

# CHAPTER 21



## Decision Analysis

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## STATISTICS *in* PRACTICE

### OHIO EDISON COMPANY\*

AKRON, OHIO

Ohio Edison Company is an operating company of FirstEnergy Corporation. Ohio Edison and its subsidiary, Pennsylvania Power Company, provide electrical service to more than 1 million customers in central and north-eastern Ohio and western Pennsylvania. Most of the electricity is generated by coal-fired power plants. Because of evolving pollution-control requirements, Ohio Edison embarked on a program to replace the existing pollution-control equipment at most of its generating plants.

To meet new emission limits for sulfur dioxide at one of its largest power plants, Ohio Edison decided to burn low-sulfur coal in four of the smaller units at the plant and to install fabric filters on those units to control particulate emissions. Fabric filters use thousands of fabric bags to filter out particles and function in much the same way as a household vacuum cleaner.

It was considered likely, although not certain, that the three larger units at the plant would burn medium- to high-sulfur coal. Preliminary studies narrowed the particulate equipment choice for these larger units to fabric filters and electrostatic precipitators (which remove particles suspended in the flue gas by passing it through a strong electrical field). Among the uncertainties that would affect the final choice were the way some air quality laws and regulations might be interpreted, potential future changes in air quality laws and regulations, and fluctuations in construction costs.

Because of the complexity of the problem, the high degree of uncertainty associated with factors affecting the decision, and the cost impact on Ohio Edison, decision analysis was used in the selection process. A graphical description of the problem, referred to as a decision tree, was developed. The measure used to evaluate the outcomes depicted on the decision tree was the annual revenue requirements for the three large units over their remaining lifetime. Revenue requirements were the monies that would have to be collected from the utility customers to recover costs resulting from the installation



Ohio Edison plants provide electrical service to more than 1 million customers. © Don Farrall Getty Images/PhotoDisc.

of the new pollution-control equipment. An analysis of the decision tree led to the following conclusions.

- The expected value of annual revenue requirements for the electrostatic precipitators was approximately \$1 million less than that for the fabric filters.
- The fabric filters had a higher probability of high revenue requirements than the electrostatic precipitators.
- The electrostatic precipitators had nearly a .8 probability of having lower annual revenue requirements.

These results led Ohio Edison to select the electrostatic precipitators for the generating units in question. Had the decision analysis not been performed, the particulate-control decision might have been based chiefly on capital cost, a decision measure that favored the fabric filter equipment. It was felt that the use of decision analysis identified the option with both lower expected revenue requirements and lower risk.

In this chapter we will introduce the methodology of decision analysis that Ohio Edison used. The focus will be on showing how decision analysis can identify the best decision alternative given an uncertain or risk-filled pattern of future events.

\*The authors are indebted to Thomas J. Madden and M. S. Hyrnick of Ohio Edison Company for providing this Statistics in Practice.

*Instructions for downloading and installing PrecisionTree are provided on the website that accompanies the text.*

Decision analysis can be used to develop an optimal decision strategy when a decision maker is faced with several decision alternatives and an uncertain or risk-filled pattern of future events. We begin the study of decision analysis by considering decision problems that involve reasonably few decision alternatives and reasonably few future events. Payoff tables are introduced to provide a structure for decision problems. We then introduce decision trees to show the sequential nature of the problems. Decision trees are used to analyze more complex problems and to identify an optimal sequence of decisions, referred to as an optimal decision strategy. In the last section, we show how Bayes' theorem, presented in Chapter 4, can be used to compute branch probabilities for decision trees. The appendix at the end of the chapter provides an introduction to PrecisionTree, an Excel add-in that can be used to develop and analyze decision trees.

## 21.1

## Problem Formulation

The first step in the decision analysis process is problem formulation. We begin with a verbal statement of the problem. We then identify the decision alternatives, the uncertain future events, referred to as **chance events**, and the **consequences** associated with each decision alternative and each chance event outcome. Let us begin by considering a construction project for the Pittsburgh Development Corporation.

Pittsburgh Development Corporation (PDC) purchased land that will be the site of a new luxury condominium complex. The location provides a spectacular view of downtown Pittsburgh and the Golden Triangle where the Allegheny and Monongahela Rivers meet to form the Ohio River. PDC plans to price the individual condominium units between \$300,000 and \$1,400,000.

PDC commissioned preliminary architectural drawings for three different-sized projects: one with 30 condominiums, one with 60 condominiums, and one with 90 condominiums. The financial success of the project depends upon the size of the condominium complex and the chance event concerning the demand for the condominiums. The statement of the PDC decision problem is to select the size of the new luxury condominium project that will lead to the largest profit given the uncertainty concerning the demand for the condominiums.

Given the statement of the problem, it is clear that the decision is to select the best size for the condominium complex. PDC has the following three decision alternatives:

- $d_1$  = a small complex with 30 condominiums
- $d_2$  = a medium complex with 60 condominiums
- $d_3$  = a large complex with 90 condominiums

A factor in selecting the best decision alternative is the uncertainty associated with the chance event concerning the demand for the condominiums. When asked about the possible demand for the condominiums, PDC's president acknowledged a wide range of possibilities but decided that it would be adequate to consider two possible chance event outcomes: a strong demand and a weak demand.

In decision analysis, the possible outcomes for a chance event are referred to as the **states of nature**. The states of nature are defined so that one and only one of the possible states of nature will occur. For the PDC problem, the chance event concerning the demand for the condominiums has two states of nature:

- $s_1$  = strong demand for the condominiums
- $s_2$  = weak demand for the condominiums

Management must first select a decision alternative (complex size), then a state of nature follows (demand for the condominiums), and finally a consequence will occur. In this case, the consequence is PDC's profit.

## Payoff Tables

Given the three decision alternatives and the two states of nature, which complex size should PDC choose? To answer this question, PDC will need to know the consequence associated with each decision alternative and each state of nature. In decision analysis, we refer to the consequence resulting from a specific combination of a decision alternative and a state of nature as a **payoff**. A table showing payoffs for all combinations of decision alternatives and states of nature is a **payoff table**.

*Payoffs can be expressed in terms of profit, cost, time, distance, or any other measure appropriate for the decision problem being analyzed.*

Because PDC wants to select the complex size that provides the largest profit, profit is used as the consequence. The payoff table with profits expressed in millions of dollars is shown in Table 21.1. Note, for example, that if a medium complex is built and demand turns out to be strong, a profit of \$14 million will be realized. We will use the notation  $V_{ij}$  to denote the payoff associated with decision alternative  $i$  and state of nature  $j$ . Using Table 21.1,  $V_{31} = 20$  indicates a payoff of \$20 million occurs if the decision is to build a large complex ( $d_3$ ) and the strong demand state of nature ( $s_1$ ) occurs. Similarly,  $V_{32} = -9$  indicates a loss of \$9 million if the decision is to build a large complex ( $d_3$ ) and the weak demand state of nature ( $s_2$ ) occurs.

## Decision Trees

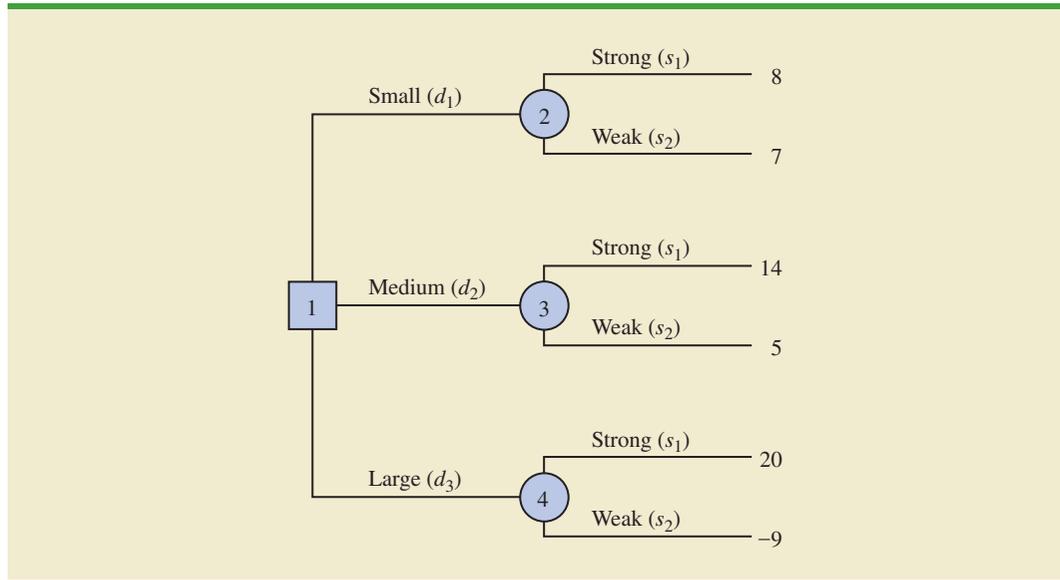
A **decision tree** graphically shows the sequential nature of the decision-making process. Figure 21.1 presents a decision tree for the PDC problem, demonstrating the natural or logical progression that will occur over time. First, PDC must make a decision regarding the size of the condominium complex ( $d_1$ ,  $d_2$ , or  $d_3$ ). Then, after the decision is implemented, either state of nature  $s_1$  or  $s_2$  will occur. The number at each end point of the tree indicates the payoff associated with a particular sequence. For example, the topmost payoff of 8 indicates that an \$8 million profit is anticipated if PDC constructs a small condominium complex ( $d_1$ ) and demand turns out to be strong ( $s_1$ ). The next payoff of 7 indicates an anticipated profit of \$7 million if PDC constructs a small condominium complex ( $d_1$ ) and demand turns out to be weak ( $s_2$ ). Thus, the decision tree shows graphically the sequences of decision alternatives and states of nature that provide the six possible payoffs.

The decision tree in Figure 21.1 has four **nodes**, numbered 1–4, that represent the decisions and chance events. Squares are used to depict **decision nodes** and circles are used to depict **chance nodes**. Thus, node 1 is a decision node, and nodes 2, 3, and 4 are chance nodes. The **branches** leaving the decision node correspond to the decision alternatives. The branches leaving each chance node correspond to the states of nature. The payoffs are shown at the end of the states-of-nature branches. We now turn to the question: How can

**TABLE 21.1** PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT  
(PAYOFFS IN \$ MILLIONS)

Decision Alternative	State of Nature	
	Strong Demand $s_1$	Weak Demand $s_2$
Small complex, $d_1$	8	7
Medium complex, $d_2$	14	5
Large complex, $d_3$	20	-9

**FIGURE 21.1** DECISION TREE FOR THE PDC CONDOMINIUM PROJECT  
(PAYOFFS IN \$ MILLIONS)



the decision maker use the information in the payoff table or the decision tree to select the best decision alternative?

### NOTES AND COMMENTS

1. Experts in problem solving agree that the first step in solving a complex problem is to decompose it into a series of smaller subproblems. Decision trees provide a useful way to show how a problem can be decomposed and the sequential nature of the decision process.
2. People often view the same problem from different perspectives. Thus, the discussion regarding the development of a decision tree may provide additional insight about the problem.

## 21.2 Decision Making with Probabilities

Once we define the decision alternatives and the states of nature for the chance events, we can focus on determining probabilities for the states of nature. The classical method, the relative frequency method, or the subjective method of assigning probabilities discussed in Chapter 4 may be used to identify these probabilities. After determining the appropriate probabilities, we show how to use the **expected value approach** to identify the best, or recommended, decision alternative for the problem.

### Expected Value Approach

We begin by defining the expected value of a decision alternative. Let

$$N = \text{the number of states of nature}$$

$$P(s_j) = \text{the probability of state of nature } s_j$$

Because one and only one of the  $N$  states of nature can occur, the probabilities must satisfy two conditions:

*The probabilities for the states of nature must satisfy the basic requirements for assigning probabilities introduced in Chapter 4.*

$$P(s_j) \geq 0 \quad \text{for all states of nature} \quad (21.1)$$

$$\sum_{j=1}^N P(s_j) = P(s_1) + P(s_2) + \cdots + P(s_N) = 1 \quad (21.2)$$

The **expected value (EV)** of decision alternative  $d_i$  is as follows.

#### EXPECTED VALUE

$$EV(d_i) = \sum_{j=1}^N P(s_j)V_{ij} \quad (21.3)$$

where

$V_{ij}$  = the value of the payoff for decision alternative  $d_i$  and state of nature  $s_j$ .

In words, the expected value of a decision alternative is the sum of weighted payoffs for the decision alternative. The weight for a payoff is the probability of the associated state of nature and therefore the probability that the payoff will occur. Let us return to the PDC problem to see how the expected value approach can be applied.

PDC is optimistic about the potential for the luxury high-rise condominium complex. Suppose that this optimism leads to an initial subjective probability assessment of .8 that demand will be strong ( $s_1$ ) and a corresponding probability of .2 that demand will be weak ( $s_2$ ). Thus,  $P(s_1) = .8$  and  $P(s_2) = .2$ . Using the payoff values in Table 21.1 and equation (21.3), we compute the expected value for each of the three decision alternatives as follows:

$$EV(d_1) = .8(8) + .2(7) = 7.8$$

$$EV(d_2) = .8(14) + .2(5) = 12.2$$

$$EV(d_3) = .8(20) + .2(-9) = 14.2$$

Thus, using the expected value approach, we find that the large condominium complex, with an expected value of \$14.2 million, is the recommended decision.

