$ yards of fencing needed.

## Squaring up to squares and rectangles

A square is wonderful to work with because you have only one measure to worry about — the length of one side is the same as all the others. A rectangle is a special four-sided figure, too. Figure 12-2 shows a rectangle with square (90-degree) corners, where the opposite sides are the same length.

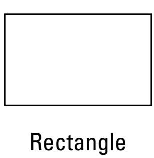



**Figure 12-2:** A shape for rooms, posters, and corrals.

To find the perimeter of a square or rectangle, use the following formulas:

- The perimeter of a *square* is four times the length of a side: $P = 4s$ (which is easier than adding $s_1 + s_2 + s_3 + s_4$).

- The perimeter of a rectangle is twice the length plus twice the width. Or you can add the length and width together and then multiply that sum by two. These formulas are easier than adding up the four sides: $P = 2l + 2w = 2(l + w)$ or $P = s_1 + s_2 + s_3 + s_4$.

The following examples illustrate using the formulas for perimeter.

An environmental group is going to search a square mile of prairie to check for toxins in beetles. What is the perimeter of that square mile in feet?

You know that 1 mile is 5,280 feet. So the perimeter is $4 \cdot 5{,}280 = 21{,}120$ feet. So, if they want to rope off the area, they need plenty of rope!

Your new garden is a rectangle measuring 85 feet long by 35 feet wide. How much fencing do you need to enclose it?

What's the perimeter? Add the 85 and 35 together and double it: $2(85 + 35) = 2(120) = 240$ feet of fencing. Of course, this doesn't include a gate — you should probably consider that, too, unless you like jumping hurdles.

## Recycling circles

A circle has a perimeter, but there's a special name for that perimeter: *circumference.* To find the circumference of a circle, all you need is the measure of the radius or the diameter. The radius is the distance from the center of the circle to any point on the circle. If you double the radius, you get the measure of the *diameter,* the distance from one side to the other through the center.

The formula for *circumference* (distance around the outside of a circle) is $C = 2\pi r = \pi d$ where $r$ is the radius, $d$ is the diameter, and $\pi$ is always about 3.14 or about $\frac{22}{7}$.

You want to construct a circular garden but you're a member of the waste-not-want-not club. The fencing you want comes in bundles of 50 feet, 100 feet, 150 feet, 200 feet, and so on, so you're going to construct your garden such that it uses every bit of the fencing around the circumference. How can you easily determine the diameter of each garden with respect to the different fencing amounts?

You should rewrite the formula so you can easily determine how wide your circular garden will be if you buy a certain size bundle of fencing to put around it and use all the fencing in the bundle.

Solving for $d$ in the formula $C = \pi d$, divide each side by $\pi$:

$$\frac{C}{\pi} = \frac{\cancel{\pi} d}{\cancel{\pi}}$$

$$\frac{C}{\pi} = d$$

The diameter is equal to the circumference divided by $\pi$.

$$d = \frac{C}{\pi}$$

If the bundle has 50 feet of fencing,

$$d = \frac{50}{3.14} \approx 15.92 \text{ feet across}$$

If the bundle has 100 feet of fencing,

$$d = \frac{100}{3.14} \approx 31.85 \text{ feet across}$$

If the bundle has 200 feet of fencing,

$$d = \frac{200}{3.14} \approx 63.69 \text{ feet across}$$

If you know the dimensions of the lot where you're putting your garden, you can determine which garden will fit.

# Making Room for Area Problems

Area is a measure of how many two-dimensional units (squares) a particular object or surface covers — how much flat space it occupies. Usually, area is given in square inches, square centimeters, square feet, or square miles, and so on.

## Ruminating about rectangles and squares

Rectangles and squares have basically the same area formulas because they both have square corners and the equal lengths on opposite sides. The general procedure here is just to multiply the measure of the length times the measure of the width. The product of two sides that are next to one another is the area.

Most rooms in homes and offices are rectangular in shape. Desks and tables and rugs are usually rectangular, also. This makes it easy to fit furniture and other objects in the room.

The area of a rectangle is its length times its width, and the area of a square is the square of the measure of any side:

Rectangle: $A = lw$

Square: $A = s^2$

A garden 85 feet long by 35 feet wide needs some fertilizer. If a bag of fertilizer covers 6 square yards, how many bags of fertilizer do you need?

