

Chapter 11

Making Algebra Tell a Story

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In This Chapter

- ▶ Setting the scene for solving a story problem
 - ▶ Getting interested in solving problems involving simple interest
 - ▶ Measuring distances with distance problems
 - ▶ Managing mixture problems
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Story problems can be one of the least favorite activities for algebra students. Although algebra and its symbols, rules, and processes act as a door to higher mathematics and logical thinking, story problems give you immediate benefits and results in real-world terms.

Algebra allows you to solve problems. Not all problems — it won't help with that noisy neighbor — but problems involving how to divvy up money equitably or make things fit in a room. In this chapter, you find some practical applications for algebra. You may not be faced with the exact situations I use in this chapter, but you should find some skills that will allow you to solve the story problems or practical applications that are special to your situation.

Making Plans to Solve Story Problems

When solving story problems, the equation you should use or how all the ingredients interact isn't always immediately apparent. It helps to have a game plan to get you started.

Sometimes, just picking up a pencil and drawing a picture can be a big help. Other times, you can just write down all the numbers involved.



You don't have to use every suggestion in the following list with every problem, but using as many as possible can make the task more manageable:

- ✓ **Draw a picture.** Label your picture with numbers or names or other information that helps you make sense of the situation. Fill it in more or change the drawing as you set up an equation for the problem.
- ✓ **Assign a variable(s) to represent *how many or number of*.** You may use more than one variable at first and refine the problem to just one variable later.
- ✓ **If you use more than one variable, go back and substitute known relationships for the extra variables.** When it comes to solving the equations, you want to solve for just one variable. You can often rewrite all the variables in terms of just one of them.
- ✓ **Look at the end of the question or problem statement.** This often gives a big clue as to what's being asked for and what the variables should represent. It can also give a clue as to what formula to use, if a formula is appropriate.
- ✓ **Translate the words into an equation.** Replace:
 - and, more than, and exceeded by with the plus sign (+)
 - less than, less, and subtract from with the minus sign (-)
 - of and times as much with the multiplication sign (\times)
 - twice with two times ($2 \times$)
 - divided by with the division sign (\div)
 - half as much with one-half times ($\frac{1}{2} \times$)
 - the verb (is or are, for example) with the equal sign (=)
- ✓ **Plug in a standard formula, if the problem lends itself to one.** When possible, use a formula as your equation

or as part of your equation. Formulas are a good place to start to set up relationships. Be familiar with what the variables in the formula stand for.

- ✓ **Check to see if the answer makes any sense.** When you get an answer, decide whether it makes sense within the context of the problem. Having an answer make sense doesn't guarantee that it's a correct answer, but it's the first check to tell if it isn't correct.

Finding Money and Interest Interesting

Figuring out how much interest you have to pay, or how much you're earning in interest, is simple with the proper formulas. And, when the formulas are incorporated into story problems, the processes just seem to fall into place.



The amount of simple interest earned is equal to the amount of the principal, P (the starting amount), times the rate of interest, r (which is written as a decimal), times the amount of time, t (usually in years). The formula to calculate simple interest is: $I = Prt$.



What is the amount of simple interest on \$10,000 when the interest rate is $2\frac{1}{2}$ percent and the time period is $3\frac{1}{2}$ years?

$$I = Prt$$

$$I = 10,000 \cdot 0.025 \cdot 3.5 = 875$$

The interest is \$875.

See! That was a *simple* story problem!

Investigating investments and interest

You can invest your money in a safe CD or savings account and get one interest rate. You can also invest in riskier ventures and get a higher interest rate, but you risk losing money.

Most financial advisors suggest that you diversify — put some money in each type of investment — to take advantage of each investment's good points.

Use the simple interest formula in each of these problems to simplify the process. (With simple interest, the interest is figured on the beginning amount only.)



Khalil had \$20,000 to invest last year. He invested some of this money at $3\frac{1}{2}$ percent interest and the rest at 8 percent interest. His total earnings in interest, for both of the investments, were \$970. How much did he have invested at each rate?