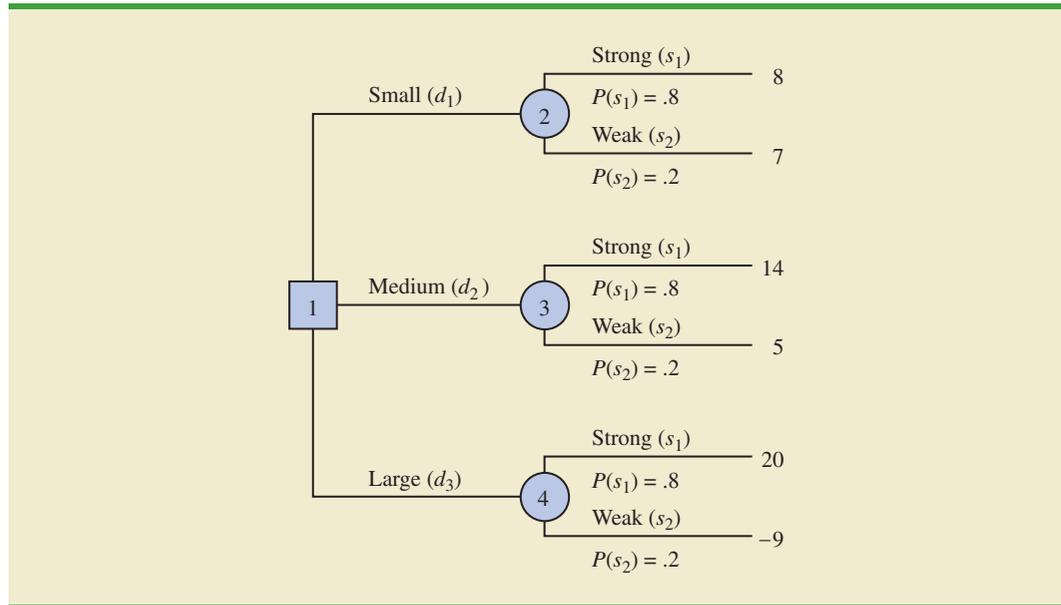
The calculations required to identify the decision alternative with the best expected value can be conveniently carried out on a decision tree. Figure 21.2 shows the decision tree for the PDC problem with state-of-nature branch probabilities. Working backward through the decision tree, we first compute the expected value at each chance node; that is, at each chance node, we weight each possible payoff by its probability of occurrence. By doing so, we obtain the expected values for nodes 2, 3, and 4, as shown in Figure 21.3.

Because the decision maker controls the branch leaving decision node 1 and because we are trying to maximize the expected profit, the best decision alternative at node 1 is  $d_3$ . Thus, the decision tree analysis leads to a recommendation of  $d_3$  with an expected value of \$14.2 million. Note that this recommendation is also obtained with the expected value approach in conjunction with the payoff table.

Other decision problems may be substantially more complex than the PDC problem, but if a reasonable number of decision alternatives and states of nature are present, you can use the decision tree approach outlined here. First, draw a decision tree consisting of decision nodes, chance nodes, and branches that describe the sequential nature of the problem. If you use the expected value approach, the next step is to determine the probabilities for

*Computer software packages are available to help in constructing more complex decision trees.*

FIGURE 21.2 PDC DECISION TREE WITH STATE-OF-NATURE BRANCH PROBABILITIES

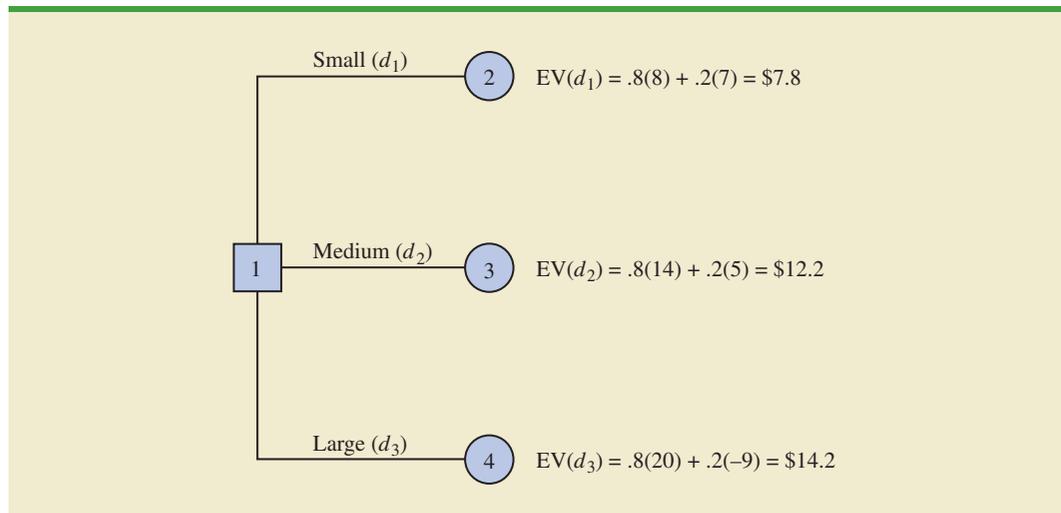


each of the states of nature and compute the expected value at each chance node. Then select the decision branch leading to the chance node with the best expected value. The decision alternative associated with this branch is the recommended decision.

### Expected Value of Perfect Information

Suppose that PDC has the opportunity to conduct a market research study that would help evaluate buyer interest in the condominium project and provide information that management could use to improve the probability assessments for the states of nature. To determine the potential value of this information, we begin by supposing that the study could provide *perfect information* regarding the states of nature; that is, we assume for the moment that

FIGURE 21.3 APPLYING THE EXPECTED VALUE APPROACH USING DECISION TREES



PDC could determine with certainty, prior to making a decision, which state of nature is going to occur. To make use of this perfect information, we will develop a decision strategy that PDC should follow once it knows which state of nature will occur. A decision strategy is simply a decision rule that specifies the decision alternative to be selected after new information becomes available.

To help determine the decision strategy for PDC, we reproduce PDC’s payoff table in Table 21.2. Note that, if PDC knew for sure that state of nature  $s_1$  would occur, the best decision alternative would be  $d_3$ , with a payoff of \$20 million. Similarly, if PDC knew for sure that state of nature  $s_2$  would occur, the best decision alternative would be  $d_1$ , with a payoff of \$7 million. Thus, we can state PDC’s optimal decision strategy if the perfect information becomes available as follows:

- If  $s_1$ , select  $d_3$  and receive a payoff of \$20 million.
- If  $s_2$ , select  $d_1$  and receive a payoff of \$7 million.

What is the expected value for this decision strategy? To compute the expected value with perfect information, we return to the original probabilities for the states of nature:  $P(s_1) = .8$  and  $P(s_2) = .2$ . Thus, there is a .8 probability that the perfect information will indicate state of nature  $s_1$  and the resulting decision alternative  $d_3$  will provide a \$20 million profit. Similarly, with a .2 probability for state of nature  $s_2$ , the optimal decision alternative  $d_1$  will provide a \$7 million profit. Thus, using equation (21.3), the expected value of the decision strategy based on perfect information is

$$.8(20) + .2(7) = 17.4$$

We refer to the expected value of \$17.4 million as the *expected value with perfect information* (EVwPI).

Earlier in this section we showed that the recommended decision using the expected value approach is decision alternative  $d_3$ , with an expected value of \$14.2 million. Because this decision recommendation and expected value computation were made without the benefit of perfect information, \$14.2 million is referred to as the *expected value without perfect information* (EVwoPI).

The expected value with perfect information is \$17.4 million, and the expected value without perfect information is \$14.2; therefore, the expected value of the perfect information (EVPI) is  $\$17.4 - \$14.2 = \$3.2$  million. In other words, \$3.2 million represents the additional expected value that can be obtained if perfect information were available about the states of nature. Generally speaking, a market research study will not provide “perfect” information; however, if the market research study is a good one, the information gathered might be worth a sizable portion of the \$3.2 million. Given the EVPI of \$3.2 million, PDC might seriously consider a market survey as a way to obtain more information about the states of nature.

*It would be worth \$3.2 million for PDC to learn the level of market acceptance before selecting a decision alternative.*

**TABLE 21.2** PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT (\$ MILLIONS)

Decision Alternative	State of Nature	
	Strong Demand $s_1$	Weak Demand $s_2$
Small complex, $d_1$	8	7
Medium complex, $d_2$	14	5
Large complex, $d_3$	20	-9

In general, the **expected value of perfect information** is computed as follows:

#### EXPECTED VALUE OF PERFECT INFORMATION

$$EVPI = |EV_{wPI} - EV_{woPI}| \quad (21.4)$$

where

$EVPI$  = expected value of perfect information

$EV_{wPI}$  = expected value *with* perfect information about the states of nature

$EV_{woPI}$  = expected value *without* perfect information about the states of nature

Note the role of the absolute value in equation (21.4). For minimization problems, information helps reduce or lower cost; thus the expected value with perfect information is less than or equal to the expected value without perfect information. In this case,  $EVPI$  is the magnitude of the difference between  $EV_{wPI}$  and  $EV_{woPI}$ , or the absolute value of the difference as shown in equation (21.4).

### Exercises

#### Methods

#### SELF test

- The following payoff table shows profit for a decision analysis problem with two decision alternatives and three states of nature.

Decision Alternative	States of Nature		
	$s_1$	$s_2$	$s_3$
$d_1$	250	100	25
$d_2$	100	100	75

- Construct a decision tree for this problem.
  - Suppose that the decision maker obtains the probabilities  $P(s_1) = .65$ ,  $P(s_2) = .15$ , and  $P(s_3) = .20$ . Use the expected value approach to determine the optimal decision.
- A decision maker faced with four decision alternatives and four states of nature develops the following profit payoff table.

Decision Alternative	States of Nature			
	$s_1$	$s_2$	$s_3$	$s_4$
$d_1$	14	9	10	5
$d_2$	11	10	8	7
$d_3$	9	10	10	11
$d_4$	8	10	11	13

The decision maker obtains information that enables the following probabilities assessments:  $P(s_1) = .5$ ,  $P(s_2) = .2$ ,  $P(s_3) = .2$ , and  $P(s_4) = .1$ .

- Use the expected value approach to determine the optimal solution.
- Now assume that the entries in the payoff table are costs. Use the expected value approach to determine the optimal decision.

**SELF test**

## Applications

3. Hudson Corporation is considering three options for managing its data processing operation: continue with its own staff, hire an outside vendor to do the managing (referred to as *outsourcing*), or use a combination of its own staff and an outside vendor. The cost of the operation depends on future demand. The annual cost of each option (in thousands of dollars) depends on demand as follows.

Staffing Options	Demand		
	High	Medium	Low
Own staff	650	650	600
Outside vendor	900	600	300
Combination	800	650	500

- If the demand probabilities are .2, .5, and .3, which decision alternative will minimize the expected cost of the data processing operation? What is the expected annual cost associated with your recommendation?
  - What is the expected value of perfect information?
4. Myrtle Air Express decided to offer direct service from Cleveland to Myrtle Beach. Management must decide between a full price service using the company's new fleet of jet aircraft and a discount service using smaller capacity commuter planes. It is clear that the best choice depends on the market reaction to the service Myrtle Air offers. Management developed estimates of the contribution to profit for each type of service based upon two possible levels of demand for service to Myrtle Beach: strong and weak. The following table shows the estimated quarterly profits (in thousands of dollars).

Service	Demand for Service	
	Strong	Weak
Full price	\$960	-\$490
Discount	\$670	\$320

- What is the decision to be made, what is the chance event, and what is the consequence for this problem? How many decision alternatives are there? How many outcomes are there for the chance event?
  - Suppose that management of Myrtle Air Express believes that the probability of strong demand is .7 and the probability of weak demand is .3. Use the expected value approach to determine an optimal decision.
  - Suppose that the probability of strong demand is .8 and the probability of weak demand is .2. What is the optimal decision using the expected value approach?
5. The distance from Potsdam to larger markets and limited air service have hindered the town in attracting new industry. Air Express, a major overnight delivery service, is considering establishing a regional distribution center in Potsdam. But Air Express will not establish the center unless the length of the runway at the local airport is increased. Another candidate for new development is Diagnostic Research, Inc. (DRI), a leading producer of medical testing equipment. DRI is considering building a new manufacturing plant. Increasing the length of the runway is not a requirement for DRI, but the planning commission feels that doing so will help convince DRI to locate their new plant in Potsdam.

Assuming that the town lengthens the runway, the Potsdam planning commission believes that the probabilities shown in the following table are applicable.

	DRI Plant	No DRI Plant
<b>Air Express Center</b>	.30	.10
<b>No Air Express Center</b>	.40	.20

For instance, the probability that Air Express will establish a distribution center and DRI will build a plant is .30.

The estimated annual revenue to the town, after deducting the cost of lengthening the runway, is as follows:

	DRI Plant	No DRI Plant
<b>Air Express Center</b>	\$600,000	\$150,000
<b>No Air Express Center</b>	\$250,000	−\$200,000

If the runway expansion project is not conducted, the planning commission assesses the probability DRI will locate their new plant in Potsdam at .6; in this case, the estimated annual revenue to the town will be \$450,000. If the runway expansion project is not conducted and DRI does not locate in Potsdam, the annual revenue will be \$0 since no cost will have been incurred and no revenues will be forthcoming.

- What is the decision to be made, what is the chance event, and what is the consequence?
- Compute the expected annual revenue associated with the decision alternative to lengthen the runway.
- Compute the expected annual revenue associated with the decision alternative to not lengthen the runway.
- Should the town elect to lengthen the runway? Explain.
- Suppose that the probabilities associated with lengthening the runway were as follows:

	DRI Plant	No DRI Plant
<b>Air Express Center</b>	.40	.10
<b>No Air Express Center</b>	.30	.20

What effect, if any, would this change in the probabilities have on the recommended decision?

- Seneca Hill Winery recently purchased land for the purpose of establishing a new vineyard. Management is considering two varieties of white grapes for the new vineyard: Chardonnay and Riesling. The Chardonnay grapes would be used to produce a dry Chardonnay wine, and the Riesling grapes would be used to produce a semi-dry Riesling wine. It takes approximately four years from the time of planting before new grapes can be harvested. This length of time creates a great deal of uncertainty concerning future demand and makes the decision concerning the type of grapes to plant difficult. Three possibilities are being considered: Chardonnay grapes only; Riesling grapes only; and both Chardonnay and Riesling grapes. Seneca management decided that for planning purposes it would be adequate to consider only two demand possibilities for each type of

wine: strong or weak. With two possibilities for each type of wine it was necessary to assess four probabilities. With the help of some forecasts in industry publications management made the following probability assessments.