Note that the measures are different. The garden is measured in feet and the fertilizer coverage is in square yards. Determine how many square feet the garden is. Then convert the fertilizer coverage to square feet per bag.

$$\text{area of garden} = l \times w = 85 \times 35 = 2{,}975 \text{ square feet}$$

Now, how many square feet are there in a square yard? If a yard is equal to 3 feet, then a square yard is 3 feet by 3 feet, so the area is $3^2 = 9$ square feet. There are 9 square feet in a square yard. A bag of fertilizer covers 6 square yards, so that's $6 \times 9 = 54$ square feet per bag.

Divide the 2,975 square feet by 54 square feet per bag:

$$\frac{2{,}975}{54} = 55\frac{5}{54} \approx 55.09 \text{ bags}$$

You can buy 56 bags and have a lot left over or buy 55 bags and skimp a little in some places.

## Taking on triangles

Finding the area of a triangle can be a bit of a challenge. Basically, a triangle's area is half that of an imaginary parallelogram that the triangle fits into. However, it isn't always easy or necessary to find the length and width of this hypothetical parallelogram — you just need a measurement or two from the triangle.

The traditional formula for finding the area of a triangle involves the length of the base, or bottom, and the height, the perpendicular distance from the base up to the *vertex* (the intersection of the other two sides).

ALGEBRA RULES

The area of a triangle is equal to half the product of the measure of the base of the triangle, $b$, times the height of the triangle, $h$: $A=\frac{1}{2}bh$.

The base is the length of the bottom that the height is drawn down to. The height is the length from the top angle down perpendicular to the base. The height forms a right angle (90 degrees) with the base. Figure 12-3 shows you a triangle with a height drawn.

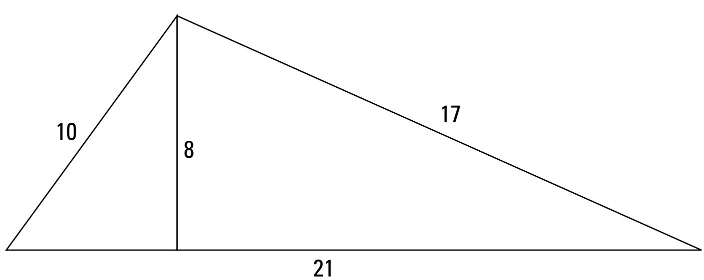



**Figure 12-3:** Triangles come in all shapes and sizes.

You use this traditional rule for area when it's possible to make these measurements — when you can draw the height perpendicular to the base and measure both of them.

EXAMPLE

Find the area of a triangle 21 feet long with a height of 8 feet. Refer to Figure 12-3 for a sketch of such a triangle.

$$A=\frac{1}{2}(21)(8)=\frac{1}{2}(168)=84 \text{ square feet}$$

### Rounding up circles

The area of a circle is tied to both the radius of the circle and the value of $\pi$.

The formula for the area of a circle is $\pi$ (about 3.14) times the radius squared: $A = \pi r^2$.

Find the area of a circular disk that is 50 feet across. First, you need to find the radius. If the circle is 50 feet across, that's the measure of the diameter, all the way across. So the radius is half that or 25 feet. Using the formula to find the area:

$$A = \pi r^2 = \pi \cdot 25^2 = 3.14 \cdot 625 = 1{,}962.5 \text{ square feet}$$

## Validating with Volume

Area is a two-dimensional figure or representation. It's a flat region. Volume is three-dimensional. To find volume, you measure across, front to back, and up and down.

With volume, you count how many cubes (picture sugar cubes) you can fit into an object. These cubes can be 1 inch on each edge, 1 centimeter on each edge, 1 foot on each edge, or however big they need to be. And, in keeping with the cube theme, you measure volume in cubic inches, cubic centimeters, cubic feet, and cubic whatevers.

### Prodding prisms and boxing boxes

The volume of a rectangular prism, better known as a box, is one of the simplest to find in the world of volume problems. The bottom and top of a prism have exactly the same measurements. The distance from the top to bottom is the same, no matter where you measure, as long as you keep that distance perpendicular to both top and bottom.

The formula for finding the volume of a prism is $V = lwh$, which means that the volume is equal to the product of the length, $l$, times the width, $w$, times the height, $h$.

EXAMPLE

If you're buying a 12-cubic-foot refrigerator, what are the dimensions (how big is it)?

There are an infinite number of ways to multiply three numbers together to get 12. Go through some integers and some fractions.

