

## Liquidity and Risk Management, Free Cash Flow, and Long-Term Finance

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### 5.1 Introduction

As ongoing entities, firms are concerned that they may in the future be deprived of the funds that would enable them to take advantage of exciting growth prospects, strengthen existing investments, or simply stay alive. Such liquidity shortages reflect an inadequacy between available resources and re-financing needs. Available resources in turn depend on the difference between the firm's income and "total payment to investors" (payouts—defined as payments to shareholders, namely, dividends and share repurchases—plus debt repayments).

For example, firms that generate a decent income but contract substantial short-term liabilities may experience a liquidity shortage. A key feature of a firm's capital structure is therefore the impact of its composition on the sequencing of payments to investors. Short-term debt, by forcing the firm to disgorge cash, and puttable securities, by allowing their holders to accelerate payments if certain covenants are violated,<sup>1</sup> exacerbate liquidity problems, while long-term debt and equity give the firm more breathing room, as do preferred stocks, a form of debt whose payments can be postponed in time.<sup>2</sup>

Besides liabilities and payouts, the potential for liquidity shortages also depends on income and its availability. For example, even in the absence of payments to investors, a liquidity shortage is quite predictable for those firms, such as R&D start-ups,

that do not generate income for a while after their inception. Income availability also depends on income variability, which in turn can be decreased or increased by diversification choices and by corporate risk management.

Unsurprisingly, liquidity planning is central to the practice of corporate finance and consumes a large fraction of chief financial officers' (CFOs') time. Income, payments to investors, and risk management are all endogenous. This chapter's task is to build an integrated account of their determinants and to rationalize some key empirical regularities discussed in Section 2.5; for instance, (i) firms with good growth prospects might be expected to take less debt for fear of compromising future investment, and (ii) highly indebted firms are more likely to borrow on a short-term and secured basis going forward.

Chapters 3 and 4 focused on a single-stage (fixed- or variable-investment) financing. This chapter analyzes multistage financing, starting with a study of corporate liquidity demand. It models liquidity demand in a straightforward way. The novelty relative to Chapters 3 and 4 is the introduction of an intermediate date (date 1) between the financing stage (date 0) and the realization of the outcome (date 2). At that intermediate date the borrower, who may or may not produce an intermediate income, experiences a liquidity shock that needs to be withstood in order for the firm to continue and possibly succeed. A simple interpretation of this liquidity shock is as a reinvestment need (an investment cost overrun), but it can be equivalently thought of as being a new investment opportunity or else a shortfall in earnings at the intermediate stage, in which case a new external cash infusion is needed in order to cover operating expenses.

1. For example, in 1995, the downgrading of KMart's debt put the company on the brink of a bankruptcy filing, as a further downgrade would have triggered the put of \$550 million in bonds, and banks had demanded covenants limiting the acceleration of payments, thus making it impossible for the firm to honor the put option. In the end, KMart reportedly paid puttable bondholders \$98 million to abandon their put option.

2. As long as dividends are not paid to shareholders: preferred stocks are senior relative to common stocks.

The question then arises as to how the firm can face this liquidity demand if it has little or no cash at the intermediate stage (it is “cash poor”), or, if it is “cash rich,” but its intermediate income has been previously committed through, say, short-term debt liabilities contracted at date 0. It must then return to the capital market and issue new securities at date 1. However, this generally proves insufficient. Indeed, we show that the borrower should not wait until the liquidity shock occurs to secure funds to withstand it. While she may be able to convince investors to renegotiate and let their claims be diluted through a new security issue if the expected return from continuation (relative to date-1 liquidation) exceeds the agency cost, the logic of credit rationing extends to the reinvestment stage as long as investors are unable to capture the entire social benefits from continuation. In our model, provided that there is moral hazard after the liquidity shock is withstood, the borrower must keep a minimum stake in the firm in order to have incentives to manage the firm properly, which prevents pledging the firm’s full value to new investors.

Thus, the borrower ought to anticipate that she will perhaps not be able to raise enough funds on the capital market to withstand the shock. It is therefore optimal for the borrower to hoard reserves either in the form of liquid securities that can be resold when the need occurs or in the form of a credit line secured from a financial institution for a cash-poor firm,<sup>3</sup> or

3. There is a wide variety of loan commitments in practice. All specify the maximum loan amount, the terms under which the loan will be made, and the commitment’s period. The borrower usually pays an up-front fee to obtain the commitment as well as fees on unused commitment balance (e.g., 25 or 50 basis points per year). The borrower is free to fully or partially “take down” the loan up to the maximum loan specified in the agreement, at an interest rate usually set at a markup above a market interest rate (e.g., a fixed add-on over the prime rate; for example, the borrower can borrow up to the specified maximum amount at LIBOR plus 50 basis points, where LIBOR is the London Interbank Offered Rate). Banks also often require that the borrower keep deposits (at below market rate) with the bank (these are the compensating balance requirements).

Loan commitments are pervasive in bank lending, with over 75% of commercial and industrial loans at large U.S. banks being take-downs under commitments (Veitch 1992). We refer to the book by Greenbaum and Thakor (1995) for further details on loan commitments.

Early theoretical work on loan commitments includes Thakor et al. (1981), Boot et al. (1987), Thakor and Udell (1987), and Greenbaum et al. (1989, 1991). These papers, as we will, view loan commitments as insurance against the borrower’s deterioration in credit worthiness. For instance, Boot et al. (1987) analyze the case of a firm that may or

in the form of retentions for a cash-rich firm. Even though the borrower is risk neutral, the hoarding of reserves is best viewed as an *insurance mechanism*. Due to credit rationing at the interim stage, the value of funds for the borrower is higher in bad states than in good ones. Reserves indeed provide an efficient cross-subsidy from good states to bad ones; for example, the borrower pays a commitment fee for the right to be able to draw on a credit line that has value only if the borrower cannot obtain funds at the interim stage, that is, in bad states of nature.

Section 5.2 provides the basics of liquidity management in the context of the fixed-investment model. Assuming, in a first stage, that the intermediate cash flow, if any, is entirely determined by events not controlled by management, it identifies the rationale of credit lines for cash-poor firms and of retentions for cash-rich ones. It also endogenizes the maturity structure of liabilities and derives the theoretical predictions relative to the empirical regularities discussed above. Section 5.3 extends the analysis to a variable investment size in order to identify a liquidity-scale tradeoff.

Section 5.4 shows how corporate risk management is part of the overall liquidity management planning, and offers some guiding principles for efficient risk management. It first shows that the rationale for hedging is to prevent the firm’s continuation and reinvestment policy from being perturbed by shocks that are exogenous to the firm. While the firm optimally insulates itself completely from these shocks in the benchmark, the subsequent analysis identifies five reasons, besides transaction costs associated with hedging contracts, why partial hedging is preferable: serial correlation of shocks, market power, aggregate risk, asymmetric information, and managerial incentives.

Section 5.5 extends the basic model of Sections 5.2 and 5.3 by assuming that the firm’s cash flow in part

may not be able to enter a standard debt agreement with prospective lenders in the future. The cause of credit rationing in their paper is the borrower’s privy information about future prospects (associated with an unobserved investment decision in their model). They show that a loan commitment setting a low borrowing rate may eliminate the welfare distortion due to credit rationing. This chapter sets up a simpler framework in which loan commitments arise even in the absence of asymmetric information at the refinancing stage. It fully endogenizes the cause of credit rationing and the optimal long-term contract.

reflects managerial decisions and not solely extraneous uncertainty. For incentives reasons, the amount of liquidity available to the firm should then increase with the realized cash flow; that is, reinvestment should be sensitive to cash flow (which corresponds well to the empirical tests of the sensitivity of investment to cash flow, which are performed on ongoing entities and demonstrate a positive association between reinvestment and cash flow). There is, however, no theoretical ground for assuming that this sensitivity decreases with the strength of the firm's balance sheet.

While Sections 5.2–5.4 emphasize the point that the capital market may *ex post* rationally, but inefficiently, deny funds to the firm, Section 5.5 also studies the opposite phenomenon of a capital market that is too lenient with the borrower. When the liquidity shock is endogenous, that is, depends on the borrower's behavior, it may be optimal to let the firm fail even for moderate liquidity shocks. The prospect of failure then acts as a disciplining device for the borrower, and induces her to better control liquidity needs. Once the need for liquidity accrues, however, it may no longer be optimal for the capital market to adhere to this tough stance. Indeed, if the expected return from continuation exceeds the agency cost, the borrower can successfully renegotiate the initial agreement and obtain more funds. This is the phenomenon of the soft budget constraint. We then show how the soft-budget-constraint problem may arise whenever more general news about poor past performance accrues at the intermediate stage.

Following Easterbrook (1984) and Jensen (1986, 1989), Section 5.6 focuses on cash-rich firms, defined as firms with cash inflows exceeding their efficient reinvestment needs or opportunities. Such firms have excess liquidity that must be “pumped out” in order not to be used on wasteful projects, unwarranted diversifications, perks, and so forth. Jensen's (1989) list of industries with potential free-cash-flow problems includes steel, chemical, television and radio broadcasting, brewing, tobacco, and wood and paper products.

Overall, the liquidity-shortage and free-cash-flow problems are two sides of the same coin. The key issue in the design of long-term financing is to ensure that, at intermediate stages, the right amount

of money is available for the payment of operating expenses and for reinvestment and the right amount is paid out to investors. Whether this results in a net inflow (the liquidity-shortage case) or outflow (the free-cash-flow case) is important for the comprehension of corporate financing, but is a pure convention as far as economic principles are concerned. And, indeed, we merely reinterpret the liquidity-shortage model in order to obtain its flip side, the free-cash-flow model.

The exposition in this chapter is based in part on joint work (in particular, Holmström and Tirole 1998, 2000) and numerous discussions with Bengt Holmström.

## 5.2 The Maturity of Liabilities

### 5.2.1 Basics

We depart from the previous sole focus on solvency by introducing the possibility that, during the implementation of the project (of size  $I$ ), the firm be hit by an adverse shock and be required to plow in some extra cash in order to be able to pursue the project. A firm has two ways of facing urgent liquidity needs if it lacks funds (either because it generates no cash in the short run (a “cash-poor firm”) or because it generates enough income in the short-run to cover reinvestment needs (“cash-rich firm”) but pays out part or all of this income and therefore has limited retentions). The first is to secure some source of cash before the liquidity shock occurs. For example, the firm may “overborrow” and keep liquid assets such as Treasury bills on its balance sheet in order to be able to absorb the shock by selling these assets when needed. Alternatively, the firm may secure a line of credit with a lender (usually a bank). In contrast, the second approach consists in waiting for the shock to occur to start raising funds.

As explained in the introduction, the wait-and-see approach generates excessive liquidity problems. That is, there are situations where the firm would be rescued under an optimal contract but neither initial lenders nor new lenders want to participate even in a coordinated rescue. This is due to the fact that the borrower's stake is incompressible, that is, a concession by the borrower (in the form of a reduction of her stake) creates moral hazard and is



and increase  $R_b$  by  $\delta R_b$ , so that the expected total reward remains constant:  $F(\rho^*)p_H\delta R_b = r_b$ . If anything, this substitution alleviates moral hazard in the case of continuation. And the suppression of the date-1 compensation does nothing to the date-1 income (which we took to be exogenous, an assumption we relax in Section 5.5).

The pledgeable income,  $\mathcal{P}$ , deflated by the investors' initial outlay,  $I - A$ , is therefore

$$\mathcal{P}(\rho^*) - (I - A) = \left[ r + F(\rho^*) \left[ p_H \left( R - \frac{B}{\Delta p} \right) \right] \right] - \left[ I + \int_0^{\rho^*} \rho f(\rho) d\rho - A \right],$$

since the entrepreneur no longer has cash and so the reinvestment must be paid out of either the investors' pocket or date-1 revenue.

Taking derivatives in  $U_b$  and  $\mathcal{P}$ , the key insights are as follows:

- The NPV ( $U_b$ ) is increasing in the cutoff  $\rho^*$  as long as  $\rho^* < p_H R$ , and is decreasing thereafter. Intuitively, one would want to salvage an investment when the cost,  $\rho$ , of a rescue is smaller than the expected payoff,  $p_H R$ , of continuing.
- By contrast, the pledgeable income increases with  $\rho^*$  for  $\rho^* < p_H(R - B/\Delta p)$  and decreases thereafter. This is again intuitive: investors have to bear the cost,  $\rho$ , of salvaging the investment and can put their hands on at most  $p_H(R - B/\Delta p)$  given that the entrepreneur must be given incentives to behave in the case of continuation.

We are then led to consider three cases. Depending on the strength of the balance sheet, there may be (i) an efficient amount of liquidation, (ii) an over-optimal amount of liquidation to satisfy investors, or (iii) no funding at all:

$$(i) \mathcal{P}(p_H R) \geq I - A.$$

In this case, the "first-best cutoff"  $\rho^* = p_H R$ , which maximizes  $U_b$ , leaves sufficient income to investors. The contract then specifies, say, no compensation  $r_b$  at date 1 for the entrepreneur, and a reward  $R_b$  in the case of continuation and success at date 2.<sup>4</sup>

4. More generally,  $r_b$  and  $R_b$  are given by the investors' breakeven condition:

$$r - r_b + F(p_H R)[p_H(R - R_b)] = I + \int_0^{p_H R} \rho f(\rho) d\rho - A,$$

as long as  $R_b \geq B/\Delta p$  and  $r_b \geq 0$ .

$$(ii) \mathcal{P}(p_H R) < I - A \leq \mathcal{P}\left(p_H \left( R - \frac{B}{\Delta p} \right)\right).$$

The optimal contract then specifies<sup>5</sup>  $r_b = 0$  and  $R_b = B/\Delta p$ . The entrepreneur receives nothing at the intermediate date and, in the case of continuation, receives the lowest compensation,  $R_b = B/\Delta p$ , that is incentive compatible. Intuitively, the entrepreneur can be paid in two currencies: cash and continuation. Cash payments are just transfers and do not affect the NPV (as long as incentive compatibility obtains); as long as  $\rho < p_H R$ , continuation is a more efficient currency since continuation increases the NPV.

The cutoff  $\rho^* \in [p_H(R - B/\Delta p), p_H R]$  is then given by<sup>6</sup>

$$r + F(\rho^*) \left[ p_H \left( R - \frac{B}{\Delta p} \right) \right] = I + \int_0^{\rho^*} \rho f(\rho) d\rho - A.$$

Figure 5.2 illustrates the determination of the cutoff in this region. The pervasive logic of credit rationing applies not only to the choice of initial investment, but also to the continuation decision. In order to be able to invest more *ex ante*, the borrower accepts a level of reinvestment below the *ex post* efficient level ( $\rho^* < p_H R$ ). The intuition is that, because incentives must be preserved, the borrower cannot pledge to the lenders the entire benefit of the reinvestment decision. Also,  $\rho^*$  exceeds the per-unit pledgeable income  $p_H(R - B/\Delta p)$ , which is the level that maximizes the borrowing capacity. A small increase in  $\rho^*$  at that level induces only a second-order decrease in

5. Here there is no indeterminacy. A positive  $r_b$  reduces  $\rho^*$ , which in turn reduces  $U_b$ .

6. An early paper emphasizing the role of the insiders' stake and the absence of maximization of the firm's value to investors in the optimal choice of an interim policy, such as continuation and restructuring, is Chang (1992). In that paper, the interim decision consists in restructuring the firm, thereby imposing a cost on insiders. It is shown that restructuring occurs less often than it would if investors had noncontingent control rights over the restructuring decision and therefore chose to restructure the firm whenever this increased the firm's interim value.

Here, abandoning the project (the analog of restructuring in Chang's paper) maximizes the investors' interim value whenever  $\rho > p_H(R - B/\Delta p)$ . However, abandoning imposes a cost on the entrepreneur, namely, the loss of rent  $p_H B/\Delta p$ . The firm continues in a broader set of circumstances than would maximize the investors' interim value, in the same way as restructuring occurs less often than would be the case if one maximized the investors' interim value in Chang's paper. Chang studies the implications for the allocation of control rights. We focus on those for liquidity management.

See also Dasgupta and Sengupta (2005) for a recent contribution to this literature.

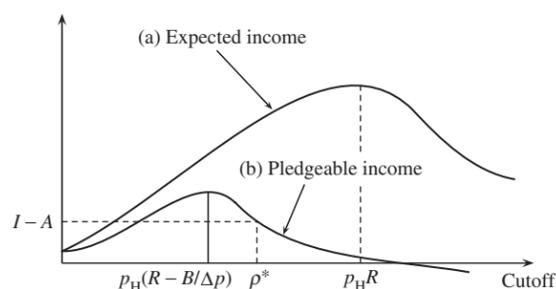


Figure 5.2 Optimal continuation policy:

$$(a) U_b + I = r + F(\rho^*)[p_H R] - \int_0^{\rho^*} \rho f(\rho) d\rho;$$

$$(b) P(\rho^*) = r + F(\rho^*)[p_H(R - B/\Delta p)] - \int_0^{\rho^*} \rho f(\rho) d\rho.$$

debt capacity, and a first-order gain in the efficiency of *ex post* refinancing.

$$(iii) P\left(p_H\left(R - \frac{B}{\Delta p}\right)\right) < I - A.$$

In this case, funding is not feasible. The value  $\rho^*$  that maximizes the pledgeable income ( $\rho^* = p_H(R - B/\Delta p)$ ) does not suffice to compensate the investors for their initial outlay.

### 5.2.2 Term Structure of Cash-Rich Firms

Let us define a cash-rich firm as one that is meant to disgorge money at the intermediate stage:  $r > \rho^*$  (in particular,  $r \geq p_H R$  suffices to ensure that the firm is cash rich). The optimal contract can be implemented through a combination of *short-term debt*,

$$d = r - \rho^*,$$

and *long-term debt* (to be paid in the case of continuation),<sup>7</sup>

$$D = R - \frac{B}{\Delta p}.$$

We thus obtain a simple theory of *maturity structure*. Note further that as the strength of the balance sheet, as measured by the value of  $A$ , changes, only  $\rho^*$  changes. In particular, as  $A$  increases,  $\rho^*$  also increases (see region (ii) in Section 5.2.1; it increases only weakly in region (i)), and so  $d$  decreases. Conversely, *a weak balance sheet implies a short maturity structure* ( $d$  large).

7. Here, long-term debt and equity are equivalent. To obtain three different claims (short-term debt, long-term debt, equity), one can proceed as in Chapter 3 and introduce a leftover value in the case of failure at date 2.