Think of the \$20,000 as being divvied up into two large containers. Let x represent the amount of money invested at $3\frac{1}{2}$ percent. The first container has the x dollars in it and a label saying $3\frac{1}{2}$. The second container has a label saying 8 percent and $20,000 - x$ dollars in it. The number \$970 is on a sign next to the containers. That's the result of multiplying the mixture (combined) percentage times the total investment of \$20,000. You don't need to know the mixture percentage — just the result.

$$3\frac{1}{2} \text{ percent}(x) + 8 \text{ percent}(20,000 - x) = 970$$

$$0.035(x) + 0.08(20,000 - x) = 970$$

$$0.035x + 1,600 - 0.08x = 970$$

Subtract 1,600 from each side and simplify on the left side:

$$-0.045x = -630$$

Dividing each side by -0.045 , you get

$$x = 14,000$$

That means that \$14,000 was invested at $3\frac{1}{2}$ percent and the other \$6,000 was invested at 8 percent.



Kathy wants to withdraw only the interest on her investment each year. She's going to put money into the account and leave it there, just taking the interest earnings. She wants to take out and spend \$10,000 each year. If she puts two-thirds of her money where it can earn 5 percent interest and the rest at 7 percent interest, how much should she put at each rate to have the \$10,000 spending money?

Let x represent the total amount of money Kathy needs to invest. Using those handy, dandy containers again, the first container has a label of 5 percent and contains $\frac{2}{3}x$ dollars. The second container has 7 percent on its label and has $\frac{1}{3}x$ dollars. The mixture has \$10,000; this is the result of the “mixed” percentage and the total amount invested.

$$5\% \left(\frac{2}{3}x \right) + 7\% \left(\frac{1}{3}x \right) = 10,000$$

Change the decimals to fractions and multiply:

$$0.05 \left(\frac{2}{3}x \right) + 0.07 \left(\frac{1}{3}x \right) = 10,000$$

$$\frac{1}{30}x + \frac{7}{300}x = 10,000$$

Find a common denominator and add the coefficients of x :

$$\frac{17}{300}x = 10,000$$

Divide each side by $\frac{17}{300}$:

$$x \approx 176,470.59$$

Kathy needs over \$176,000 in her investment account. Two-thirds of it, about \$117,647, has to be invested at 5 percent and the rest, about \$58,824, at 7 percent.

Greening up with money

Money is everyone’s favorite topic. When you’re combining money and algebra, you have to consider the number of coins or bills and their worth or denomination. Other situations involving money can include admission prices, prices of different pizzas in an order, or any commodity with varying prices.



Chelsea has five times as many quarters as dimes, three more nickels than dimes, and two fewer than nine times as many pennies as dimes. If she has \$15.03 in coins, how many of them are quarters?

Some containers work here, too. There will be four of them added together, labeled: *dimes*, *quarters*, *nickels*, and *pennies*. Also, on the label, is the value of each coin. Every coin count refers to dimes in this problem, so let the number of dimes be represented by x and compare everything else to it.

The first container would contain dimes; put 0.10 and x on the label. The second container contains quarters; put 0.25 and $5x$ on the label. The third container contains nickels; so put 0.05 and $x + 3$ on the label. The fourth container contains pennies; so put 0.01 and $9x - 2$ on the label. A mixture container, on the right, has \$15.03 on it.

$$0.10(x) + 0.25(5x) + 0.05(x + 3) + 0.01(9x - 2) = 15.03$$
$$0.10x + 1.25x + 0.05x + 0.15 + 0.09x - 0.02 = 15.03$$

Simplifying on the left, you get

$$1.49x + 0.13 = 15.03.$$

Subtracting 0.13:

$$1.49x = 14.90$$

And, after dividing by 1.49,

$$x = 10$$

Because x is the number of dimes, there are 10 dimes, five times as many or 50 quarters, three more or 13 nickels and two fewer than nine times as many or 88 pennies. The question was, “How many quarters?” There were 50 quarters; use the other answers to check to see if this comes out correctly.

Formulating Distance Problems

You travel, I travel, everybody travels, and at some point everybody asks, “Are we there yet?” Algebra can’t answer that question for you, but it can help you estimate how long it takes to get there — wherever “there” is.