Try to picture what the refrigerator would look like with these dimensions:

- $12 = 1(1)(12)$. That's 1 foot long, 1 foot wide, and 12 feet tall!
- $12 = 2(1)(6)$. That's 2 feet long, 1 foot wide, and 6 feet tall.
- $12 = 2(3)(2)$. That's 2 feet long, 3 feet wide, and 2 feet tall.
- $12 = 1\frac{1}{2}\left(1\frac{1}{2}\right)\left(5\frac{1}{3}\right)$. That's $1\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and $5\frac{1}{3}$ feet tall.

Which refrigerator would you want? How tall are you? How far can you reach into the back?

## Cycling cylinders

Cylinders were my brother's favorite shape when he was in the Navy on the aircraft carrier USS *Guadalcanal*. Being the wonderful sister that I am, I would send him chocolate chip cookies that fit exactly into a 3-pound coffee can. Imagine a stack of chocolate chip cookies coming to you every couple of weeks. Was he ever popular on *that* ship!

ALGEBRA RULES

The formula for the volume of a cylinder is $V = \pi r^2 h$. The volume is equal to π times the radius (halfway across a circle) squared times the height.

To find the volume of a cylinder, you need the radius of the top and bottom, and you need the height. This formula tells you how many cubes will fit in the cylinder — like putting square pegs in a round hole — just trim them a bit.

EXAMPLE

Find the volume of an above-ground swimming pool that has a radius of 12 feet and a height of 4 feet.

Using the formula for the volume of a cylinder:

$V = \pi r^2 h = \pi(12^2)(4) = \pi(576) \approx 3.14(576) = 1{,}808.64$ cubic feet of water

## Pointing to pyramids and cones

A pyramid is an easy thing to describe because everyone has a mental picture of what a pyramid looks like. Technically, a pyramid is an object with a base (bottom) and triangles coming up from each side of the base to meet at a point. The base can be any polygon: a triangle, rectangle, square, and so on.

Cones are also very familiar. You see those orange shapes along the road in construction zones, and you hold them very carefully when they're full of drippy ice cream. I put these two figures together, because their volume formulas are so similar. Both volume formulas are essentially one-third of their height times the area of their base.

The formula for the volume of a pyramid is $V = \frac{1}{3}(\text{area of base}) \cdot h$.

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

Find the original volume of the Great Pyramid, which originally had a square base with each side measuring 756 feet and a height of 480 feet.

The base is a square, so the area of the base is $s^2$:

$$V = \frac{1}{3}s^2 \cdot h = \frac{1}{3}(756)^2 \cdot 480 = 91{,}445{,}760 \text{ cubic feet}$$

What is the volume of a cone-shaped tent that has a diameter of 18 feet and a height of 20 feet?

If the diameter is 18 feet, then the radius is 9 feet:

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(9)^2 \cdot 20 = 540\pi \approx 1{,}696 \text{ cubic feet}$$

# Chapter 13

# Grappling with Graphing

### In This Chapter

- Looking at lines and their special features
- Investigating intersections of lines
- Taking a peek at parabolas

In this chapter, I present the basics for working with lines and their equations. You find lines determined by two points and then other lines determined by a slope and a point. You see lines that meet and lines that avoid one another forever.

I also throw you a curve or two! Circles and parabolas are the most recognizable of the algebraic curves and have the most respectable equations. The basics for drawing these curves is found here.

## Preparing to Graph a Line

A straight *line* is the set of all the points on a graph that satisfy a linear equation. When any two points on a line are chosen, the *slope* of the segment between those two points is always the same number.

To graph a line, you need only two points. A rule in geometry says that only one line can go through two particular points. Even though only two points are needed to graph a line, it's usually a good idea to graph at least three points to be sure that you graphed the line correctly.

An equation whose graph is a straight line is said to be *linear.* A linear equation has a standard form of $ax + by = c$, where $x$ and $y$ are variables and $a$, $b$, and $c$ are real numbers. A point

$(x, y)$ lies on the line if the $x$ and $y$ make the equation true. When graphing a line, you can find some pairs of numbers that make the equation true and then connect them. Connect the dots!

Graphing lines from their equations just takes finding enough points on the line to convince you that you've drawn the graph correctly.

Find a point on the line $x - y = 3$.

1. **Choose a random value for one of the variables, either *x* or *y*.**

   To make the arithmetic easy for yourself, pick a large enough number so that, when you subtract $y$ from that number, you get a positive 3. In $x - y = 3$, you can let $x = 8$, so $8 - y = 3$.