This helps us to understand why *highly indebted firms are more likely to borrow on a short-term basis*. Highly leveraged firms can be viewed as firms with a weak balance sheet,<sup>8</sup> and so must accept shorter maturities.

Similarly, if we added another margin of concession in the form of costly collateral pledging (thus combining this section with the modeling in Section 4.3), one would find that firms with weak balance sheets borrow on a short-term and secured basis.

*Discussion.* While we emphasize the short-term debt interpretation, this payment can actually be interpreted either as a short-term debt as in Jensen (1986) or as a dividend as in Easterbrook (1984). Note, though, that the dividend interpretation must be accompanied by a covenant concerning maximal dividend distribution.<sup>9</sup> Otherwise, investors would want to pay dividends up to  $r - \rho_0 > d$ , where  $\rho_0 = p_H(R - B/\Delta p)$ , in order to prevent the entrepreneur from reinvesting whenever the liquidity shock exceeds the date-1 pledgeable income  $\rho_0$ . With this interpretation, we see that covenants specifying maximal amounts of dividends serve to protect the *entrepreneur* against excessive liquidation.<sup>10</sup>

8. Suppose that the firm already owes  $D_0$  at date 2. The income in the case of success is then  $R - D_0$ . The analysis above shows that in the constrained region (ii),  $\rho^*$  decreases as  $D_0$  increases. And so the short-term debt  $d$  increases.

Things get more complex when the initial debt is short-term debt ( $d_0$ ). Then the disposable short-term debt revenue becomes  $r - d_0$ . The cutoff  $\rho^*$  decreases with  $d_0$ . Total short-term debt ( $d_0 + d$ ) increases with  $d_0$ , but the sign of the impact on *new* short-term debt  $d$  depends on distributional assumptions.

This analysis presumes, as in Section 3.3, that initial short-term ( $d_0$ ) or long-term ( $D_0$ ) debts are not renegotiated. The analysis is different if initial debtholders can be brought to the bargaining table, but the general point that their presence weakens the firm's balance sheet remains.

9. In practice, dividends may also be limited because managers have some control over their level (this alternative story is more complex to analyze than the covenant one because it relies on the drivers' of managerial "real authority" (see Chapter 10 for the concept of real authority)).

10. This insight complements the standard, and important explanation for the existence of such covenants. As discussed in Chapter 2, they are usually viewed as protecting *creditors* against expropriation by the equityholders, who could use dividend distributions and share repurchases to leave long-term creditors with an "empty shell." In this part, we focus on the conflict between the entrepreneur and the securityholders, and so the introduction of conflicts among securityholders would serve no purpose.

This study focuses on the conflict between the entrepreneur and the investors concerning payments to investors out of cash-flow, without going into the details of whether the payment  $d$  must be interpreted as short-term debt or as a (constrained) dividend. That is, it is general enough to encompass the theories of Easterbrook and Jensen, but ought to be refined in order to motivate a diversity of securities. Note also that by predicting a fixed payment  $d$ , it does not do justice to the rich range of conditional payments observed in practice, that endow investors with more or less flexibility in pumping cash out of the firm: dividend, preferred dividend, putable securities, renegotiated short-term debt, short-term debt (we will return to this point in Section 5.6.2).

### 5.2.3 Credit Lines for Cash-Poor Firms

Suppose in contrast that the investment “takes a long time” to produce income. At the extreme, there is no short-term profit:  $r = 0$ .

Can the entrepreneur just “wait and see,” that is, borrow  $I$  at date 0 in exchange for shares in the firm and return to the capital market at date 1 if need occurs? Let us thus assume that the entrepreneur does not plan her liquidity in advance and that the liquidity shock occurs at date 1. To raise cash on the capital market to pay  $\rho$ , the entrepreneur must issue new shares and thereby dilute historical investors.

Letting  $\rho_0 \equiv p_H(R - B/\Delta p)$ , and to illustrate this dilution, suppose that the entrepreneur faces a liquidity shock  $\rho = \frac{1}{2}\rho_0$ . The value of external shares held by initial investors is equal to  $\rho_0$ . Suppose that the number of shares is doubled.<sup>11</sup> That is, as many shares are sold to new investors as already exist. So the value of each share is halved. The firm thereby raises  $\frac{1}{2}\rho_0 = \rho$  in cash and can withstand the liquidity shock. Are initial investors willing to let themselves be diluted? The value of their shares is, of course, reduced to  $\frac{1}{2}\rho_0$ . But contemplate the alternative of liquidating the investment, under which the initial investors receive nothing! Thus, initial investors are willing to accept the dilution.<sup>12</sup>

11. Including for internal shares, so as to keep the entrepreneur's stake  $R_0$  in success constant and therefore preserve incentive compatibility.

12. Note the analogy with the incentives for debt forgiveness when there is a debt overhang (see Chapter 3).

Similarly, to meet a liquidity shock equal to  $\frac{3}{4}\rho_0$ , the firm must quadruple the number of shares, and so on. But there is an upper bound to this process: investors will never pay more than the firm is worth to them. Hence, even in a frictionless capital market, the firm cannot raise more than  $\rho_0$ . Going back to the capital market at date 1 then at best allows the firm to withstand a liquidity shock of magnitude

$$\rho \leq \rho_0 = p_H \left( R - \frac{B}{\Delta p} \right).$$

Because the optimal financing arrangement specifies<sup>13</sup>

$$p_H \left( R - \frac{B}{\Delta p} \right) < \rho^* \leq p_H R,$$

the entrepreneur must secure a line of credit or hoard liquidity in order to face the date-1 liquidity shock.<sup>14</sup> We will shortly describe how to do so, but there are basically two alternatives and combinations thereof: a credit line or liquid assets of magnitude  $\rho^*$  with no right to dilute existing claimholders by issuing new claims at date 1 (so the entrepreneur borrows  $I + \rho^*$ ); or a smaller credit line or amount of liquid assets, equal to  $[\rho^* - p_H(R - B/\Delta p)]$  with a right to dilute claimholders as needed to ensure continuation. Either way, the entrepreneur must plan liquidity management.

The optimum can be implemented by a nonrevokable line of credit granted by, say, one of the lenders (a bank) at level  $\rho^*$ . It is important that this line of credit be nonrevokable (in a broad sense: see below). Otherwise the lender would have an incentive not to abide by his promise to rescue the firm if  $\rho > \rho_0$ , that is, if the liquidity shock exceeds the date-1 pledgeable income  $\rho_0$ . In practice, lenders often prefer to keep discretion over the extension of credit by making the line revokable, or delivering promises such as “comfort or highly confident letters,” which are legally hard to enforce and are only a moral promise to provide credit. This discretion potentially has a cost to the borrower, as, whenever  $\rho_0 < \rho < \rho^*$ , the lender would like to renege on his promise to provide funds to the firm unless he tries to maintain a reputation for “fairness” by extending credit even when this is not strictly profitable for him (see Boot

13. Except in the nongeneric case where  $\mathcal{P}(p_H(R - B/\Delta p)) = I - A$ .

14. Note that if  $A > 0$ , the hoarding of liquidity can in part come from the retention of  $A$  (this is a matter of accounting).

et al. 1987, 1993). In practice, a bank may offer a formally revokable line of credit, but have a reputation for abiding by its promise unless the borrower has behaved in an egregious way that yet was not ruled out by a negative covenant.

We also implicitly assume that there is no concern over the lender's *ability* (as opposed to *willingness*) to abide by his commitment. However, the lender may himself face liquidity and solvency problems in the future. In practice, only well-capitalized and safe institutions are able to make a firm promise of this type (banks and some other financial institutions obviously have a comparative advantage in doing so, due to the close monitoring of their solvency and liquidity by the regulators as well as, at least for large ones, an explicit or implicit backing of their on- and off-balance-sheet liabilities by the state).

*Remark (capital market frictions).* Note here that the suboptimality of reinvestment under the wait-and-see policy is independent of the debt-overhang phenomenon discussed in Section 3.3. Indeed, the assumption that liquidity shocks below  $\rho_0$  can be withstood through the dilution of existing claims implies either that lending is concentrated among a few lenders, or that the initial agreement is structured so as to facilitate renegotiation,<sup>15</sup> or else that the entrepreneur receives rights to dilute existing claims by issuing senior claims (as in Hart and Moore (1995)). If some claims proved difficult to renegotiate, the firm would be able to raise even less than  $\rho_0$  by turning to the capital market at date 1, and its demand for liquidity would be even higher than that derived here.

*Remark (renegotiation).* Could this line of credit be renegotiated to the parties' mutual advantage once the fraction  $\rho$  is realized? First, note that if  $\rho \leq \rho^*$ , then *a fortiori*  $\rho < p_H R$  and therefore it is *ex post* efficient to continue; so there is no scope for a renegotiation in which the lender would compensate the borrower for not using the credit line, as this renegotiation would reduce total surplus and therefore at least one of the parties would be strictly better off not renegotiating. Second, could the two parties both benefit from an increase in the line of credit

to  $\rho$  when  $\rho^* < \rho \leq p_H R$ ? Even though this increase would yield the *ex post* efficient reinvestment policy, there is no way for the borrower to compensate the lender, again because the borrower's stake is incompressible. One can show that the lender will turn down any request for an increase in the credit line.<sup>16</sup> So will any alternative lender (other lenders may have even less incentives to refinance, because unlike the initial lender they do not have a vested stake to lose).

*Remark (role played by uncertainty about liquidity needs).* We can now explain why *ex ante* uncertainty about the liquidity need is a key ingredient of the demand for liquidity. Suppose, in contrast, that  $\rho$  is deterministic. If  $\rho \geq \rho_0 = p_H(R - B/\Delta p)$ , then investors do not want to lend at date 0, since they know that they will have to cover at date 1 a liquidity shock that exceeds the income that can be pledged to them in period 2. If  $\rho < \rho_0$ , then the firm is always solvent at date 1, in that new claims can be issued at date 1 (that partially dilute existing ones) in order to meet the liquidity shock and continue; hence, there is no need to hoard reserves.

Again, a good way of thinking about this issue is in terms of insurance. A high liquidity shock is similar to an illness or an accident, and a low liquidity shock is similar to the absence of such a mishap. There is no scope for insurance if it is known in advance whether there will be an illness or an accident.

#### 5.2.4 A Reinterpretation: Growth Prospects

In the basic model, the firm is liquidated if it does not meet the liquidity shock. In a straightforward reinterpretation, it continues as is, but cannot take advantage of a profitable growth opportunity if it does not come up with enough cash to reinvest.

Suppose that, at date 1, the firm still receives deterministic income  $r$ , but, in the absence of cash reinjection at date 1, continues and succeeds with probability  $p = p_H$  or  $p_L$ , depending on whether the entrepreneur behaves or misbehaves at date 1. At date 1, though, the firm can raise its date-2 expected profit by reinvesting. One way of formalizing

15. See Section 5.5.3 for a discussion of factors hindering and facilitating renegotiation of claims.

16. More formally, the lender turns down the request because  $\rho > \rho^* > \rho_0$ .

this is to assume that the payoffs in the cases of success and failure remain  $R$  and  $0$ , respectively, but the probability of success in the case of reinvestment becomes  $p + \tau$ , where  $\tau > 0$  and  $p = p_H$  or  $p = p_L$ , depending on whether the entrepreneur behaves or misbehaves. This separable form is handy as it implies that the entrepreneur's incentive compatibility constraint is not affected by the reinvestment:

$$(p_H + \tau)R_b \geq (p_L + \tau)R_b + B \iff (\Delta p)R_b \geq B.$$

The reinvestment cost  $\rho$  is drawn at date 1 from the cumulative distribution function  $F(\rho)$  with density  $f(\rho)$  on  $[0, \infty)$ .

It is clearly optimal to reinvest if and only if  $\rho$  is below some cutoff  $\rho^*$ . The entrepreneur's utility, equal to the NPV, is

$$U_b(\rho^*) = [r + [p_H + F(\rho^*)\tau]R] - \left[ I + \int_0^{\rho^*} \rho f(\rho) d\rho \right].$$

As earlier, the interesting case arises when the firm is financially constrained but nonetheless can raise funds (a situation equivalent to that labeled region (ii) in Section 5.2.1); the cutoff is then given by the investors' breakeven condition:

$$r + [p_H + F(\rho^*)\tau] \left( R - \frac{B}{\Delta p} \right) = [I - A] + \int_0^{\rho^*} \rho f(\rho) d\rho$$

and

$$\tau \left( R - \frac{B}{\Delta p} \right) \leq \rho^* < \tau R.$$

The latter set of inequalities expresses the fact that reinvestment is first-best suboptimal ( $\rho^* < \tau R$ ), but occurs whenever it boosts pledgeable income ( $\rho^* \geq \tau[R - (B/\Delta p)]$ ).

In this model, growth opportunities are measured by the parameter  $\tau$ . Let us look at the impact of growth opportunities on the maturity structure by differentiating the investors' breakeven condition:

$$\begin{aligned} \frac{d(d)}{d\tau} &= \frac{d(r - \rho^*)}{d\tau} \\ &= -\frac{F(\rho^*)}{f(\rho^*)} \frac{R - B/\Delta p}{\rho^* - \tau(R - B/\Delta p)} < 0. \end{aligned}$$

Thus, *firms with better growth opportunities should go for longer maturities*. Relatedly, there is substantial evidence that firms with growth opportunities have lower leverage ratios.<sup>17</sup>

17. See Section 2.5. Recall that equity here can be viewed as debt with a long maturity.

### 5.3 The Liquidity-Scale Tradeoff

The fixed-investment model is handy to illustrate the optimal term structure of debt for cash-rich firms and credit line for cash-poor ones. But, for other purposes, it is too simple, in that there is no other "margin" that the entrepreneur can trade off against liquidity. When, for example, investment size is variable, as we now assume, the entrepreneur faces a choice between a larger investment and more liquidity.<sup>18</sup>

This section focuses on cash-poor firms and extends the model of Section 5.2 to include a variable investment size in order to identify the liquidity-scale tradeoff (which also applies to cash-rich firms): the firm must sacrifice scale in order to benefit from more liquidity.

#### 5.3.1 The Two-Shock Case

We consider the variable-investment model and add a liquidity shock at an intermediate stage. This liquidity shock amounts to a cost overrun that is proportional to the initial investment. To develop our intuition, let us begin with the case in which there are only two possible values for the (per-unit) liquidity shock:  $0$  with probability  $1 - \lambda$  and  $\rho$  with probability  $\lambda$  (see Figure 5.3). We will say that the firm is "intact" when it does not need to reinvest and "in distress" when it needs to reinvest  $\rho$  per unit of investment.

Except for this random shock, the model is identical to the variable-investment version of Section 3.4. Continuation (which is contingent on reinvesting  $\rho I$  if the firm is in distress) is subject to moral hazard. The probability of success is  $p_H$  if the entrepreneur behaves and  $p_L$  if she misbehaves. The private benefit of misbehaving is  $BI$ . The project yields  $RI$  in the case of success and  $0$  in the case of failure. Note that we focus on policies that rescue either the entire investment or none of it in the case of distress.<sup>19</sup>

18. More generally, the entrepreneur would face a tradeoff between more liquidity and fewer control rights granted to investors (see Chapter 10), and so forth.

19. Quite generally, we could allow partial reinvestments. That is, a reinvestment  $\rho x I$  allows the firm to salvage a fraction  $x \in [0, 1]$  of the investment. In this case, the private benefit of misbehaving,  $BxI$ , is proportional to the salvaged investment  $xI$ ; and so is the profit  $RxI$  in the case of success. But it turns out that one can focus without loss of generality on policies that either rescue the entire investment ( $x = 1$ ) or rescue none ( $x = 0$ ) in the case of distress.



shock if

- it is low ( $\rho$  low),
- it is likely ( $\lambda$  high).

The first conclusion is obvious; but the second may be less so since a high probability of a liquidity shock increases both the benefit and the cost of withstanding it.

As in the case of a fixed investment size, we can draw the implications of this analysis for liquidity management. If the optimal policy is not to rescue the investment in the case of distress, nothing needs to be done at date 0 besides signing a contract and investing  $I$ . In contrast, if the optimal policy is to pursue the project even in the case of distress, the entrepreneur must be able to avail herself of the amount  $\rho I$  if a shock occurs.

If  $\rho > \rho_0$  (which is not inconsistent with the condition  $(1 - \lambda)\rho \leq 1$  obtained earlier), then liquidity necessarily must be planned in advance. Waiting exposes the firm to credit rationing at date 1. (As the analysis for a continuum of liquidity shock will demonstrate, this case is in a sense the “generic case.”) For example, the firm may contract a credit line to the level of  $\rho I$  with a bank; alternatively, it can contract for a credit line corresponding only to the shortfall  $(\rho - \rho_0)I$  and also acquire the right to dilute initial investors (so as to obtain  $\rho_0 I$ ). More on this in Section 5.3.3.

### 5.3.2 Continuum of Liquidity Shocks

We now generalize the analysis to a continuum of possible values for the liquidity shock. This continuous-investment, continuous-shock version will be used in the rest of the chapter.

After the (endogenous size) investment  $I$  is sunk at date 0 and before the borrower works on the project, some exogenous shock occurs at date 1 that determines a per-unit-of-investment level  $\rho \in [0, \infty)$  of “cost overruns.” That is, a cash infusion equal to  $\rho I$  is needed in order for the project to continue. If  $\rho I$  is not invested, the project is abandoned altogether and thus yields no income. As in Section 5.2, the fraction  $\rho$  is *a priori* distributed according to the continuous distribution  $F(\rho)$  on  $[0, \infty)$ , with density  $f(\rho)$ . (As we already observed, the model of Section 3.4 is therefore a special case, with  $F$  being a spike at  $\rho = 0$ .)

Regardless of the required amount of the cash infusion, the project, if pursued, is still a project of size  $I$ , in that the income in the case of success is  $RI$  and the borrower’s private benefit from misbehaving is  $BI$ . One cannot increase the size of the project after the initial stage.

The timing is summarized in Figure 5.4.

