

## Aggregate Liquidity Shortages and Liquidity Asset Pricing

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### 15.1 Introduction

#### 15.1.1 The Investors' Commitment Problem and the Demand for Stores of Value

As stressed repeatedly throughout this book, agency problems deprive firms of a proper access to finance. Despite the many strategies designed to boost pledgeable income, firms often cannot invest as much and under the same conditions as they would if they did not need outside funding.

This chapter shows that agency may generate an additional source of inefficiency. Namely, production plans that generate a positive present discounted value (PDV) of *pledgeable* (i.e., investor) income, estimated at the investors' intertemporal marginal rate of substitution (IMRS), may not be feasible.

This new departure from the Arrow-Debreu paradigm arises when firms face sequential financing needs and agency will in the future continue to create a wedge between value and pledgeable income, that is, when refinancing is not a foregone conclusion.

A key difference between credit rationing at the initial financing stage and credit rationing at later refinancing stages is that the latter can be planned and addressed.

In Chapter 5, though, we assumed either (a) that investors could commit to refinance the firm out of their own future income as specified in the initial contract or, alternatively, (b) that stores of value could be set aside that could be called upon to allow investors to fulfill their contractual refinancing obligations. By contrast, in a world in which investors cannot pledge their future human capital (or, for some of them, are not yet born), investors may be

unable to commit to inject new funds as required in the future unless there exists in the economy a sufficient quantity of stores of value that enable investor commitment.

In an *efficient production plan*, each entrepreneur maximizes her utility subject to the investors' intertemporal budget constraint: the PDV of investors' income net of investments, assessed at the investors' IMRS, must be nonnegative.

An efficient production plan may require a transfer of wealth across states of nature or across time: the firm may need cash injections in the future in adverse states of nature, while being a source of cash for the investors in more favorable ones; or the firm may have excess cash in some period that it would like to carry over to later periods if earnings and investment opportunities are asynchronized.

Two kinds of stores of value or liquid assets allow firms to transfer wealth across states of nature or across time:

*Inside liquidity*, namely, liquidity created by the corporate sector through the issuance of claims on its future cash flow—equity and debt claims in firms that other firms can use as stores of value and resell when funds are needed. Among other things, this chapter asks whether the corporate sector as a whole creates enough stores of value on its own.

*Outside liquidity*, namely, liquidity generated “outside” the corporate sector—land or other natural resources, or rents already existing in the economy;<sup>1</sup> or, as will be discussed in Section 15.3.3, government-created liquidity such as Treasury securities.

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1. The distinction between inside and outside liquidity is not as clear cut as it would seem. In practice, some of the existing rents have been created by the corporate sector. In the end, though, what matters is the total amount of stores of value that can be harnessed to operate the future wealth transfers.

### 15.1.2 Chapter Outline

Section 15.2 analyzes transfers of wealth across states of nature. To measure inside liquidity, it embeds the corporate-liquidity-demand framework of Chapter 5 in a general equilibrium setting. As in Chapter 5, a borrowing firm anticipates the accrual of liquidity needs later on. Concerned about being rationed by the credit market in the future, it optimally demands some insurance against it; that is, it secures liquidity that it will be able to use in adverse circumstances. As we just discussed, Chapter 5, however, assumed either that current investors were able to commit to bring the funds even when reinvesting augments their losses, or that there existed a sufficient amount of stores of value to allow investors to abide by their promise.

Section 15.2 therefore raises the sufficiency question: does the volume of equity and debt claims on the corporate sector suffice to resolve the investors' commitment problem and thereby to allow entrepreneurs to achieve their efficient production plan? A simple and general self-sufficiency result emerges: even in the absence of outside stores of value, efficient production plans can be implemented provided that (a) the liquidity needs are independently distributed across firms (there is no aggregate shock), and (b) the existing liquidity is pooled among firms and dispatched through a system of credit lines (liquidity is not wasted).

Section 15.3, in contrast, shows that aggregate uncertainty creates scope for a shortage of inside liquidity even if it is pooled and dispatched properly. This lack of self-sufficiency introduces a role for outside liquidity and generates liquidity premia (i.e., a market return below the interest rate predicted by the IMRS) for assets that are used as stores of value. Asset prices are then determined not solely by the assets' stochastic yields and the consumers' IMRS, but also by their consumption or supply of liquidity services.

Finally, Section 15.4 shows that similar insights arise even in the absence of aggregate uncertainty provided that firms have asynchronized income and projects and are at times net lenders, so that they must transfer wealth forward in time.

Most of this chapter, except the end of Section 15.4, will focus on a three-period setting:  $t = 0$ ,

1, 2. All economic agents (entrepreneurs, investors) will have preferences over consumption streams  $\{c_0, c_1, c_2\}$  such that  $c_t \geq 0$  for all  $t$ :

$$U = c_0 + c_1 + c_2.$$

In particular, the investors' IMRS is equal to 1, i.e., consumers demand a rate of return equal to 0. Any return above or below 0 will therefore be attributable to a liquidity service or consumption. (The end of Section 15.4 will consider an extension to the infinite-horizon setting, with the natural generalization of preferences:  $U = \sum_t \beta^t c_t$  with discount factor  $\beta < 1$ ; the IMRS is then equal to  $\beta$ .)

(This chapter borrows particularly heavily from my joint work with Bengt Holmström (see Holmström and Tirole 1996, 1998, 2001, 2002, 2005) and from the many discussions about the literature that I have had with him.)

### 15.2 Moving Wealth across States of Nature: When Is Inside Liquidity Sufficient?

This section argues that the corporate sector as a whole creates enough liquidity to sustain an efficient production plan provided that

- the corporate sector is a net borrower,
- there is no economy-wide shock, and
- liquidity is dispatched properly within the corporate sector.

The third assumption will be discussed in this section, while the first and the second will be relaxed in Sections 15.4 and 15.3, respectively.

#### 15.2.1 The Sufficiency Result

##### 15.2.1.1 Model

We first illustrate the sufficiency result in the context of the two-shock, variable-investment version of Section 15.3.1,<sup>2</sup> and then point at the generality of the result.

There are three periods,  $t = 0, 1, 2$ . Investors are risk neutral and the market rate of interest in the economy is 0. The economy is also populated by a

2. Although the reader may want to return to Section 5.3.1 in order to refresh his/her memory, the presentation here is entirely self-contained.

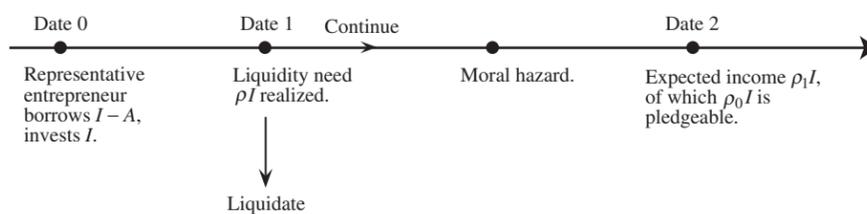


Figure 15.1

large number—technically a continuum of mass 1—of *ex ante* identical, risk-neutral entrepreneurs. The representative entrepreneur at date 0 has wealth  $A$ , borrows  $I - A$ , and invests  $I$ . At date 1, a given firm faces liquidity shock  $\rho I$ , with

$$\rho = \begin{cases} \rho_L & \text{with probability } 1 - \lambda \text{ (healthy firm),} \\ \rho_H > \rho_L & \text{with probability } \lambda \text{ (firm in distress).} \end{cases}$$

The firm can continue only if it finds funds to defray its liquidity shock; otherwise, it is liquidated. We normalize the liquidation value at 0.

In the case of continuation, the firm's date-2 expected income is denoted by  $\rho_1 I$ , of which only  $\rho_0 I < \rho_1 I$  is pledgeable to investors (see Figure 15.1).<sup>3</sup>

#### 15.2.1.2 Efficient Allocation

Let us assume that

$$\rho_L < \rho_0 < \rho_H < \rho_1 \quad (15.1)$$

and

$$(1 - \lambda)(\rho_H - \rho_L) < 1. \quad (15.2)$$

Let us first discuss condition (15.1). Note that a firm can, at date 1, raise  $\rho_0$  per unit of investment by returning to the capital market and by issuing new claims on date-2 profit (i.e., by diluting the claims of the date-0 investors in the firm). The inequality  $\rho_H > \rho_0$  means that in the bad (high-shock) state of nature, the “wait-and-see” policy of returning to the capital market if needed will not suffice to cover the high realization of the liquidity shock. The condition  $\rho_H < \rho_1$  means that continuation is *ex post*

socially desirable even in the case of a high liquidity shock. Lastly, we assume that  $\rho_L < \rho_0$ . Otherwise the liquidity shock would always exceed the pledgeable income and so investors could never recoup their date-0 investment and would not lend, which would violate the net-borrowing assumption.