		Riesling Demand	
		Weak	Strong
Chardonnay Demand	Weak	.05	.50
	Strong	.25	.20

Revenue projections show an annual contribution to profit of \$20,000 if Seneca Hill only plants Chardonnay grapes and demand is weak for Chardonnay wine, and \$70,000 if they only plant Chardonnay grapes and demand is strong for Chardonnay wine. If they only plant Riesling grapes, the annual profit projection is \$25,000 if demand is weak for Riesling grapes and \$45,000 if demand is strong for Riesling grapes. If Seneca plants both types of grapes, the annual profit projections are shown in the following table.

		Riesling Demand	
		Weak	Strong
Chardonnay Demand	Weak	\$22,000	\$40,000
	Strong	\$26,000	\$60,000

- a. What is the decision to be made, what is the chance event, and what is the consequence? Identify the alternatives for the decisions and the possible outcomes for the chance events.
  - b. Develop a decision tree.
  - c. Use the expected value approach to recommend which alternative Seneca Hill Winery should follow in order to maximize expected annual profit.
  - d. Suppose management is concerned about the probability assessments when demand for Chardonnay wine is strong. Some believe it is likely for Riesling demand to also be strong in this case. Suppose the probability of strong demand for Chardonnay and weak demand for Riesling is .05 and that the probability of strong demand for Chardonnay and strong demand for Riesling is .40. How does this change the recommended decision? Assume that the probabilities when Chardonnay demand is weak are still .05 and .50.
  - e. Other members of the management team expect the Chardonnay market to become saturated at some point in the future, causing a fall in prices. Suppose that the annual profit projections fall to \$50,000 when demand for Chardonnay is strong and Chardonnay grapes only are planted. Using the original probability assessments, determine how this change would affect the optimal decision.
7. The Lake Placid Town Council has decided to build a new community center to be used for conventions, concerts, and other public events, but considerable controversy surrounds the appropriate size. Many influential citizens want a large center that would be a show-case for the area, but the mayor feels that if demand does not support such a center, the community will lose a large amount of money. To provide structure for the decision process, the council narrowed the building alternatives to three sizes: small, medium, and large. Everybody agreed that the critical factor in choosing the best size is the number of people who will want to use the new facility. A regional planning consultant provided demand estimates under three scenarios: worst case, base case, and best case. The worst-case scenario corresponds to a situation in which tourism drops significantly; the base-case scenario corresponds to a situation in which Lake Placid continues to attract visitors at

current levels; and the best-case scenario corresponds to a significant increase in tourism. The consultant has provided probability assessments of .10, .60, and .30 for the worst-case, base-case, and best-case scenarios, respectively.

The town council suggested using net cash flow over a five-year planning horizon as the criterion for deciding on the best size. A consultant developed the following projections of net cash flow (in thousands of dollars) for a five-year planning horizon. All costs, including the consultant's fee, are included.

Center Size	Demand Scenario		
	Worst Case	Base Case	Best Case
Small	400	500	660
Medium	-250	650	800
Large	-400	580	990

- What decision should Lake Placid make using the expected value approach?
- Compute the expected value of perfect information. Do you think it would be worth trying to obtain additional information concerning which scenario is likely to occur?
- Suppose the probability of the worst-case scenario increases to .2, the probability of the base-case scenario decreases to .5, and the probability of the best-case scenario remains at .3. What effect, if any, would these changes have on the decision recommendation?
- The consultant suggested that an expenditure of \$150,000 on a promotional campaign over the planning horizon will effectively reduce the probability of the worst-case scenario to zero. If the campaign can be expected to also increase the probability of the best-case scenario to .4, is it a good investment?

## 21.3

## Decision Analysis with Sample Information

In applying the expected value approach, we showed how probability information about the states of nature affects the expected value calculations and thus the decision recommendation. Frequently, decision makers have preliminary or **prior probability** assessments for the states of nature that are the best probability values available at that time. However, to make the best possible decision, the decision maker may want to seek additional information about the states of nature. This new information can be used to revise or update the prior probabilities so that the final decision is based on more accurate probabilities for the states of nature. Most often, additional information is obtained through experiments designed to provide **sample information** about the states of nature. Raw material sampling, product testing, and market research studies are examples of experiments (or studies) that may enable management to revise or update the state-of-nature probabilities. These revised probabilities are called **posterior probabilities**.

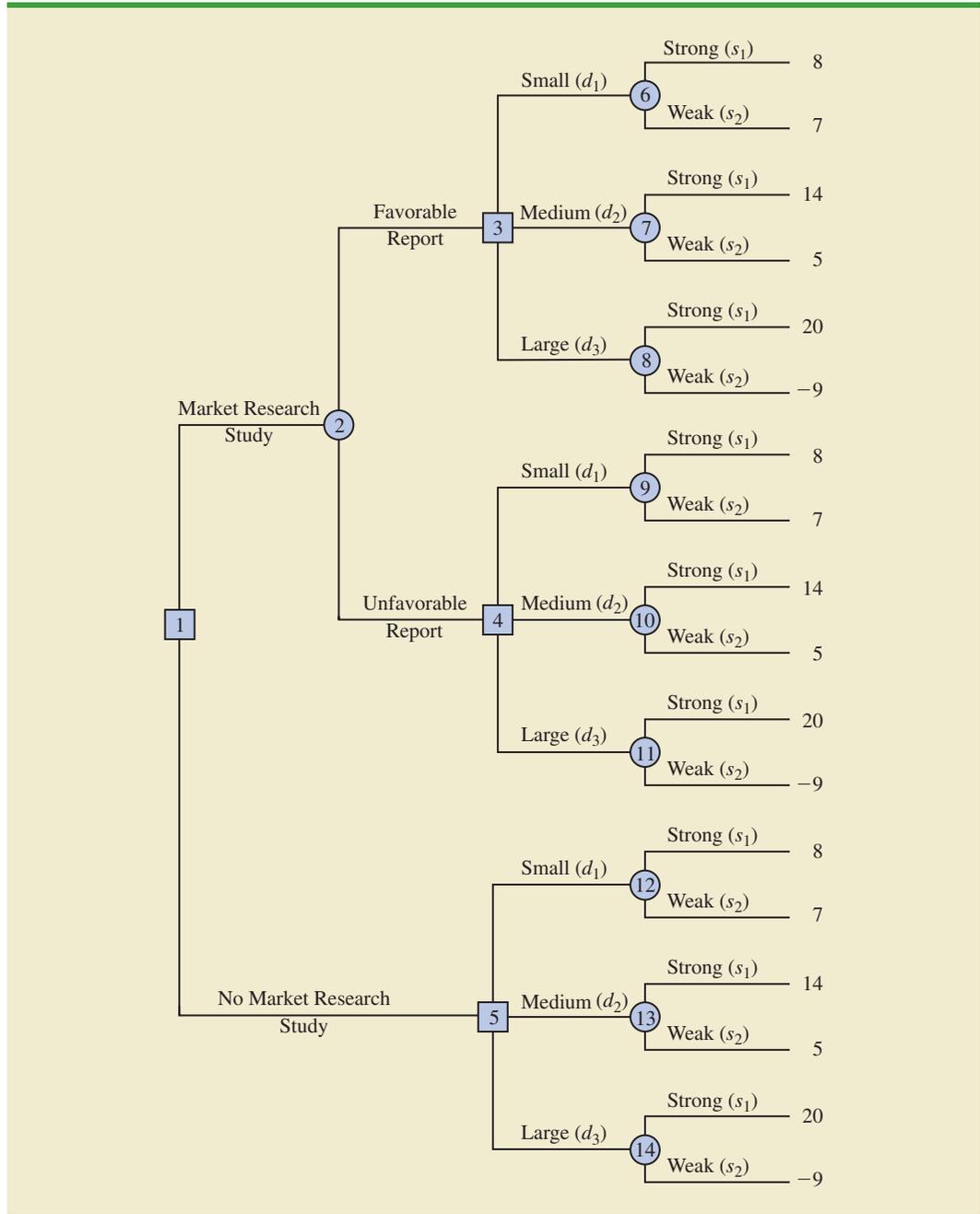
Let us return to the PDC problem and assume that management is considering a six-month market research study designed to learn more about potential market acceptance of the PDC condominium project. Management anticipates that the market research study will provide one of the following two results:

- Favorable report: A significant number of the individuals contacted express interest in purchasing a PDC condominium.
- Unfavorable report: Very few of the individuals contacted express interest in purchasing a PDC condominium.

### Decision Tree

The decision tree for the PDC problem with sample information shows the logical sequence for the decisions and the chance events in Figure 21.4. First, PDC’s management must decide whether the market research should be conducted. If it is conducted, PDC’s management must be prepared to make a decision about the size of the condominium project if the market research report is favorable and, possibly, a different decision about the size of the condominium project if the market research report is unfavorable.

**FIGURE 21.4** THE PDC DECISION TREE INCLUDING THE MARKET RESEARCH STUDY



In Figure 21.4, the squares are decision nodes and the circles are chance nodes. At each decision node, the branch of the tree that is taken is based on the decision made. At each chance node, the branch of the tree that is taken is based on probability or chance. For example, decision node 1 shows that PDC must first make the decision whether to conduct the market research study. If the market research study is undertaken, chance node 2 indicates that both the favorable report branch and the unfavorable report branch are not under PDC's control and will be determined by chance. Node 3 is a decision node, indicating that PDC must make the decision to construct the small, medium, or large complex if the market research report is favorable. Node 4 is a decision node showing that PDC must make the decision to construct the small, medium, or large complex if the market research report is unfavorable. Node 5 is a decision node indicating that PDC must make the decision to construct the small, medium, or large complex if the market research is not undertaken. Nodes 6 to 14 are chance nodes indicating that the strong demand or weak demand state-of-nature branches will be determined by chance.

*We explain in Section 21.4 how these probabilities can be developed.*

Analysis of the decision tree and the choice of an optimal strategy requires that we know the branch probabilities corresponding to all chance nodes. PDC developed the following branch probabilities.

If the market research study is undertaken,

$$\begin{aligned} P(\text{Favorable report}) &= P(F) = .77 \\ P(\text{Unfavorable report}) &= P(U) = .23 \end{aligned}$$

If the market research report is favorable,

$$\begin{aligned} P(\text{Strong demand given a favorable report}) &= P(s_1|F) = .94 \\ P(\text{Weak demand given a favorable report}) &= P(s_2|F) = .06 \end{aligned}$$

If the market research report is unfavorable,

$$\begin{aligned} P(\text{Strong demand given an unfavorable report}) &= P(s_1|U) = .35 \\ P(\text{Weak demand given an unfavorable report}) &= P(s_2|U) = .65 \end{aligned}$$

If the market research report is not undertaken, the prior probabilities are applicable.

$$\begin{aligned} P(\text{Strong demand}) &= P(s_1) = .80 \\ P(\text{Weak demand}) &= P(s_2) = .20 \end{aligned}$$

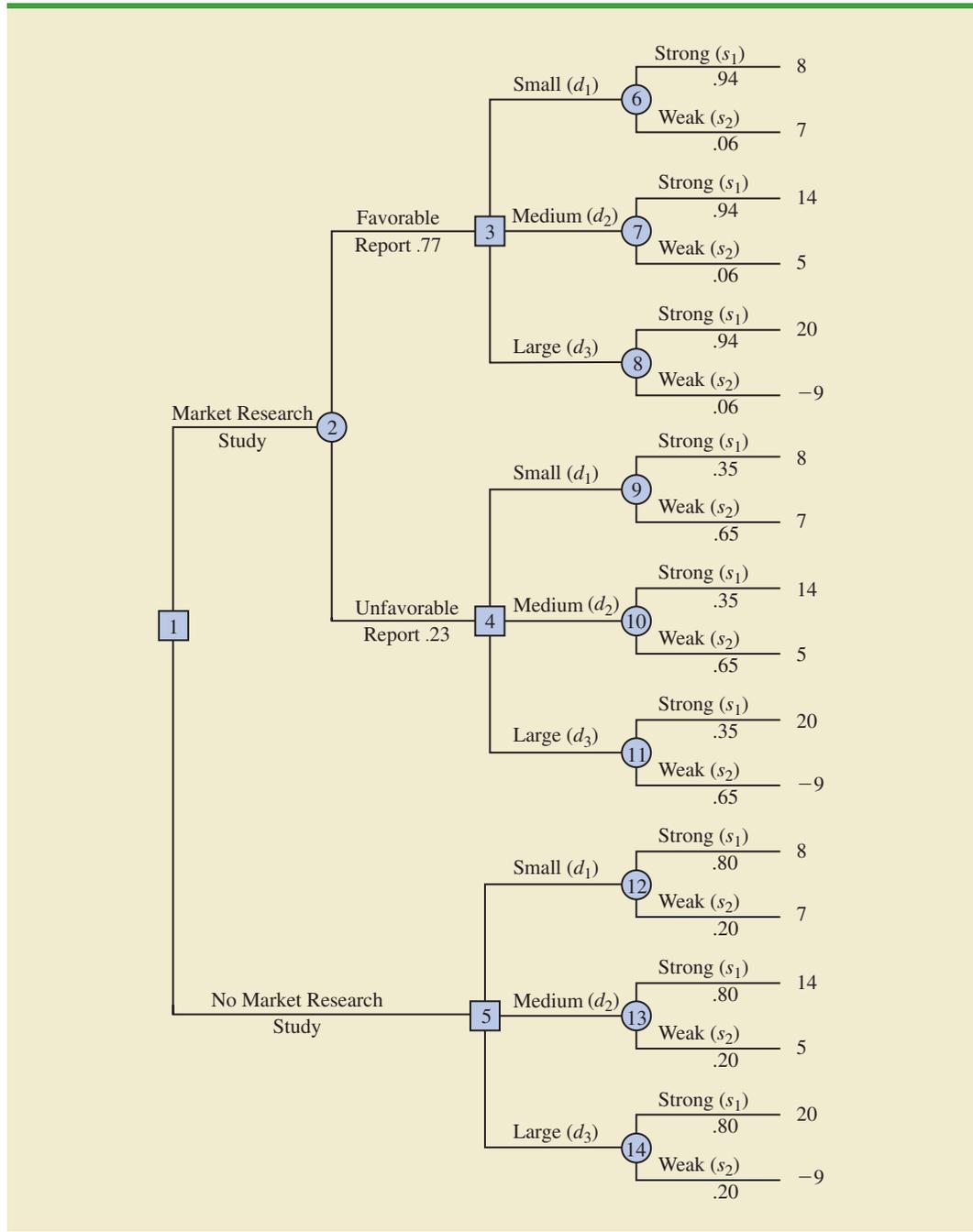
The branch probabilities are shown on the decision tree in Figure 21.5.