We assume that investment has positive NPV. That is, under a rule that specifies that the project is abandoned if and only if  $\rho \geq \tilde{\rho}$  for at least some threshold  $\tilde{\rho}$ , the expected payoff per unit of investment is strictly positive. This positive-NPV condition under liquidity shocks is

$$\max_{\tilde{\rho}} \left\{ F(\tilde{\rho})p_H R - 1 - \int_0^{\tilde{\rho}} \rho f(\rho) d\rho \right\} > 0. \quad (5.1)$$

We first look for the optimal loan agreement. The next subsection will discuss its implementation. It is easy to show that it is optimal to have a “cutoff rule” for infusing cash. There exists an optimal threshold  $\rho^*$  such that one should continue if and only if

$$\rho \leq \rho^*. \quad (5.2)$$

The incentive constraint in the case of continuation is the same as in the absence of a liquidity shock (see Section 3.4):

$$(\Delta p)R_b \geq BI. \quad (IC_b)$$

The breakeven condition is slightly altered by the presence of liquidity shocks:

$$F(\rho^*)[p_H(RI - R_b)] \geq I - A + \int_0^{\rho^*} \rho I f(\rho) d\rho. \quad (IR_1)$$

That is, the lenders receive a return only if the project is pursued, which has probability  $F(\rho^*)$ . The left-hand side of (IR<sub>1</sub>) is the expected pledgeable income. Furthermore, there is a new term, representing the expected outlay on overruns, on the right-hand side. From these two constraints, we deduce the borrowing capacity (or, more precisely, the maximum investment that allows the lenders to break even):

$$I = k(\rho^*)A,$$

where

$$\begin{aligned} k(\rho^*) &= \frac{1}{1 + \int_0^{\rho^*} \rho f(\rho) d\rho - F(\rho^*)[p_H R - p_H B / \Delta p]} \\ &= \frac{1}{1 + \int_0^{\rho^*} \rho f(\rho) d\rho - F(\rho^*)\rho_0} \end{aligned} \quad (5.3)$$

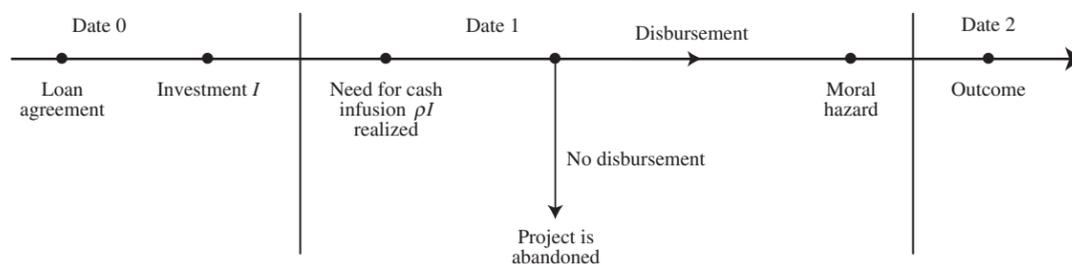


Figure 5.4

involves a straightforward modification relative to the no-liquidity-shock multiplier  $k$ . Reduced profitability implies that the multiplier is smaller than that in the absence of liquidity shocks:  $k(\rho^*) < k = 1/(1 - \rho_0)$ . Note that the borrower's borrowing capacity is maximal when the threshold  $\rho^*$  is equal to the expected per-unit pledgeable income  $\rho_0 \equiv p_H(R - B/\Delta p)$ .

Given that the competitive lenders make no profits, the borrower's net utility is as usual the social surplus brought about by the project, namely,

$$U_b = m(\rho^*)I = m(\rho^*)k(\rho^*)A, \quad (5.4)$$

where

$$m(\rho^*) \equiv F(\rho^*)p_H R - 1 - \int_0^{\rho^*} \rho f(\rho) d\rho$$

is the margin per unit of investment.

What is the optimal continuation rule? Ideally, one would want to continue if and only if this is *ex post* efficient, that is, if and only if  $\rho \leq p_H R$ . Indeed,  $\rho^* = p_H R$  maximizes the margin  $m(\rho^*)$ . However, at  $\rho^* = p_H R$ , the multiplier  $k$  is decreasing in  $\rho^*$ . So one actually ought to choose a lower threshold in comparison to the *ex post* efficient one. It is easily seen from (5.3) and (5.4) that

$$U_b = \frac{p_H R - (1 + \int_0^{\rho^*} \rho f(\rho) d\rho)/F(\rho^*)}{(1 + \int_0^{\rho^*} \rho f(\rho) d\rho)/F(\rho^*) - p_H(R - B/\Delta p)} A,$$

and so the optimal threshold minimizes the expected unit cost  $c(\rho^*)$  of effective investment:

$$\rho^* \text{ minimizes } c(\rho^*) \equiv \frac{1 + \int_0^{\rho^*} \rho f(\rho) d\rho}{F(\rho^*)} \quad (5.5)$$

or

$$\int_0^{\rho^*} F(\rho) d\rho = 1. \quad (5.6)$$

Condition (5.6) can be obtained, for example, by integrating by parts and rewriting the expected unit cost of effective investment as

$$c(\rho^*) = \rho^* + \frac{1 - \int_0^{\rho^*} F(\rho) d\rho}{F(\rho^*)}.$$

This expression also shows that *at the optimum*,<sup>20</sup> *the threshold liquidity shock is equal to the expected unit cost of effective investment*:<sup>21</sup>

$$c(\rho^*) = \rho^*.$$

This in turn implies that

$$U_b = \frac{\rho_1 - \rho^*}{\rho^* - \rho_0} A. \quad (5.7)$$

Next, we observe that *this optimal threshold lies between the expected per-unit-of-investment pledgeable income and income*:

$$\rho_0 = p_H \left( R - \frac{B}{\Delta p} \right) < \rho^* < \rho_1 = p_H R. \quad (5.8)$$

This follows from the fact that the margin  $m(\rho^*)$  and the multiplier  $k(\rho^*)$  are both decreasing above  $\rho_1$  and both increasing below  $\rho_0$  (see Figure 5.5).<sup>22</sup> Condition (5.8) is consistent with (5.7): if  $\rho^*$  were to exceed  $\rho_1$ , the project could not be financed profitably. And if  $\rho^*$  were to be lower than  $\rho_0$ , the borrowing capacity and the borrower's utility would be infinite.

Equation (5.8) implies, as in Section 5.2.3, that a wait-and-see policy, under which the borrower tries

20. It is easy to show that  $c(\cdot)$  is quasi-convex ( $c''(\rho^*) > 0$  if  $c'(\rho^*) = 0$ ).

21. Note that  $\rho^*$  is here independent of  $A$ . The constant-returns-to-scale model is a limit case in that the probability of continuation and all per-unit-of-investment variables are independent of  $A$ : all firms are alike up to a scale factor.

22. Indeed,  $m(\cdot)$  is quasi-concave with a maximum at  $\rho_1$  and  $k(\cdot)$  is quasi-concave with a maximum at  $\rho_0$ .

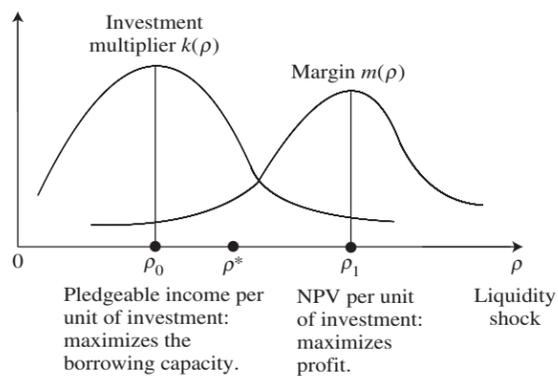


Figure 5.5

to raise funds from the lenders on the capital market at date 1 in order to cover the liquidity shock, is suboptimal. Even under perfect coordination among lenders at date 1 (there is no “debt-overhang” phenomenon), the lenders will provide new credit only if the pledgeable income exceeds the amount of re-investment, that is, only if

$$\rho \leq \rho_0.$$

Because  $\rho_0 < \rho^*$ , it is optimal for the borrower to get more assurance against the firm’s shortage of funds than is provided by a wait-and-see policy. This creates a corporate demand for liquidity.

*Remark (effect of an increase in risk on liquidity hoarding).* Condition (5.6) has a simple implication. An increase in the riskiness of the liquidity shock in the sense of a mean-preserving spread of  $F^{23}$  raises the left-hand side of (5.6) and thus reduces the threshold  $\rho^*$ . So, the borrower should hoard more liquidity when the liquidity shock incurs a mean-preserving reduction in risk.<sup>24</sup>

23. See, for example, Rothschild and Stiglitz (1970, 1971). The distribution  $G(\rho)$  (with density  $g(\rho)$ , say) is a mean-preserving spread of distribution  $F(\rho)$  if (i)  $\int_0^\infty G(\rho) d\rho = \int_0^\infty F(\rho) d\rho$  ( $\Leftrightarrow \int_0^\infty \rho g(\rho) d\rho = \int_0^\infty \rho f(\rho) d\rho$ , so the means are the same), and (ii)  $\int_0^{\rho^*} G(\rho) d\rho \geq \int_0^{\rho^*} F(\rho) d\rho$  for all  $\rho^*$ .

24. This, however, does *not* imply that the firm should hoard a lot of liquidity when uncertainty disappears: suppose that the distribution  $F$  converges to a spike at  $\rho > \rho_0$ . Then, the investors’ breakeven condition cannot be satisfied and there is no borrowing. More generally, an empirical analysis of the impact of liquidity risk on liquidity hoarding will confront a selection bias: because continuation is akin to an option value, a decrease in the uncertainty about  $\rho$  affects pledgeable income and NPV (more on this shortly) and thereby impacts the investment size or the very existence of investment.

*Liquidation value.* We have assumed that no money is recovered if the project is abandoned at date 1. Let us generalize the model slightly by assuming that the assets in place have a salvage value  $LI \geq 0$ , that is,  $L$  per unit of investment if the firm is liquidated at date 1. The salvage value is a monetary value that can be transferred to the lenders if the project is abandoned. We let the reader follow the steps of the previous analysis and show the following: the equity multiplier and the margin become

$$k(\rho^*) = \frac{1}{[1 - L + \int_0^{\rho^*} \rho f(\rho) d\rho] - F(\rho^*)(\rho_0 - L)}, \quad (5.3')$$

$$m(\rho^*) = F(\rho^*)(\rho_1 - L) - \left[1 - L + \int_0^{\rho^*} \rho f(\rho) d\rho\right]. \quad (5.4')$$

These modifications can be understood in the following way. First, there is a fictitious reduction of  $L$  in the unit cost of investment. Were the project always abandoned at date 1, the lenders would collect  $L$  and thus the net unit cost of investment would be equal to  $1 - L$ . Second, and with this convention, the decision to continue at date 1 implies a loss  $L$  per unit of investment. This monetary loss must be subtracted both from the expected payoff  $\rho_1 = p_H R$  and from the expected pledgeable income  $\rho_0 = p_H(R - B/\Delta p)$ . This yields (5.3') and (5.4').

Next,  $U_b = m(\rho^*)k(\rho^*)A$  and so the threshold  $\rho^*$  still minimizes the (modified) expected unit cost of effective investment:

$$\begin{aligned} \rho^* \text{ minimizes } c(\rho^*) &\equiv \frac{1 - L + \int_0^{\rho^*} \rho f(\rho) d\rho}{F(\rho^*)} \\ &= \rho^* + \frac{1 - L - \int_0^{\rho^*} F(\rho) d\rho}{F(\rho^*)}. \end{aligned} \quad (5.5')$$

And so, at the optimum,

$$\begin{aligned} \int_0^{\rho^*} F(\rho) d\rho &= 1 - L, \\ c(\rho^*) &= \rho^*, \end{aligned} \quad (5.6')$$

and

$$U_b = \frac{(\rho_1 - L) - \rho^*}{\rho^* - (\rho_0 - L)} A. \quad (5.7')$$

As the margin and the multiplier are both decreasing above  $\rho_1 - L$  and increasing below  $\rho_0 - L$ , we have

$$\rho_0 - L < \rho^* < \rho_1 - L.$$

We can thus generalize the insight that liquidity has to be secured in advance. Under a wait-and-see strategy, the lenders (or the capital market more generally) do not want to reinvest more than the net gain of continuation, namely,  $\rho_0 - L$  per unit of investment. And so the borrower should hoard liquidity at date 0.

From (5.6'), we also infer that

$$\frac{d\rho^*}{dL} = -\frac{1}{F(\rho^*)}.$$

That is, a unit increase in the salvage value reduces the threshold by more than 1 unit. The gap between the optimal stopping rule and the wait-and-see outcome narrows as the salvage value increases. This result will have an interesting implication when we apply the model to cash-rich firms in Section 5.6.

### 5.3.3 Application to Liquidity Management

We now pursue in more detail the analysis of Section 5.2 concerning whether common institutions can implement the optimal reinvestment policy.

The optimum can be implemented by a nonrevokable line of credit granted by a lender (a bank) at level  $\rho^*I$ . The borrower, who is always better off continuing, will always take advantage of this line of credit as long as  $\rho \leq \rho^*$ , although she will need only part of it. (In practice, lines of credit are actually often unused. Their value is essentially an option value.)

Alternatively, the lenders can grant a smaller line of credit, namely,  $(\rho^* - \rho_0)I$ , and give the borrower the right to dilute their claims at date 1 in order to finance the liquidity shock. The value of external claims in the case of continuation, that is, the *date-1* pledgeable income, is equal to  $\rho_0I$  and therefore the borrower can raise up to  $\rho_0I$  in a perfect capital market (by issuing new equity or new debt, depending on the interpretation given to external claims). So, overall, the borrower can gather  $(\rho^* - \rho_0)I + \rho_0I = \rho^*I$  in order to withstand the liquidity shock.

An alternative to providing a credit line for the future is for the lenders (especially if they are dispersed) to lend more money today, which the borrower will be able to use in the case of a liquidity

shock. That is, the lenders can invest  $I(1 + \rho^*) - A$  in the firm at the start. We now observe that the lenders should not let the borrower allocate resources freely between liquid and illiquid assets (illiquid assets are here the investment), but rather should demand that a liquidity ratio (which we will define as the ratio of liquid assets over total assets) be kept equal to  $\rho^*/(1 + \rho^*)$  until the liquidity shock accrues. The borrower then invests  $I$  and keeps  $\rho^*I$  in safe, liquid claims (which bear no interest by convention).

*Monitoring overinvestment in illiquid assets.* Recall from Chapter 2 that loan agreements do not focus solely on the borrower's solvency, that is, on the relationship between the firm's total indebtedness and its assets, but also strictly constrain the borrower's liquidity. For example, many loan agreements require that the borrower maintain a minimum level of working capital. To the extent that liquidity crises are ultimately solvency problems, it is not *a priori* clear why this is so. Let us bring one answer to this puzzle, and show that it may be optimal for lenders to *simultaneously* impose gearing (leverage) and liquidity ratios.

In the absence of a liquidity requirement, the borrower may want to invest more than  $I$  initially into illiquid assets. To develop our intuition for this, suppose that the borrower invests the full  $I(1 + \rho^*) \equiv I^*$  in illiquid assets; despite the lack of cash left for reinvestment, the project will often be continued, as the lenders, facing the fait accompli of an overinvestment in illiquid assets, have an incentive to rescue the firm as long as it is profitable for them to do so at date 1:  $\rho \leq \rho_0$ .

An interesting issue relates to whether the investors should renegotiate the borrower's compensation scheme so as to account for the unexpectedly high scale of operations. The answer to this question depends on the way the managerial compensation contract was initially drawn, namely, on whether the entrepreneur was granted a share of the final profit or a fixed bonus in the case of success (the two specifications are equivalent when the investment size is fixed, but no longer are so when investment, and therefore profit, can be scaled up or down). If the borrower owns a share in the firm's final profit, then managerial compensation scales up with investment, and the initial incentive scheme remains

incentive compatible as investment increases and is not renegotiated by lenders to account for the altered firm size.

Alternatively, the entrepreneur may have been granted in the initial agreement a fixed reward for “success”; because the private benefit scales up with investment, the initial incentive scheme is then no longer incentive compatible. Lenders then offer to increase the borrower’s reward in the case of “success” and so they raise the borrower’s payoff in the case of success to  $BI^*/\Delta p$  in order to make sure the borrower behaves.<sup>25</sup>

The lenders might, of course, want to claim initially that they will not put any more money into the venture, but this is not a credible commitment. Anticipating this *soft budget constraint*, the borrower may overinvest. Indeed, the borrower, who, regardless of the design of her initial compensation contract, receives expected rent  $p_H B/(\Delta p)$  per unit of illiquid assets, prefers investing  $I^*$  rather than  $I$  if

$$F(\rho^*)p_H\left(\frac{B}{\Delta p}I\right) < F(\rho_0)p_H\left(\frac{B}{\Delta p}I^*\right)$$

or

$$F(\rho^*) < F(\rho_0)(1 + \rho^*). \quad (5.9)$$

Condition (5.9) is satisfied as long as  $B$  lies below some threshold:  $\rho_0$  is decreasing in  $B$ , and, for  $\rho_0$  just below  $\rho^*$ , (5.9) is necessarily satisfied, and it is optimal for the borrower to deviate from investment  $I$ . Because the borrower is then strictly better off overinvesting, the lender should rationally anticipate to lose money overall.<sup>26</sup> Hence, the rationale for a liquidity requirement.

*Monitoring overhoarding of liquid assets.* As mentioned earlier, lenders may also need to verify that the borrower does not underinvest in illiquid assets in order to overinsure against liquidity shocks. The

analysis is contingent on several assumptions and we select a specific set of assumptions for the sole purpose of illustrating a possible incentive to underinvest in illiquid assets. Suppose (i) that the borrower can use the excess liquidity in order to withstand the liquidity shock, (ii) that the borrower and investors receive shares of the date-2 profit with share  $(B/\Delta p)/R = (\rho_1 - \rho_0)/\rho_1$  held by the borrower and share  $(R - B/\Delta p)/R = \rho_0/\rho_1$  held by the investors (all-equity firm), and (iii) that unused liquidity is returned to investors. Suppose further that the borrower invests  $I' \leq I$  in illiquid assets and thus hoards liquidity equal to  $\rho^*I + [I - I']$ . She can then withstand liquidity shocks  $\rho$  such that

$$\rho I' \leq \rho^*I + [I - I'].$$

Letting  $\varepsilon \equiv (I - I')/I'$ , and using the all-equity-firm assumption, the borrower prefers to underinvest if and only if

$$F(\rho^* + (1 + \rho^*)\varepsilon)I' > F(\rho^*)I$$

or

$$F(\rho^* + (1 + \rho^*)\varepsilon) > F(\rho^*)(1 + \varepsilon).$$

For small underinvestments, this condition is satisfied if and only if

$$\frac{(1 + \rho^*)f(\rho^*)}{F(\rho^*)} > 1.$$

Roughly, if liquidity shocks around the threshold  $\rho^*$  are quite likely, hoarding a bit more liquidity than allowed is privately profitable for the borrower.