2. **Solve for the value of the other variable.**

   Subtract 8 from each side to get $-y = -5$.

   Multiply each side by $-1$ to get $y = 5$.

3. **Write an ordered pair for the coordinates of the point.**

   You chose 8 for $x$ and solved to get $y = 5$, so your first ordered pair is (8, 5).

You can find more ordered pairs by choosing another number to substitute for either $x$ or $y$.

Find a point that lies on the line $2x + 3y = 12$.

1. **Solve the equation for one of the variables.**

   Solving for $y$ in the sample problem, $2x + 3y = 12$, you get

   $$3y = 12 - 2x$$

   $$y = \frac{12 - 2x}{3}$$

   With multipliers involved, you often get a fraction.

2. **Choose a value for the other variable and solve the equation.**

   Try to pick values so that the result in the numerator is divisible by the 3 in the denominator — giving you an integer.

   For example, let $x = 3$. Solving the equation:

   $$y = \frac{12 - 2 \cdot 3}{3} = \frac{6}{3} = 2$$

   So, the point (3, 2) lies on the line.

# Incorporating Intercepts

An *intercept* of a line is a point where the line crosses an axis. Unless a line is vertical or horizontal, it crosses both the $x$ and $y$ axes, so it has two intercepts: an $x$-intercept and a $y$-intercept. Horizontal lines have just a $y$-intercept, and vertical lines have just an $x$-intercept. The exceptions are when the horizontal line is actually the $x$-axis or the vertical line is the $y$-axis. Intercepts are quick and easy to find and can be a big help when graphing.

The $x$-intercept of a line is where the line crosses the $x$-axis. To find the $x$-intercept, let the $y$ in the equation equal 0 and solve for $x$.

Find the $x$-intercept of the line $4x - 7y = 8$.

First, let $y = 0$ in the equation. Then:

$$\begin{aligned} 4x - 0 &= 8 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

The $x$-intercept of the line is (2, 0). The line goes through the $x$-axis at that point.

The $y$-intercept of a line is where the line crosses the $y$-axis. To find the $y$-intercept, let the $x$ in the equation equal 0 and solve for $y$.

Find the $y$-intercept of the line $3x - 7y = 28$.

First, let $x = 0$ in the equation. Then:

$$0 - 7y = 28$$
$$-7y = 28$$
$$y = -4$$

The $y$-intercept of the line is (0, –4).

As long as you're careful when graphing the $x$- and $y$-intercepts and get them on the correct axes, the intercepts are often all you need to graph a line.

# Sliding the Slippery Slope

The slope of a line is a number that describes the steepness and direction of the graph of the line. The slope is a positive number if the line moves upward from left to right; the slope is a negative number if the line moves downward from left to right. The steeper the line, the greater the absolute value of the slope (the farther the number is from 0).

Knowing the slope of a line beforehand helps you graph the line. You can find a point on the line and then use the slope and that point to graph it. A line with a slope of 6 goes up steeply. If you know what the line should look like (that is, whether it should go up or down) — information you get from the slope — you'll have an easier time graphing it correctly.

Figure 13-1 shows some lines with their slopes. The lines are all going through the origin just for convenience.

What about a horizontal line — one that doesn't go upward or downward? A horizontal line has a 0 slope. A vertical line has no slope; the slope of a vertical line (it's so steep) is undefined.