Making the distance formula work for you

You've been on a slow boat to China for a couple of days and want to know how far you've come. Or you want to figure out how long it'll take for the rocket to reach Jupiter. Or maybe you want to know how fast a train travels if it gets you from Toronto, Ontario, to Miami, Florida, in 18 hours. The distance = rate · time formula can help you find the answer to all these questions.



The formula $d = rt$ means the distance traveled is equal to the rate r (the speed) times how long it takes, t (the time). Solving the formula for either the rate or the time, you get: $r = \frac{d}{t}$ and $t = \frac{d}{r}$. Given any two of the values, you can solve for the third. You change the original formula to one that you can use.



If a plane travels 2,000 miles in 4.8 hours, then what was the average speed of the plane during the trip?

You're looking for the speed or *rate*, r , so you use this formula:

$$r = \frac{d}{t}$$

So, plugging in the numbers, $r = \frac{2,000}{4.8} = 416\frac{2}{3}$ miles per hour



Always be sure that the units are the same: Miles per day and total number of miles go together, but miles per hour and total number of days would take some adjusting.



How far did Alberto travel in his triathlon if he swam at 2 miles per hour for 30 minutes, bicycled at 25 miles per hour for 45 minutes, and then ran at 6 miles per hour for 6 minutes?

The distance formula $d = rt$ is used three times and the results added together to get the total distance.

You need to change 2 mph for 30 minutes to 2 mph for $\frac{1}{2}$ hour. Then change 25 mph for 45 minutes to 25 mph for $\frac{3}{4}$ hour.

Finally, change 6 mph for 6 minutes to 6 mph for $\frac{1}{10}$ hour. All those fractions of hours come from dividing the number of minutes by 60.

$$\begin{aligned}
 \left(2 \times \frac{1}{2}\right) + \left(25 \times \frac{3}{4}\right) + \left(6 \times \frac{1}{10}\right) &= 1 + \frac{75}{4} + \frac{6}{10} \\
 &= 1 + 18\frac{3}{4} + \frac{3}{5} \\
 &= 19 + \left(\frac{3}{4} + \frac{3}{5}\right) \\
 &= 19 + \left(\frac{15}{20} + \frac{12}{20}\right) \\
 &= 19 + \frac{27}{20} \\
 &= 19 + 1\frac{7}{20} \\
 &= 20\frac{7}{20}
 \end{aligned}$$

Alberto traveled over 20 miles.

Figuring distance plus distance

A basic distance problem involves one object traveling a certain distance, a second object traveling another distance, and the two distances getting added together.



Deirdre and Donovan are in love and will be meeting in Kansas City to get married. Deirdre boarded a train at noon traveling due east toward Kansas City. Two hours later, Donovan boarded a train traveling due west, also heading for Kansas City, and going at a rate of speed 20 miles per hour faster than Deirdre. At noon, they were 1,100 miles apart. At 9 p.m., they both arrived in Kansas City. How fast were they traveling?

distance of Deirdre from Kansas City + distance of
Donovan from Kansas City = 1,100

(rate \times time) + (rate \times time) = 1,100

Let the speed (rate) of Deirdre's train be represented by r . Donovan's train was traveling 20 miles per hour faster than Deirdre's, so the speed of Donovan's train is $r + 20$.

Let the time traveled by Deirdre's train be represented by t . Donovan's train left two hours after Deirdre's, so the time traveled by Donovan's train is $t - 2$. Substituting the expressions into the first equation,

$$rt + (r + 20)(t - 2) = 1,100$$

Deirdre left at noon and arrived at 9, so $t = 9$ hours for Deirdre's travels and $t - 2 = 7$ hours for Donovan's. Replacing these values in the equation,

$$r(9) + (r + 20)(7) = 1,100$$

Now distribute the 7:

$$9r + 7r + 140 = 1,100$$

Combine the two terms with r :

$$16r = 960$$

Divide each side by 16:

$$r = 60$$

Deirdre's train is going 60 mph; Donovan's is going $r + 20 = 80$ mph.