The borrower would always prefer underinvesting to investing  $I$  if she had a fixed claim (namely,  $BI/\Delta p$  in the case of success).

## 5.4 Corporate Risk Management

Risk management is ranked by financial executives, CEOs, and investors as one of their most important concerns (see, for example, Rawls and Smithson 1990; Froot 1995). Firms can hedge against risk in a variety of ways. They can trade in forward/futures markets or enter swap agreements (which are over-the-counter deals that oblige two parties to exchange well-defined cash flows at specified dates) in order to cover their exposure to price variations: multinationals and financial institutions routinely obtain such

25. As long as

$$p_H\left(R - \frac{B}{\Delta p}\right)I^* \geq p_L RI^* - p_L \frac{B}{\Delta p} I \\ \Leftrightarrow (\Delta p)\left(R - \frac{B}{\Delta p}\right) \geq p_L \frac{B}{\Delta p} \frac{\rho^*}{1 + \rho^*},$$

which holds at least if  $p_L$  is small. (We here assume that the reward is not canceled when the firm succeeds and the profit is higher than what it would have been in the case of success.)

26. That the lender loses money results from the facts that the borrower deviates from investment  $I$  to obtain more than  $U_b$ , and that  $U_b$  is the maximum utility for the borrower consistent with a nonnegative profit for the lender.

insurance against currency or interest rate fluctuations, and producers or buyers of raw material or agricultural products similarly insure against price fluctuations by trading in commodity futures. Other hedging instruments include securitization, in which the issuer sells part of her portfolio of loans, assets, or intellectual property (or at least reduces the risk borne on the corresponding assets if she keeps some liability), and straight insurance against specific risks (theft, fire, death of key employee, guarantee of a financial institution against default on a claim such as a receivable, and so forth).

Corporate risk management is not driven by the desire to provide claimholders with insurance. There are two ways to see this: first, claimholders can obtain this insurance by diversifying their own portfolio; second, and relatedly, an insurance contract transfers risk from one party to another and therefore does not affect the aggregate uncertainty, which, according to standard asset pricing theory (the consumption-based capital asset pricing model), is the key driver of asset prices. By contrast, corporate risk management can be rationalized by agency-based (credit-rationing) considerations. We have seen that, even in a world of universal risk neutrality, firms ought to obtain some insurance against liquidity shocks as long as capital market imperfections prevent them from pledging the entire value of their activity to new investors. Following Froot, Scharfstein, and Stein (1993), we therefore derive an elementary explanation of corporate hedging from agency-based considerations.<sup>27</sup>

Froot et al. study risk management and financial structure in a sequential contracting context. In a first stage, an entrepreneur who has not yet issued securities to investors faces an uncertain short-term income. This short-term income serves, in the absence of hedging, as cash on hand for the second-stage investment; the second-stage investment is

financed by resorting to borrowing from investors but, as in Chapter 3, agency costs may expose the entrepreneur to credit rationing. The entrepreneur in the first stage can choose to stabilize her short-term income, and therefore her net worth in the subsequent borrowing stage.

As Froot et al. point out, the absence of financial design in a sequential contracting context makes it difficult to make general predictions as to whether the entrepreneur should hedge. Exercise 3.21, in part adapted from Froot et al., indeed presented a number of situations in which the entrepreneur preferred either to hedge against an exogenous risk or to use this risk to gamble. For example, if the agency cost is linear in investment, hedging is optimal when the production function is strictly concave, while gambling is optimal if there are indivisibilities in investment (as is the case in the fixed-investment model of Section 3.2) and hedging does not allow the entrepreneur to reach the funding threshold of cash on hand. In the variable-investment model of Section 3.4, the entrepreneur is indifferent between hedging and gambling, and would prefer hedging (gambling) if the private benefit were convex (concave) instead of linear in investment.

When risk management is not integrated with a choice of financial structure (the entrepreneur is still residual claimant when choosing whether to hedge), risk management is a “jack of all trades and a master of none”: because the level of liquidity cannot be separately controlled, the choice of its riskiness must also make up for the missing optimization of the financial structure. Indeed, hedging is always optimal in the environments presented in Exercise 3.21 under simultaneous liquidity and risk management. The following treatment therefore builds on Froot et al.’s seminal work by integrating liquidity and risk management.

#### 5.4.1 The Rationale for Hedging

Let us assume that some shock exogenous to the firm affects the firm’s date-1 net revenue, which we here normalize to 0. Let  $\varepsilon$  denote this income shock, where

$$E(\varepsilon | \rho) = 0.$$

For example,  $I$  might stand for a foreign investment, and  $\varepsilon$  might represent a foreign exchange risk. Let

27. Other explanations have been offered in the literature. Stulz (1984) argues that corporate hedging allows managers to obtain some insurance for their risky portfolio (stock options, etc.) against shocks that they have no control over. While this point is well-taken, Froot et al. (1993) note that managers could obtain such diversification by going to the corresponding markets themselves, and so Stulz’s argument relies on a transaction cost differential. Tax reasons have also been discussed in the literature. See Mason (1995) for a more complete discussion.

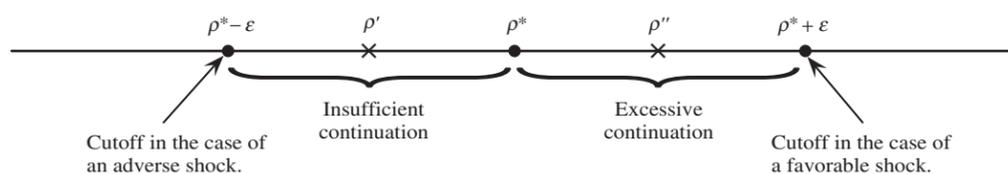


Figure 5.6

us furthermore assume that the firm can costlessly obtain insurance against this exogenous shock. As was the case for liquidity management, we envision an *ex ante* contract between borrower and investors and thereby obtain an unambiguous answer to the question: “Should the firm neutralize the cash flow variability by entering hedging arrangements?”<sup>28</sup>

Intuitively, a random liquidity garbles the reinvestment policy. Suppose, for example, that the shock can take values  $\varepsilon$  and  $-\varepsilon$  with equal probabilities; the firm’s need for a given  $\rho$  becomes  $\rho + \varepsilon$  and  $\rho - \varepsilon$ , respectively. Relative to the deterministic reinvestment policy obtained by eliminating the shock (i.e., reinvest if and only if  $\rho \leq \rho^*$ ), the firm reinvests too little in the case of an adverse shock and too much in the case of a favorable one (see Figure 5.6). For example, the firm has enough cash to continue when  $\rho = \rho''$  and the income shock is favorable and not enough when  $\rho = \rho' < \rho''$  and the income shock is adverse.

This reasoning is, however, too simplistic as the cutoff  $\rho^*$  itself depends on the risk management policy. Let us now provide a more rigorous proof. This proof is the same for a fixed and a variable investment. Let us, for instance, consider the variable-investment model and assume that the income shock (an earnings shortfall if it is positive, a gain if negative) is  $\varepsilon I$ , proportional to investment and distributed according to an arbitrary continuous distribution.

If the firm *hedges*, then for a given amount of liquidity hoarding the threshold under which the firm can continue,  $\rho^*$ , is deterministic and the analysis

28. “*Ex post*,” that is, once financing has been secured, borrower and investors do not have perfectly congruent views on risk management, and therefore the notion of “optimal risk management” at that point in time depends on whose standpoint one takes. Similarly, different classes of investors (e.g., debtholders and shareholders), if any, have conflicting objectives regarding risk management.

of Section 5.3.2 shows that the borrower’s utility is

$$U_b = \frac{\rho_1 - c(\rho^*)}{c(\rho^*) - \rho_0} A$$

for an arbitrary threshold  $\rho^*$ .

In the *absence of corporate hedging*, the threshold is now random: if the firm hoards just enough liquidity to withstand liquidity shocks below some  $\rho^*$  when  $\varepsilon = 0$ , then for an arbitrary realization  $\varepsilon$  the firm can withstand liquidity shocks  $\rho$  such that<sup>29</sup>

$$\rho + \varepsilon \leq \rho^*,$$

and so the state-contingent threshold is  $\rho^* - \varepsilon$ . Writing (IR<sub>1</sub>) and (5.4) as expectations with respect to the random variable  $\varepsilon$ , the reader will check that the borrower’s utility in the absence of corporate hedging is

$$\hat{U}_b \equiv \frac{\rho_1 - \hat{c}(\rho^*)}{\hat{c}(\rho^*) - \rho_0},$$

where  $\rho^*$  denotes the threshold when  $\varepsilon = 0$ ,

$$\hat{c}(\rho^*) \equiv \frac{1 + E_\varepsilon[\int_0^{\rho^* - \varepsilon} \rho f(\rho) d\rho]}{E_\varepsilon[F(\rho^* - \varepsilon)]},$$

and  $E_\varepsilon$  denotes an expectation with respect to  $\varepsilon$ .

Using the Arrow-Pratt Theorem (see Arrow 1965; Pratt 1964),<sup>30</sup> it is easy to see that, for each  $\rho^*$ , there

29. We here ignore the possibility of renegotiation (see Section 5.5), which arises for large  $\varepsilon$ : if  $\rho \in (\rho^* - \varepsilon, \rho_0)$ , then the liquidity shock is smaller than the pledgeable income and investors are willing to bring in new cash (see the treatment of the soft budget constraint). Similarly, when  $\rho \in (\rho_1, \rho^* - \varepsilon)$ , continuation is inefficient and investors optimally offer a bribe to the borrower for not continuing.

Two remarks are in order here. First, our analysis can be amended to reflect the possibility of renegotiation. Second, renegotiation is irrelevant if the exogenous shock  $\varepsilon$  remains small.

30. Let  $H(x) \equiv 1 + \int_0^x \rho f(\rho) d\rho$ . Let us first show that  $H$  is “more convex than  $F$ ,” in the sense that  $H$  is a convex transform of  $F$ , that is,  $H \circ F^{-1}$  is convex. A straightforward computation shows that  $(H \circ F^{-1}(y))' = F^{-1}(y)$ , and so  $(H \circ F^{-1}(y))'' > 0$ , where  $y \equiv F(x)$ .

Second, for a given threshold  $\rho^*$ , define  $\bar{\rho}$  such that

$$F(\bar{\rho}) = E_\varepsilon[F(\rho^* - \varepsilon)].$$

That is,  $\bar{\rho}$  is the certainty equivalent of the random variable  $\rho^* - \varepsilon$  for function  $F$ . Because  $H$  is more convex than  $F$ , the Arrow-Pratt Theorem (which states that the risk premium is smaller for the more convex

exists a  $\bar{\rho}$  such that

$$c(\bar{\rho}) \leq \hat{c}(\rho^*),$$

which implies that

$$U_b \geq \hat{U}_b.$$

In words, corporate risk management lowers the expected unit cost of effective investment and adds value.<sup>31</sup>

*Remark (substitutes to corporate hedging: alternative risk transfer).* Note that insurance could be provided by means other than hedging on a market. In particular, a bank could offer a *conditional* credit line, such that the maximal amount varies one-to-one (and positively) with  $\varepsilon$ .<sup>32</sup> Namely, the maximal commitment is equal to  $(\rho^* + \varepsilon)I$ , and so the firm can withstand liquidity shocks  $\rho I \leq (\rho^* + \varepsilon)I - \varepsilon I = \rho^* I$ . In the absence of transaction costs, conditional credit lines and corporate hedging are perfect substitutes. Such contingent credit lines do exist,<sup>33</sup> but they are less pervasive than corporate hedging. Contingent credit lines may substitute for corporate hedging when either the insurance contract must be tailored to the borrower's specific needs (and so there is no market for the corresponding claims) or when it is difficult to write formal hedging contracts because the underlying shock cannot be well-described *ex ante* or objectively measured *ex post*. The contingent credit line

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function) implies that  $H(\bar{\rho}) \leq E_\varepsilon[H(\rho^* - \varepsilon)]$ , and so  $c(\bar{\rho}) \leq c(\rho^*)$ , as announced.

31. As is the case for the allocation of the initial credit between liquid and illiquid assets (see Section 5.3.3), the borrower's compliance with corporate hedging must be monitored. We invite the reader to check that it may not be in the borrower's best interest to indeed purchase the associated insurance policy once she has obtained the financing for the investment and secured the associated amount of liquidity.

32. Alternatively, the firm could issue debt with interest payments indexed on the shock  $\varepsilon$ . For example, an oil producer could issue debt whose interest payment increases with the market price of oil.

33. Standby loan commitments are informal arrangements, generally backing the issue of commercial paper by large firms. Under a standby loan agreement, the bank promises to refinance the firm during disruptions in the commercial paper market (Veitch 1992). Incidentally, it is interesting to note that the usage rate on standby commitments is low relative to other categories of loan commitments (Veitch 1992).

In the area of international finance, a number of authors have proposed that reimbursement of sovereign debt be made contingent on observable shocks, such as GDP or exchange rate fluctuations, or (better as this does not give rise to government moral hazard) to world prices of raw materials and other competitive exports of the country.

must in the latter circumstances rest on the bank's reputation for abiding by its implicit promises.

In circumstances in which the risk can be insured against in deep markets, corporate hedging is likely to be a lower-transaction-cost alternative; for, and as we will see in future chapters, the credit line is only one of several variables that must be indexed to exogenous shocks such as macroeconomic shocks. (For example, managerial compensation should not depend on shocks over which managers have no control. Hence, bonuses and stock options should be indexed on currency and interest rate fluctuations and on several other exogenous risks. Similarly, the allocation of control rights among claimholders should be indexed on such variables.)

While it may be simpler to have the firm engage in corporate hedging rather than index many contracts and covenants, further study is needed before drawing such a conclusion. As a matter of fact, financing arrangements known under the heading of alternative risk transfer (ART) have developed over the years although they have still much scope for growth. Such products blend elements of corporate finance and insurance. A case in point is catastrophe bonds (cat bonds) such as the ones issued by Vivendi Universal to cover its movie studios in Los Angeles against earthquakes, or the bonds that are contingent on the occurrence of a hurricane.<sup>34</sup>

#### 5.4.2 When Is Incomplete Hedging Optimal? Another Look at the Sensitivity of Investment to Cash Flow

We just obtained a stark result of *full hedging*: any exogenous income fluctuation perturbs optimal liquidity management by making the firm sometimes reinvest when the reinvestment cost is high while it sometimes is unable to reinvest for low reinvestment costs. Even leaving aside the *transaction costs* involved in entering hedging contracts (including those associated with the monitoring of the counterparty's solvency), there are several reasons why firms, or countries for that matter, should not, and actually do not in practice, fully hedge.

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34. As another example, a few years ago Michelin secured a bank credit line and an insurance facility for five years, contingent on a simultaneous fall in GDP in its various markets and in tyre sales.

(a) *Market power.* Consider the producer of a raw material (copper, oil, etc.) with market power. The market price then depends not only on uncertainty that is exogenous to the firm (e.g., demand shifts), but also on the firm's supply decisions. Thus, suppose for illustrative purposes that there are two dates, 0 and 1 (these two dates are meant to correspond to the risk-management-choice and the risk-income dates of the model). And suppose for simplicity that the firm is a monopolist in the market for the raw material. The monopolist at date 0 sells  $f$  units forward at predetermined price  $p^f$ . This amounts to writing an insurance contract that pays the firm at date 1  $f$  times the (positive or negative) difference between  $p^f$  and the date-1 spot price. Once the monopolist has sold these  $f$  units, though, they are no longer hers, and therefore the monopolist has at date 1 decreased incentives to restrain output to keep the spot price up. From the point of view of the monopolist at date 1, output withholding raises the price on her extra production only (her inframarginal units do not include the forward sales). Forward sales overall result in an output that exceeds the monopoly output and therefore reduce revenue.<sup>35</sup>

*Example.* Suppose that the date-1 spot price is  $\tilde{a} - q$ , where  $\tilde{a}$  is an exogenous demand shock realized at date 1 and  $q$  is output, and that the marginal cost is 0. In the absence of forward sales, the monopolist chooses  $q$  at date 1 so as to maximize  $q(\tilde{a} - q)$ , yielding  $q = \frac{1}{2}\tilde{a}$  and a revenue that is random at date 0:  $r = \frac{1}{4}\tilde{a}^2$ . The expected profit is thus  $\frac{1}{4}E[\tilde{a}^2]$ , where  $E[\cdot]$  denotes an expectation with respect to  $\tilde{a}$ .

Suppose now that the monopolist sells  $f$  units at price  $p^f$  at date 0. At date 1, the monopolist chooses an extra output  $q$  (to be added to the  $f$  units that she committed to deliver) so as to maximize  $q[\tilde{a} - (q + f)]$ , and so  $q = \frac{1}{2}(\tilde{a} - f)$ .<sup>36</sup> Under rational expectations, the forward price must be equal to the

35. This reasoning is reminiscent of that underlying the "Coase conjecture," which states that a durable-good monopolist tends to create its own competition and to "flood the market" (see, for example, Tirole 1988, Chapter 1), although the setting is slightly different (the good is here nondurable).

36. We assume that the price is always positive. Otherwise,

$$q = \max\{\frac{1}{2}(\tilde{a} - f), 0\};$$

but the gist of the analysis remains the same.

expected spot price:

$$p^f = E[\tilde{a} - (q + f)] = E[\frac{1}{2}(\tilde{a} - f)].$$

Total (date-0 plus date-1) profit,

$$\frac{1}{4}(E[\tilde{a}^2] - f^2),$$

decreases with  $f$ .

More generally, forward sales reduce monopoly power, and so, in the absence of date-1 reinvestment need, it is strictly optimal not to hedge at all ( $f = 0$ ).<sup>37</sup> When one combines the corporate risk management motive of this chapter with the exercise of market power, the optimal degree of hedging is partial hedging.

(b) *Serial correlation of profits.* An important assumption behind the full-hedging result of Section 5.4.1 is that the date-1 profit realization conveys no information about the firm's prospects: it is a transitory shock. Suppose in contrast that a high date-1 profit is good news about date-2 profitability. For example, the price of a crop may reflect permanent shocks such as the reduction of trade barriers, the entry of competing offers, or a change in consumer preferences.