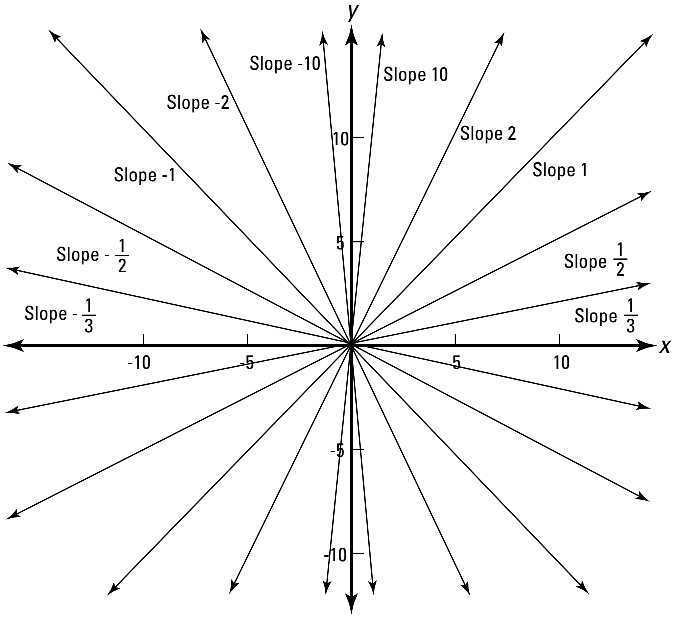



**Figure 13-1:** Pick a line — see its slope.

TIP

One way of referring to the slope, when it's written as a fraction, is rise over run. If the slope is $\frac{3}{2}$, it means that for every 2 units the line runs left to right along the *x*-axis, it rises 3 units along the *y*-axis. A slope of $\frac{-1}{8}$ indicates that as the line runs 8 units horizontally, parallel to the *x*-axis left to right, it drops (negative rise) 1 unit vertically.

# Computing slope

If you know two points on a line, you can compute the number representing the slope of the line.

ALGEBRA RULES

The slope of a line, denoted by the small letter *m*, is found when you know the coordinates of two points on the line, $(x_1, y_1)$ and $(x_2, y_2)$:

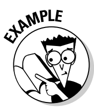
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Subscripts are used here to identify which is the first point and which is the second point. There's no rule as to which is which; you can name the points any way you want. It's just a good idea to identify them to keep things in order. Reversing the points in the formula gives you the same slope (when you subtract in the opposite order):

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

You just can't mix them and do $(y_1 - y_2)$ over $(x_2 - x_1)$.

Now, you can see how to compute slope with the following examples.




Find the slope of the line going through (3, 4) and (2, 10).

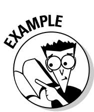
Let (3, 4) be $(x_1, y_1)$ and (2, 10) be $(x_2, y_2)$. Substitute into the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{2 - 3}$$

Simplify:

$$m = \frac{6}{-1} = -6$$

This line is pretty steep as it falls from left to right.




Find the slope of the line going through (4, 2) and (–6, 2).

Let (4, 2) be $(x_1, y_1)$ and (–6, 2) be $(x_2, y_2)$. Substitute into the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{-6 - 4}$$

Simplify:

$$m = \frac{0}{-10} = 0$$

These points are both 2 units above the *x*-axis and determine a horizontal line. That's why the slope is 0.

Find the slope of the line going through (2, 4) and (2, –6).

Let (2, 4) be $(x_1, y_1)$ and (2, –6) be $(x_2, y_2)$. Substitute into the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 4}{2 - 2}$$

Simplify:

$$m = \frac{-10}{0}$$

Oops! You can't divide by 0. There is no such number. The slope doesn't exist or is undefined. These two points are on a vertical line.

Watch out for these common errors when working with the slope formula:

- **Be sure that you subtract the *y* values on the top of the division formula.** A common error is to subtract the *x* values on the top.
- **Be sure to keep the numbers in the same order when you subtract.** Decide which point is first and which point is second. Then take the second *y* minus the first *y* and the second *x* minus the first *x*. Don't do the top subtraction in a different order from the bottom.

## Combining slope and intercept

An equation of a single line can take many forms. Just as you can solve for one variable or another in a formula, you can solve for one of the variables in the equation of a line. This change of format can help you find the points to graph the line or find the slope of a line.

A common and popular form of the equation of a line is the *slope-intercept form.* It's given this name because the slope of the line and the *y*-intercept of the line are obvious on sight. When a line is written $6x + 3y = 5$, you can find points by plugging in numbers for *x* or *y* and solving for the other coordinate. But, by using methods for solving linear equations (see Chapter 6), the same equation can be written $y = -2x + \frac{5}{3}$, which tells you that the slope is –2 and the place where the line crosses the *y*-axis (the *y*-intercept) is $\left(0, \frac{5}{3}\right)$.

Where *y* and *x* represent coordinates of a point on the line, *m* is the slope of the line, and *b* is the *y*-intercept of the line, the slope-intercept form is $y = mx + b$.

In every case shown next, the equation is written in the slope-intercept form. The coefficient of *x* is the slope of the line and the constant gives the *y*-intercept.

- **$y = 2x + 3$:** The slope is 2; the *y*-intercept is (0, 3).
- **$y = \frac{1}{3}x - 2$:** The slope is $\frac{1}{3}$; the *y*-intercept is (0, –2).
- **$y = 7$:** The slope is 0; the *y*-intercept is (0, 7). You can read this equation as being $y = 0 \cdot x + 7$.