## Decision Strategy

A **decision strategy** is a sequence of decisions and chance outcomes where the decisions chosen depend on the yet to be determined outcomes of chance events. The approach used to determine the optimal decision strategy is based on a backward pass through the decision tree using the following steps:

1. At chance nodes, compute the expected value by multiplying the payoff at the end of each branch by the corresponding branch probability.
2. At decision nodes, select the decision branch that leads to the best expected value. This expected value becomes the expected value at the decision node.

FIGURE 21.5 THE PDC DECISION TREE WITH BRANCH PROBABILITIES



Starting the backward pass calculations by computing the expected values at chance nodes 6 to 14 provides the following results.

$$\begin{aligned}
 \text{EV(Node 6)} &= .94(8) + .06(7) = 7.94 \\
 \text{EV(Node 7)} &= .94(14) + .06(5) = 13.46 \\
 \text{EV(Node 8)} &= .94(20) + .06(-9) = 18.26 \\
 \text{EV(Node 9)} &= .35(8) + .65(7) = 7.35 \\
 \text{EV(Node 10)} &= .35(14) + .65(5) = 8.15 \\
 \text{EV(Node 11)} &= .35(20) + .65(-9) = 1.15 \\
 \text{EV(Node 12)} &= .80(8) + .20(7) = 7.80 \\
 \text{EV(Node 13)} &= .80(14) + .20(5) = 12.20 \\
 \text{EV(Node 14)} &= .80(20) + .20(-9) = 14.20
 \end{aligned}$$

Figure 21.6 shows the reduced decision tree after computing expected values at these chance nodes.

Next move to decision nodes 3, 4, and 5. For each of these nodes, we select the decision alternative branch that leads to the best expected value. For example, at node 3 we have the choice of the small complex branch with  $\text{EV}(\text{Node 6}) = 7.94$ , the medium complex branch with  $\text{EV}(\text{Node 7}) = 13.46$ , and the large complex branch with  $\text{EV}(\text{Node 8}) = 18.26$ . Thus, we select the large complex decision alternative branch and the expected value at node 3 becomes  $\text{EV}(\text{Node 3}) = 18.26$ .

For node 4, we select the best expected value from nodes 9, 10, and 11. The best decision alternative is the medium complex branch that provides  $\text{EV}(\text{Node 4}) = 8.15$ . For node 5, we select the best expected value from nodes 12, 13, and 14. The best decision alternative is the large complex branch that provides  $\text{EV}(\text{Node 5}) = 14.20$ . Figure 21.7 shows the reduced decision tree after choosing the best decisions at nodes 3, 4, and 5.

The expected value at chance node 2 can now be computed as follows:

$$\begin{aligned}
 \text{EV}(\text{Node 2}) &= .77\text{EV}(\text{Node 3}) + .23\text{EV}(\text{Node 4}) \\
 &= .77(18.26) + .23(8.15) = 15.93
 \end{aligned}$$

This calculation reduces the decision tree to one involving only the two decision branches from node 1 (see Figure 21.8).

Finally, the decision can be made at decision node 1 by selecting the best expected values from nodes 2 and 5. This action leads to the decision alternative to conduct the market research study, which provides an overall expected value of 15.93.

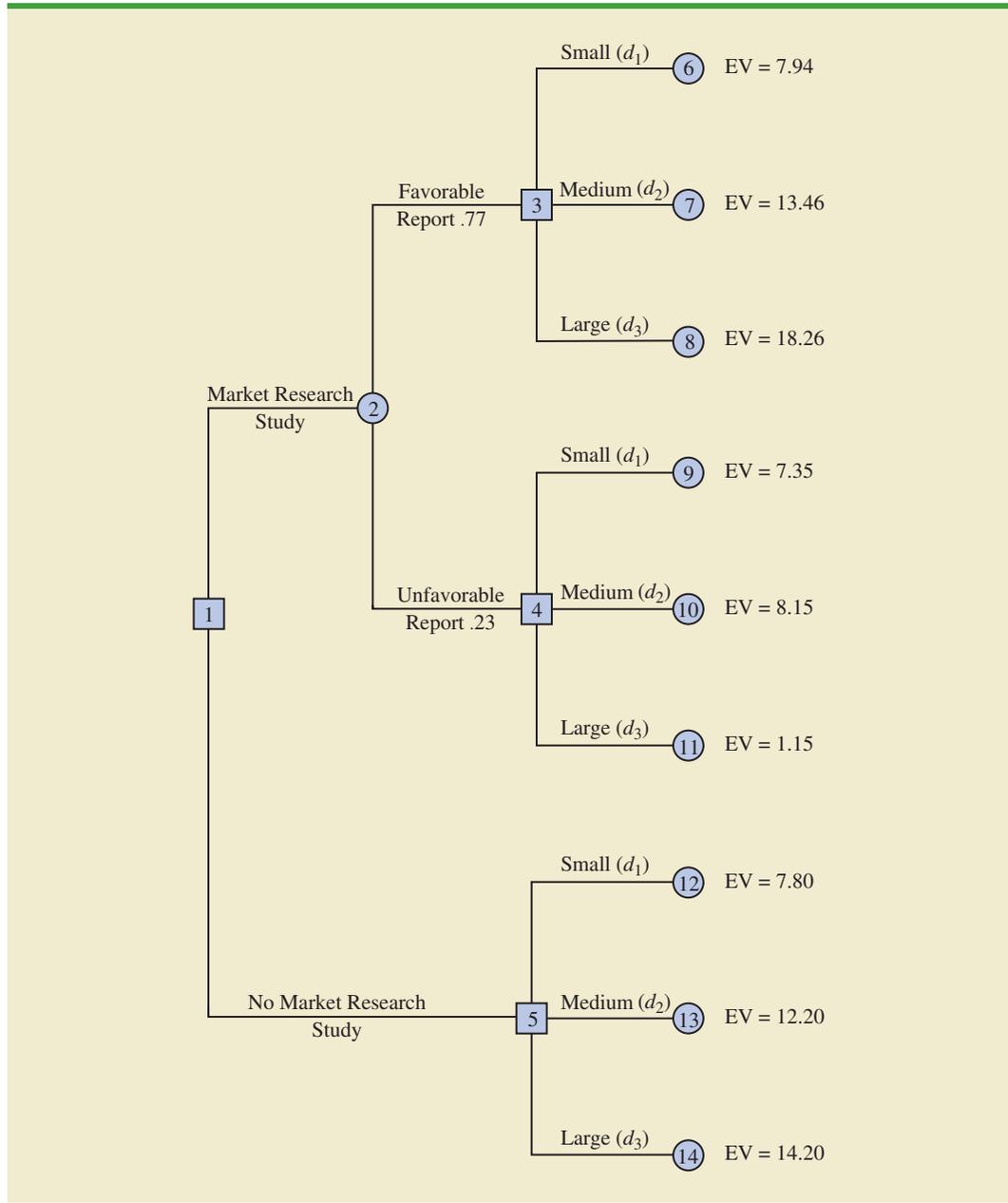
The optimal decision for PDC is to conduct the market research study and then carry out the following decision strategy:

If the market research is favorable, construct the large condominium complex.

If the market research is unfavorable, construct the medium condominium complex.

The analysis of the PDC decision tree illustrates the methods that can be used to analyze more complex sequential decision problems. First, draw a decision tree consisting of decision and chance nodes and branches that describe the sequential nature of the problem. Determine the probabilities for all chance outcomes. Then, by working backward through the tree, compute expected values at all chance nodes and select the best decision branch at all decision nodes. The sequence of optimal decision branches determines the optimal decision strategy for the problem.

**FIGURE 21.6** PDC DECISION TREE AFTER COMPUTING EXPECTED VALUES AT CHANCE NODES 6 TO 14

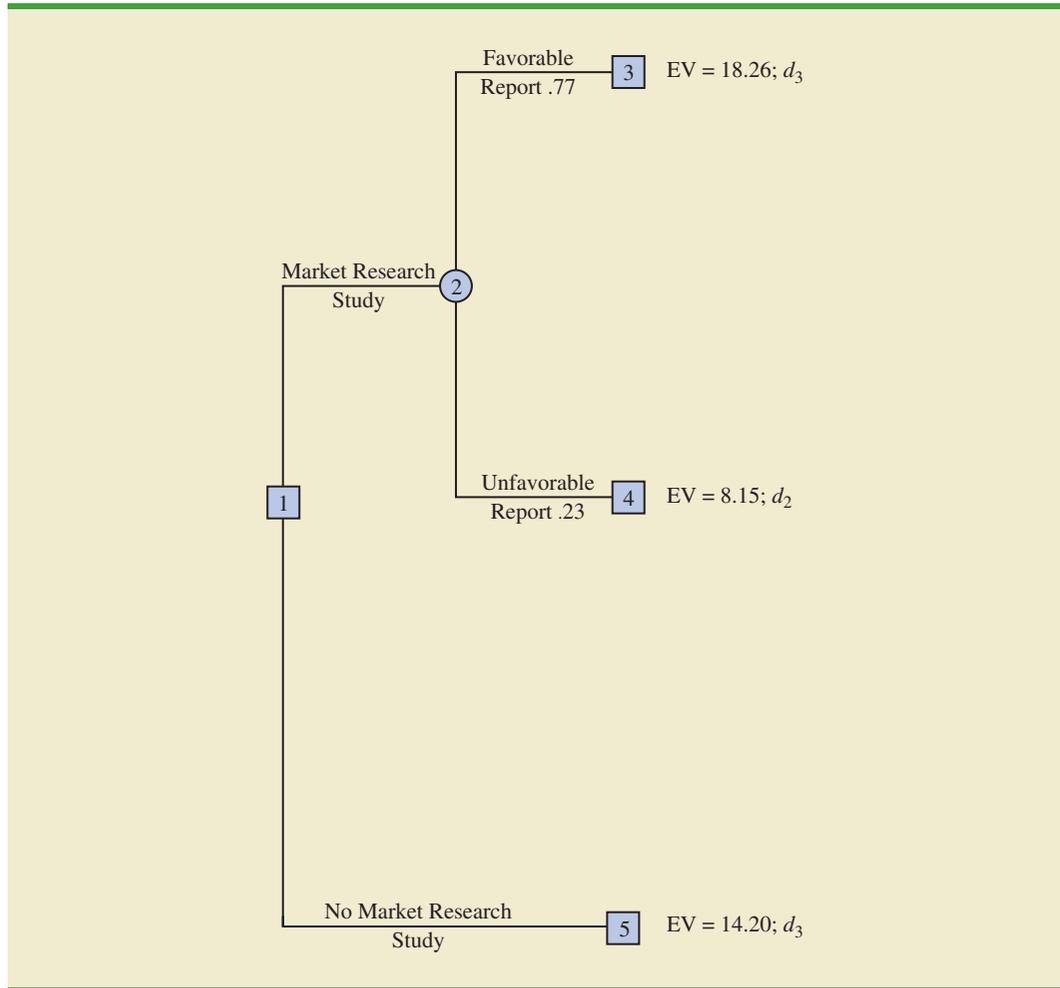


### Expected Value of Sample Information

*The EVSI = \$1.73 million suggests PDC should be willing to pay up to \$1.73 million to conduct the market research study.*

In the PDC problem, the market research study is the sample information used to determine the optimal decision strategy. The expected value associated with the market research study is \$15.93. In Section 21.3 we showed that the best expected value if the market research study is *not* undertaken is \$14.20. Thus, we can conclude that the difference, \$15.93 – \$14.20 = \$1.73, is the **expected value of sample information (EVSI)**. In other words,

**FIGURE 21.7** PDC DECISION TREE AFTER CHOOSING BEST DECISIONS AT NODES 3, 4, AND 5



conducting the market research study adds \$1.73 million to the PDC expected value. In general, the expected value of sample information is as follows:

#### EXPECTED VALUE OF SAMPLE INFORMATION

$$EVSI = |EV_{wSI} - EV_{woSI}| \quad (21.5)$$

where

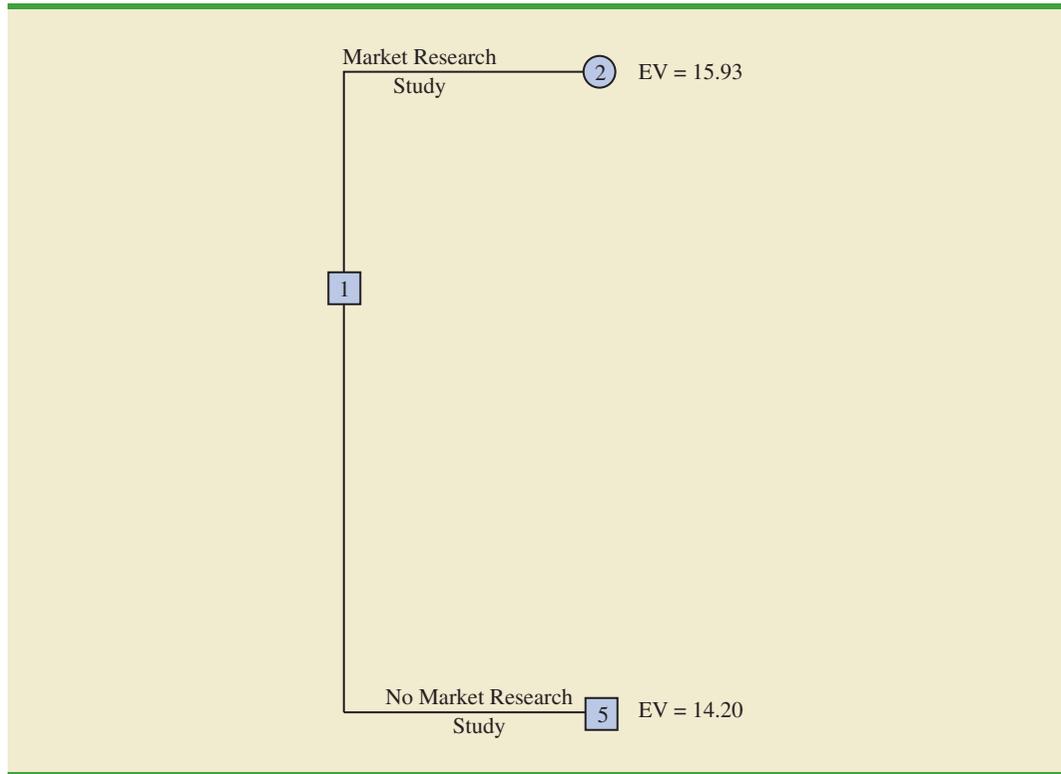
$EVSI$  = expected value of sample information

$EV_{wSI}$  = expected value *with* sample information about the states of nature

$EV_{woSI}$  = expected value *without* sample information about the states of nature

Note the role of the absolute value in equation (21.5). For minimization problems the expected value with sample information is always less than or equal to the expected value without

**FIGURE 21.8** PDC DECISION TREE REDUCED TO TWO DECISION BRANCHES



sample information. In this case, EVSI is the magnitude of the difference between EVwSI and EVwoSI; thus, by taking the absolute value of the difference as shown in equation (21.5), we can handle both the maximization and minimization cases with one equation.