With positive serial correlation of profits, a high current profit is associated with attractive reinvestment opportunities. This suggests that the liquidity available to the borrower at date 1 should covary with the date-1 profit (so, for example, the farmer's debt contract should not be fully indexed to the price of the crop). Things are, however, more complex than this first argument suggests, because better prospects also make it easier for the borrower to return to the capital market at the intermediate stage. The *attractive-reinvestment-opportunities*

37. This basic insight must be amended a bit in the case of oligopoly. A large literature, starting with Allaz and Vila (1993), has shown that firms that compete *à la Cournot* (in quantities) partially hedge despite the absence of reinvestment need. The intuition is that forward markets induce each oligopolist to try to act as a "Stackelberg leader" and to thereby force its rivals to cut output on the spot market (see, for example, Chao et al. (2005), Creti and Manca (2005), and Willems (2005) for recent contributions to this literature). As usual, this conclusion is reversed if firms compete in prices rather than quantities (see Mahenc and Salanié 2004); under price competition, oligopolists would like to "commit" to set high prices so as to induce others to also set high prices. Buying (i.e., gambling) on the forward market is a commitment for suppliers to set high prices in the spot market.

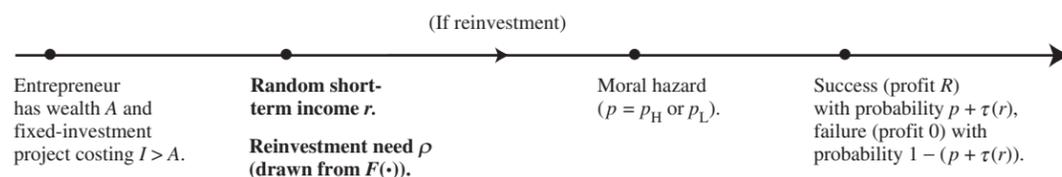


Figure 5.7

effect, however, in general dominates the *easier-refinancing effect*, and so the firm should not be fully insured against exogenous profit shocks, as we now illustrate.

Let us consider the fixed-investment model of Section 5.2, but with two twists:

- The short-term income,  $r$ , is random, with mean  $\bar{r}$ .
- The probability of success in the case of continuation is an increasing function of  $r$ ,

$$p + \tau(r), \quad \text{with } \tau' > 0,$$

where  $p = p_H$  or  $p_L$  depending on the entrepreneur's date-1 behavior. (The separable form of the probability-of-success function as usual guarantees that the incentive constraint is invariant.) We assume that the realizations of  $r$  and  $\rho$  are independent.

These twists are depicted in bold in Figure 5.7.

Let us follow the steps of Section 5.2 and determine the optimal state-contingent cutoff  $\rho^*(r)$  (so continuation occurs if and only if  $\rho \leq \rho^*(r)$ ). Letting  $E[\cdot]$  denote expectations with respect to  $r$ , the NPV is

$$U_b = \bar{r} + E[F(\rho^*(r))[p_H + \tau(r)]R] - I - E\left[\int_0^{\rho^*(r)} \rho f(\rho) d\rho\right].$$

The investors' breakeven constraint similarly is

$$\bar{r} + E\left[F(\rho^*(r))[p_H + \tau(r)]\left[R - \frac{B}{\Delta p}\right]\right] \geq I - A + E\left[\int_0^{\rho^*(r)} \rho f(\rho) d\rho\right].$$

Letting  $\mu$  denote the shadow price of the breakeven constraint (we assume that the constraint is binding, i.e.,  $\mu > 0$ ), the first-order condition with respect to

$\rho^*(r)$  yields, for each  $r$ ,

$$\rho^*(r) = \frac{[p_H + \tau(r)][R + \mu(R - B/\Delta p)]}{1 + \mu}.$$

Let us now investigate the implementation of the optimal contract. A *fully indexed debt* can be defined as a date-1 liability  $d(r)$  such that

$$d(r) = d_0 + r,$$

for some constant  $d_0$ . That is, in the absence of refinancing in the capital market, a fully indexed debt insulates the firm's retained earnings against its cash-flow risk. We, however, want to allow the firm to return to the capital market: insulation of retained earnings against the cash-flow risk does not imply insulation of the reinvestment policy. The amount it can raise in the capital market at date 1,

$$[p_H + \tau(r)]\left(R - \frac{B}{\Delta p}\right),$$

is increasing with the date-1 profit as  $\tau' > 0$  (this was referred to earlier as the "easier-refinancing effect"). The optimal policy is implemented when the cutoff is equal to the cash cushion plus the refinancing capacity:

$$\rho^*(r) = [r - d^*(r)] + [p_H + \tau(r)]\left(R - \frac{B}{\Delta p}\right),$$

or

$$d^*(r) = r - \frac{p_H + \tau(r)}{1 + \mu} \left(\frac{B}{\Delta p}\right).$$

In the presence of an agency cost ( $B > 0$ ), the debt is not fully indexed. The easier-refinancing effect is at play, but the existence of a managerial rent puts a limit on what can be achieved by returning to the capital market. Put differently, *the firm should keep some of its cash flow as retained earnings*.

The source of this cash-flow sensitivity of debt is the monotonicity of managerial rents<sup>38</sup> with the

38. Here, this rent is  $[p_H + \tau(r)](B/\Delta p)$ .

resolution of uncertainty. Thus, the credit rationing problem at the seasoned offering stage is more severe, the more favorable the resolution of uncertainty. While this monotonicity is often a reasonable assumption, one can, of course, envision cases where it does not hold. To check our intuition, Exercise 5.11 considers the case of a permanent price shock  $P$ : the date-1 income is  $Pr$  (where  $r$  is now known and  $P$  is a random variable realized at date 1) and the date-2 income in the case of success is  $PR$ . The managerial rent in the case of continuation<sup>39</sup> is then insensitive to the state of nature. While the date-1 cash flow affects reinvestment through its informational content, there should not be any cash-flow sensitivity of retained earnings; put differently, debt due at date 1 is perfectly indexed to the output price ( $d(P) = Pr - l_0$  for some positive  $l_0$ ).

When, in contrast, a high profit today announces low profits tomorrow (negative serial correlation,  $\tau' < 0$ ), the conclusions are reversed. Suppose, for example, that an industry is subject to cycles and furthermore that investments made at the peak (trough) mature at the trough (peak); one possible story is that the other firms in the industry are subject to poor governance and that they invest when they have large cash flows rather than when investments are profitable. How should a (well-governed) firm behave in such an industry? By analogy with the formula above, it should retain less money in net terms when its profit grows.<sup>40</sup>

(c) *Aggregate risk.* Hedging markets often involve economic variables, such as interest rates or exchange rates, that respond to macroeconomic shocks. As is well-known and reflected, for instance, in the capital asset pricing model (CAPM), aggregate risk must be borne by and is optimally shared among economic agents; insuring against it therefore involves a risk premium. Put differently, economic agents cannot insulate themselves from such risks at a “fair price.”

We invite the reader to return to the analysis of Section 5.4.1, focusing for simplicity on linear insurance schemes and assuming that eliminating a fraction  $\theta$  of the income shock (which therefore becomes

$(1 - \theta)\varepsilon$  in net terms) costs  $\sigma\theta$  (proportional to  $\theta$ ). It is easy to see<sup>41</sup> that it is suboptimal to fully hedge; that is, the optimal  $\theta$  is less than 1. Intuitively, a small risk ( $\theta$  close to, but smaller than 1) induces only small deviations from the optimal risk management and reinvestment policy, and therefore a second-order NPV loss; in contrast, the cost of this insurance is first order and proportional to  $\theta$ .

We thus conclude that firms should hedge less against shocks involving larger macroeconomic risk premia.

(d) *Asymmetric information.* Asymmetric information may limit the development of hedging markets. Consider, for example, the potential market for five-year hedges against variations in the overall power prices and in zonal price differences in the U.S. electricity Midwest market. The value of such derivatives depends on very complex predictions of the evolution of supply and demand as well as of likely changes in incentive regulation for both generators and transmission grid owners.

Generators, load-serving entities, and transmission owners, who are keen on hedging their positions, may find few counterparts who have the necessary expertise. And even if some employees of financial institutions do have this expertise, their bosses probably do not and will be reluctant to let them gamble large amounts of money on such long-term derivative markets.

(e) *Incentives.* Finally, borrowers may need to be made somewhat accountable for fluctuations in an exogenous variable, because the quality of their investments depends on how well they predict the future value of this variable. For example, the oil manager of a small oil company has no impact on the oil price; however, the choice of how much to invest in oil rather than in other activities depends on her forecast of the future price of oil. In this case, insulating the borrower from fluctuations in the oil price provides poor incentives for accurate prediction and therefore for efficient investment.

Forecasting future exogenous variables can be modeled in the basic framework as a date-0 moral hazard. The next section studies the implications for liquidity management of such *ex ante* moral hazard.

39. Equal to  $p_H(B/\Delta p)$ .

40. This policy may be difficult to implement, especially if the firm can hide profits (see Chapter 7).

41. See Holmström and Tirole (2000) for a more rigorous proof.

There, it will be shown that borrowers should not be rewarded for good short-term performance solely through monetary compensation and that liquidity should be sensitive to cash flow. This implies, in particular, that the liquidity of an oil company should not be fully insulated from fluctuations in the stock price even if the company has no market power.

## 5.5 Endogenous Liquidity Needs, the Sensitivity of Investment to Cash Flow, and the Soft Budget Constraint

### 5.5.1 Endogenous Liquidity Shocks

Starting with Dewatripont and Maskin (1995), the economics literature has stressed the perverse incentive effects of bailouts and other insurance devices: a state-owned enterprise that knows that it will be bailed out by the government if it loses money has little incentive to reduce its costs or generate revenue.<sup>42</sup> A project manager who knows that the company will be keen on completing the project once large fixed costs have been sunk may “goldplate” the project or spend time on other activities. Hardening the budget constraint may therefore improve incentives.<sup>43</sup>

In the context of corporate financing, liquidity hoarding and credit line commitments become less attractive when liquidity shocks are endogenous, that is, when they depend on the borrower’s actions. For incentive purposes, it is not optimal to commit to rescue the borrower often. The borrower has sub-optimal incentives to avoid adverse shocks if she knows that she can easily raise cash to cover such shocks. In such circumstances the borrower must be kept “on a short leash.” We will discuss how this can be done.

To illustrate in a stark way the point that one may want to commit to a “hard budget constraint,” suppose that, after the loan agreement is signed but before the reinvestment need parameter  $\rho$  is realized, the borrower can by incurring private effort cost  $c$

prevent any cost overrun:  $\rho = 0$  with probability 1 (as in Section 3.4). On the other hand,  $\rho$  is drawn from distribution  $F(\rho)$  (as in this section) if the borrower does not incur this cost. Suppose further that  $c$  is small enough that it is optimal to induce the borrower to incur the cost.

Assuming for example that the firm has no date-1 income (and so is cash-poor), the optimal policy then obviously consists in letting the borrower invest  $I$  and promising never to plow back any money into the firm. In this case the borrower knows that if she does not spend  $c$ , the project will be discontinued with probability 1 (provided that the cumulative distribution  $F$  has no atom at 0). This threat obviously keeps her on her toes.

The crux of the matter is then, How can we make this hard budget constraint credible? For, we have seen that, in the case of “reasonable” overrun ( $\rho \leq \rho_0$ ), the lenders have an *ex post* incentive to renege on their promise not to rescue the firm. Anticipating this, the borrower may not bother to incur cost  $c$  to prevent overruns.

#### 5.5.1.1 A Broader Perspective

When is the firm’s budget constraint likely to be soft? The basic idea of long-term financing is, as we have seen in Sections 5.2 and 5.3, that the intermediate stage (date-1) exhibits rationing of credit for reinvestment and so it is optimal for the firm to secure *ex ante* (at date 0) more liquidity than it will obtain by going to the capital market at the intermediate stage. Thus the problem is not that the capital market is too soft but rather that it is too tough at the intermediate stage. Hence, the soft-budget-constraint problem does not arise.

This need not be so, however, when information accrues at date 1 that sheds light on some activity subject to earlier (date-0) moral hazard.<sup>44</sup> It is then optimal to commit at date 0 to punish the entrepreneur if the information “signals” that the borrower has not acted in the lenders’ interest.

42. See Kornai (1980) for a study of the soft budget constraint in centrally planned economies and its macroeconomic consequences.

43. Hardening the budget constraint may, however, induce short-termism, that is, a managerial focus on immediate performance, to the detriment of long-term goals, as was demonstrated by von Thadden (1995). See Chapter 7 for a study of short-termism.

44. Or to adverse selection for that matter. For example, if information accrues at date 1 that the entrepreneur is likely to be a bad borrower, notwithstanding claims to the contrary at the contracting stage, it is in general optimal to commit at date 0 to punish the firm for such bad news at date 1, in order to screen borrowers more efficiently. See Chapter 6 for a treatment of adverse selection.

The key to the soft-budget-constraint phenomenon is that monetary punishments may be limited because they are costly. In our model, the entrepreneur's incompressible stake implies that monetary punishments are limited in the case of continuation. So, *liquidation may be the only feasible punishment for the entrepreneur when bad signals about her activity accrue at date 1*. In contrast with monetary punishments, which are simple transfers from the entrepreneur to the lenders, nonmonetary punishments may be *ex post* Pareto-inefficient. The soft budget constraint arises from the fact that while the punishment serves a purpose at date 0 (it deters bad date-0 behavior), it may no longer serve a purpose at date 1. And so it is likely to be renegotiated away if it is *ex post* Pareto-inefficient.<sup>45</sup> In the present case, a Pareto-inefficient liquidation, namely, one that occurs for liquidity shocks below the pledgeable income, is not credible.

Two types of news about date-0 moral hazard can accrue at date 1. The first involves "bygones," namely, variables that, in the absence of considerations relative to punishing or rewarding past behavior, should have no impact on decision making because they no longer affect payoffs. Such a variable is date-1 income.<sup>46</sup> It does not impact the optimal date-1 policy in the absence of considerations of reward or punishment.

Variables in the second set both convey information about managerial performance and impact date-1 decision making. The level of date-1 liquidity shock, news about the prospects for date 2 in the case of continuation (say, news about the probability of success or about income in the case of success),

45. The literature on mutually advantageous renegotiation is based on the same principle: an *ex ante* contract between a principal and an agent creates distortions in order to provide the agent with incentives to act in the principal's interest. Once the agent has acted, the distortion no longer serves a purpose and tends to be renegotiated away, thus reducing the agent's *ex ante* incentives. For example, in the standard moral-hazard model, the agent receives suboptimal insurance, which is then partly renegotiated away (see Fudenberg and Tirole 1990; Ma 1991). There is also a large literature, initiated by Dewatripont (1989), on renegotiation when the initial contract is plagued by adverse selection (see, for example, Hart and Tirole 1988; Laffont and Tirole 1990; Rey and Salanié 1996).

46. An almost equivalent example is a separable date-2 revenue that will accrue independently of date-1 decisions (such as liquidation versus continuation) and is publicly learned at date 1. Indeed, if the corresponding claim is securitized, it becomes a date-1 revenue for the firm.

and the level of the date-1 salvage value of the assets in the case of liquidation all belong to this second category.

In the next section, we focus on the case of an endogenous intermediate revenue in order to identify the punishment aspect and the soft budget constraint in the simplest manner. It is straightforward, though, to extend the analysis to the second set of variables (see Exercises 5.3 and 5.4). These exercises show that the results obtained in Section 5.5.2 carry over to news about date-2 prospects and about the salvage value. In particular, the soft-budget-constraint problem always arises when news is bad, that is, when performance is poor.

### 5.5.2 Endogenous Intermediate Income

Let us generalize the model of Section 5.3.2 by introducing an endogenous short-term revenue.<sup>47</sup> The investment of variable size  $I$  generates a non-negative date-1 revenue  $rI$ . This (verifiable) date-1 income is subject to date-0 moral hazard. The distribution of the per-unit income  $r$  on an interval  $[0, r^+]$  is  $G(r)$  with density  $g(r)$  if the entrepreneur works at date 0, and  $\tilde{G}(r)$  with density  $\tilde{g}(r)$  if the entrepreneur shirks at date 0. Let

$$\ell(r) \equiv \frac{g(r) - \tilde{g}(r)}{g(r)}$$

denote the likelihood ratio.<sup>48</sup> As usual, we assume that a high date-1 revenue signals that the entrepreneur is likely to have worked at date 0.

*Monotone likelihood ratio property:*  $\ell(r)$  weakly increases with  $r$ .

This property implies, in particular, that the distribution of the date-1 income improves, in the sense of first-order stochastic dominance, if the entrepreneur works:  $G(r) \leq \tilde{G}(r)$  for all  $r$ . To avoid technical difficulties, we will further assume that the likelihood ratio is constant past some level of  $r$  lower than  $r^+$ .<sup>49</sup>

47. The analysis in this section is modeled after that in Section 3 of Rochet and Tirole (1996). This article has quite a different purpose. It studies systemic risk generated by interbank exposures. Interbank lending is motivated by the benefits from peer monitoring among banks. The date-1 income of this section corresponds to (minus) the loss in the interbank market in Rochet and Tirole.

48. There are, of course, several equivalent ways of defining this ratio. Another common one is  $g(r)/\tilde{g}(r)$ .

49. In the absence of this assumption and given risk neutrality, it may be optimal to give the entrepreneur an extra rent beyond her

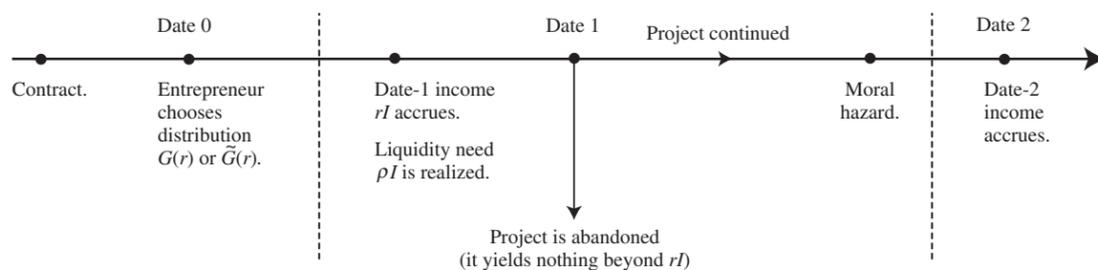


Figure 5.8

This is a purely technical assumption, which has no serious consequence for the analysis.

The entrepreneur enjoys private benefit  $B_0I$  at date 0 if she shirks, and 0 if she works. The modified timing is summarized in Figure 5.8.

As earlier, we let

$$\rho_1 = p_H R \quad \text{and} \quad \rho_0 = p_H \left( R - \frac{B}{\Delta p} \right)$$

denote the per-unit expected income and pledgeable income, respectively. (Recall that  $\rho_0$  embodies the date-1 moral hazard, and so there is no need for including the corresponding incentive constraint (IC<sub>b</sub>) in the program below.)