As was stressed in Chapter 5, there is a trade-off between investment scale and continuation. That continuation in the high-liquidity-shock state is *ex post* socially desirable ( $\rho_1 > \rho_H$ ) does not imply that it is *ex ante* optimal for the entrepreneur. Continuation in the high-liquidity-shock state is costly to investors ( $\rho_H > \rho_0$ ), making them less eager to fund investment at a given investment scale and forcing the entrepreneur to reduce investment size. Condition (15.2) implies, as we will show, that the high liquidity shock is small enough and sufficiently frequent that the entrepreneur is willing to accept a lower investment scale in exchange for being able to continue when  $\rho = \rho_H$ .

Because investors lose money (even abstracting from any contribution to the initial investment) in the event of an adverse shock ( $\rho_H > \rho_0$ ), they would never by themselves refinance the firm at date 1 in that state of nature. Let us in a first step ignore this difficulty (which is, however, central to the insights of this chapter), and assume that somehow investors can commit to any probability  $x$  in  $[0, 1]$  of continuation in the adverse state of nature and that they do not demand an extra return for this (that is, they just want to recoup the extra loss  $x(\rho_H - \rho_0)I$  induced by continuation in that state). One can, for example, imagine that there exists in the economy a sufficient quantity of stores of value that in exchange for 1 unit of good at date 0 deliver 1 unit of good at date 1.<sup>4</sup>

3. The wedge between date-2 value and pledgeable income can, as in Chapter 5, be motivated by moral-hazard considerations: the firm yields  $RI$  with probability  $p$  and 0 with probability  $1 - p$ . The probability  $p$  of success is equal to  $p_H$  if the entrepreneur behaves and  $p_L = p_H - \Delta p$  if she misbehaves. Letting  $BI$  denote the entrepreneur's private benefit in the case of misbehavior, the entrepreneur must be given reward  $R_b$  in the case of success such that  $(\Delta p)R_b \geq BI$ , and so  $\rho_1 \equiv p_H R$  and  $\rho_0 \equiv p_H [R - B/\Delta p]$ .

4. Or, for that matter, at date 2, since consumers at date 1 are willing to pay 1 for an asset that yields 1 at date 2.

The investors' outlay at date 0 is equal to  $I - A$ , their expected outlay at date 1 is  $((1 - \lambda)\rho_L + \lambda\rho_H x)I$ , and their expected income is  $(1 - \lambda + \lambda x)\rho_0 I$ . Hence, the investors' breakeven constraint is

$$[1 + (1 - \lambda)\rho_L + \lambda\rho_H x]I - A \leq [1 - \lambda + \lambda x]\rho_0 I.$$

The *efficient allocation* is defined as the one that maximizes the representative entrepreneur's utility subject to the constraint that investors break even at their IMRS. Note that such an allocation is efficient given the existence of an agency cost (put differently, it is "constrained efficient"). This efficient allocation solves

$$\max_{\{I, x\}} \{(1 - \lambda + \lambda x)(\rho_1 - \rho_0)I\}$$

s.t.

$$[1 + (1 - \lambda)\rho_L + \lambda\rho_H x]I - A \leq [1 - \lambda + \lambda x]\rho_0 I.$$

Using the investors' breakeven constraint (satisfied with equality), we can compute  $I$  as a function of  $x$ :

$$I = \frac{1}{(1 + (1 - \lambda)\rho_L + \lambda\rho_H x) - (1 - \lambda + \lambda x)\rho_0} A.$$

And so the efficient allocation is given by

$$\max_{\{x\}} \left\{ \frac{(1 - \lambda + \lambda x)(\rho_1 - \rho_0)}{(1 + (1 - \lambda)\rho_L + \lambda\rho_H x) - (1 - \lambda + \lambda x)\rho_0} A \right\}.$$

As in Chapter 5, consider the "unit cost of effective investment," that is, the average cost of bringing 1 unit of investment to completion. If  $x$  is the probability of continuation at date 1 in the high-shock state, then the total—investment plus reinvestment—cost per unit of initial investment is  $1 + (1 - \lambda)\rho_L + \lambda\rho_H x$ , yielding a total probability of continuation<sup>5</sup> equal to  $1 - \lambda + \lambda x$ . And so the unit cost of effective investment is

$$c(x) \equiv \frac{1 + (1 - \lambda)\rho_L + \lambda\rho_H x}{1 - \lambda + \lambda x}.$$

The program yielding the efficient allocation becomes

$$\max_{\{x\}} \left\{ \frac{(\rho_1 - \rho_0)A}{c(x) - \rho_0} \right\}.$$

The optimal  $x$  must therefore minimize  $c(x)$ , which, together with condition (15.2), implies that

$$x = 1.$$

5. Alternatively,  $x$  could denote the fraction of the initial investment that is not liquidated (i.e.,  $1 - x$  is the downsizing intensity).

Let us now write the investors' breakeven constraint for the policy of never liquidating:

$$[(1 - \lambda)(\rho_0 - \rho_L) + \lambda(\rho_0 - \rho_H)]I = I - A$$

or

$$(\rho_0 - \bar{\rho})I = I - A,$$

where

$$\bar{\rho} \equiv (1 - \lambda)\rho_L + \lambda\rho_H$$

is the expected liquidity shock per unit of investment.

Lastly, we assume that

$$\rho_1 > 1 + \bar{\rho},$$

and so entrepreneurs prefer investing to consuming  $A$  (the project's NPV is positive).

To sum up, the efficient allocation is given by

$$x = 1 \quad \text{and} \quad I = \frac{A}{(1 + \bar{\rho}) - \rho_0}.$$

### 15.2.1.3 The Sufficiency Result

Next we assume that investors cannot pledge their future earnings and therefore cannot directly commit to reinject cash at date 1 in the firm when they lose money on this reinjection ( $\rho_H > \rho_0$ ); we ask whether the efficient allocation can nevertheless be implemented.

Assume that the shocks are drawn independently across firms, and so there is no macroeconomic uncertainty. Because the corporate sector is a net borrower,

$$I - A > 0,$$

the investors' breakeven condition implies that investors' profit in the healthy firms (those facing a low shock at date 1) more than offsets the loss that they incur in the others:

$$(1 - \lambda)(\rho_0 - \rho_L) > \lambda(\rho_H - \rho_0). \quad (15.3)$$

We can define (gross) *inside* liquidity as the value,  $(1 - \lambda)(\rho_0 - \rho_L)I$ , of healthy firms. Condition (15.3) states that the (gross) inside liquidity exceeds the net refinancing need,  $\lambda(\rho_H - \rho_0)I$ , of firms facing a high liquidity shock (and so the net amount of inside liquidity, namely, the difference between gross outside liquidity and the net refinancing need, is positive).

Put differently, the corporate sector's long-term investments create enough stores of value in the form of tradable rights to pledgeable date-2 profits that the policy that is optimal when the future is

discounted at the investors' rate of time preference can be implemented:

- in the absence of outside stores of value,
- without any need for the corporate sector to create inefficient stores of value.<sup>6</sup>

The reasoning is completely general: as long as the corporate sector is a net borrower, the net value of investors' date-1 claims on the corporate sector must be strictly positive, which implies that reinvestments can be financed through the value of existing shares.

As we will see, this property need not hold in the presence of an aggregate shock.

### 15.2.2 Wasting Inside Liquidity

The sufficiency result by itself only states that the corporate sector produces enough inside liquidity to support its optimal reinvestment policy. It is silent on *how* the latter can be implemented.

Let us first point out that the "natural implementation," namely, the policy that consists in each firm holding the stock index, that is, a representative portfolio of claims on all firms in the economy, in general does not work. The date-1 value of the index is

$$(\rho_0 - \bar{\rho})I.$$

To see this, note that the corporate sector will return  $\rho_0 I$  to investors at date 2. The average reinvestment cost, however, is  $((1 - \lambda)\rho_L + \lambda\rho_H)I = \bar{\rho}I$ , which must be financed at date 1 by issuing new shares on the corporate sector date-2 income and thereby diluting existing shareholders.