## Creating the slope-intercept form

If the equation of the line isn't already in the slope-intercept form, solving for *y* changes the equation to slope-intercept form.

Put the equation $5x - 2y = 10$ in slope-intercept form.

1. **Get the *y* term by itself on the left.**

   Subtract 5*x* from each side to get the *y* term alone:

   $$-2y = -5x + 10$$

2. **Solve for *y*.**

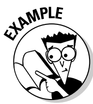
   Divide each side by –2 and simplify the two terms on the right:

   $$\frac{-2y}{-2} = \frac{(-5x + 10)}{-2}$$
   $$y = \frac{-5x}{-2} + \frac{10}{-2}$$
   $$y = \frac{5}{2}x - 5$$

   The slope is $\frac{5}{2}$ and the *y*-intercept is at (0, –5).

## Graphing with slope-intercept

One advantage to having an equation in the slope-intercept form is that graphing the line can be a fairly quick task, as the following example shows.




Graph $y = \frac{3}{2}x + 1$.

The slope of this line is $\frac{3}{2}$ and the *y*-intercept is the point (0, 1). First, graph the *y*-intercept (see Figure 13-2). Then use the rise-over-run interpretation of slope to count spaces to another point on the line. To do this, do the run, or bottom, movement first. In this sketch, move 2 units to the right of (0, 1). From there, rise (or go up) 3 units, which should get you to (2, 4).

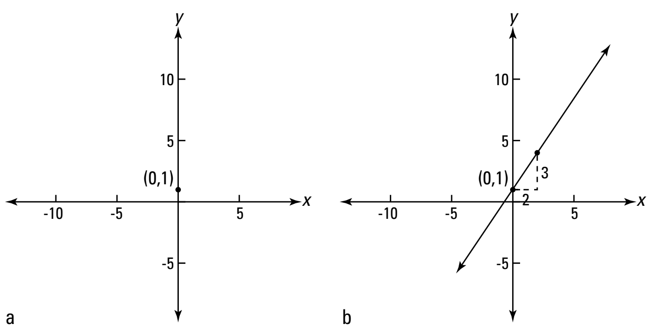



**Figure 13-2:** The *y*-intercept is located; use run and rise to find another point.

It's sort of like going on a treasure hunt: "Two steps to the east; three steps to the north; now dig in!" Only our "dig in" is to put a point there and connect that point with the starting point — the intercept. Look at the right-hand side (the b side) of Figure 13-2 to see how it's done.

# Making Parallel and Perpendicular Lines Toe the Line

The slope of a line gives you information about a particular characteristic of the line. It tells you if it's steep or flat and if it's rising or falling as you read from left to right. The slope of a line can also tell you if one line is parallel or perpendicular to another line.

Parallel lines never touch. They're always the same distance apart and never share a common point. They have the same slope.

Perpendicular lines form a 90-degree angle (a *right angle*) where they cross. They have slopes that are negative reciprocals of one another. For example, the $x$-axis and $y$-axis are perpendicular lines.

If line $l_1$ has a slope of $m_1$, and if line $l_2$ has a slope of $m_2$, then the lines are parallel if $m_1 = m_2$. If line $l_1$ has a slope of $m_1$, and if line $l_2$ has a slope of $m_2$, then the lines are perpendicular if $m_1 = -\frac{1}{m_2}$ or if they are horizontal or vertical lines.

The following examples show you how to determine whether lines are parallel or perpendicular by just looking at their slopes:

- The line $y = 3x + 2$ is parallel to the line $y = 3x - 7$ because their slopes are both 3.
- The line $3x + 2y = 8$ is parallel to the line $6x + 4y = 7$ because their slopes are both $\frac{-3}{2}$. Write each line in the slope-intercept form to see this: $3x + 2y = 8$ can be written $y = \frac{-3}{2}x + 4$ and $6x + 4y = 7$ can be written $y = \frac{-3}{2}x + \frac{7}{4}$.
- The line $y = \frac{3}{4}x + 5$ is perpendicular to the line $y = \frac{-4}{3}x + 6$ because their slopes are negative reciprocals of one another.
- The line $y = -3x + 4$ is perpendicular to the line $y = \frac{1}{3}x - 8$ because their slopes are negative reciprocals of one another.

# Criss-Crossing Lines

If two lines *intersect,* or cross one another, then they intersect exactly once and only once. The place they cross is the point of intersection and that common point is the only one both lines share. Careful graphing can sometimes help you to find the point of intersection.