Let us, in a first step, ignore the credibility issue. Letting “NSBC” stand for “no soft budget constraint,” we maximize the project’s NPV subject to the constraints that lenders break even and that the entrepreneur has an incentive to work at date 0. A *contract* specifies a state-contingent threshold  $\rho^*(r)$  and a per-unit “extra rent”  $\Delta(r)$ .

A word of explanation is called for here. This per-unit extra rent is equal to the entrepreneur’s expected rent per unit of investment when the state of nature is  $r$ , minus either the minimal per-unit rent,  $p_H B / \Delta p$  that is necessary to induce good behavior in the case of continuation, or 0 in the case of liquidation. So, if the entrepreneur receives  $R_b \geq B / \Delta p$  in the case of success at date 2, then

$$\Delta(r) = p_H \left( R_b - \frac{B}{\Delta p} \right).$$

And, in the case of liquidation,  $\Delta(r) \geq 0$  represents the cash payment made to the entrepreneur at date 1.

incentive-compatible stake, entirely at the highest possible income  $r^+$ , in the form of a “spike” at  $r^+$ .

Section 3.4 showed that in the absence of date-0 moral hazard, it is optimal to set this extra rent  $\Delta(r)$  equal to 0, so as to pledge as much income as is feasible to the lenders and thus to boost debt capacity. As we will see, this no longer needs to be the case in the presence of date-0 moral hazard. The flip side of punishing the entrepreneur for bad performance, that is, for a low date-1 income, by liquidating the firm even for low liquidity shocks, is that it is optimal to reward her for high date-1 income with continuation even for high liquidity shocks. But, for  $\rho > \rho_1$ , continuation is inefficient and it is optimal, as we will see, to convert the reward into monetary rewards and thus into extra rents  $\Delta(r) > 0$ .

Ignoring for simplicity the choice of investment size  $I$ , we can now write the program when there is no credibility issue.

*Program NSBC:*

$$\begin{aligned} \max_{\{\rho^*(\cdot), \Delta(\cdot) \geq 0\}} & \left\{ \int_0^{r^+} \left[ r + F(\rho^*(r)) \rho_1 \right. \right. \\ & \left. \left. - \int_0^{\rho^*(r)} \rho f(\rho) d\rho - 1 \right] g(r) dr \right\} I \\ \text{s.t.} & \left\{ \int_0^{r^+} \left[ r + F(\rho^*(r)) \rho_0 - \Delta(r) \right. \right. \\ & \left. \left. - \int_0^{\rho^*(r)} \rho f(\rho) d\rho \right] g(r) dr \right\} I \geq I - A \end{aligned} \quad (\text{IR}_1)$$

and

$$\begin{aligned} & \left\{ \int_0^{r^+} [F(\rho^*(r))(\rho_1 - \rho_0) + \Delta(r)] \right. \\ & \left. \times [g(r) - \tilde{g}(r)] dr \right\} I \geq B_0 I, \end{aligned} \quad (\text{IC}'_b)$$

recalling that  $B_0I$  is the *date-0* private benefit of misbehaving.

Note that (IC'<sub>b</sub>) can be rewritten by highlighting the role of the likelihood ratio:

$$\int_0^{r^+} [F(\rho^*(r))(\rho_1 - \rho_0) + \Delta(r)]\ell(r)g(r) dr \geq B_0. \quad (\text{IC}'_b)$$

Letting  $\mu$  and  $\nu$  denote the (nonnegative) multipliers of constraints (IR<sub>1</sub>) and (IC'<sub>b</sub>), the necessary (and sufficient) conditions for program NSBC yield

$$\rho^*(r) = \frac{\rho_1 + \mu\rho_0 + \nu(\rho_1 - \rho_0)\ell(r)}{1 + \mu}$$

and

$$\begin{aligned} \Delta(r) = 0 &\Rightarrow \nu\ell(r) \leq \mu \Rightarrow \rho^*(r) \leq \rho_1, \\ \Delta(r) > 0 &\Rightarrow \nu\ell(r) = \mu \Rightarrow \rho^*(r) = \rho_1. \end{aligned}$$

Note that the latter inequalities imply that there is never a negative-NPV continuation ( $\rho > \rho_1$ ). And, as we suggested earlier, there is no extra rent as long as  $\rho^*(r) < \rho_1$ . The explanation is that for  $\rho < \rho_1$ , continuation maximizes net payoff and thus it is better to reward the entrepreneur with continuation than with (nonincentive-based) cash. In contrast, for  $\rho > \rho_1$ , continuation is inefficient and so, if  $\rho^*(r) > \rho_1$ , one can improve the welfare of all parties by liquidating the firm and providing the entrepreneur with more cash.

Next, we analyze the optimal continuation rule. Because likelihood ratios are equal to 0 in expectation, one has

$$E[\rho^*(r)] = \frac{\rho_1 + \mu\rho_0}{1 + \mu},$$

where  $E[\cdot]$  denotes the expectation operator (with respect to density  $g$ ). And so, “on average,” the threshold is a convex combination of  $\rho_1$  and  $\rho_0$ , as in the absence of date-0 moral hazard. The state-contingent threshold can be rewritten as

$$\rho^*(r) - E[\rho^*(r)] = \lambda\ell(r),$$

where

$$\lambda \equiv \frac{\nu(\rho_1 - \rho_0)}{1 + \mu}.$$

Because the likelihood ratio is increasing, the continuation rule is more lenient, the higher the date-1 income.

Figure 5.9 summarizes the analysis. The coefficient  $\lambda$  is small when date-0 moral hazard is relatively unimportant. This arises either if the date-0

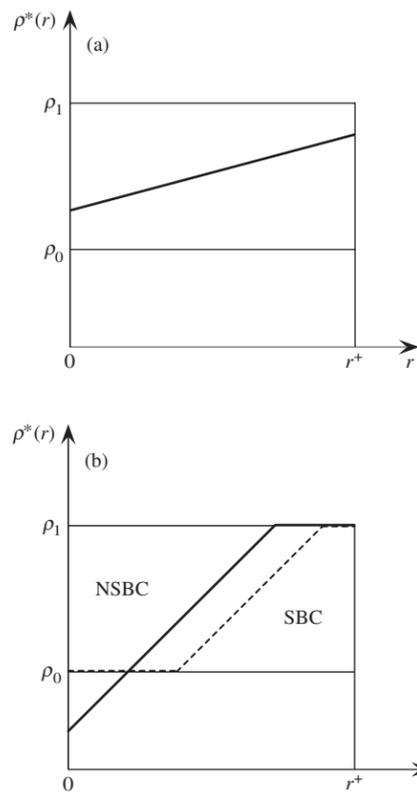


Figure 5.9 (a)  $\lambda$  small; (b)  $\lambda$  large.

per-unit-of-investment private benefit  $B_0$  is small or if the date-1 income is mainly determined by external demand and cost shocks that lie beyond the control of the entrepreneur (and so  $\ell(\cdot)$  remains close to 0; see part (a) of the figure).<sup>50</sup> When date-0 moral hazard is more substantial ( $\lambda$  large), two new phenomena can arise. First, the “constraint”  $\rho^*(r) \leq \rho_1$  may become binding for  $r$  large. Second,  $\rho^*(r)$  may fall below the pledgeable income  $\rho_0$  for  $r$  low. The solution, ignoring renegotiation, is depicted in bold.

We are now set for a discussion of the soft budget constraint. If the entrepreneur can renegotiate Pareto-suboptimal liquidation, then the relevant program becomes

*Program SBC* = Program NSBC with added  
constraint  $\rho^*(r) \geq \rho_0$  for all  $r$ .

50. In the latter case, though, it may become optimal to let the entrepreneur take her private benefit  $B_0$  at date 0.

If date-0 moral hazard is small enough ( $\lambda$  small) so that  $\rho^*(0) \geq \rho_0$ , the soft-budget-constraint problem does not arise. If date-0 moral hazard is substantial ( $\lambda$  large), then  $\rho^*(r) < \rho_0$  for  $r < r_0$  (see Figure 5.9(b)).

We leave it to the reader to check that, for any level of investment  $I$ , the solution to Program SBC is depicted by the dashed curve in Figure 5.9(b).

Lastly, note that—and this is obviously a general property—the borrower's *ex ante* welfare is always (weakly) lower when renegotiation is feasible, since the soft-budget-constraint problem adds an extra constraint to the optimization program.

### 5.5.3 Keeping Commitment Credible

Several devices that might allow lenders to commit not to plow back money into the firm have been considered in the literature (in contexts that differ from the one studied here, but which have in common the need for such a commitment). Following the debt overhang literature (see Section 3.3), Hart and Moore (1995) assume that the initial lenders are *dispersed* and cannot participate in a claim restructuring;<sup>51</sup> and, to prevent refinancing by new investors, Hart and Moore restrict the availability of new capital by putting limits on the dilution of the claims of initial lenders. In particular, making initial lenders senior and new lenders junior strongly reduces the incentive of new lenders to provide refinancing (in the absence of renegotiation, the senior lenders' stake is another incompressible stake on top of the entrepreneur's. So there is hardly any pledgeable income).<sup>52</sup>

Another possibility is to create a diversity of claims with different control rights, and to give, in states of financial trouble, control to “tough” claimholders who have a strong incentive to impose abandonment of the project or risk reduction in such states. In Dewatripont and Tirole (1994), those tough claimholders are debtholders rendered conservative

by their concave return stream, (outside) equityholders being softer. Berglöf and von Thadden (1994) argue that the short-term debtholders can be used to play the role of the “tough guy,” with the long-term debtholders being softer. In Burkart et al. (1995), a bank receives senior, secured claims in order to have a strong incentive to liquidate the firm in case of trouble (see also Gorton and Kahn 2000).

The use of a tough claimholder with control rights in the case of financial straits can provide a hard budget constraint only if one of the following two conditions holds:

- (i) either the tough claimholder is unable to renegotiate with other claimholders and the entrepreneur;
- (ii) or renegotiation is feasible, but some concession can be extracted from the entrepreneur in the bargaining process through the threat of tough intervention in the case of disagreement.

It is important to note that this second possibility could not be a motivation for the diversity of claims in the model of this section. While the claimholders can obtain a concession from the entrepreneur in the form of a lower stake through the threat of abandoning the project, this concession destroys the entrepreneur's incentives sufficiently that it actually does not benefit the claimholders. The concession story can only be valid in a situation where the entrepreneur is able to make concessions that do not substantially impair her incentives.

To sum up, there is no surefire way of imposing a hard budget constraint; at this stage we mainly have at our disposal methods that in specific circumstances should, but need not, harden the budget constraint.

### 5.5.4 Sensitivity of Investment to Cash Flow

As discussed in Section 2.5.2, the empirical finding that firms' investments are sensitive to their cash flow can be either rationalized by optimal contracting considerations or viewed as evidence that managers take advantage of poor governance in order to engage in wasteful investments when they have the ability to do so. While both explanations seem relevant, we pursue the first one here.

51. This assumption is commonly made for public debt in particular. For example, building on Bulow and Shoven (1978) and White (1980), Gertner and Scharfstein (1991) emphasize the difficulty of rescheduling debt when there are many creditors.

52. As Hart and Moore show, it may be optimal to allow some dilution of existing claims because new profitable investment opportunities may arrive and need to be financed (in our model small overruns may occur even if the entrepreneur incurs cost  $B_0$ , so that it is worth allowing some reinvestment on the equilibrium path).

Recall also the debate between Fazzari et al. (1988) and Kaplan and Zingales (1997) as to whether firms with a weak balance sheet exhibit a higher sensitivity of investment to cash flow. We took a first look at this prediction in Section 3.2.7 by interpreting “cash flow” as “net worth” and observed that the theory makes no clear prediction in this regard. In that section, though, we argued that this first look has drawbacks and that firms are better viewed as ongoing entities.

The relationship

$$\rho^*(r) - E[\rho^*(r)] = \lambda \ell(r)$$

indicates that (re)investment should indeed be sensitive to cash flow: continuation or investment (in the reinterpretation in which retentions are used to finance growth prospects) are part of an optimal carrot-and-stick scheme designed to encourage the production of cash flow.<sup>53</sup>

The issue of whether the sensitivity of investment to cash flow increases with the intensity of financial constraints is more complex. In the case of small date-0 moral hazard (implying  $\Delta(r) \equiv 0$ ), and letting  $\hat{\rho} \equiv E[\rho^*(r)]$ , the constraint (IC<sub>b</sub>) can be rewritten as

$$E_r[F(\hat{\rho} + \lambda \ell(r))\ell(r)] = \frac{B_0}{\rho_1 - \rho_0}.$$

For a uniform distribution ( $F(\rho) = f \cdot \rho$ ) and using the fact that the expectation of the likelihood ratio is equal to 0, we obtain

$$\lambda E_r[\ell^2(r)] = \frac{B_0}{f(\rho_1 - \rho_0)} = \text{constant}.$$

The financial constraint impacts only the average liquidity in that, as earlier, a tighter financial constraint in general results in a shorter maturity structure:<sup>54</sup>

$$\rho^*(r | A) = \hat{\rho}(A) + \lambda \ell(r).$$

53. As discussed in Chapter 2,  $\rho^*(r)$  alternatively should increase with  $r$  even in the absence of date-0 moral hazard, if the first- and second-period revenues are correlated. A simple way to introduce this learning effect in our model would be to assume that the date-2 probability of success is  $p + \tau(r)$ , where (i)  $p = p_L$  or  $p_H$  depends, as usual, on the entrepreneur's date-1 behavior, and (ii)  $\tau$  is increasing in  $r$  (see (b) in Section 5.4.2).

54. (IR<sub>1</sub>), in the case of a uniform distribution and normalizing  $f = 1$ , can be rewritten as

$$\hat{\rho}\rho_0 - \frac{1}{2}\hat{\rho}^2 - \lambda^2 E[\frac{1}{2}\ell^2(r)] = I - A - \bar{r}.$$

Because  $\lambda$  is independent of  $A$  and  $\hat{\rho} > \rho_0$ ,  $\hat{\rho}$  increases with  $A$ .

Thus, for a uniform distribution, the sensitivity of investment to cash flow is independent of the financial constraint.<sup>55</sup> More generally, with nonuniform distributions, the sensitivity parameter  $\lambda$  may increase or decrease with  $A$ . We thus conclude that no strong prediction emerges as to the relationship between financial constraint and sensitivity of investment to cash flow.

## 5.6 Free Cash Flow

As we discussed in the introduction to this chapter, the free-cash-flow problem faced by firms with excess liquidity is the mirror image of the liquidity shortage problem faced by cash-poor ones. While the latter must contract on the provision of liquidity beyond the level provided *ex post* by the capital market, the former must design a mechanism that forces them to pay out excess cash in the future.

We first review the relationship between the liquidity shortage and the free-cash-flow problems. The problem of preventing inefficient liquidation of cash-poor firms becomes one of preventing inefficient continuation of the cash-rich firm. This results in a theory of claim maturity. The optimal contract takes the form of a mandatory payment to claimholders at date 1. As in Section 5.2.2, this payment, which can be interpreted either as a dividend as in Easterbrook (1984)<sup>56</sup> or as short-term debt as in Jensen (1986), forces the borrower to pay out the excess cash and prevents her from wasting it on sub-optimal reinvestments.

Section 5.6.2 goes beyond this reinterpretation of the liquidity shortage model by considering more complex settings in which a fixed payment is not optimal. As has been emphasized in the literature, rough instruments such as short-term debt then simultaneously allow some undesirable reinvestments and prevent some desirable ones. As we explain, optimal contracting requires the firm to use market information more fully in order to properly manage the firm's liquidity.

55. The constant-returns-to-scale model, as usual, is not appropriate to study the impact of the intensity of financial constraints on the sensitivity of investment to cash flow, since all firms are scaled-up or scaled-down versions of each other (Program NSBC depends only on  $A/I$ ). But suppose that  $I$  is fixed in Program NSBC (more generally, returns could be decreasing).

56. An early paper on dividends with a similar idea is Rozeff (1982).

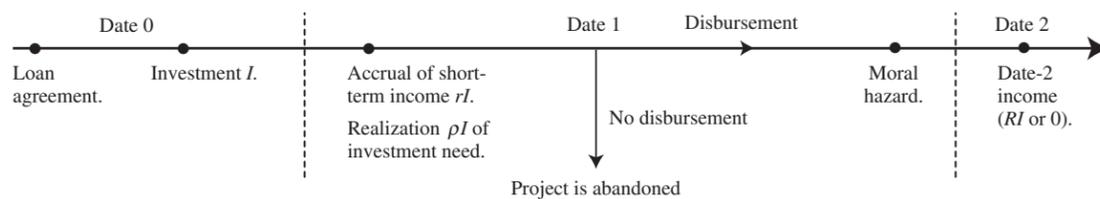


Figure 5.10

### 5.6.1 Optimal Claim Maturity

Let us return to the continuous-investment, continuous-shock version of Section 5.3.2, but with a short-term income (the analysis is not really new and is therefore only sketched): see Figure 5.10. Because the short-term income  $rI$  is fully pledgeable to the lenders, everything is *as if* the unit investment cost were equal to  $1 - r$  instead of 1. The lenders' breakeven condition, that is, the equality between expected revenue and expected investment cost, becomes

$$rI + F(\rho^*)\rho_0 I = I - A + \left[ \int_0^{\rho^*} \rho f(\rho) d\rho \right] I$$

and so

$$k(\rho^*) = \frac{1}{1 + \int_0^{\rho^*} \rho f(\rho) d\rho - [r + F(\rho^*)\rho_0]}. \quad (5.3'')$$

The margin (expected profit of the firm per unit of investment) becomes

$$m(\rho^*) = [r + F(\rho^*)\rho_1] - \left[ 1 + \int_0^{\rho^*} \rho f(\rho) d\rho \right]. \quad (5.4'')$$

And thus the borrower's (gross) utility becomes

$$U_b = m(\rho^*)k(\rho^*)A = \frac{\rho_1 - c(\rho^*)}{c(\rho^*) - \rho_0} A,$$

where the expected unit cost of effective investment,  $c(\rho^*)$ , is given by

$$c(\rho^*) = \frac{1 - r + \int_0^{\rho^*} \rho f(\rho) d\rho}{F(\rho^*)}. \quad (5.5'')$$

So the optimal threshold is given by

$$\int_0^{\rho^*} F(\rho) d\rho = 1 - r \quad (5.6'')$$

and the borrower's utility by

$$U_b = \frac{\rho_1 - \rho^*}{\rho^* - \rho_0} A. \quad (5.7'')$$

It is important to note that *the short-term income, even though it is deterministic and fully pledgeable,*

*is not equivalent to an increase in the borrower's cash on hand A.* Such an increase in equity would result in a larger investment (as is the case here), but not in a modification of the continuation rule. By contrast, condition (5.6'') shows that the larger the short-term profit, the lower the optimal threshold  $\rho^*$ . To understand this point, recall the tradeoff between increasing borrowing capacity (by choosing  $\rho^*$  close to  $\rho_0$ ) and increasing the probability of continuation (by choosing  $\rho^*$  close to  $\rho_1$ ). The short-term revenue (like a salvage value) makes it worth sacrificing continuation more in order to boost borrowing capacity. Lastly, note that the distinction between a short-term revenue and a salvage value is that the salvage value is obtained only if the investment is liquidated at date 1. And so the net expected date-2 profit and date-1 pledgeable income are  $\rho_1 - L$  and  $\rho_0 - L$  in the case of a salvage value, and  $\rho_1$  and  $\rho_0$  in the case of a short-term income. This explains the difference between, say, (5.7') and (5.7'').