Thus, if all firms hold equal shares in the index,<sup>7</sup> those with a high liquidity shock can meet it by reselling shares if and only if<sup>8</sup>

$$(\rho_H - \rho_0)I \leq (\rho_0 - \bar{\rho})I$$

6. For example, such inefficient inside liquidity could take the form of short-term investments that deliver less than 1 unit of good at date 1 per unit of investment at date 0.

7. Implementing the optimal policy would be even more difficult with unequal shares, since the firms with fewer-than-average shares would have a harder time satisfying (15.4) below.

8. A related way to derive the same inequality goes as follows. The stock index at the end of date 1, as we noted, has value  $\rho_0 I$ . And so the total value for investors of a firm that holds the index is its own value plus the index, or  $2\rho_0 I$ . However, it must sell some of this stake to meet its liquidity shock  $\rho_i I$ , where  $i \in \{L, H\}$ . Hence, the firm's pledgeable wealth at the end of date 1,  $2\rho_0 I - E(\rho_i I) = (2\rho_0 - \bar{\rho})I$ , must exceed the high liquidity need,  $\rho_H I$ .

or

$$\rho_H + \bar{\rho} \leq 2\rho_0. \quad (15.4)$$

If  $\rho_H + \bar{\rho} > 2\rho_0$ , then holding the stock index does not allow the firms facing the high liquidity shock to continue.<sup>9</sup>

Why does this "self-provision" of liquidity result in a waste of liquidity? Firms that face a low liquidity shock at date 1 have excess cash for two reasons: first (reasoning in terms of per unit of investment), they can raise up to  $\rho_0$  when they need only  $\rho_L$ ; second, they have invested in the stock index, with resulting value  $\rho_0 - \bar{\rho}$ . This excess liquidity is, of course, not fully wasted as the extra profit is partly reappropriated by distressed firms that own a fraction of the stock index and therefore own part of the healthy firms. Still, healthy firms do have excess liquidity. At date 1, they either redistribute the excess cash to or invest it on behalf of their owners; they have no incentive to invest in distressed firms, in which the reinvestment cost exceeds the pledgeable income.

Readers familiar with the treatment of corporate liquidity demand in Chapter 5, on the one hand, and with the Diamond-Dybvig model of consumer liquidity demand of Chapter 12, on the other, will intuit the rational response to this potential waste: in order to force healthy firms to redispense cash to distressed ones at date 1, firms must at date 0 pool their liquidity and organize a system of (or akin to) *credit lines*. For example,<sup>10</sup> shares are deposited with one (or an arbitrary number of) financial institutions; each firm is then entitled to draw on a credit line up to a cap of  $\rho_H I$ .<sup>11</sup> The financial institution can raise the cash needed to honor the credit lines by selling at date 1 shares it holds in firms to date-1 investors.<sup>12</sup> From the net-borrower assumption, the

9. Or forces them to downsize if part of the investment can be abandoned without impacting the rest of the investment.

10. We assume that firms cannot misuse their credit lines, say, by demanding more than what they need and investing at date 1 in inefficient projects; see Holmström and Tirole (2005) for an analysis of what happens when they can engage in such misuse. Also, there is some indeterminacy here as to the way in which the efficient allocation can be collectively implemented. The key feature shared by these implementations is the centralized dispatch of liquidity.

11. Healthy firms draw only  $\rho_L I$ .

12. As will be discussed in more detail below, we assume that individual investors have at date 1 cash on hand that they can use to buy shares in the firms. That cash, however, cannot be committed in the

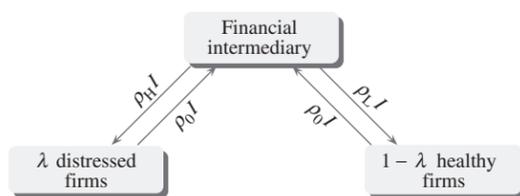


Figure 15.2

proceeds of such sales can cover the financing of the credit lines:<sup>13</sup>

$$(1 - \lambda)(\rho_0 - \rho_L)I > \lambda(\rho_H - \rho_0)I$$

(see Figure 15.2).

*Discussion.* The implications of a self-provision of liquidity have been investigated in detail in an international context by Caballero and Krishnamurthy (2001, 2003, 2004a,b). Their models are richer than the one presented here, in (at least) two ways. First, they involve two goods (tradables, nontradables) rather than one; a liquidity shortage can then be interpreted as a lack of international liquidity (technically, a shortfall in tradables) for the country. Second, the supply of liquidity is not fixed, but increases with its price; namely, firms can create liquidity by investing in (low-yield) short-term investments (on this see Chapter 12).

Caballero and Krishnamurthy, for example, consider settings in which domestic borrowers must borrow in dollars (tradables) at date 0 to produce pesos (nontradables) at date 2. A fraction of firms experience a high liquidity shock at date 1, formalized as the need to reinject dollar-denominated investment into the firm. Each firm thus makes two uses of dollars at date 0: long-term investment and reserves for a possible date-1 liquidity shock. Caballero and Krishnamurthy show that, from a social viewpoint, firms (which do not coordinate their liquidity provision) overinvest in the illiquid asset and underinvest in reserves (hence, the use of “underinsurance”

form of credit lines granted by the consumers to the firms at date 0 either because consumers “were not yet born” or because their date-1 cash comes from labor income and human capital is inalienable.

13. Alternatively, the intermediary can grant a credit line equal to  $(\rho_H - \rho_0)I$  per firm, and allow the firm to raise further income by issuing new securities. Firms in distress raise  $\rho_0 I$  by diluting their owner and complement this amount by drawing on the credit line. Healthy firms do not need to draw on the credit line and may just issue enough securities to raise  $\rho_L I$ .

in the title of their 2003 paper). Healthy firms resell their extra dollars at date 1 to distressed ones who pledge nontradable collateral in exchange. But they need not appropriate the full surplus of continuation and so, at date 0, there is underinvestment in reserves.<sup>14</sup>

In practice, liquidity may be wasted in other ways. For example, a lack of coordination may result in too many asset sales in a recession. One may have in mind here banks disposing of their large commercial and residential real estate portfolios when their capital adequacy becomes insufficient. With a downward-sloping demand for the corresponding assets, such “fire sales” depress the price; put differently, the sellers could be better off agreeing to limit the amount of asset sales in bad times.<sup>15</sup>

Lorenzoni (2003) develops a setting in which there is no firm-specific uncertainty and thus only an aggregate shock (so wasting liquidity by failing to pool it is not an issue) and in which financiers can write detailed state-contingent contracts with entrepreneurs. Workers (risk-neutral ones), however, are hired after the aggregate shock is realized. While wages are determined *ex post* through the labor market clearing equation and therefore are state contingent (they are lower in recessions), their evolution does not mimic an optimal *ex ante* labor contract. Put differently, workers *ex post* do share some of the risk with firms, but they do not contribute in the *ex ante* optimal way to the optimal sharing of risk in the economy (here, the efficient provision of liquidity to firms, due to worker risk neutrality). This reasoning is reminiscent of the observation that investors *ex post* do not provide the *ex ante* socially efficient volume of funds to a firm that has not planned its liquidity management. Under worker risk neutrality,

14. Another contribution that builds on self-provision of liquidity (but in a closed-economy context) is Kiyotaki and Moore (2001), which develops an infinite-horizon model in which a store of value commands a liquidity premium. In each period only a fraction of entrepreneurs have an investment opportunity. In order to be able to borrow and invest, credit-rationed entrepreneurs must carry net worth from the previous period through holding stores of value (a bit like in the Kiyotaki-Moore model reviewed in Section 14.3). Entrepreneurs self-provide their liquidity, i.e., they do not pool. This waste of liquidity creates a shortage of liquidity even in the absence of aggregate liquidity shock.

15. The analysis is similar to that of the impact of cartelization of the asset resale market on pledgeable income (see the analysis in Exercise 4.16).

an *ex ante* labor contract is a way of committing “funds” in the form of a substantial wage reduction during hard times in exchange for a rent (relative to a spot labor market outcome) for the workers in good times. Lorenzoni shows that the firms’ balance sheet may be overexposed to the aggregate shock.