**Exercises**

**Methods**

**SELF test**

8. Consider a variation of the PDC decision tree shown in Figure 21.5. The company must first decide whether to undertake the market research study. If the market research study is conducted, the outcome will either be favorable (*F*) or unfavorable (*U*). Assume there are only two decision alternatives  $d_1$  and  $d_2$  and two states of nature  $s_1$  and  $s_2$ . The payoff table showing profit is as follows:

Decision Alternative	State of Nature	
	$s_1$	$s_2$
$d_1$	100	300
$d_2$	400	200

- a. Show the decision tree.
- b. Use the following probabilities. What is the optimal decision strategy?

$$\begin{array}{llll}
 P(F) = .56 & P(s_1 | F) = .57 & P(s_1 | U) = .18 & P(s_1) = .40 \\
 P(U) = .44 & P(s_2 | F) = .43 & P(s_2 | U) = .82 & P(s_2) = .60
 \end{array}$$

## Applications

9. A real estate investor has the opportunity to purchase land currently zoned residential. If the county board approves a request to rezone the property as commercial within the next year, the investor will be able to lease the land to a large discount firm that wants to open a new store on the property. However, if the zoning change is not approved, the investor will have to sell the property at a loss. Profits (in thousands of dollars) are shown in the following payoff table.

Decision Alternative	State of Nature	
	Rezoning Approved $s_1$	Rezoning Not Approved $s_2$
Purchase, $d_1$	600	-200
Do not purchase, $d_2$	0	0

- If the probability that the rezoning will be approved is .5, what decision is recommended? What is the expected profit?
- The investor can purchase an option to buy the land. Under the option, the investor maintains the rights to purchase the land anytime during the next three months while learning more about possible resistance to the rezoning proposal from area residents. Probabilities are as follows.

Let  $H$  = high resistance to rezoning

$L$  = low resistance to rezoning

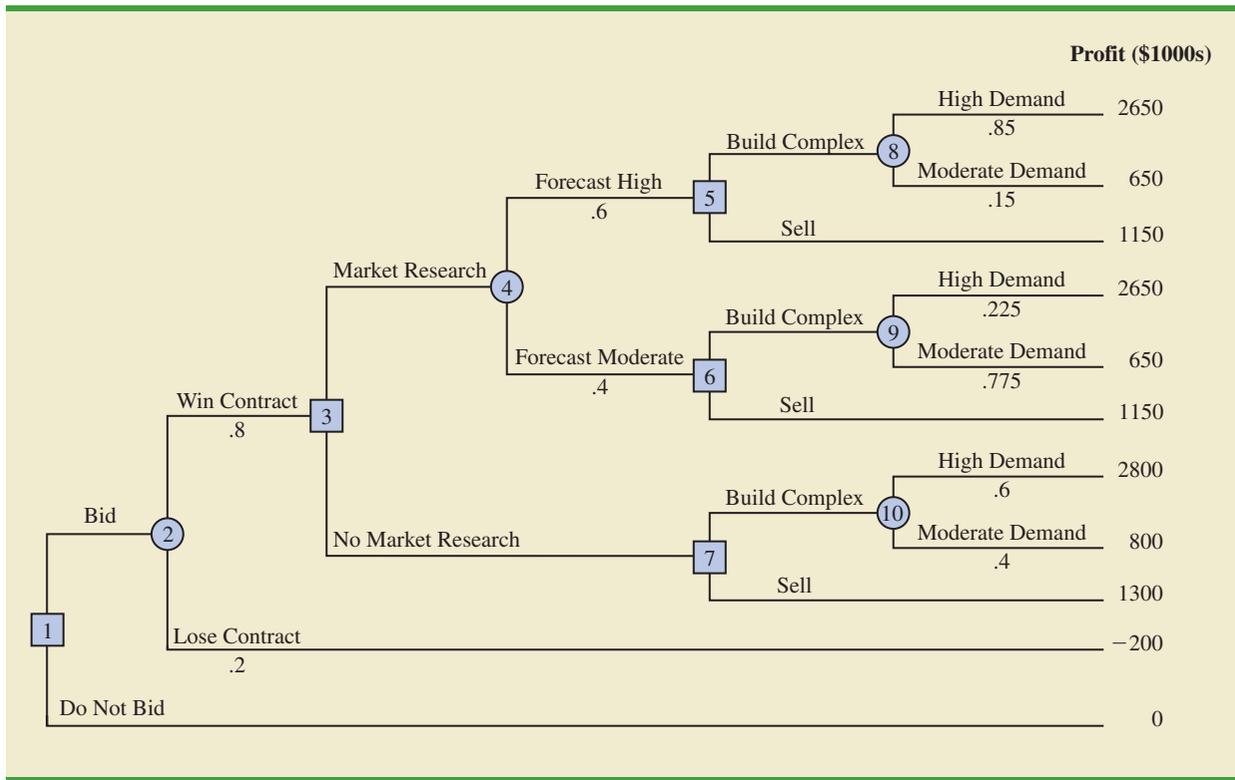
$$P(H) = .55 \quad P(s_1 | H) = .18 \quad P(s_2 | H) = .82$$

$$P(L) = .45 \quad P(s_1 | L) = .89 \quad P(s_2 | L) = .11$$

What is the optimal decision strategy if the investor uses the option period to learn more about the resistance from area residents before making the purchase decision?

- If the option will cost the investor an additional \$10,000, should the investor purchase the option? Why or why not? What is the maximum that the investor should be willing to pay for the option?
10. Dante Development Corporation is considering bidding on a contract for a new office building complex. Figure 21.9 shows the decision tree prepared by one of Dante's analysts. At node 1, the company must decide whether to bid on the contract. The cost of preparing the bid is \$200,000. The upper branch from node 2 shows that the company has a .8 probability of winning the contract if it submits a bid. If the company wins the bid, it will have to pay \$2,000,000 to become a partner in the project. Node 3 shows that the company will then consider doing a market research study to forecast demand for the office units prior to beginning construction. The cost of this study is \$150,000. Node 4 is a chance node showing the possible outcomes of the market research study.
- Nodes 5, 6, and 7 are similar in that they are the decision nodes for Dante to either build the office complex or sell the rights in the project to another developer. The decision to build the complex will result in an income of \$5,000,000 if demand is high and \$3,000,000 if demand is moderate. If Dante chooses to sell its rights in the project to another developer, income from the sale is estimated to be \$3,500,000. The probabilities shown at nodes 4, 8, and 9 are based on the projected outcomes of the market research study.
- Verify Dante's profit projections shown at the ending branches of the decision tree by calculating the payoffs of \$2,650,000 and \$650,000 for first two outcomes.
  - What is the optimal decision strategy for Dante, and what is the expected profit for this project?
  - What would the cost of the market research study have to be before Dante would change its decision about conducting the study?

FIGURE 21.9 DECISION TREE FOR THE DANTE DEVELOPMENT CORPORATION



11. Hale’s TV Productions is considering producing a pilot for a comedy series in the hope of selling it to a major television network. The network may decide to reject the series, but it may also decide to purchase the rights to the series for either one or two years. At this point in time, Hale may either produce the pilot and wait for the network’s decision or transfer the rights for the pilot and series to a competitor for \$100,000. Hale’s decision alternatives and profits (in thousands of dollars) are as follows:

Decision Alternative	State of Nature		
	Reject, $s_1$	1 Year, $s_2$	2 Years, $s_3$
Produce pilot, $d_1$	-100	50	150
Sell to competitor, $d_2$	100	100	100

The probabilities for the states of nature are  $P(s_1) = .2$ ,  $P(s_2) = .3$ , and  $P(s_3) = .5$ . For a consulting fee of \$5000, an agency will review the plans for the comedy series and indicate the overall chances of a favorable network reaction to the series. Assume that the agency review will result in a favorable ( $F$ ) or an unfavorable ( $U$ ) review and that the following probabilities are relevant.

$$\begin{aligned}
 P(F) &= .69 & P(s_1 | F) &= .09 & P(s_1 | U) &= .45 \\
 P(U) &= .31 & P(s_2 | F) &= .26 & P(s_2 | U) &= .39 \\
 & & P(s_3 | F) &= .65 & P(s_3 | U) &= .16
 \end{aligned}$$

- Construct a decision tree for this problem.
- What is the recommended decision if the agency opinion is not used? What is the expected value?

- c. What is the expected value of perfect information?
  - d. What is Hale's optimal decision strategy assuming the agency's information is used?
  - e. What is the expected value of the agency's information?
  - f. Is the agency's information worth the \$5000 fee? What is the maximum that Hale should be willing to pay for the information?
  - g. What is the recommended decision?
12. Martin's Service Station is considering entering the snowplowing business for the coming winter season. Martin can purchase either a snowplow blade attachment for the station's pick-up truck or a new heavy-duty snowplow truck. After analyzing the situation, Martin believes that either alternative would be a profitable investment if the snowfall is heavy. Smaller profits would result if the snowfall is moderate, and losses would result if the snowfall is light. The following profits/losses apply.

Decision Alternatives	State of Nature		
	Heavy, $s_1$	Moderate, $s_2$	Light, $s_3$
Blade attachment, $d_1$	3500	1000	-1500
New snowplow, $d_2$	7000	2000	-9000

The probabilities for the states of nature are  $P(s_1) = .4$ ,  $P(s_2) = .3$ , and  $P(s_3) = .3$ . Suppose that Martin decides to wait until September before making a final decision. Assessments of the probabilities associated with a normal ( $N$ ) or unseasonably cold ( $U$ ) September are as follows:

$$\begin{array}{lll}
 P(N) = .8 & P(s_1 | N) = .35 & P(s_1 | U) = .62 \\
 P(U) = .2 & P(s_2 | N) = .30 & P(s_2 | U) = .31 \\
 & P(s_3 | N) = .35 & P(s_3 | U) = .07
 \end{array}$$

- a. Construct a decision tree for this problem.
  - b. What is the recommended decision if Martin does not wait until September? What is the expected value?
  - c. What is the expected value of perfect information?
  - d. What is Martin's optimal decision strategy if the decision is not made until the September weather is determined? What is the expected value of this decision strategy?
13. Lawson's Department Store faces a buying decision for a seasonal product for which demand can be high, medium, or low. The purchaser for Lawson's can order 1, 2, or 3 lots of the product before the season begins but cannot reorder later. Profit projections (in thousands of dollars) are shown.

Decision Alternative	State of Nature		
	High Demand $s_1$	Medium Demand $s_2$	Low Demand $s_3$
Order 1 lot, $d_1$	60	60	50
Order 2 lots, $d_2$	80	80	30
Order 3 lots, $d_3$	100	70	10

- a. If the prior probabilities for the three states of nature are .3, .3, and .4, respectively, what is the recommended order quantity?
- b. At each preseason sales meeting, the vice president of sales provides a personal opinion regarding potential demand for this product. Because of the vice president's enthusiasm and optimistic nature, the predictions of market conditions have always been

either “excellent” ( $E$ ) or “very good” ( $V$ ). Probabilities are as follows. What is the optimal decision strategy?

$$\begin{array}{lll} P(E) = .7 & P(s_1 | E) = .34 & P(s_1 | V) = .20 \\ P(V) = .3 & P(s_2 | E) = .32 & P(s_2 | V) = .26 \\ & P(s_3 | E) = .34 & P(s_3 | V) = .54 \end{array}$$

- c. Compute EVPI and EVSI. Discuss whether the firm should consider a consulting expert who could provide independent forecasts of market conditions for the product.

## 21.4

## Computing Branch Probabilities Using Bayes' Theorem

In Section 21.3 the branch probabilities for the PDC decision tree chance nodes were specified in the problem description. No computations were required to determine these probabilities. In this section we show how **Bayes' theorem**, a topic covered in Chapter 4, can be used to compute branch probabilities for decision trees.