(a) *Liquidity management.* Let us now turn to the implementation of the optimum and thus to the claim maturity. To this purpose we make the following assumption.<sup>57</sup>

*Free-cash-flow assumption:*  $r > \rho^*$ . Under the free-cash-flow assumption, and given that the entrepreneur cannot steal the intermediate income, the entrepreneur would reinvest excessively if she were not asked to pay out money to investors at date 1. Namely, she would reinvest as long as  $\rho \leq r$ .

To obtain the optimal amount of reinvestment, an amount  $P_1 \equiv (r - \rho^*)I$  must be pumped out of the firm, and the entrepreneur must be denied the right to dilute initial investors.

57. Of course, it must also be the case that  $\rho^* > \rho_0$  (otherwise, the borrower's borrowing capacity and utility would be infinite in this constant-returns-to-scale model). Because  $d\rho^*/dr < -1$ , we must thus also assume that  $r$  is not "too large."

*Remark (salvage value).* The analysis is again extended straightforwardly to allow for a salvage value  $L$  for the assets if the project is discontinued at date 1. The threshold  $\rho^*$  is then given by

$$\int_0^{\rho^*} F(\rho) d\rho = 1 - r - L.$$

We thus conclude that the short-term payment  $P_1 = (r - \rho^*)I$  grows faster than the salvage value.

### 5.6.2 Liquidity Management in More General Settings

The previous section considered a somewhat special setting, in which short-term debt suffices to fine-tune the firm's cash at date 1. As one might imagine, a fixed payment at date 1 in general is unlikely to be quite the right way to manage a cash-rich firm's liquidity (neither is a fixed credit line for a cash-poor firm). More instruments are needed in order to obtain the optimal state-contingent reinvestment policy. A sizeable literature has developed that shows that with rough instruments such as short-term debt there is in general a tradeoff between allowing more undesirable reinvestments and preventing more desirable ones (see, for example, Harris and Raviv 1990; Hart and Moore 1995; Stulz 1991).<sup>58</sup> The literature has not yet, to the best of my knowledge, come to grips with a general theory of liquidity management. Although we will not provide such a theory, we can make a number of observations relative to it.

*Investors' date-1 control of liquidity is unlikely to be optimal.* One might think that date-1 control by investors provides the flexibility required when a fixed payment (or a fixed credit line) does not properly adjust the firm's liquidity. We have seen, however, that investors tend to liquidate excessively (to reinvest too little), and so investors' control is unlikely to be optimal.

58. In Harris and Raviv and Stulz, short-term debt reduces free cash flow. Hart and Moore allow a more complex management of liquidity (they allow the amount of cash used at date 1 to be contingent on the date-2 revenue, which is deterministic at date 1 in their model). They do not, however, allow the firm's liquidity to be fully contingent on the market's information about variables that are realized at date 1 and which could be obtained from the value of securities or the money raised in a security issuance.

*Make full use of market information.* Consider a general environment in which a number of variables besides the liquidity shock are random and are realized and publicly observed at date 1: the first-period income  $r$ , the salvage value  $L$ , the second-period expected payoff in the case of continuation  $\rho_1$  and the date-1 pledgeable income in the case of continuation  $\rho_0$ . Suppose in a first step that these variables are verifiable by a court of law. Then the optimal contract should specify a state-contingent threshold  $\rho^*(r, L, \rho_1, \rho_0)$  beyond which reinvestment does not take place. This state-contingent threshold is straightforwardly computed by generalizing the previous analysis to random payoff values (see below for an example of such a computation).

At date 1, though, only the first-period income  $r$  is directly verifiable. The implementation of the optimal state-contingent rule requires extracting the values of  $L$ ,  $\rho_1$ , and  $\rho_0$  from the capital market. The date-1 values of the securities provide such information; in this respect, we should note that a diversity of tradable securities creates more market valuations and may be able to "span" a larger state space. But reading from market valuations is not the only way to extract information about the state of nature. For example, the acceptance of an exchange offer by a secured creditor with unpaid short-term debt (that is, an offer of securities or cash in exchange for debt forgiveness) reveals information about the salvage value  $L$  of the assets that are collateralized. Similarly, the renegotiation of existing claims embodies available information at date 1.<sup>59</sup>

Again, our aim here is not to develop a general theory of liquidity management, but rather to point out that optimal liquidity management should make use of the wealth of information held by the capital market about current and future asset values. We now illustrate this point through an example.

#### 5.6.2.1 An Illustration: *Ex Ante* Uncertainty about the Second-Period Income

Let us assume that there is *ex ante* uncertainty not only about the liquidity shock  $\rho$  but also about the

59. David (2001), for example, argues that the renegotiation of puttable securities enables the payment to their holders to be contingent on the state of nature.

second-period income in the case of success (see Exercise 5.8 for a different illustration of the use of market valuations for liquidity management). More precisely, suppose that the second-period income is equal to  $RI$  with probability  $\alpha$  and to  $(R + \Delta R)I$  (with  $\Delta R > 0$ ) with probability  $\bar{\alpha} = 1 - \alpha$ . (All of our results generalize to a continuum of possible values for the income in the case of success.) The second-period income in the case of failure is always equal to 0. So, in terms of our general modeling, there is uncertainty of magnitude  $p_H \Delta R$  with regards to both  $\rho_0$  and  $\rho_1$ . For notational simplicity, we set  $L = 0$  (no salvage value).

One can show that if all variables were verifiable at date 1, the optimal liquidity management would specify two thresholds,  $\rho^*$  when the second-period income is  $RI$  and  $\bar{\rho}^*$  when the second-period income is  $(R + \Delta R)I$ , where<sup>60</sup>

$$\bar{\rho}^* = \rho^* + p_H \Delta R.$$

As one would expect, the optimal threshold moves one-to-one with the realized increment in expected second-period income and pledgeable income.

It is clear that *short-term debt is no longer sophisticated enough to provide the firm with the appropriate amount of liquidity*. Assuming away any right for the entrepreneur to dilute initial investors, a fixed payment  $P_1$  defines a threshold,

$$\rho^* \equiv r - P_1,$$

that is independent of the news about date-2 income.

It is also easy to illustrate in this model a tradeoff that has been highlighted repeatedly in the literature. Suppose one constrains oneself to the use of short-term debt and that the firm is not allowed to conduct a seasoned offering at date 1. The optimal level of short-term debt defines a threshold equal to

the value given in (5.6''),<sup>61</sup> and satisfying

$$\bar{\rho}^* > \rho^* > \rho^*.$$

So, under the restriction to liquidity management through short-term debt, the contract must trade off insufficient reinvestment in the state in which prospects are good and excessive reinvestment in the state in which the prospects are mediocre. And, indeed, at the constrained optimum, there is excessive reinvestment when  $\rho \in (\rho^*, \bar{\rho}^*]$  and the second-period per-unit income in the case of success is  $R$  and insufficient reinvestment when  $\rho \in (\rho^*, \bar{\rho}^*]$  and the second-period per-unit income in the case of success is  $(R + \Delta R)$ .

A similar point can, of course, be made for a random date-1 income, as a fixed  $P_1$  does not pump the proper amount of money out of the firm as long as either  $r$  or  $\rho^*$  is random. This tradeoff thus suggests why (nonindexed) *debt is a more appropriate instrument for firms with safe cash flows* (regulated public utilities, banks, firms in mature industries).<sup>62</sup>

To let the reinvestment policy respond to future prospects, it is necessary to use market information about these prospects. There are several ways of doing so. Here is a simple way of relying efficiently on market information in the context of an unknown payoff in the case of success: force the entrepreneur to pay out

$$P_1 = [r - (\rho^* - \rho_0)]I$$

(if  $P_1$  is positive; otherwise contract at date 0 for a credit line at level  $-P_1$ ); and give the entrepreneur the right to dilute at date 1 existing securities in order to withstand a liquidity shock. Because the pledgeable income is equal to  $\rho_0 I$  in the mediocre state and  $(\rho_0 + p_H \Delta R)I$  in the good state,

61. The reader will check that

$$U_b = \frac{(\rho_1 + \bar{\alpha} p_H \Delta R) - \rho^*}{\rho^* - (\rho_0 + \bar{\alpha} p_H \Delta R)} A,$$

where  $\rho^*$  is given by (5.6'').

62. Jensen and Meckling (1976) argue on different grounds that firms with safe cash flows should have more debt. They are interested in the conflict of interests between shareholders and debtholders, and observe that high debt levels may induce shareholders to pursue highly risky strategies if the riskiness of income can be easily manipulated. Note that the definitions of "safe cash flows" are not quite the same in both arguments. We use "safe" in the sense of "nonrisky" while Jensen and Meckling emphasize the absence of moral hazard in the choice of riskiness.

60. We leave it to the reader to check that

$$k(\rho^*, \bar{\rho}^*) = \frac{1}{(1-r) - \alpha[F(\rho^*)\rho_0 - \int_0^{\rho^*} \rho f(\rho) d\rho] - \bar{\alpha}[F(\bar{\rho}^*)(\rho_0 + p_H \Delta R) - \int_0^{\bar{\rho}^*} \rho f(\rho) d\rho]}$$

and

$$m(\rho^*, \bar{\rho}^*) = \alpha \left[ F(\rho^*)\rho_1 - \int_0^{\rho^*} \rho f(\rho) d\rho \right] + \bar{\alpha} \left[ F(\bar{\rho}^*)(\rho_1 + p_H \Delta R) - \int_0^{\bar{\rho}^*} \rho f(\rho) d\rho \right] - (1-r).$$

the entrepreneur is able to withstand shocks up to

$$rI - [r - (\rho^* - \rho_0)]I + \rho_0 I = \rho^* I$$

in the mediocre state and

$$rI - [r - (\rho^* - \rho_0)]I + (\rho_0 + p_H \Delta R)I = \bar{\rho}^* I$$

in the good state. We have thus verified that the use of market information about date-2 income allows the implementation of the optimal state-contingent reinvestment policy.

It is clear that more sophisticated mechanisms are required to fine-tune the firm's liquidity when there is also uncertainty about the first-period income,<sup>63</sup> the salvage value, and the entrepreneur's minimum stake (which defines  $\rho_1 - \rho_0$ ). But the general message is clear: market mechanisms can supply the information that is required to implement an optimal liquidity management policy.

## 5.7 Exercises

**Exercise 5.1 (long-term contract and loan commitment).** Consider the two-project, two-period version of the fixed-investment model of Section 3.2 and a unit discount factor. Assume, say, that the borrower initially has no equity ( $A = 0$ ). Show the following.

(i) If  $p_H(p_H R - I) + (p_H R - I - p_H B / \Delta p) \geq 0$ , then the optimal long-term contract specifies a loan commitment in which the second-period project is financed at least if the first-period project is successful. Show that if  $p_H(p_H R - I) + (p_H R - I - p_H B / \Delta p) > 0$ , then the optimal long-term contract specifies that the second-period project is implemented with probability 1 in the case of first-period success, and with probability  $\xi \in (0, 1)$  in the case of failure.

(ii) In question (i), look at how  $\xi$  varies with various parameters.

(iii) Is the contract "renegotiation proof," that is, given the first-period outcome, would the parties

63. If this first-period income  $rI$  is random but exogenous (that is, not affected by moral-hazard or adverse-selection considerations), it suffices to distribute it, so as not to create a spurious dependence of the reinvestment policy on the particular realization of date-1 income. The optimal policy is clearly more complex if moral-hazard or adverse-selection considerations imply that the entrepreneur should be rewarded for high date-1 income by receiving more liquidity.

want to modify the contract to their mutual advantage?

(iv) Investigate whether the long-term contract outcome can be implemented through a sequence of short-term contracts where the first-period contract specifies that the borrower receives  $\bar{A} = I - p_H(R - B / \Delta p)$  with probability 1 in the case of success and with probability  $\xi$  in the case of failure.

**Exercise 5.2 (credit rationing, predation, and liquidity shocks).** (i) Consider the fixed-investment model. An entrepreneur has cash  $A$  and can invest  $I_1 > A$  in a project. The project's payoff is  $R_1$  in the case of success and 0 otherwise. The entrepreneur can work, in which case her private benefit is 0 and the probability of success is  $p_H$ , or shirk, in which case her private benefit is  $B_1$  and the probability of success is  $p_L$ . The project has positive NPV ( $p_H R_1 > I_1$ ), but will not be financed if the contract induces the entrepreneur to shirk. The (expected) rate of return demanded by investors is 0.

What is the threshold value of  $A$  such that the project is financed?

In the following, let

$$\rho_0^1 \equiv p_H \left( R_1 - \frac{B_1}{\Delta p} \right).$$

The next three questions add a prior period, period 0, in which the entrepreneur's equity  $A$  is determined. The discount factor between dates 0 and 1 is equal to 1.

(ii) In this question, the entrepreneur's date-1 (entire) equity is determined by her date-0 profit. This profit can take one of two values,  $a$  or  $A$ , such that

$$a < I_1 - \rho_0^1 < A.$$

At date 0, the entrepreneur faces a competitor in the product market. The competitor can "prey" or "not prey." The entrepreneur's date-0 profit is  $a$  in the case of predation and  $A$  in the absence of predation. Preying reduces the competitor's profit at date 0, but by an amount smaller than the competitor's date-1 gain from the entrepreneur's date-1 project not being funded.

- What happens if the entrepreneur waits until date 1 to go to the capital market?
- Can the entrepreneur avoid this outcome? You may want to think about a credit line from a

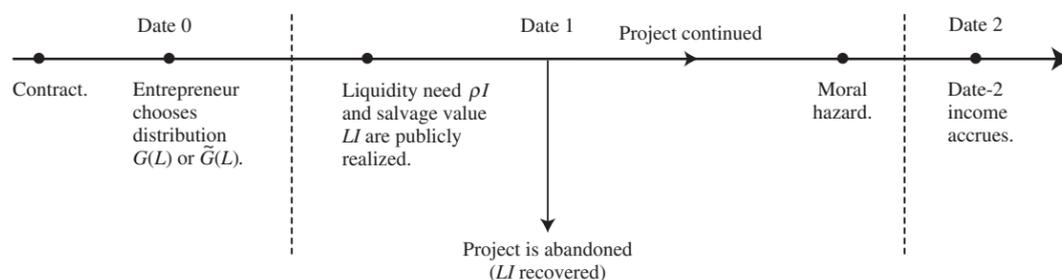


Figure 5.11

bank. Would such a credit line be credible, that is, would it be renegotiated to the mutual advantage of the entrepreneur and his investors at the end of date 0?

(iii) Forget about the competitor, but keep the assumption that the entrepreneur's date-0 profit can take the same two values,  $a$  and  $A$ . We now introduce a date-0 moral-hazard problem on the entrepreneur's side.

Assume that the entrepreneur's date-0 production involves an investment cost  $I_0$  and that the entrepreneur initially has no cash. The entrepreneur can work or shirk at date 0. Working yields no private benefit and probability of profit  $A$  equal to  $q_H$  (and probability  $1 - q_H$  of obtaining profit  $a$ ). Shirking yields private benefit  $B_0$  to the entrepreneur, but reduces the probability of profit  $A$  to  $q_L = q_H - \Delta q$  ( $0 < q_L < q_H < 1$ ). Assume that

$$I_1 + I_0 - (q_L A + (1 - q_L)a) > \rho_0^1.$$

- Interpret this condition.

Consider the following class of long-term contracts between the entrepreneur and investors. "The date-1 project is financed with probability 1 if the date-0 profit is  $A$  and with probability  $x < 1$  if this profit is  $a$ . The entrepreneur receives  $R_b = B_1/\Delta p$  if the date-1 project is financed *and* succeeds, and 0 otherwise." Assume that such contracts are not renegotiated.

- What is the optimal probability  $x^*$ ? (Assume that  $(\Delta q)p_H B_1 \geq (\Delta p)B_0$ .)
- Assuming that  $\rho_0^1 > I_1$ , is the previous contract robust to (a mutually advantageous) renegotiation?

(iv) Show that the entrepreneur cannot raise sufficient funds at date 0 if renegotiation at the end of date 0 cannot be prevented, if  $\rho_0^1 > I_1$ , and if

$$I_0 + I_1 - (q_H A + (1 - q_H)a) > \rho_0^1 - \left( \frac{q_H B_0}{\Delta q} \right).$$

**Exercise 5.3 (asset maintenance and the soft budget constraint).** Consider the variable-investment framework of Section 5.3.2, except that the date-0 moral hazard affects the per-unit salvage value  $L$ . Date-1 income is now equal to a constant (0, say). Assets are resold at price  $LI$  in the case of date-1 liquidation. The distribution of  $L$  on  $[0, \bar{L}]$  is  $G(L)$ , with density  $g(L)$ , if the borrower works at date 0, and  $\tilde{G}(L)$ , with density  $\tilde{g}(L)$ , if the borrower shirks at date 0. We assume the monotone likelihood ratio property:

$$\frac{g(L)}{\tilde{g}(L)} \text{ is increasing in } L.$$

The borrower enjoys date-0 private benefit  $B_0 I$  if she shirks, and 0 if she shirks. The timing is summarized in Figure 5.11.

As usual, let  $\rho_1 \equiv p_H R$  and  $\rho_0 = p_H(R - B/\Delta p)$ . And let

$$\ell(L) \equiv \frac{g(L) - \tilde{g}(L)}{g(L)}.$$

(i) Determine the optimal contract  $\{\rho^*(L), \Delta(L)\}$  (where  $\rho^*(L)$  and  $\Delta(L)$  are the state-contingent threshold and extra rent (see Section 5.5.2)) in the absence of the soft budget constraint (that is, the commitment to the contract is credible). Show that

- $\rho^*(L) = -L + (\rho_1 + \mu\rho_0 + \nu(\rho_1 - \rho_0)\ell(L))/(1 + \mu)$  for some positive  $\mu$  and  $\nu$ ;
- $\Delta(L) = 0$  as long as  $\rho^*(L) \leq \rho_1 - L$ ;
- conclude as to when rewards take the form of an increased likelihood of continuation or cash (or both).