Yet another way in which liquidity may be wasted is when consumers themselves demand liquidity. For example, consumers may demand liquid assets more when fearing unemployment. But the consumer demand for liquidity drains the liquidity available to corporations, which may need to lay workers off in a recession. To avoid this, complex coordination between firms and their employees might be needed.<sup>16</sup>

### 15.3 Aggregate Liquidity Shortages and Liquidity Asset Pricing

#### 15.3.1 Aggregate Shocks and the Value of Outside Liquidity

Suppose now that liquidity shocks are perfectly correlated across firms in the model of Section 15.2 (that firms face the same shock is the starkest way of introducing an aggregate shock; Exercise 15.3 studies the more general case of imperfectly correlated shocks). In this polar case, and in the absence of outside liquidity, firms cannot continue when  $\rho = \rho_H > \rho_0$ :

$$x = 0.$$

We will assume that the entrepreneurs prefer investing to consuming even if the investment is liquidated in the bad state of nature, i.e., that, per unit of investment, the expected output  $(1 - \lambda)\rho_1$  exceeds the sum of the initial investment cost, 1, and the expected reinvestment cost,  $(1 - \lambda)\rho_L$ :

$$(1 - \lambda)\rho_1 > 1 + (1 - \lambda)\rho_L.$$

(This positive-NPV assumption is more stringent than the previous one:  $\rho_1 > 1 + \bar{\rho}$ .)

Because all firms are valueless in the bad state of nature, distressed firms cannot meet liquidity shocks by selling (even indirectly through a financial intermediary) shares in healthy ones.

The problem is that money cannot be moved across states of nature at date 1. The corporate

sector has a high value in the good state of nature (when  $\rho = \rho_L$ ) but, according to the continuation strategy defined in Section 15.2, is a “sink” in the bad state of nature (when  $\rho = \rho_H$ ). In the latter state, investors are unwilling to bring more than  $\rho_0 I$  when  $\rho_H I$  would be needed.

The key source of inefficiency is the inability of investors to commit to transfer the large profit that they can make in the good state of nature to subsidize firms in the bad state of nature. This inability of investors to commit to refinancing firms in a recession may have two sources:

- consumers with income to invest at date 1 are not yet born at date 0, or
- consumers with income to invest at date 1 are already present at date 0, but they are unable to pledge their future income (say, the income derived from their human capital) at dates 1 and 2.

In practice, two factors may lead to a less drastic conclusion:

- (a) First, there may be alternative stores of value. There may exist exogenous or outside stores of value (e.g., land). Alternatively, the corporate sector itself may create stores of value, for example by investing in “inefficient projects” that have a low yield, but support reinvestment in the more efficient projects in a recession.
- (b) Second, the government’s regalian power of taxation may help harness the investors’ otherwise unpledgeable income in the bad state of nature at date 1.

Here we introduce outside stores of value into the picture. For simplicity, there are  $L^S$  such stores of value. A store of value yields 1 unit of good at date 1 for certain. Its date-0 price is  $q$ ; because the market rate of interest is equal to 0 and investors can buy the store of value,

$$q \geq 1.$$

Furthermore, if  $q = 1$ , then the corporate sector is able to implement its efficient allocation. If  $q > 1$ , then the corporate sector must hold all stores of value.<sup>17</sup>

16. For more on the waste of liquidity, see Holmström and Tirole (2005).

17. This is, of course, extreme. We could allow consumers to face liquidity shocks themselves (as in Chapter 12) and hold some of the liquid assets.

In the bad state of nature, the representative firm continues with probability  $x$ . Equivalently, a fraction  $1 - x$  of its investment is liquidated. For ease of exposition, we will work under the latter interpretation.

The liquidity need in the bad state,  $\rho_H x I$ , must not exceed the sum of the amount  $\rho_0 x I$ , which can be raised by returning to the capital market at date 1, and of the income  $L$  associated with a date-0 purchase of outside liquidity in amount  $L$ . And so

$$(\rho_H - \rho_0)xI \leq L.$$

The investors' breakeven constraint condition states that the total investor date-0 outlay to pay for illiquid and liquid assets should be recouped from the pledgeable income:

$$[I + qL] - A \leq L + (1 - \lambda)(\rho_0 - \rho_L)I + \lambda(\rho_0 - \rho_H)xI.$$

To make things interesting, we will assume that  $L^S$  is not so large that  $q = 1$  (we will later provide a condition for this to be the case).  $q > 1$  only if firms compete with each other for the scarce liquidity. The date-0 price  $q$  adjusts to the level at which the demand for liquid assets is equal to the supply:

$$L = L^S.$$

Let

$$U_b = [(1 - \lambda)(\rho_1 - \rho_L) + \lambda(\rho_1 - \rho_H)x]I - (q - 1)(\rho_H - \rho_0)xI - I \quad (15.5)$$

denote the representative borrower's utility or net present value, given that an amount of liquidity  $L = (\rho_H - \rho_0)xI$  is required to bring a fraction  $x$  of investment to completion in the bad state of nature.

The investors' breakeven constraint can be rewritten as

$$[(1 - \lambda)(\rho_0 - \rho_L) + \lambda(\rho_0 - \rho_H)x]I - (q - 1)(\rho_H - \rho_0)xI \geq I - A. \quad (15.6)$$

This constraint yields the investment level  $I$  as a function of  $x$  and  $q$ . Substituting into (15.5),

$$U_b = \frac{\rho_1 - c(x, q)}{c(x, q) - \rho_0} A,$$

where

$$c(x, q) = \frac{1 + (1 - \lambda)\rho_L + \lambda\rho_H x + (q - 1)(\rho_H - \rho_0)x}{1 - \lambda + \lambda x}$$

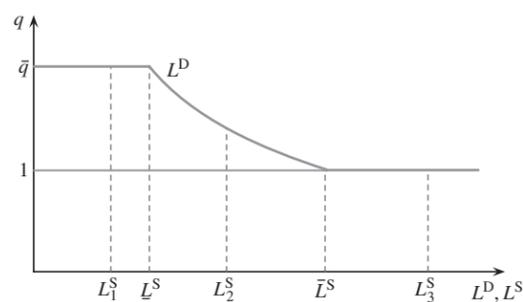


Figure 15.3

is the cost of effective investment, that is, the average cost of bringing 1 unit of investment to completion. Let  $\bar{q}$  be given by

$$\begin{aligned} \frac{\partial c}{\partial x}(x, \bar{q}) &= 0 \\ \Leftrightarrow (1 - \lambda)(\rho_H - \rho_L) + \frac{(1 - \lambda)}{\lambda}(\rho_H - \rho_0)(\bar{q} - 1) &= 1. \end{aligned}$$

Then

$$\begin{aligned} x &= 1 && \text{if } q < \bar{q} \\ &= 0 && \text{if } q > \bar{q} \\ &\in [0, 1] && \text{if } q = \bar{q}. \end{aligned}$$

For  $q < \bar{q}$ , the demand for liquid assets is given by

$$L^D = (\rho_H - \rho_0)I = \frac{\rho_H - \rho_0}{c(1, q) - \rho_0} A.$$

The equilibrium in the market for liquidity is depicted in Figure 15.3, where

$$\bar{L}^S \equiv \frac{(\rho_H - \rho_0)A}{1 + \bar{\rho} - \rho_0}$$

is the lowest amount of outside liquidity such that the market clearing price for liquidity is  $q = 1$ .

Figure 15.3 illustrates the three regions. When liquidity is very scarce ( $L^S = L_1^S$  for example), there is liquidation ( $x < 1$ ); the price of liquid assets is then the highest price,  $\bar{q}$ , that firms are willing to pay in order to hold this liquidity and be able to salvage their assets in the bad state of nature. As the supply of liquidity  $L^S$  increases,  $x$  increases. Once  $x$  reaches 1, the price of liquid assets adjusts. A lower price increases the firms' borrowing capacity and, indirectly, the demand for liquid assets, which are a complementary input into the production process. This region is illustrated by  $L^S = L_2^S$  in

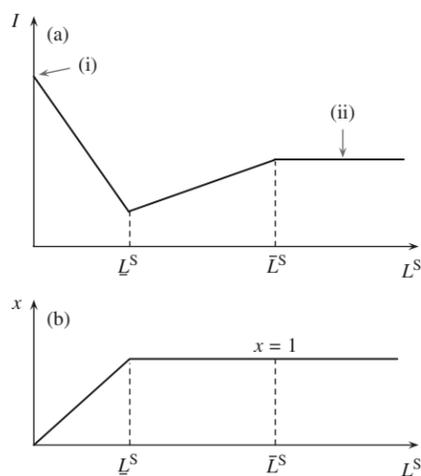


Figure 15.4 (a) (i)  $I = A / (1 + (1 - \lambda)\rho_L - (1 - \lambda)\rho_0)$ ;  
(ii)  $I = A / (1 + \bar{\rho} - \rho_0)$ .