The PDC decision tree is shown again in Figure 21.10. Let

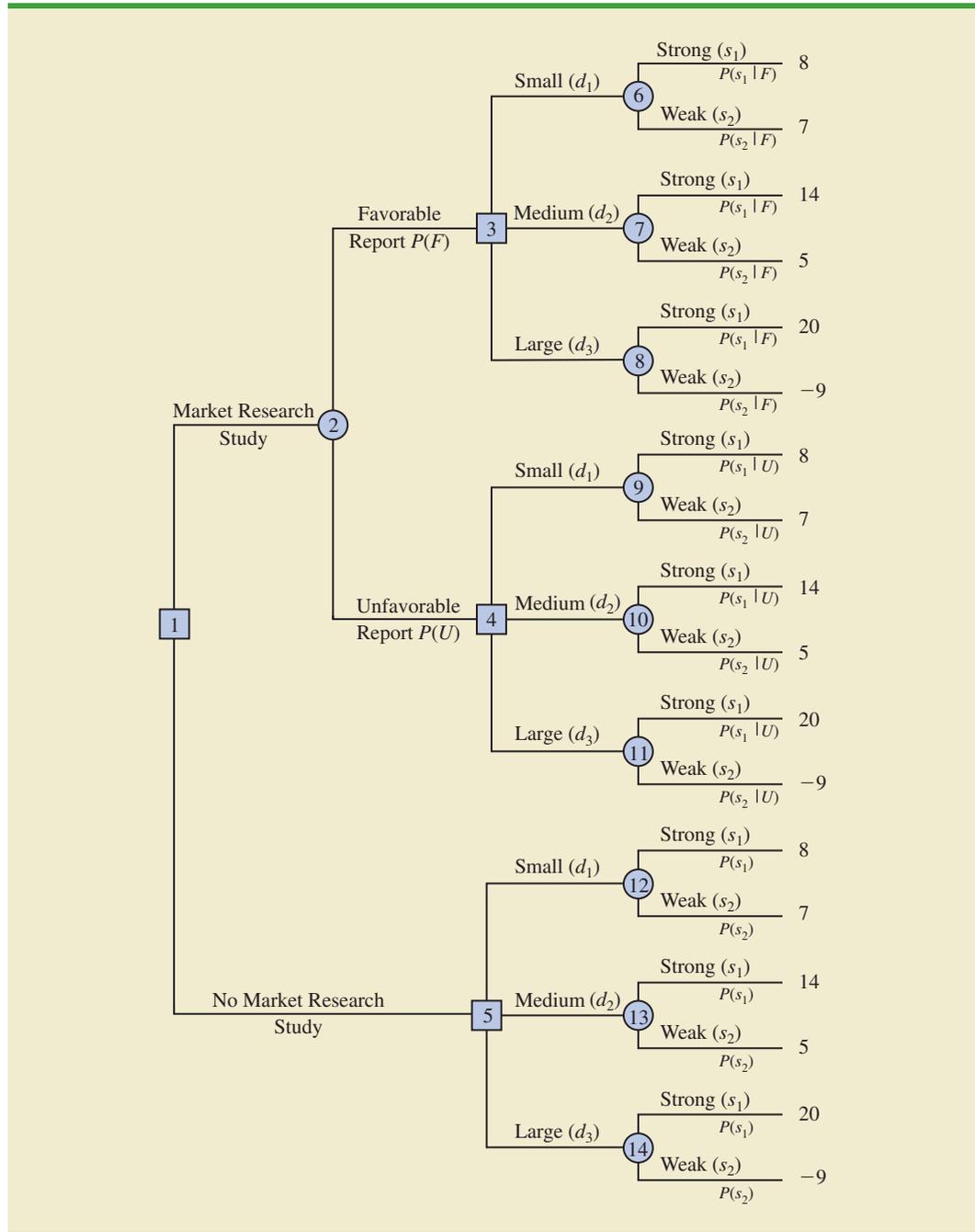
- $F$  = Favorable market research report
- $U$  = Unfavorable market research report
- $s_1$  = Strong demand (state of nature 1)
- $s_2$  = Weak demand (state of nature 2)

At chance node 2, we need to know the branch probabilities  $P(F)$  and  $P(U)$ . At chance nodes 6, 7, and 8, we need to know the branch probabilities  $P(s_1 | F)$ , the probability of state of nature 1 given a favorable market research report, and  $P(s_2 | F)$ , the probability of state of nature 2 given a favorable market research report.  $P(s_1 | F)$  and  $P(s_2 | F)$  are referred to as *posterior probabilities* because they are conditional probabilities based on the outcome of the sample information. At chance nodes 9, 10, and 11, we need to know the branch probabilities  $P(s_1 | U)$  and  $P(s_2 | U)$ ; note that these are also posterior probabilities, denoting the probabilities of the two states of nature *given* that the market research report is unfavorable. Finally at chance nodes 12, 13, and 14, we need the probabilities for the states of nature,  $P(s_1)$  and  $P(s_2)$ , if the market research study is not undertaken.

In making the probability computations, we need to know PDC's assessment of the probabilities for the two states of nature,  $P(s_1)$  and  $P(s_2)$ , which are the prior probabilities as discussed earlier. In addition, we must know the **conditional probability** of the market research outcomes (the sample information) *given* each state of nature. For example, we need to know the conditional probability of a favorable market research report given that strong demand exists for the PDC project; note that this conditional probability of  $F$  given state of nature  $s_1$  is written  $P(F | s_1)$ . To carry out the probability calculations, we will need conditional probabilities for all sample outcomes given all states of nature, that is,  $P(F | s_1)$ ,  $P(F | s_2)$ ,  $P(U | s_1)$ , and  $P(U | s_2)$ . In the PDC problem, we assume that the following assessments are available for these conditional probabilities.

State of Nature	Market Research	
	Favorable, $F$	Unfavorable, $U$
Strong demand, $s_1$	$P(F   s_1) = .90$	$P(U   s_1) = .10$
Weak demand, $s_2$	$P(F   s_2) = .25$	$P(U   s_2) = .75$

FIGURE 21.10 THE PDC DECISION TREE



Note that the preceding probability assessments provide a reasonable degree of confidence in the market research study. If the true state of nature is  $s_1$ , the probability of a favorable market research report is .90, and the probability of an unfavorable market research report is .10. If the true state of nature is  $s_2$ , the probability of a favorable market research report is .25, and the probability of an unfavorable market research report is .75. The reason for a .25 probability of a potentially misleading favorable market research report for state of nature  $s_2$  is that when some potential buyers first hear about the new condominium

**TABLE 21.3** BRANCH PROBABILITIES FOR THE PDC CONDOMINIUM PROJECT BASED ON A FAVORABLE MARKET RESEARCH REPORT

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
$s_j$	$P(s_j)$	$P(F   s_j)$	$P(F \cap s_j)$	$P(s_j   F)$
$s_1$	.8	.90	.72	.94
$s_2$	<u>.2</u>	.25	<u>.05</u>	<u>.06</u>
	1.0		$P(F) = .77$	1.00

project, their enthusiasm may lead them to overstate their real interest in it. A potential buyer's initial favorable response can change quickly to a "no thank you" when later faced with the reality of signing a purchase contract and making a down payment.

In the following discussion, we present a tabular approach as a convenient method for carrying out the probability computations. The computations for the PDC problem based on a favorable market research report ( $F$ ) are summarized in Table 21.3. The steps used to develop this table are as follows.

- Step 1.** In column 1 enter the states of nature. In column 2 enter the *prior probabilities* for the states of nature. In column 3 enter the *conditional probabilities* of a favorable market research report ( $F$ ) given each state of nature.
- Step 2.** In column 4 compute the **joint probabilities** by multiplying the prior probability values in column 2 by the corresponding conditional probability values in column 3.
- Step 3.** Sum the joint probabilities in column 4 to obtain the probability of a favorable market research report,  $P(F)$ .
- Step 4.** Divide each joint probability in column 4 by  $P(F) = .77$  to obtain the revised or *posterior probabilities*,  $P(s_1 | F)$  and  $P(s_2 | F)$ .

Table 21.3 shows that the probability of obtaining a favorable market research report is  $P(F) = .77$ . In addition,  $P(s_1 | F) = .94$  and  $P(s_2 | F) = .06$ . In particular, note that a favorable market research report will prompt a revised or posterior probability of .94 that the market demand of the condominium will be strong,  $s_1$ .

The tabular probability computation procedure must be repeated for each possible sample information outcome. Thus, Table 21.4 shows the computations of the branch probabilities of the PDC problem based on an unfavorable market research report. Note that the probability of obtaining an unfavorable market research report is  $P(U) = .23$ . If an

**TABLE 21.4** BRANCH PROBABILITIES FOR THE PDC CONDOMINIUM PROJECT BASED ON AN UNFAVORABLE MARKET RESEARCH REPORT

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
$s_j$	$P(s_j)$	$P(U   s_j)$	$P(U \cap s_j)$	$P(s_j   U)$
$s_1$	.8	.10	.08	.35
$s_2$	<u>.2</u>	.75	<u>.15</u>	<u>.65</u>
	1.0		$P(U) = .23$	1.00

unfavorable report is obtained, the posterior probability of a strong market demand,  $s_1$ , is .35 and of a weak market demand,  $s_2$ , is .65. The branch probabilities from Tables 21.3 and 21.4 were shown on the PDC decision tree in Figure 21.5.

The discussion in this section shows an underlying relationship between the probabilities on the various branches in a decision tree. To assume different prior probabilities,  $P(s_1)$  and  $P(s_2)$ , without determining how these changes would alter  $P(F)$  and  $P(U)$ , as well as the posterior probabilities  $P(s_1 | F)$ ,  $P(s_2 | F)$ ,  $P(s_1 | U)$ , and  $P(s_2 | U)$ , would be inappropriate.

*Exercise 14 asks you to compute posterior probabilities.*

## Exercises

### Methods

#### SELF test

14. Suppose that you are given a decision situation with three possible states of nature:  $s_1$ ,  $s_2$ , and  $s_3$ . The prior probabilities are  $P(s_1) = .2$ ,  $P(s_2) = .5$ , and  $P(s_3) = .3$ . With sample information  $I$ ,  $P(I | s_1) = .1$ ,  $P(I | s_2) = .05$ , and  $P(I | s_3) = .2$ . Compute the revised or posterior probabilities:  $P(s_1 | I)$ ,  $P(s_2 | I)$ , and  $P(s_3 | I)$ .
15. In the following profit payoff table for a decision problem with two states of nature and three decision alternatives, the prior probabilities for  $s_1$  and  $s_2$  are  $P(s_1) = .8$  and  $P(s_2) = .2$ .

Decision Alternative	State of Nature	
	$s_1$	$s_2$
$d_1$	15	10
$d_2$	10	12
$d_3$	8	20

- a. What is the optimal decision?
- b. Find the EVPI.
- c. Suppose that sample information  $I$  is obtained, with  $P(I | s_1) = .20$  and  $P(I | s_2) = .75$ . Find the posterior probabilities  $P(s_1 | I)$  and  $P(s_2 | I)$ . Recommend a decision alternative based on these probabilities.

### Applications

16. To save on expenses, Rona and Jerry agreed to form a carpool for traveling to and from work. Rona preferred to use the somewhat longer but more consistent Queen City Avenue. Although Jerry preferred the quicker expressway, he agreed with Rona that they should take Queen City Avenue if the expressway had a traffic jam. The following payoff table provides the one-way time estimate in minutes for traveling to and from work.

Decision Alternative	State of Nature	
	Expressway Open	Expressway Jammed
Queen City Avenue, $d_1$	$s_1$ 30	$s_2$ 30
Expressway, $d_2$	25	45

Based on their experience with traffic problems, Rona and Jerry agreed on a .15 probability that the expressway would be jammed.

In addition, they agreed that weather seemed to affect the traffic conditions on the expressway. Let

$C$  = clear

$O$  = overcast

$R$  = rain

The following conditional probabilities apply.

$$\begin{aligned} P(C | s_1) &= .8 & P(O | s_1) &= .2 & P(R | s_1) &= .0 \\ P(C | s_2) &= .1 & P(O | s_2) &= .3 & P(R | s_2) &= .6 \end{aligned}$$

- a. Use Bayes' theorem for probability revision to compute the probability of each weather condition and the conditional probability of the expressway open,  $s_1$ , or jammed,  $s_2$ , given each weather condition.
  - b. Show the decision tree for this problem.
  - c. What is the optimal decision strategy, and what is the expected travel time?
17. The Gorman Manufacturing Company must decide whether to manufacture a component part at its Milan, Michigan, plant or purchase the component part from a supplier. The resulting profit is dependent upon the demand for the product. The following payoff table shows the projected profit (in thousands of dollars).

Decision Alternative	State of Nature		
	Low Demand $s_1$	Medium Demand $s_2$	High Demand $s_3$
Manufacture, $d_1$	-20	40	100
Purchase, $d_2$	10	45	70

The state-of-nature probabilities are  $P(s_1) = .35$ ,  $P(s_2) = .35$ , and  $P(s_3) = .30$ .

- a. Use a decision tree to recommend a decision.
- b. Use EVPI to determine whether Gorman should attempt to obtain a better estimate of demand.
- c. A test market study of the potential demand for the product is expected to report either a favorable ( $F$ ) or unfavorable ( $U$ ) condition. The relevant conditional probabilities are as follows:

$$\begin{aligned} P(F | s_1) &= .10 & P(U | s_1) &= .90 \\ P(F | s_2) &= .40 & P(U | s_2) &= .60 \\ P(F | s_3) &= .60 & P(U | s_3) &= .40 \end{aligned}$$

What is the probability that the market research report will be favorable?

- d. What is Gorman's optimal decision strategy?
- e. What is the expected value of the market research information?

## Summary

Decision analysis can be used to determine a recommended decision alternative or an optimal decision strategy when a decision maker is faced with an uncertain and risk-filled pattern of future events. The goal of decision analysis is to identify the best decision alternative

or the optimal decision strategy given information about the uncertain events and the possible consequences or payoffs. The uncertain future events are called chance events and the outcomes of the chance events are called states of nature.