The point (5, 1) is the point of intersection of the two lines $x + y = 6$ and $2x - y = 9$ because the coordinates make each equation true:

- If $x + y = 6$, then substituting the values $x = 5$ and $y = 1$ give you $5 + 1 = 6$, which is true.
- If $2x - y = 9$, then substituting the values $x = 5$ and $y = 1$ give $2 \cdot 5 - 1 = 10 - 1 = 9$, which is also true.

This is the only point that works for both the lines.

One way to find the intersection of two lines is to graph both lines (very carefully) and observe where they cross. This technique is not very helpful when the intersection has fractional coordinates, though.

Another way to find the point where two lines intersect is to use a technique called *substitution* — you substitute the *y* value from one equation for the *y* value in the other equation and then solve for *x*. Because you're looking for the place where *x* and *y* of each line are the same — that's where they intersect — then you can write the equation $y = y$, meaning that the *y* from the first line is equal to the *y* from the second line. Replace the *y*'s with what they're equal to in each equation, and solve for the value of *x* that works.

Find the intersection of the lines $3x - y = 5$ and $x + y = -1$.

1. **Put each equation in the slope-intercept form, which is a way of solving each equation for *y*.**

   $3x - y = 5$ is written as $y = 3x - 5$, and $x + y = -1$ is written as $y = -x - 1$. (The lines are not parallel, and their slopes are different, so there will be a point of intersection.)

2. **Set the *y* points equal and solve.**

   From $y = 3x - 5$ and $y = -x - 1$, you substitute what *y* is equal to in the first equation with the *y* in the second equation: $3x - 5 = -x - 1$.

**3. Solve for the value of *x*.**

Add $x$ to each side and add 5 to each side:

$$3x + x - 5 + 5 = -x + x - 1 + 5$$
$$4x = 4$$
$$x = 1$$

Substitute that 1 for $x$ into either equation to find that $y = -2$. The lines intersect at the point (1, –2).

# Turning the Curve with Curves

A circle is a most recognizable shape. A circle is basically all the points that are a set distance from the point called the circle's *center.* A parabola isn't quite as recognizable as a circle, but it's represented by a quadratic equation and fairly easy to graph.

## Going around in circles with a circular graph

An example of an equation of a circle is $x^2 + y^2 = 25$. The circle representing this equation goes through an infinite number of points. Here are just some of those points:

| | | | | |
|---|---|---|---|---|
| (0, 5) | (0, –5) | (5, 0) | (–5, 0) | (3, 4) |
| (4, 3) | (4, –3) | (–3, 4) | (–3, –4) | (–4, –3) |

I haven't finished all the possible points with integer coordinates, let alone points with fractional coordinates, such as $\left(\frac{25}{13}, \frac{60}{13}\right)$.

TIP: When graphing an equation, you don't expect to find all the points. You just want to find enough points to help you sketch in all the others without naming them.

## Putting up with parabolas

Parabolas are nice, U-shaped curves. They're the graphs of quadratic equations where either an $x$ term is squared or a $y$ term is squared, but not both are squared at the same time. Parabolas have a highest point or a lowest point (or the farthest left point or the farthest right point) called the *vertex.*

### Trying out the basic parabola

My favorite example of a parabola is $y = x^2$, the basic parabola. Figure 13-3 shows a graph of this formula. This equation says that the $y$-coordinate of every point on the parabola is the square of the $x$-coordinate.

The vertex of the parabola in Figure 13-3 is at the origin, (0, 0), and the graph curves upward.

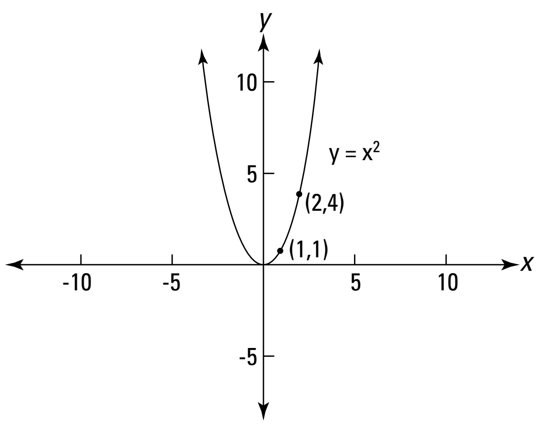



**Figure 13-3:** The simplest parabola.

You can make this parabola steeper or flatter by multiplying the $x^2$ by certain numbers. If you multiply the squared term by numbers bigger than 1, it makes the parabola steeper. If you multiply by numbers between 0 and 1 (which are proper fractions), it makes the parabola flatter.