Figure 15.3. Finally, for  $L^S \geq \bar{L}^S$  (for instance  $L^S = \bar{L}^S$  in Figure 15.3), there is an excess supply of liquidity. The market price is then  $q = 1$ , and the excess liquidity,  $L^S - \bar{L}^S$ , is absorbed by individual investors, who demand a rate of return equal to 0.

*Does liquidity crowd investment in or out?* Figure 15.4 illustrates the relationship between liquid and illiquid assets: a higher stock of liquid assets first depresses and then boosts corporate investment.<sup>18</sup>

Whether liquid assets crowd in or out illiquid ones depends on whether the entrepreneurs are willing to invest without continuing in the adverse state of nature ( $(1 - \lambda)\rho_1 > 1 + (1 - \lambda)\rho_L$ ) or not. In the former case, which we have just analyzed, the purchase of liquid assets first comes at the expense of investment: liquid assets *crowd out* illiquid ones. For larger amounts of liquidity ( $L \geq \bar{L}_S$ ),  $x = 1$  and liquid assets and investment are necessarily *complements*: a larger stock of liquid assets lowers the liquidity premium ( $q - 1$ ) and thereby the cost of investment.<sup>19</sup>

18. For  $L^S < \bar{L}^S$ ,  $x < 1$  and  $q = \bar{q}$ . Investment is then given by

$$[1 + (1 - \lambda)\rho_L - (1 - \lambda)\rho_0]I + [(q - 1) + \lambda]L^S = A.$$

For  $L^S \in [\bar{L}^S, \bar{L}^S]$ ,  $x = 1$  and  $q \leq \bar{q}$ . Furthermore,

$$L^S = (\rho_H - \rho_0)I \quad \text{and} \quad I = \frac{A}{1 + \bar{\rho} + (q - 1)(\rho_H - \rho_0) - \rho_0}.$$

19. Note that our choice of consumer preferences ( $c_0 + c_1 + c_2$ ) implies that stores of value cannot crowd out investment through an

When investing yields a negative NPV per unit of investment when the investment is liquidated in the adverse state ( $(1 - \lambda)\rho_1 < 1 + (1 - \lambda)\rho_L$ ), entrepreneurs invest only if they can complement their illiquid investment with liquidity. In this case, liquid and illiquid investments are always *complements* (see Exercise 15.4).

*Remark (using foreign liquidity).* It might appear that a shortage of liquidity at the domestic level could be compensated by resorting to international liquidity. After all, aggregate shocks are likely to be smaller in relative size at the world level than at a country's level. For example, Thai banks and firms could obtain liquidity through a credit line from a consortium of international banks or by holding shares in the U.S. S&P500 index. This resort to international liquidity is unfortunately limited by the country's own pledgeability problem (also called the "shortage of international collateral"). Thus the conclusions reached in this section carry over to an environment of capital account liberalization.<sup>20</sup>

### 15.3.2 Liquidity Asset Pricing

The analysis in the previous subsection focused on the pricing of safe claims, namely, claims that deliver a constant yield at date 1 regardless of the state of the economy. We now note that risky claims can also be priced out by invoking the value of their "liquidity service" (or disservice).<sup>21</sup>

In our example, there are only two aggregate states of nature:  $\omega \in \{L, H\}$ . Let  $m(\omega)$  denote (one plus) the liquidity service of the safe claim, i.e., the marginal utility of one more unit of good available at date 1 in state of nature  $\omega$ . Because there is no demand for liquidity in the good state of nature,

$$m_L = 1.$$

By contrast, 1 unit of good available in the bad state of nature generally has a value in excess of 1:

$$m_H \geq 1.$$

increase in the rate of interest demanded by consumers. An elastic savings function would add a factor of substitutability between stores of value and investment as in Diamond (1965) and Tirole (1985).

20. On this, see Caballero and Krishnamurthy (2001, 2003, 2004a,b) and Holmström and Tirole (2002).

21. For more details, see Holmström and Tirole (2002, 2005).

$m_H$  can be computed from the price of the safe asset:

$$\begin{aligned} q &= E[m(\omega) \cdot 1] \\ &= 1 - \lambda + \lambda m_H, \end{aligned}$$

and so

$$q - 1 = \lambda(m_H - 1). \quad (15.7)$$

The liquidity premium,  $q - 1$ , is equal to the product of the probability  $\lambda$  that the asset will perform a liquidity service and the net value,  $m_H - 1$ , of this service.

$m(\omega)$  is the liquidity asset pricing model (LAPM) analog of the stochastic discount factor in the consumption-based capital asset pricing model (CCAPM).<sup>22</sup> Like the stochastic discount factor, it allows a systematic pricing of assets with arbitrary return streams. Using the expression of  $m_H$  given in (15.7), one can then find the date-0 price, per unit of expected return, of an arbitrary asset  $i$  with flow return  $\{y^i(\omega)\}_{\{\omega=L,H\}}$ :

$$q^i = \frac{E[m(\omega)y^i(\omega)]}{E[y^i(\omega)]}.$$

Consider, for example, the representative firm with investment  $I$  whose shares are acquired by a financial institution that also hoards liquid assets and grants the firm the right to draw enough liquidity in order to continue in the bad state of nature. From the point of view of the financial institution, this firm yields

$$y^{\text{firm}}(L) = (\rho_0 - \rho_L)I \quad \text{and} \quad y^{\text{firm}}(H) = -(\rho_H - \rho_0)I.$$

Hence, its price is

$$\begin{aligned} q^{\text{firm}} &= \frac{[(1 - \lambda)(\rho_0 - \rho_L) - \lambda(\rho_H - \rho_0)m_H]I}{[(1 - \lambda)(\rho_0 - \rho_L) - \lambda(\rho_H - \rho_0)]I} \\ &= 1 - \frac{\lambda(\rho_H - \rho_0)(m_H - 1)}{E[y^{\text{firm}}(\omega)/I]} \\ &= 1 - \frac{(q - 1)[(\rho_H - \rho_0)I]}{E[y^{\text{firm}}(\omega)]}. \end{aligned}$$

Because it consumes rather than supplies liquidity in the bad state of nature, the firm is valued below par, that is, it exhibits an equity premium. In this simple model, the equity premium is perfectly negatively correlated with the liquidity premium  $q - 1$ , as  $q$  varies between 1 and  $\bar{q}$  (due to variations in the supply of outside liquidity, say).

22. For expositions of the CCAPM, see, in particular, Campbell et al. (1996), Cochrane (2005), and Duffie (2001).

### 15.3.3 Government Provision of Outside Liquidity

We saw that, whenever liquidity is properly dispatched within the corporate sector, the failure to achieve the efficient allocation stems from the investors' inability to promise income to the corporate sector in the bad state of nature.<sup>23</sup> In that state of nature and under condition (15.2), continuation is desirable, but the corporate sector is *ex post* unable to convince investors to bring cash, as only part of the benefits from continuation can be returned to them.

The government's unique ability to tax consumers can make up for the latter's inability to pledge money to the corporate sector. Ideally, the government would like to boost the corporate sector's solvency in the bad state of nature by taxing consumers and transferring the proceeds to the corporate sector. Such a policy need not be to the detriment of consumers, though: the government can tax the corporate sector in the good state of nature and thereby compensate (in expectation) consumers for the loss they incur in the bad state of nature. But optimal liquidity provision is *contingent* liquidity provision: the government must operate a redistribution from the households to corporations in those states of nature in which the latter encounter hardship.

In practice, the creation of outside liquidity by the government takes a variety of other forms, among which only a richer model can distinguish. One has the government issue Treasury bonds (at date 0 in our model). These bonds are akin to the stores of value studied in Section 15.3.1, in that they can be used by the corporate sector to overcome the high liquidity shock. The government can create liquidity and thereby affect the allocation because it has access to consumers' date 1 (and 2) endowments and can thereby back the bond issue through this tax "collateral."

It is also important to stress that liquidity is created by forcing consumers to redistribute toward the corporate sector in bad times. Were the coupons of the Treasury bonds financed through a corporate tax, the Treasury bonds would do nothing to boost

23. This feature underlies the non-Ricardian properties discussed below.

the corporate sector solvency in bad times: the logic of this argument elaborates on that underlying the result that an investment subsidy financed through a corporate income tax has no effect on investment (see Exercise 3.19).

Other ways in which the government creates liquidity and supports economic activity in bad times include a countercyclical monetary policy; deposit insurance premia that are not indexed to the business cycle (banks are riskier in a recession and therefore market-based deposit insurance premia would then adjust upwards) and use of the discount window; publicly provided unemployment insurance (in a recession, layoffs are more frequent and workers remain unemployed for a longer period of time, so a market-based, private layoff insurance scheme would yield high premia in recessions); implicit guarantees to private pension funds; and so forth.