We showed how payoff tables and decision trees could be used to structure a decision problem and describe the relationships among the decisions, the chance events, and the consequences. With probability assessments provided for the states of nature, the expected value approach was used to identify the recommended decision alternative or decision strategy.

In cases where sample information about the chance events is available, a sequence of decisions can be made. First we decide whether to obtain the sample information. If the answer to this decision is yes, an optimal decision strategy based on the specific sample information must be developed. In this situation, decision trees and the expected value approach can be used to determine the optimal decision strategy.

The Excel add-in PrecisionTree can be used to set up the decision trees and solve the decision problems presented in this chapter. Instructions for downloading and installing the PrecisionTree software on your computer are provided on the website that accompanies the text. An example showing how to use PrecisionTree for the PDC problem in Section 21.1 is provided in the end-of-chapter appendix.

## Glossary

**Chance event** An uncertain future event affecting the consequence, or payoff, associated with a decision.

**Consequence** The result obtained when a decision alternative is chosen and a chance event occurs. A measure of the consequence is often called a payoff.

**States of nature** The possible outcomes for chance events that affect the payoff associated with a decision alternative.

**Payoff** A measure of the consequence of a decision, such as profit, cost, or time. Each combination of a decision alternative and a state of nature has an associated payoff (consequence).

**Payoff table** A tabular representation of the payoffs for a decision problem.

**Decision tree** A graphical representation of the decision problem that shows the sequential nature of the decision-making process.

**Node** An intersection or junction point of an influence diagram or a decision tree.

**Decision nodes** Nodes indicating points where a decision is made.

**Chance nodes** Nodes indicating points where an uncertain event will occur.

**Branch** Lines showing the alternatives from decision nodes and the outcomes from chance nodes.

**Expected value approach** An approach to choosing a decision alternative that is based on the expected value of each decision alternative. The recommended decision alternative is the one that provides the best expected value.

**Expected value (EV)** For a chance node, it is the weighted average of the payoffs. The weights are the state-of-nature probabilities.

**Expected value of perfect information (EVPI)** The expected value of information that would tell the decision maker exactly which state of nature is going to occur (i.e., perfect information).

**Prior probabilities** The probabilities of the states of nature prior to obtaining sample information.

**Sample information** New information obtained through research or experimentation that enables an updating or revision of the state-of-nature probabilities.

**Posterior (revised) probabilities** The probabilities of the states of nature after revising the prior probabilities based on sample information.

**Decision strategy** A strategy involving a sequence of decisions and chance outcomes to provide the optimal solution to a decision problem.

**Expected value of sample information (EVSI)** The difference between the expected value of an optimal strategy based on sample information and the “best” expected value without any sample information.

**Bayes’ theorem** A theorem that enables the use of sample information to revise prior probabilities.

**Conditional probabilities** The probability of one event given the known outcome of a (possibly) related event.

**Joint probabilities** The probabilities of both sample information and a particular state of nature occurring simultaneously.

## Key Formulas

### Expected Value

$$EV(d_i) = \sum_{j=1}^N P(s_j)V_{ij} \quad (21.3)$$

### Expected Value of Perfect Information

$$EVPI = |EV_{wPI} - EV_{woPI}| \quad (21.4)$$

### Expected Value of Sample Information

$$EVSI = |EV_{wSI} - EV_{woSI}| \quad (21.5)$$

## Supplementary Exercises

18. An investor wants to select one of seven mutual funds for the coming year. Data showing the percentage annual return for each fund during five typical one-year periods are shown here. The assumption is that one of these five-year periods will occur again during the coming year. Thus, years A, B, C, D, and E are the states of nature for the mutual fund decision.

Mutual Fund	State of Nature				
	Year A	Year B	Year C	Year D	Year E
Large-Cap Stock	35.3	20.0	28.3	10.4	−9.3
Mid-Cap Stock	32.3	23.2	−0.9	49.3	−22.8
Small-Cap Stock	20.8	22.5	6.0	33.3	6.1
Energy/Resources Sector	25.3	33.9	−20.5	20.9	−2.5
Health Sector	49.1	5.5	29.7	77.7	−24.9
Technology Sector	46.2	21.7	45.7	93.1	−20.1
Real Estate Sector	20.5	44.0	−21.1	2.6	5.1

- a. Suppose that an experienced financial analyst reviews the five states of nature and provides the following probabilities: .1, .3, .1, .1, and .4. Using the expected value

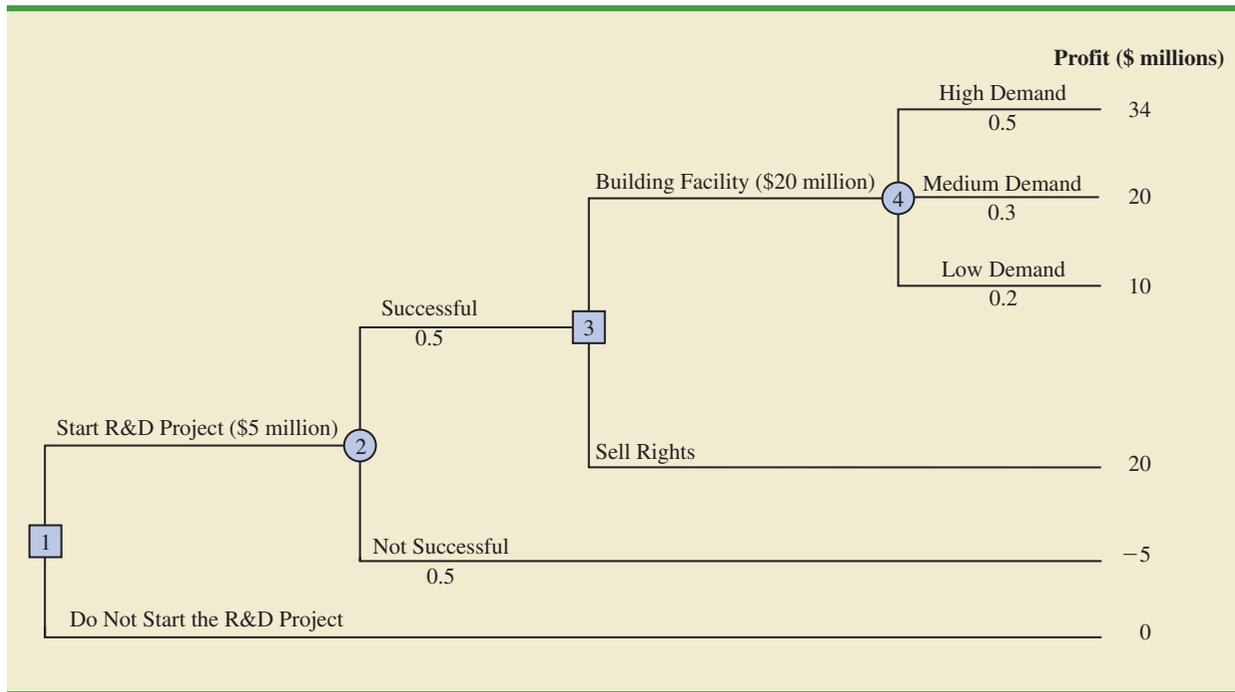
- approach, what is the recommended mutual fund? What is the expected annual return? Using this mutual fund, what are the minimum and maximum annual returns?
- A conservative investor notes that the Small-Cap mutual fund is the only fund that does not have the possibility of a loss. In fact, if the Small-Cap fund is chosen, the investor is guaranteed a return of at least 6%. What is the expected annual return for this fund?
  - Considering the mutual funds recommended in parts (a) and (b), which fund appears to have more risk? Why? Is the expected annual return greater for the mutual fund with more risk?
  - What mutual fund would you recommend to the investor? Explain.
19. Warren Lloyd is interested in leasing a new car and has contacted three automobile dealers for pricing information. Each dealer offered Warren a closed-end 36-month lease with no down payment due at the time of signing. Each lease includes a monthly charge and a mileage allowance. Additional miles receive a surcharge on a per-mile basis. The monthly lease cost, the mileage allowance, and the cost for additional miles follow:

Dealer	Monthly Cost	Mileage Allowance	Cost per Additional Mile
Forno Automotive	\$299	36,000	\$0.15
Midtown Motors	\$310	45,000	\$0.20
Hopkins Automotive	\$325	54,000	\$0.15

- Warren decided to choose the lease option that will minimize his total 36-month cost. The difficulty is that Warren is not sure how many miles he will drive over the next three years. For purposes of this decision he believes it is reasonable to assume that he will drive 12,000 miles per year, 15,000 miles per year, or 18,000 miles per year. With this assumption Warren estimated his total costs for the three lease options. For example, he figures that the Forno Automotive lease will cost him \$10,764 if he drives 12,000 miles per year, \$12,114 if he drives 15,000 miles per year, or \$13,464 if he drives 18,000 miles per year.
- What is the decision, and what is the chance event?
  - Construct a payoff table.
  - Suppose that the probabilities that Warren drives 12,000, 15,000, and 18,000 miles per year are 0.5, 0.4, and 0.1, respectively. What dealer should Warren choose?
  - Suppose that after further consideration, Warren concludes that the probabilities that he will drive 12,000, 15,000 and 18,000 miles per year are 0.3, 0.4, and 0.3, respectively. What dealer should Warren select?
20. Hemmingway, Inc., is considering a \$50 million research and development (R&D) project. Profit projections appear promising, but Hemmingway's president is concerned because the probability that the R&D project will be successful is only 0.50. Secondly, the president knows that even if the project is successful, it will require that the company build a new production facility at a cost of \$20 million in order to manufacture the product. If the facility is built, uncertainty remains about the demand and thus uncertainty about the profit that will be realized. Another option is that if the R&D project is successful, the company could sell the rights to the product for an estimated \$25 million. Under this option, the company would not build the \$20 million production facility.

The decision tree is shown in Figure 21.11. The profit projection for each outcome is shown at the end of the branches. For example, the revenue projection for the high demand outcome is \$59 million. However, the cost of the R&D project (\$5 million) and the cost of the production facility (\$20 million) show the profit of this outcome to be  $\$59 - \$5 - \$20 = \$34$  million. Branch probabilities are also shown for the chance events.

FIGURE 21.11 DECISION TREE FOR HEMMINGWAY, INC.



- Analyze the decision tree to determine whether the company should undertake the R&D project. If it does, and if the R&D project is successful, what should the company do? What is the expected value of your strategy?
  - What must the selling price be for the company to consider selling the rights to the product?
21. Embassy Publishing Company received a six-chapter manuscript for a new college textbook. The editor of the college division is familiar with the manuscript and estimated a 0.65 probability that the textbook will be successful. If successful, a profit of \$750,000 will be realized. If the company decides to publish the textbook and it is unsuccessful, a loss of \$250,000 will occur.

Before making the decision to accept or reject the manuscript, the editor is considering sending the manuscript out for review. A review process provides either a favorable ( $F$ ) or unfavorable ( $U$ ) evaluation of the manuscript. Past experience with the review process suggests probabilities  $P(F) = 0.7$  and  $P(U) = 0.3$  apply. Let  $s_1$  = the textbook is successful, and  $s_2$  = the textbook is unsuccessful. The editor's initial probabilities of  $s_1$  and  $s_2$  will be revised based on whether the review is favorable or unfavorable. The revised probabilities are as follows.

$$\begin{aligned} P(s_1 | F) &= 0.75 & P(s_1 | U) &= 0.417 \\ P(s_2 | F) &= 0.25 & P(s_2 | U) &= 0.583 \end{aligned}$$

- Construct a decision tree assuming that the company will first make the decision of whether to send the manuscript out for review and then make the decision to accept or reject the manuscript.
- Analyze the decision tree to determine the optimal decision strategy for the publishing company.

- c. If the manuscript review costs \$5000, what is your recommendation?
- d. What is the expected value of perfect information? What does this EVPI suggest for the company?

## Case Problem Lawsuit Defense Strategy

John Campbell, an employee of Manhattan Construction Company, claims to have injured his back as a result of a fall while repairing the roof at one of the Eastview apartment buildings. In a lawsuit asking for damages of \$1,500,000, filed against Doug Reynolds, the owner of Eastview Apartments, John claims that the roof had rotten sections and that his fall could have been prevented if Mr. Reynolds had told Manhattan Construction about the problem. Mr. Reynolds notified his insurance company, Allied Insurance, of the lawsuit. Allied must defend Mr. Reynolds and decide what action to take regarding the lawsuit.

Following some depositions and a series of discussions between the two sides, John Campbell offered to accept a settlement of \$750,000. Thus, one option is for Allied to pay John \$750,000 to settle the claim. Allied is also considering making John a counteroffer of \$400,000 in the hope that he will accept a lesser amount to avoid the time and cost of going to trial. Allied's preliminary investigation shows that John has a strong case; Allied is concerned that John may reject their counteroffer and request a jury trial. Allied's lawyers spent some time exploring John's likely reaction if they make a counteroffer of \$400,000.

The lawyers concluded that it is adequate to consider three possible outcomes to represent John's possible reaction to a counteroffer of \$400,000: (1) John will accept the counteroffer and the case will be closed; (2) John will reject the counteroffer and elect to have a jury decide the settlement amount; or (3) John will make a counteroffer to Allied of \$600,000. If John does make a counteroffer, Allied has decided that they will not make additional counteroffers. They will either accept John's counteroffer of \$600,000 or go to trial.

If the case goes to a jury trial, Allied considers three outcomes possible: (1) the jury rejects John's claim and Allied will not be required to pay any damages; (2) the jury finds in favor of John and awards him \$750,000 in damages; or (3) the jury concludes that John has a strong case and awards him the full amount of \$1,500,000.

Key considerations as Allied develops its strategy for disposing of the case are the probabilities associated with John's response to an Allied counteroffer of \$400,000, and the probabilities associated with the three possible trial outcomes. Allied's lawyers believe the probability that John will accept a counteroffer of \$400,000 is .10, the probability that John will reject a counteroffer of \$400,000 is .40, and the probability that John will, himself, make a counteroffer to Allied of \$600,000 is .50. If the case goes to court, they believe that the probability the jury will award John damages of \$1,500,000 is .30, the probability that the jury will award John damages of \$750,000 is .50, and the probability that the jury will award John nothing is .20.