Figuring distance and fuel

My son, Jim, sent me this problem when he was stationed in Afghanistan with the Marines. He was always a whiz at story problems — doing them in his head and not wanting to show any work. He must have been listening to me, because, at the end of his contribution, he added, “Don't forget to show your work!”



A CH-47 troop-carrying helicopter can travel 300 miles if there aren't any passengers. With a full load of passengers, it can travel 200 miles before running out of fuel. If Camp Tango is 120 miles away from Camp Sierra, can the CH-47 carry a full load of Special Forces members from Tango to Sierra, drop off the troops, and return safely to Tango before running out of fuel? If so, what percentage of fuel will it have left?

I felt a little nervous, working on this problem, with so much at stake. So I took my own advice and drew a picture, tried some scenarios with numbers, and assigned a variable to an amount.

Let x represent the number of gallons of fuel available in the helicopter, and write expressions for the amount used during each part of the operation.

When the helicopter is loaded, it can travel 200 miles on a full tank of fuel. The camps are 120 miles apart, so the helicopter uses $\frac{120}{200}x$ gallons for that part of the trip.

When there are no passengers, the helicopter can travel 300 miles on a full tank. So it uses $\frac{120}{300}x$ gallons for the return flight.

Adding the two amounts together:

$$\frac{120}{200}x + \frac{120}{300}x = \frac{3}{5}x + \frac{2}{5}x = \frac{5}{5}x = x$$

It looks like there's no room for a scenic side-trip. And I haven't figured in the fuel needed for landing and taking off. Hopefully, there's a reserve tank.

Stirring Things Up with Mixtures

Mixture problems can take on many different forms. There are the traditional types, in which you can actually mix one solution and another, such as water and antifreeze. There are the types in which different solid ingredients are mixed, such as in a salad bowl or candy dish. Another type is where different investments at different interest rates are mixed together. I covered some of these mixture problems in the earlier section, "Finding Money and Interest Interesting." Here I show you a process you can use to solve most mixture problems — very much like I used in the interest and money problems.

Drawing a picture helps with all mixture problems. The same picture can work for all: liquids, solids, and investments. Figure 11-1 shows three sample containers — two added together to get a third (the mixture). In each case, the containers are labeled with the quality and quantity of the contents.

These two values get multiplied together before adding. The quality is the strength of the antifreeze or the percentage of the interest or the price of the ingredient. The quantity is the amount in quarts or dollars or pounds. You can use the same picture for the containers in every mixture problem or you can change to bowls or boxes. It doesn't matter — you just want to visualize the way the mixture is going together.

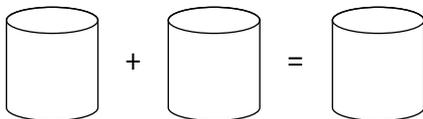


Figure 11-1: Visualizing containers can help with mixture problems.

A traditional mixture-type problem involves solutions — where you mix water and antifreeze to create a particular strength of antifreeze. When the liquids are mixed, the strengths of the two liquids begin to average out.



How many quarts of 80 percent antifreeze have to be added to 8 quarts of 20 percent antifreeze to get a mixture of 60 percent antifreeze?

First, label your containers. The first would be labeled 80 percent on the top and x on the bottom. (I don't know yet how many quarts have to be added.) The second container would be labeled with 20 percent on the top and 8 quarts on the bottom. The third container, which represents the final mixture, would be labeled 60 percent on top and $x + 8$ quarts on the bottom. To solve this, multiply each “quality” or percentage strength of antifreeze times its “quantity” and put these in the equation:

$$\begin{aligned} 80\%(x) \text{ quarts} + 20\%(8) \text{ quarts} &= 60\%(x + 8) \text{ quarts} \\ (0.8)(x) + (0.2)(8) &= (0.6)(x + 8) \\ 0.8x + 1.6 &= 0.6x + 4.8 \end{aligned}$$

Subtracting $0.6x$ from each side and subtracting 1.6 from each side, I get

$$0.2x = 3.2$$

Dividing each side of the equation by 0.2, I get

$$x = 16$$

So 16 quarts of 80 percent antifreeze have to be added.

You can use the liquid mixture rules with salad dressings, mixed drinks, and all sorts of sloshy concoctions.