These injections of liquidity are either discretionary (e.g., countercyclical monetary policy) or part of an automatic stabilization mechanism (e.g., non-indexed deposit insurance premia). As Sundaresan and Wang (2004) point out, explicit preannouncements of liquidity provision are rare because the timing of liquidity crises is uncertain. These authors, though, identify one episode in which the government offered state-contingent liquidity: the century change date. There was a fear that a Y2K computer bug might provoke widespread difficulties and a severe liquidity crisis. Sundaresan and Wang first present evidence for the United States of high liquidity premia associated with this concern. They then describe and assess how private sector concerns were partially alleviated by the central bank's provision of state-contingent liquidity. For example, the Federal Reserve auctioned off call options on the ability to borrow from the discount window at dates around January 1, 2000, at a strike set at 150 basis points above the prevailing Federal funds rate; other auctions related to the right to enter overnight repo transactions with the New York Fed at a preset strike price (also 150 basis points above the prevailing Federal funds rate).

Finally, this informal treatment of government creation of outside liquidity misses a discussion of the cost of this creation. For example, taxing

consumers involves a deadweight loss of taxation.<sup>24</sup> Clearly, the government must engage in a cost-benefit analysis when choosing how much liquidity to create. The market for liquid assets may help guide the government in this respect, as liquidity premia reflect the corporate demand for stores of value and their scarcity. Similarly, along the intertemporal dimension, the design of the term structure of public debt can be guided by the liquidity premia embodied in the bonds of various maturities.

#### 15.4 Moving Wealth across Time: The Case of the Corporate Sector as a Net Lender

Historically, the enabling of transfers of wealth by stores of value was first stressed in environments where wealth had to be moved across periods rather than across states of nature. It has, for example, figured prominently in the overlapping generations (OLG) literature,<sup>25</sup> in which consumers want to save some of the income earned when young for their old-age consumption. To give this older literature a corporate finance connotation, let us follow Woodford (1990) in assuming that entrepreneurs' income and investment opportunities are asynchronized.

We still consider three dates,  $t = 0, 1, 2$ . Assume a continuum of mass 1 of identical entrepreneurs. The representative entrepreneur is, as earlier, born with endowment  $A$  at date 0. She no longer has any meaningful investment opportunity at that date, however. By contrast, she anticipates that she will at date 1 have a variable-investment-size project: by investing  $I \in [0, \infty)$  at date 1, the entrepreneur will create an expected income equal to  $\rho_1 I$  at date 2, of which only  $\rho_0 I$  is pledgeable to date-1 investors. As in Section 3.4, we assume that

$$\rho_1 > 1 > \rho_0,$$

so that investment has a positive NPV, but pledgeable income per unit of investment is lower than unity.

24. The size of the deadweight loss may further depend on whether taxes are levied during the recession (in which case they may impose further hardships on households, who may be laid off by their firm) or delayed through the use of government borrowing.

25. First developed by Allais (1947), Samuelson (1958), and Diamond (1965).

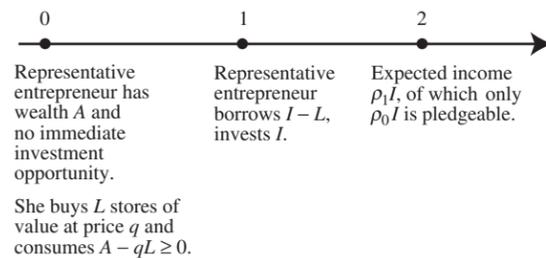


Figure 15.5

We further assume that there exist  $L^S$  stores of value at date 0, each of which deliver 1 unit of good at date 1.

The timing is summarized in Figure 15.5.

We obtain the equilibrium outcome by working backwards in time. Assume that, at date 1, the representative entrepreneur has wealth  $L$  (she has then consumed  $A - qL$  at date 0). The analysis is identical to that of Section 3.4, for that level of net worth. The entrepreneur's borrowing capacity is determined by the date-1 investors' breakeven constraint:

$$I - L = \rho_0 I \iff I = \frac{L}{1 - \rho_0}.$$

The NPV—which, due to the breakeven condition, goes to the entrepreneur—corresponds to the non-pledgeable part:

$$(\rho_1 - \rho_0)I = \frac{\rho_1 - \rho_0}{1 - \rho_0} L.$$

Turning now to the market for liquid assets at date 0, note that the representative entrepreneur's intertemporal utility is

$$[A - qL] + \left[ \frac{\rho_1 - \rho_0}{1 - \rho_0} L \right],$$

where  $L$  must satisfy  $qL \leq A$ .

Qualitatively, there can be two equilibrium configurations.

*Excess liquidity:*  $L = A \leq L^S$  and  $q = 1$ . When there is a large number of stores of value ( $L^S \geq A$ ), the latter command no liquidity premium. Entrepreneurs save their entire endowment and invest it at date 1.

*Scarce liquidity:* because entrepreneurs are willing to pay up to

$$\bar{q} \equiv \frac{\rho_1 - \rho_0}{1 - \rho_0} > 1$$

per unit of liquidity, as  $L^S$  falls below  $A$ , the price first adjusts so as to clear the supply and the

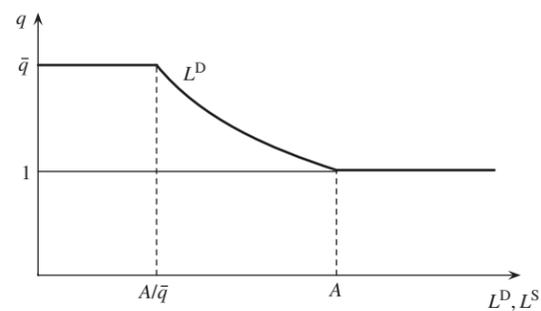


Figure 15.6

demand for liquidity,

$$A = qL^S,$$

until  $L^S$  reaches the level  $A/\bar{q}$ . As  $L^S$  falls further, the price of liquidity stabilizes at the entrepreneurs' willingness to pay  $\bar{q}$  (see Figure 15.6).

In the region in which liquidity is very scarce ( $L^S < A/\bar{q}$ ), *financial development*, interpreted as an increase in the extent of pledgeability  $\rho_0$  (keeping  $\rho_1$  constant), makes liquidity more valuable and raises its price (that is,  $\bar{q}$  increases).

*Creation of liquid instruments by the corporate sector.* Suppose now that  $L^S = 0$ , but that each entrepreneur can, at date 0 and at increasing and convex cost  $C(L)$ , create  $L$  units of income at date 1. The privately optimal investment is given by<sup>26</sup>

$$C'(L) = \bar{q} = \frac{\rho_1 - \rho_0}{1 - \rho_0}.$$

Note, in particular, that the marginal unit of inside liquidity thus created has a negative return ( $1/\bar{q} < 1$ ). Indeed, if  $C'(0) \geq 1$ , all units of liquidity have a negative return. Yet, each entrepreneur is willing to invest in these inefficient projects in order to benefit from the attractive investment opportunities at date 1.

*Infinite-horizon versions.* There are at least two ways of extending these ideas to infinite-horizon settings. First, we can follow Woodford (1990) in assuming multiple categories of infinitely lived entrepreneurs. Woodford's model has two groups of entrepreneurs. A group-1 entrepreneur receives an endowment  $A$  (of a nondurable good) in each

26. Provided that  $C'(0) < \bar{q} < C'(C^{-1}(A))$ .

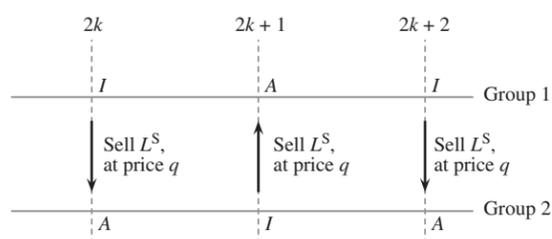


Figure 15.7

odd period; she has investment opportunities only in even periods. A group-2 entrepreneur in contrast receives endowment  $A$  (of a nondurable good) in each even period and has investment opportunities in odd periods.

For expositional simplicity, Woodford assumes that the return on investment is immediate (accrues at the period in which the investment is made), and that none of it is pledgeable ( $\rho_0 = 0$ ).

The only means of transferring wealth from periods of endowment to periods of investment is ownership of a store of value. There are  $L^S$  consols or consol bonds<sup>27</sup>, each yielding 1 unit of nondurable good per period, forever.