### Managerial Report

Perform an analysis of the problem facing Allied Insurance and prepare a report that summarizes your findings and recommendations. Be sure to include the following items:

1. A decision tree
2. A recommendation regarding whether Allied should accept John's initial offer to settle the claim for \$750,000
3. A decision strategy that Allied should follow if they decide to make John a counteroffer of \$400,000
4. A risk profile for your recommended strategy

## Appendix An Introduction to PrecisionTree

PrecisionTree is an Excel add-in that can be used to develop and analyze decision trees. In this appendix we show how to install and use PrecisionTree to solve the PDC problem presented in Section 21.1.

### Installing and Opening PrecisionTree

Instructions for downloading and installing the PrecisionTree software on your computer are provided on the website that accompanies the text. After installing the PrecisionTree software, perform the following steps to use it as an Excel add-in.

- Step 1.** Click the **Start** button on the taskbar and then point to **All Programs**
- Step 2.** Point to the folder entitled **Palisade Decision Tools**
- Step 3.** Click **PrecisionTree for Excel**

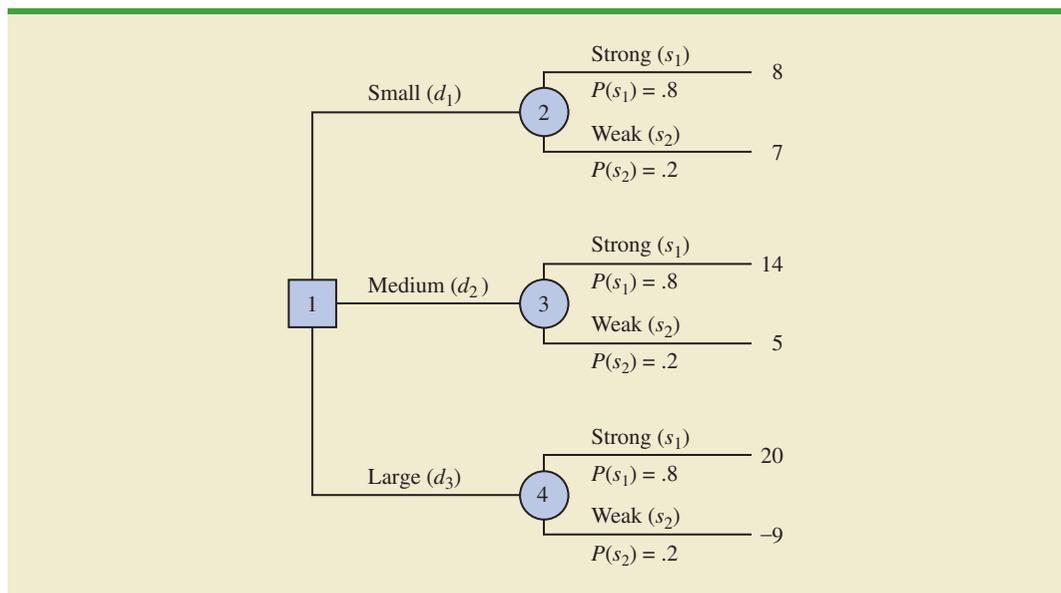
These steps will open Excel and add the PrecisionTree tab next to the Add-Ins tab on the Excel Ribbon. Alternately, if you are already working in Excel, these steps will make PrecisionTree available.

### Getting Started: An Initial Decision Tree

We assume that PrecisionTree has been installed, an Excel workbook is open, and a worksheet that will contain the decision tree has been selected. To create a PrecisionTree version of the PDC decision tree (see Figure 21.12), proceed as follows:

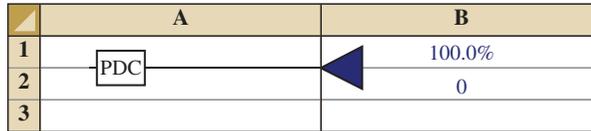
- Step 1.** Click the **PrecisionTree** tab on the Ribbon
- Step 2.** In the **Create New** group, click **Decision Tree**
- Step 3.** When the PrecisionTree for Excel dialog box appears:
  - Click cell **A1**
  - Click **OK**

**FIGURE 21.12** PDC DECISION TREE



- Step 4.** When the PrecisionTree-Model Settings dialog box appears, Enter PDC in the **Name** box  
Click **OK**

An initial decision tree with an end note and no branches will appear.



### Adding a Decision Node and Branches

The initial tree shown above contains a name and one triangle-shaped end node. Recall that the PDC decision tree has one decision node with three branches, one for each decision alternative (small, medium, and large complexes). The following steps show how to change the end node to a decision node and add the three decision alternative branches.

- Step 1.** Click the triangle shaped end note
- Step 2.** When the PrecisionTree-Decision Tree Node Settings dialog box appears:
  - Click the **Decision** button under Node Type
  - Click the Branches tab
  - Click **Add**
  - Click **OK**

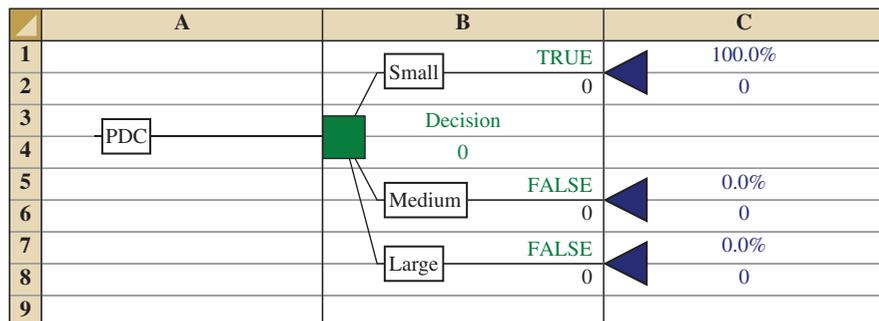
An expanded decision tree with a decision node and three branches will appear.

### Naming the Decision Alternatives

Each of the three decision branches has the generic name **branch** followed by a number to identify it. We want to rename the branches **Small**, **Medium**, and **Large**. Let us start with Branch#1.

- Step 1.** Click the name Branch#1
- Step 2.** When the PrecisionTree for Excel dialog box appears:
  - Replace Branch#1 with **Small**
  - Click **OK**

Continue by applying the same two steps to name the other two decision branches. After naming the branches, the PDC decision tree with three branches appears as follows:



## Adding Chance Nodes and Branches

The chance event for the PDC problem is the demand for the condominiums, which may be either strong or weak. Thus, a chance node with two branches must be added at the end of each decision alternative branch.

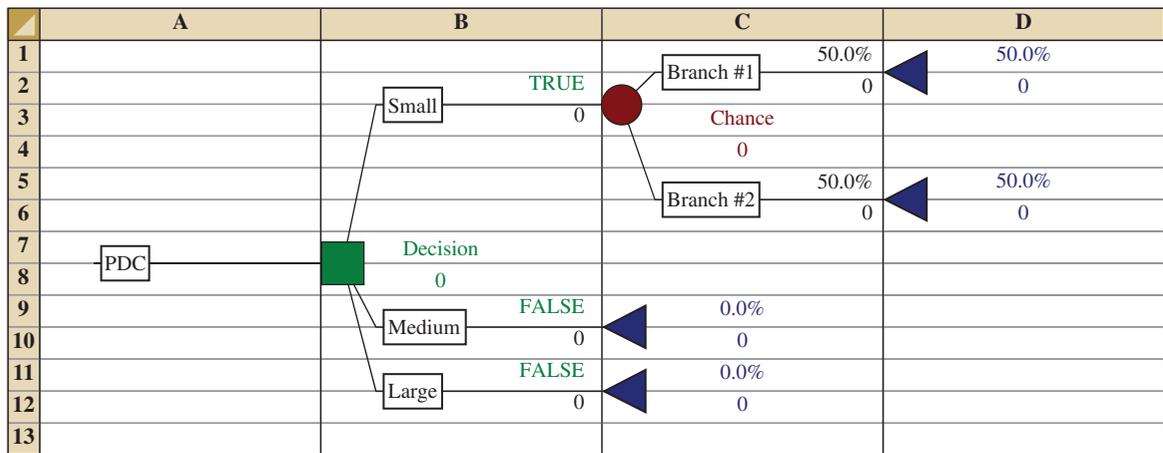
**Step 1.** Click the end node for the Small decision alternative branch

**Step 2.** When the PrecisionTree-Decision Tree Node Settings dialog box appears:

Click the **Chance** button under Node Type

Click **OK**

In step 2, the default value for the number of branches in the Decision Tree Node Settings dialog box is 2. As a result, for the PDC problem we did not need to specify the number of branches for the chance node we just created. The decision tree now appears as follows:



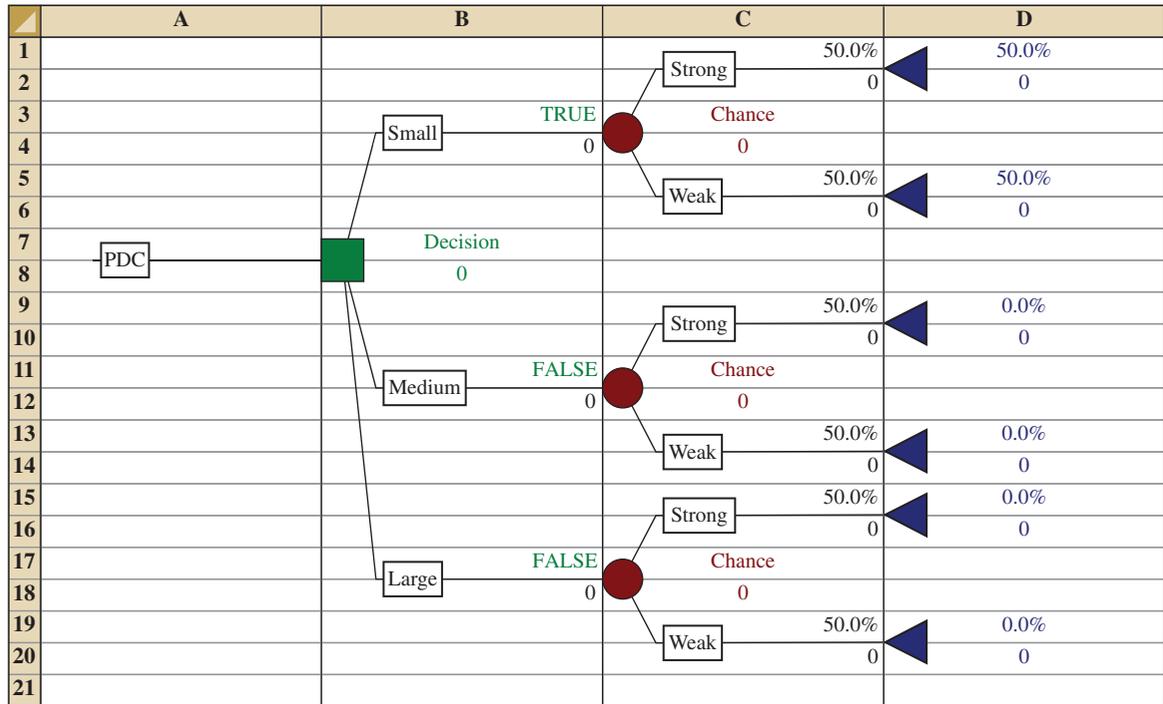
We can now rename the chance node branches as Strong and Weak by using the same procedure we did for the decision branches. Chance nodes can now be inserted at the end of the other two decision branches in a similar fashion.\* Doing so leads to the PDC decision tree shown in Figure 21.13.

## Inserting Probabilities and Payoffs

PrecisionTree provides the capability of inserting probabilities and payoffs into the decision tree. In Figure 21.13, we see that PrecisionTree automatically assigned an equal probability .5 (shown as 50%) to each branch from a chance node. For PDC, the probability of strong demand is .8 and the probability of weak demand is .2. We can select cells C1, C5, C9, C13, C15, and C19 and insert the appropriate probabilities. The payoffs for the chance outcomes are inserted in cells C2, C6, C10, C14, C16, and C20. After inserting the PDC probabilities and payoffs, the PDC decision tree appears as shown in Figure 21.14.

\*PrecisionTree also has a capability for copying nodes that could be used to create the other two chance nodes. Just right-click on the first chance node created, and click Copy SubTree. Then right-click on one of the other end nodes, and click Past Subtree. Do the same for the other end node.

FIGURE 21.13 PDC DECISION TREE DEVELOPED BY PRECISIONTREE



### Interpreting the Result

When probabilities and payoffs are inserted, PrecisionTree automatically makes the backward pass computations necessary to compute expected values and determine the optimal solution. Optimal decisions are identified by the word **True** on the decision branch. Nonoptimal decision branches are identified by the word **False**. Note that the word **True** appears on the Large decision branch. Thus, decision analysis recommends that PDC construct the Large condominium complex. The expected value of this decision appears just to the right of the decision node at the beginning of the tree. Thus, we see that the maximum expected value is \$14.2 million. The expected values of the other decision alternatives are displayed just to the right of the chance nodes at the end of the decision alternative branches. We see that the expected value of the decision to build the small complex is \$7.8 million and the expected value of the decision to build the medium complex is \$12.2 million.

### Other Options

We have used PrecisionTree with a maximization objective. This is the default. If you have a decision tree with a minimization objective, follow these steps:

- Step 1.** Click on the decision tree name (at the beginning of the tree)
- Step 2.** When the PrecisionTree-Model Settings dialog box appears:
  - Click the Calculation tab
  - Select **Minimum Payoff** in the **Optimum Path** box
  - Click **OK**

FIGURE 21.14 PDC DECISION TREE WITH BRANCH PROBABILITIES AND PAYOFFS