You can make the parabola open downward by multiplying the $x^2$ by a negative number, and make it steeper or flatter than the basic parabola — in a downward direction.

### Putting the vertex on an axis

The basic parabola, $y = x^2$, can be slid around — left, right, up, down — placing the vertex somewhere else on an axis and not changing the general shape.

If you change the basic equation by adding a constant number to the $x^2$ — such as $y = x^2 + 3$, $y = x^2 + 8$, $y = x^2 - 5$, or $y = x^2 - 1$ — then the parabola moves up and down the $y$-axis. Note that adding a negative number is also part of this rule. These manipulations help make a parabola fit the model of a certain situation.

If you change the basic parabolic equation by adding a number to the $x$ first and then squaring the expression — such as $y = (x + 3)^2$, $y = (x + 8)^2$, $y = (x - 5)^2$, or $y = (x - 1)^2$ — you move the graph to the left or right of where the basic parabola lies. Using +3, as in the equation $y = (x + 3)^2$, moves the graph to the left, and using –3, as in the equation $y = (x - 3)^2$, moves the graph to the right. It's the opposite of what you might expect, but it works this way consistently.

EXAMPLE

The following equations and their graphs are shown in Figure 13-4:

- $\mathbf{y = 3x^2 - 2}$**:** The 3 multiplying the $x^2$ makes the parabola steeper, and the –2 moves the vertex down to (0, –2).
- $\mathbf{y = \frac{1}{4}x^2 + 1}$**:** The $\frac{1}{4}$ multiplying the $x^2$ makes the parabola flatter, and the +1 moves the vertex up to (0, 1).
- $\mathbf{y = -5x^2 + 3}$**:** The –5 multiplying the $x^2$ makes the parabola steeper and causes it to go downward, and the +3 moves the vertex to (0, 3).
- $\mathbf{y = 2(x - 1)^2}$**:** The 2 multiplier makes the parabola steeper, and subtracting 1 moves the vertex right to (1, 0).

- $y = -\frac{1}{3}(x+4)^2$**:** The $-\frac{1}{3}$ makes the parabola flatter and causes it to go downward, and adding 4 moves the vertex left to (–4, 0).
- $y = -\frac{1}{20}x^2 + 5$**:** The $-\frac{1}{20}$ multiplying the $x^2$ makes the parabola flatter and causes it to go downward, and the +5 moves the vertex to (0, 5).

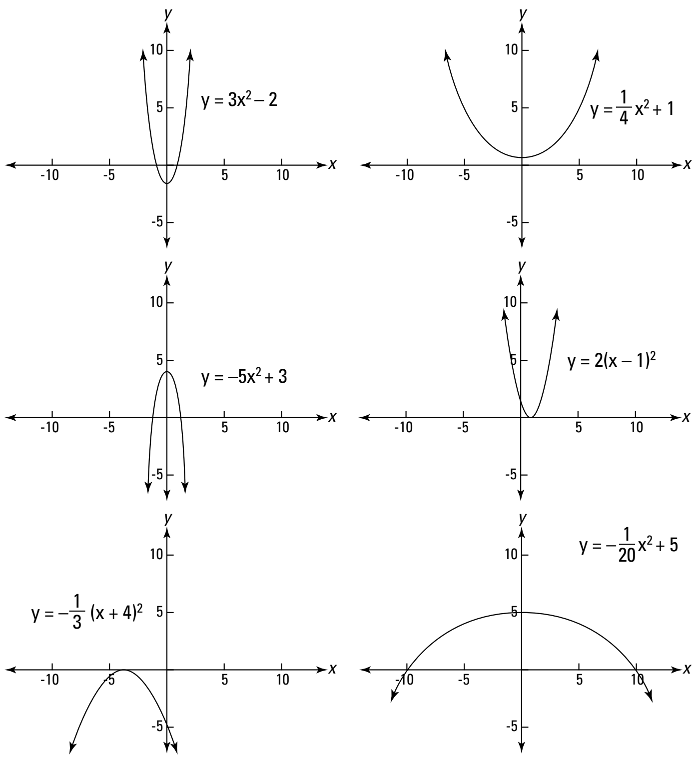



**Figure 13-4:** Parabolas galore.