In equilibrium, group-1 entrepreneurs purchase the store of value from group 2 in odd periods; and vice versa in even periods (see Figure 15.7).

All have preferences

$$\sum_0^{\infty} \beta^t c_t,$$

where  $\beta < 1$  is the discount factor and  $c_t \geq 0$  is consumption. In a period with an investment opportunity, investing  $I$  yields  $\rho_1 I$  (of which, recall, nothing is pledgeable). We assume that  $\beta\rho_1 > 1$ , so delaying consumption in order to invest is worthwhile. Focusing for conciseness on the case in which there are few consols, the price of consols,  $q$ , is given by

$$q = \beta(1 + q)\rho_1,$$

since one consol purchased at date  $t$  yields dividend 1 and generates resale price  $q$ , and this net worth  $(1 + q)$  enables the entrepreneur to invest at the same level (due to  $\rho_0 = 0$ ), yielding  $(1 + q)\rho_1$ .

27. A bond is a consol bond if it does not have a maturity and pays a fixed coupon perpetually.

Note, in particular, that the rate of return on consols,  $1/q$ , is smaller than the agents' rate of preference  $(1 - \beta)/\beta$ : the liquid asset sells at a *discount*.<sup>28</sup>

An alternative approach is to posit an OLG structure. In order to facilitate comparison with the Woodford model, let us assume that investment  $I$  yields  $\rho_1 I$  within the same period and that none of this income is pledgeable to investors.<sup>29</sup>

Generation  $t$  ( $G_t$ ) comprises a unit mass of entrepreneurs.  $G_t$ 's representative entrepreneur receives exogenous and nondurable endowment  $A$  and can use it either to consume at date  $t$  or to purchase  $L_t$  consols from generation  $G_{t-1}$ , which will enable her to invest at  $t + 1$  (see Figure 15.8). There are  $L^S$  consols in the economy, each delivering 1 unit of perishable good per period, forever. In equilibrium,  $L_t$  must be equal to  $L^S$  for all  $t$ .

Let  $\beta$  denote the discount factor between the two periods of a generation's life.  $G_t$ 's representative entrepreneur's utility from consumption ( $c_t, c_{t+1}$ ) at dates  $t$  and  $t + 1$  is thus  $u_t = c_t + \beta c_{t+1}$ .

Focusing again on the case in which there are few consols, the market price of a consol is given by the same condition as in the Woodford model:

$$q = \beta(1 + q)\rho_1.$$

Every generation but the first has utility  $u_t = A$ , since it is in equilibrium indifferent between buying consols and consuming the endowment. The initial generation (that born with the consols, which start delivering income at date 1, say) has utility  $u_0 = A + \beta(1 + q)L^S\rho_1 = A + qL^S$ .<sup>30</sup>

28. Put differently, the rate of interest on a consol,  $r^c$ , is given by  $q \equiv 1/r^c$ . The rate of time preference,  $r$ , satisfies  $\beta \equiv 1/[1 + r]$ . And so  $r^c < r$ .

29. This total lack of pledgeability makes the analogy with the Allais-Samuelson-Diamond OLG model particularly striking. For, an old consumer in this model consumes but cannot borrow.

30. Under the OLG structure and under certain circumstances, all generations can be made better off through a sequence of transfers from each generation to the previous one. Suppose, for example, that  $L^S = 0$ , and so all generations including the initial one have utility  $u_t = A$ . Suppose that the initial generation sells a "bubble" (i.e., an asset paying no dividend and with rate of return equal to the market rate of interest)  $b = A$  to the second generation, and so forth. Then each generation but the first has utility  $u_t = \beta\rho_1 A > A$ . The first generation also gains as it gets  $u_0 = A + \beta\rho_1 A$ . (With consols  $L^S > 0$ , the feasibility of such schemes depends on the rate of growth of the economy and on whether the consols appear over time or are capitalized at the initial date (see, for example, Tirole 1985).)

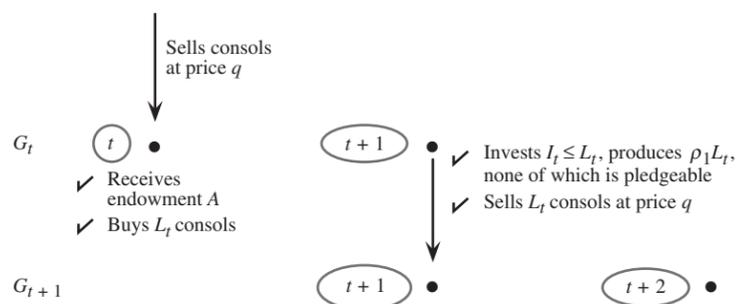


Figure 15.8

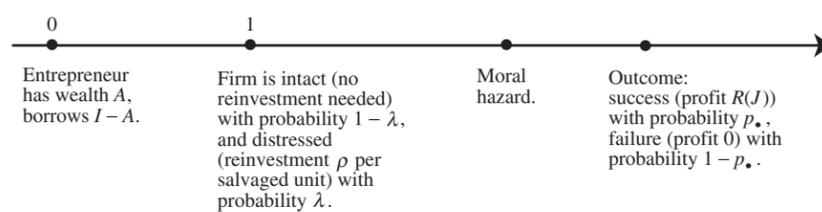


Figure 15.9

## 15.5 Exercises

**Exercise 15.1 (downsizing and aggregate liquidity).** Consider the variable-investment model with *decreasing* returns to scale and a liquidity shock. There is a unit mass of identical entrepreneurs. The timing for a given entrepreneur is in Figure 15.9.

At date 1, an amount  $J$ ,  $0 \leq J \leq I$ , is rescued. In the absence of a liquidity shock (event has probability  $1 - \lambda$ ), of course  $J = I$ . But in the face of a liquidity shock (which has probability  $\lambda$ ), the investment is downsized to  $J \leq I$  (the cost of continuation is then  $\rho J$ ). The shock is verifiable. Let  $R(J)$  denote the profit in the case of success.

The moral-hazard stage is described as it usually is: the probability of success is  $p_H$  if the entrepreneur works and  $p_L = p_H - \Delta p$  if she shirks. The entrepreneur obtains private benefit  $BJ$  by misbehaving and 0 otherwise. Investors and entrepreneur are risk neutral, and the latter is protected by limited liability.

Economic agents do not discount the future (which does not imply that rates of interest are always 0!).

From now on, use  $J$  for the amount that is salvaged when there is a liquidity shock (as we noted, the corresponding amount is  $I$  in the absence of shock).

Assume that  $R(0) = 0$ ,  $R' > 0$ ,  $R'' < 0$ ,  $R'(0) = \infty$ ,  $R'(\infty) = 0$ .

(i) Assume that there is plenty of liquidity in the economy, so that the firms have access to a store of value (by paying  $q = 1$  at date 0, they receive 1 at date 1).

Show that downsizing occurs in the case of a liquidity shock,

$$J^* < I^*,$$

if and only if

$$\rho > \frac{1}{1 - \lambda}.$$

(Hints: (1) write the incentive constraints (the sharing rule can be adjusted to the realization of the shock) and infer the pledgeable income; (2) maximize the entrepreneur's utility (employ the usual trick) subject to the investors' breakeven condition, ignoring the constraint  $J \leq I$ ; let  $\mu$  denote the shadow price of the constraint; (3) derive the stated result.)

(ii) Suppose that the liquidity shocks are *perfectly correlated*.

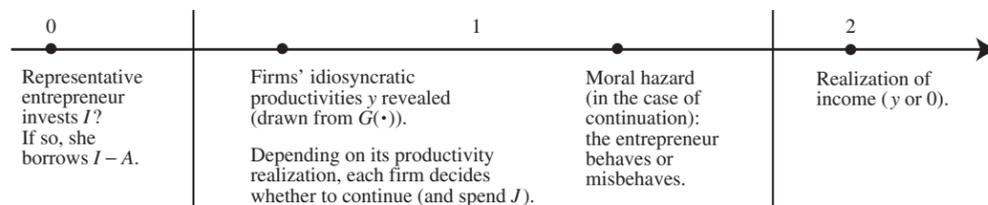


Figure 15.10

- What is the minimal number  $L^*$  of outside stores of value (delivering 1 unit of good each at date 1) needed to support the allocation described in (i)?
- Argue that if  $L < L^*$ , then  $q > 1$  and  $J < I$  *a fortiori* if  $\rho > 1/(1 - \lambda)$ . Derive the equations giving the liquidity premium  $(q - 1)$  under these assumptions.
- (iii) Suppose now that the liquidity shocks are independent across firms.
  - Argue that (provided that the entrepreneurs borrow at date 0) there is enough liquidity to support the allocation derived in (i).
  - Suppose that each entrepreneur holds the stock index. When will this provide enough liquidity? How can one prevent this potential waste of liquidity?