(ii) When would the investors want to rescue the firm at date 1 if it has insufficient liquidity? Draw  $\rho^*(L)$  and use a diagram to provide a heuristic description of the soft-budget-constraint problem. Show that the soft budget constraint arises for  $L \leq L_0$  for some  $L_0 \geq 0$ .

**Exercise 5.4 (long-term prospects and the soft budget constraint).** Perform the same analysis as in Exercise 5.3, with the difference that the date-0 choice of the entrepreneur does not affect the salvage value, which is always equal to 0. Rather, the date-0 moral hazard refers to the choice of the distribution of the second-period income in the case of continuation. This income is  $R_L \geq 0$  or  $R_H = R_L + R$ , where  $R \geq 0$  is a constant. The distribution of  $R_L$ ,  $G(R_L)$ , or  $\bar{G}(R_L)$  is determined at date 0. Assume that  $g(R_L)/\bar{g}(R_L)$  is increasing in  $R_L$ . As usual, let  $p_H$  and  $p_L$  denote the probabilities of  $R_H$  when the entrepreneur works or shirks *ex post*. And let  $\rho_1 \equiv p_H R$  and  $\rho_0 = p_H(R - B/\Delta p)$ . Assume that  $R_L$  is publicly revealed at date 1 before the continuation decision. Solve for the optimal state-contingent policy in the absence of the soft-budget-constraint problem. Show that the soft-budget-constraint problem arises (if it arises at all) under some threshold value of  $R_L$ .

**Exercise 5.5 (liquidity needs and pricing of liquid assets).** Consider the liquidity-needs model with a *fixed investment* and *two possible liquidity shocks*. The borrower has cash  $A$  and wants to finance a fixed-size investment  $I > A$  at date 0. At date 1, a cash infusion equal to  $\rho$  is needed in order for the project to continue. If  $\rho$  is not invested at date 1, the project stops and yields nothing. If  $\rho$  is invested, the borrower chooses between working (no private benefit, probability of success  $p_H$ ) and shirking (private benefit  $B$ , probability of success  $p_L = p_H - \Delta p$ ). The project then yields, at date 2,  $R$  in the case of success and 0 in the case of failure.

The liquidity shock is equal to  $\rho_L$  with probability  $(1 - \lambda)$  and to  $\rho_H$  with probability  $\lambda$ , where

$$\rho_L < \rho_0 < \rho_H < \rho_1,$$

where  $\rho_1 \equiv p_H R$  and  $\rho_0 \equiv p_H(R - B/\Delta p)$ . Assume further that

$$\rho_0 - \rho_L > I - A. \quad (1)$$

There is a single liquid asset, Treasury bonds. A Treasury bond yields 1 unit of income for certain at date 1 (and none at dates 0 and 2). It is sold at date 0 at price  $q \geq 1$ . (The investors' rate of time preference is equal to 0.)

(i) Suppose that the firm has the choice between buying enough Treasury bonds to withstand the high liquidity shock and buying none. Show that it chooses to hoard liquidity if

$$(q - 1)(\rho_H - \rho_0) \leq (1 - \lambda)(\rho_0 - \rho_L) - \lambda(\rho_H - \rho_0) - I + A \quad (2)$$

and

$$(q - 1)(\rho_H - \rho_0) \leq \lambda(\rho_1 - \rho_H). \quad (3)$$

(ii) Suppose that the economy is composed of a continuum, with mass 1, of identical firms with characteristics as described above. The liquidity shocks of the firms are perfectly correlated. There are  $T$  Treasury bonds in the economy, with  $T < \rho_H - \rho_0$ . Show that when  $\lambda$  is small, the liquidity premium  $(q - 1)$  commanded by Treasury bonds is proportional to the probability of a high liquidity shock. (Hint: show that either (2) or (3) must be binding, and use (1) to conclude that (3) is binding.)

(iii) Suppose that, in the economy considered in the previous subquestion, the government issues at date 0 not only the  $T$  Treasury bonds, but also a security that yields at date 1 a payoff equal to 1 in the good state (the firms experience liquidity shock  $\rho_L$ ) and 0 in the bad state (the firms experience liquidity shock  $\rho_H$ ). What is the equilibrium date-0 price  $q'$  of this new asset? (Prices of the Treasury bonds and of this new asset are market clearing prices.)

**Exercise 5.6 (continuous entrepreneurial effort; liquidity needs).** (i) An entrepreneur with initial cash  $A$  and protected by limited liability wants to invest in a fixed-size project with investment cost  $I > A$ . After the investment is made, the entrepreneur chooses the probability  $p$  of success ( $0 \leq p \leq 1$ ); the disutility of effort is  $g(p) = \frac{1}{2}p^2$ . (The entrepreneur enjoys no private benefit in this model.) In question (i) only, the profit is  $R = 2\sqrt{I - A}$  in the case of success and 0 in the case of failure. (We assume that  $R < 1$  to avoid considering probabilities of success exceeding 1.  $R$  takes an arbitrary value in question (ii).) As usual, the uninformed investors demand

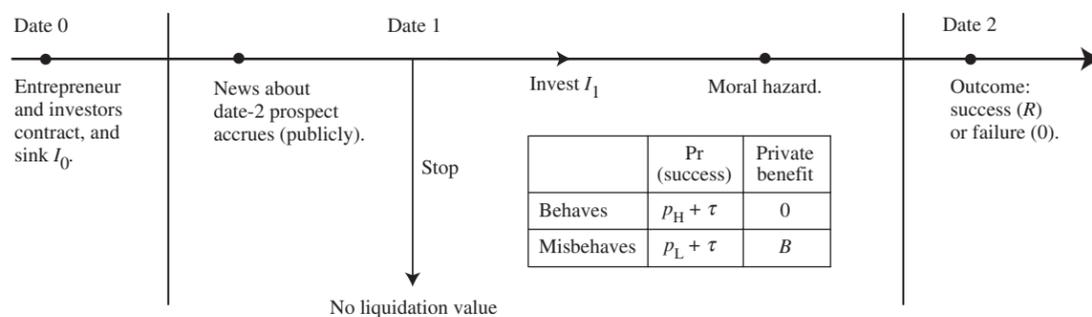


Figure 5.12

an expected rate of interest equal to 0 and everyone is risk neutral. Let  $R_b$  denote the entrepreneur's reward in the case of success.

Solve for the optimal contract ( $R_b$ ). Show that

$$R_b = \frac{1}{2}R.$$

(ii) Now introduce an intermediate liquidity shock  $\rho$  (for a now arbitrary level  $A$  of cash on hand). The cumulative distribution of  $\rho$  is  $F(\rho)$  on  $[0, \infty)$  and the density  $f(\rho)$ . The effort decision is made *after* the value of  $\rho$  is realized and, of course, *conditional* on the choice of continuing (incurring reinvestment cost  $\rho$ ). Suppose that the entrepreneur's stake in the case of continuation ( $R_b$ ) is independent of  $\rho$ . Write the investors' breakeven condition. Write the optimization program yielding  $(R_b, \rho^*)$ , where  $\rho^*$  is the cutoff liquidity shock.

**Exercise 5.7 (decreasing returns to scale).** Extend the treatment of Section 5.6.1 to the case of decreasing returns to scale: the payoff in the case of continuation and success is  $R(I)$ , with  $R(0) = 0$ ,  $R' > 0$ ,  $R'' < 0$ ,  $R'(0) = \infty$ , and  $R'(\infty) = 0$ . The rest is unchanged (the short-term income is  $rI$ , the reinvestment need is  $\rho I$ , and the private benefit is  $BI$ ).

(i) What are the first-order conditions yielding the optimal investment level  $I$  and cutoff  $\rho^*$ ?

(ii) Assuming that  $r > \rho^*$ , and that  $(R(I)/I - R'(I))$  is increasing in  $I$  (a condition satisfied, for example, by  $R(I)$  quadratic), derive the impact of the strength of the balance sheet as measured, say, by  $A$  on debt maturity.

**Exercise 5.8 (multistage investment with interim accrual of information about prospects).** In this

chapter we have focused mostly on the case of shocks about the reinvestment need (cost overruns, say). Consider, instead, the case of news about the final profitability. In the two-outcome framework, news can accrue about either the probability of success or the payoff in the case of success. We consider both, in sequence. The investment is a multistage one: let

$$I = I_0 + I_1,$$

where  $I_0$  is the date-0 investment and  $I_1$  is the date-1 reinvestment. In contrast with  $I_0$ ,  $I_1$  is not incurred if the firm decides to stop. The timing is as in Figure 5.12.

As usual, the entrepreneur has initial wealth  $A$ , is risk neutral, and protected by limited liability. Investors are risk neutral. The discount rate is equal to 0. If reinvestment cost  $I_1$  is sunk at date 1, then the firm can continue. Misbehavior reduces the probability of success by  $\Delta p$ , but yields private benefit  $B$  to the entrepreneur.

Assume

$$B < (\Delta p)R.$$

As announced, we consider two variants.

(a) *News about the probability of success.*  $R$  is known at date 0, but the probability of success is  $p_H + \tau$  in the case of good behavior and  $p_L + \tau$  in the case of misbehavior, where  $\tau$  is publicly learned at the beginning of date 1. The random variable  $\tau$  is distributed according to the distribution function  $F(\tau)$  with density  $f(\tau)$  on  $[\underline{\tau}, \bar{\tau}] = [-p_L, 1 - p_H]$  (to keep probabilities in the interval  $[0, 1]$ ). Let  $\tau^e$  denote the expectation of  $\tau$ .

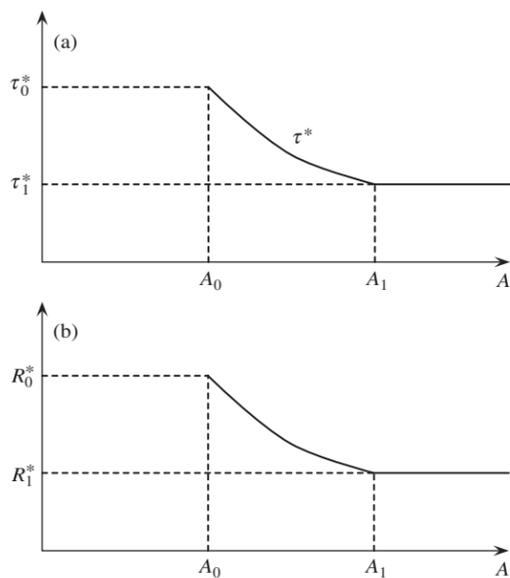


Figure 5.13

(b) *News about the payoff in the case of success.* The probabilities of success are known:  $p_H$  and  $p_L$  (normalize:  $\tau = 0$ ). By contrast, the profit  $R$  in the case of success is drawn from distribution  $G(R)$  with density  $g(R)$  on  $(0, \infty)$ . (The profit in the case of failure is always equal to 0.)

(i) For each variant, show that there exist two thresholds,  $A_0$  and  $A_1$ ,  $A_0 < A_1$ , such that the first best prevails for  $A \geq A_1$  and financing is secured if and only if  $A \geq A_0$ . Show that the continuation rules take the form of cutoffs, as described in Figure 5.13.

Determine  $\tau_0^*$ ,  $\tau_1^*$ ,  $R_0^*$ ,  $R_1^*$ .

(ii) For each variant, assume that  $A = A_0$ . Let  $y \equiv (p_H + \tau)R$  denote the expected income, and  $\mathcal{R}(y)$  denote the entrepreneur's rent in the case of continuation. Show that (above the threshold  $y^*$ )

- $0 < \mathcal{R}'(y) < 1$  in variant (a);
- $\mathcal{R}$  is constant in variant (b).

**Exercise 5.9 (the priority game: uncoordinated lending leads to a short-term bias).** This chapter, like Chapters 3 and 4, has assumed that the firm's balance sheet is transparent. In particular, each investor has perfect knowledge of loans made by other lenders and of the firm's obligations to them.

This exercise argues that uncoordinated lending leads to financing that is too oriented to the short term. In a nutshell, lenders, by cashing out early, exert a negative externality on other lenders. Because this externality is not internalized, the resulting financial structure contains too much short-term debt.

We consider a three-period model:  $t = 0, 1, 2$ . The entrepreneur has no cash ( $A = 0$ ), is risk neutral, and is protected by limited liability. At date 0, a fixed investment  $I$  is made. The project yields a known return  $r > 0$  at date 1, and an uncertain return ( $R$  or 0) at date 2. Because the point is quite general and does not require credit constraints, we assume away moral hazard; or, equivalently, the private benefit from misbehaving is 0. The probability of a date 2 success is

$$p + \tau(I_1),$$

where  $I_1$  is the date-1 deepening investment, equal to  $r$  minus the level of short-term debt repaid to lenders and the date-1 payment to the entrepreneur (the firm does not return to the capital market at date 1), and  $\tau$  is an increasing and concave function (with  $\tau'(0) = \infty$ ). Assume that  $\tau'(r)R < 1$ .

We assume that the entrepreneur cannot engage in "fraud," that is, cannot fail to honor the short-term debt and, if the project succeeds at date 2, the long-term debt. By contrast, obligations to lenders, and in particular  $I_1$ , cannot be verified as the firm's balance sheet is opaque.

(i) Derive the first-best investment  $I_1^*$ . Show how this allocation can be implemented by a mixture of short- and long-term debt (note that in this model without moral hazard the structure of compensation for the entrepreneur exhibits a degree of indeterminacy).

(ii) Assume that  $r - I_1^* < I$  (creditors must hold long-term debt). Suppose next that financing is not transparent. Start from the first-best solution, with a large number (a continuum of mass 1) of lenders, with the representative lender owning short-term claim  $\tau_1$  and contingent long-term claim  $R_1$  on the firm.

Show that the entrepreneur has an incentive to secretly collude with any lender to increase the latter's short-term claim in exchange for a smaller long-term claim.

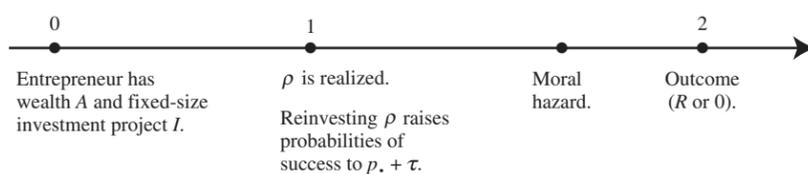


Figure 5.14

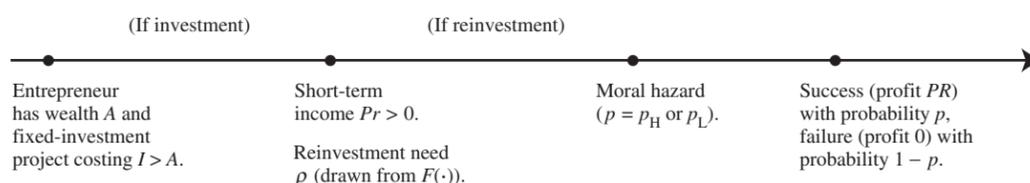


Figure 5.15

Given the constraint that financing is provided by many lenders and that the latter do not observe each other's contracts, is the indeterminacy mentioned in question (i) resolved?

**Exercise 5.10 (liquidity and deepening investment).** (i) Consider the fixed-investment model. The entrepreneur has cash  $A$  and can invest  $I > A$  in a project. The project's return in the case of success (respectively, failure) is  $R$  (respectively,  $0$ ). The probability of success is  $p_H$  if the entrepreneur behaves (she then gets no private benefit) and  $p_L = p_H - \Delta p$  if she misbehaves (in which case she gets private benefit  $B$ ). In this subquestion and in the subsequent extension, one will assume that the project is viable only if the incentive scheme induces the entrepreneur to behave. The entrepreneur and the capital market are risk neutral; the entrepreneur is protected by limited liability; and the market rate of interest is equal to  $0$ .

Let

$$\rho_1 \equiv p_H R \quad \text{and} \quad \rho_0 \equiv p_H [R - B/\Delta p],$$

and assume  $\rho_1 > I > \rho_0$ .

What is the necessary and sufficient condition for the project to be financed?

(ii) Now add an intermediate stage, in which there is an option to make a deepening investment. This investment increases the probability of success to  $p_H + \tau$  (in the case of good behavior) and  $p_L + \tau$  (in the case of misbehavior).

If the deepening investment is not made, the probabilities of success remain  $p_H$  and  $p_L$ , respectively. This deepening investment costs  $\rho$ , where  $\rho$  is unknown *ex ante* and distributed according to distribution  $F(\rho)$  and density  $f(\rho)$  on  $[0, \infty)$ . The timing is summarized in Figure 5.14.

Let  $\mu \equiv \tau/p_H$ ,  $\hat{\rho}_1 \equiv \mu\rho_1$ , and  $\hat{\rho}_0 \equiv \mu\rho_0$ .

Write the incentive compatibility constraint and (for a given cutoff  $\rho^*$ ) the investors' breakeven condition.

(iii) What is the optimal cutoff  $\rho^*$ ? (Hint: consider three cases, depending on whether

$$\rho_0 [1 + \mu F(\hat{\rho}_k)] \leq I - A + \int_0^{\hat{\rho}_k} \rho f(\rho) d\rho,$$

with  $k = 0, 1$ .)

(iv) Should the firm content itself with returning to the capital market at date 1 in order to finance the deepening investment (if any)?

**Exercise 5.11 (should debt contracts be indexed to output prices?).** This exercise returns to optimal corporate risk management when profits are positively serially correlated (see Section 5.4.2). The source of serial correlation is now a permanent shift in the market price of output, as summarized in Figure 5.15. The model is the fixed-investment model, except that the date-1 and date-2 incomes depend on an exogenous market price  $P$ , with mean  $\bar{P}$ , that is realized at date 1. The realizations of  $P$  and  $\rho$  are independent.

The rest of the model is otherwise the same as in Section 5.2. Following the steps of Section 5.4.2:

(i) Determine the optimal reinvestment policy  $\rho^*(P)$ .

(ii) Show that, accounting for seasoned offerings, the optimal debt is *fully indexed debt*:

$$d(P) = Pr - \ell_0,$$

where  $\ell_0$  is a positive constant.

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