**Exercise 15.2 (news about prospects and aggregate liquidity).** Consider an economy with a continuum of identical risk-neutral entrepreneurs. The representative entrepreneur has a fixed-size investment project costing  $I$ , and limited personal wealth  $A < I$ . The project, if undertaken, will deliver a random but verifiable income  $y \in [0, 1]$ , with cumulative distribution function  $G(y)$  and density  $g(y)$ , provided that a reinvestment  $J$  is made after  $y$  is learned, but before  $y$  is produced. The project yields nothing if it is interrupted.

Moreover, in the case of “continuation” (that is, if  $J$  is sunk), and regardless of the value of  $y$ , the entrepreneur may behave, in which case income is  $y$  for certain, or misbehave, in which case income is  $y$  with probability  $p_L$  and 0 with probability  $1 - p_L$ . The entrepreneur, who is protected by limited liability, obtains private benefit  $B$  when misbehaving (and no private benefit otherwise). Let

$$\mathcal{R} \equiv \frac{B}{1 - p_L}$$

(one will assume that  $B$  is small enough that, in the relevant range, it is worth inducing the entrepreneur to behave in the case of continuation).

The timing is summarized in Figure 15.10.

The rate of interest in the economy is equal to 0.

(i) Compute the NPV and the investors’ net income as functions of the threshold  $y^*$  for continuation.

(ii) Let  $y_0^* \equiv J$  and  $y_1^* = J + \mathcal{R}$ .

Define  $A_0^*$  and  $A_1^*$  by

$$I - A_k^* \equiv \int_{y_k^*}^1 y \, dG(y) - [1 - G(y_k^*)][J + \mathcal{R}],$$

for  $k \in \{0, 1\}$ .

• What are the date-0 investment policy (investment/no investment) and the date-1 reinvestment policy (the threshold  $y^*$ ) as functions of  $A$ ? (Hint: distinguish three regions  $A \geq A_0^*$ ,  $A \leq A_1^*$ , and  $A_1^* < A < A_0^*$ .)

• Argue that, for  $A > A_1^*$ , the entrepreneur must arrange at date 0 for her firm’s date-1 liquidity.

(iii) • Is there enough inside liquidity if productivities are drawn *independently* from the distribution  $G(\cdot)$ ? Why?

(iv) Suppose, in contrast, that there is a macroeconomic shock  $\theta$  that is revealed at the beginning of date 1. (One will denote by  $E_\theta[\cdot]$  the date-0 expectations over the random variable  $\theta$ .) Let  $y^*(\theta)$  denote the state-contingent threshold.

• Write the date-0 financing constraint.

• Show that the optimal threshold when liquidity is abundant is actually state independent: there exists  $y^*$  such that

$$y^*(\theta) = y^* \quad \text{for all } \theta.$$

• Show that the second-best allocation can be implemented when there are at least

$$\min_{\{\theta\}} \int_{y^*}^1 (y - J - \mathcal{R}) \, dG(y \mid \theta)$$

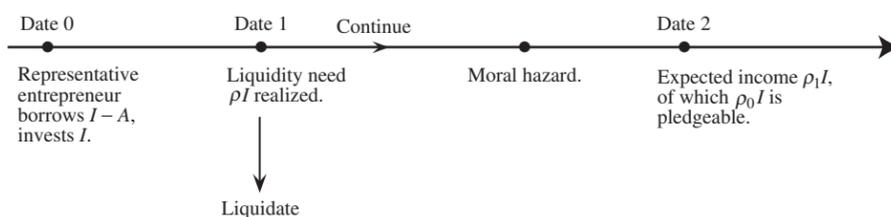


Figure 15.11

units of outside liquidity delivering 1 unit of good for certain at date 1.

- What would happen if there were few such stores of value?

**Exercise 15.3 (imperfectly correlated shocks).** This exercise extends the analysis of Section 15.3 to allow for imperfect correlation among the shocks faced by the firms. As in Section 15.3.1, there is a mass 1 of *ex ante* identical entrepreneurs. Each entrepreneur has a constant-returns-to-scale project. An investment of size  $I$  at date 0 yields  $\rho_1 I$  at date 2, of which  $\rho_0 I$  is pledgeable, provided that the liquidity shock  $\rho I$  is met at date 1.  $\rho$  is equal to  $\rho_L$  with probability  $(1 - \lambda)$  and  $\rho_H$  with probability  $\lambda$ , with  $\rho_L < \rho_0 < \rho_H < \rho_1$  and  $(1 - \lambda)(\rho_H - \rho_0) < 1$ . As usual, entrepreneurs and investors are risk neutral, and the latter demand a rate of return equal to 0 (see Figure 15.11).

The new feature is that shocks are imperfectly correlated: for a fraction  $1 - \theta$  of entrepreneurs, shocks are drawn independently ( $\theta = 0$  in Section 15.2.1). A fraction  $\theta$  of entrepreneurs face the same shock,  $\rho_L$  with probability  $(1 - \lambda)$  and  $\rho_H$  with probability  $\lambda$  ( $\theta = 1$  in Section 15.3.1).

There is no outside store of value, and the long-term projects are the only investment projects available to the corporate sector.

Show that the private sector is self-sufficient (i.e., the efficient allocation can be implemented using the inside liquidity created by the long-term projects) if and only if  $\theta \leq \theta^*$ , where

$$(1 - \theta^*)(I - A) = \theta^*(\rho_H - \rho_0)I,$$

where  $I$  is independent of  $\theta$ .

**Exercise 15.4 (complementarity between liquid and illiquid assets).** Go through the analysis of Section 15.3.1 assuming that entrepreneurs do not want

to invest in projects that are discontinued in the adverse state of nature:

$$(1 - \lambda)\rho_1 < 1 + (1 - \lambda)\rho_L.$$

Show that an increase in the supply  $L^S$  of liquid assets increases the investment  $I$  in illiquid ones.

## References

- Allais, M. 1947. *Economie et Intérêt*. Paris: Imprimerie Nationale.
- Caballero, R. and A. Krishnamurthy. 2001. International and domestic collateral constraints in a model of emerging market crises. *Journal of Monetary Economics* 48:513–548.
- . 2003. Excessive dollar debt: financial development and underinsurance. *Journal of Finance* 58:867–893.
- . 2004a. A “vertical” analysis of monetary policy in emerging markets. Mimeo, MIT.
- . 2004b. Smoothing sudden stops. *Journal of Economic Theory* 119:104–127.
- Campbell, J., A. W. Lo, and C. MacKinlay. 1996. *The Econometrics of Financial Markets*. Princeton University Press.
- Cochrane, J. 2005. *Asset Pricing*, 2nd edn. Princeton University Press.
- Diamond, P. 1965. National debt in a neo-classical growth model. *American Economic Review* 55:1126–1150.
- Duffie, D. 2001. *Dynamic Asset Pricing Theory*, 3rd edn. Princeton University Press.
- Holmström, B. and J. Tirole. 1996. Modeling aggregate liquidity. *American Economic Review, Papers & Proceedings* 86:187–191.
- . 1998. Private and public supply of liquidity. *Journal of Political Economy* 106:1–40.
- . 2001. LAPM: a liquidity-based asset pricing model. *Journal of Finance* 56:1837–1867.
- . 2002. Domestic and international supply of liquidity. *American Economic Review* 92:42–45.
- . 2005. *Inside and Outside Liquidity*. Mimeo, MIT and IDEI, Toulouse. Cambridge, MA: MIT Press, in press.

- Kiyotaki, N. and J. Moore. 2001. Liquidity and asset prices. Mimeo, Edinburgh University and London School of Economics.
- Lorenzoni, G. 2003. The costs of credit booms. Mimeo, Princeton University.
- Samuelson, P. 1958. An exact consumption-loan model of interest with or without the contrivance of money. *Journal of Political Economy* 66:467-482.
- Sundaresan, S. M. and Z. Wang. 2004. Public provision of private liquidity: evidence from the millennium date change. Mimeo, Columbia Business School.
- Tirole, J. 1985. Asset bubbles and overlapping generations. *Econometrica* 53:1071-1100 (reprint 1499-1528).
- Woodford, M. 1990. Public debt as private liquidity. *American Economic Review, Papers & Proceedings* 80:382-388.