



CHAPTER 11

Inferences About Population Variances

CONTENTS

STATISTICS IN PRACTICE:
U.S. GOVERNMENT
ACCOUNTABILITY OFFICE

11.1 INFERENCES ABOUT A
POPULATION VARIANCE
Interval Estimation
Hypothesis Testing

11.2 INFERENCES ABOUT TWO
POPULATION VARIANCES



STATISTICS *in* **PRACTICE**

U.S. GOVERNMENT ACCOUNTABILITY OFFICE*
 WASHINGTON, D.C.

The U.S. Government Accountability Office (GAO) is an independent, nonpolitical audit organization in the legislative branch of the federal government. GAO evaluators determine the effectiveness of current and proposed federal programs. To carry out their duties, evaluators must be proficient in records review, legislative research, and statistical analysis techniques.

In one case, GAO evaluators studied a Department of Interior program established to help clean up the nation's rivers and lakes. As part of this program, federal grants were made to small cities throughout the United States. Congress asked the GAO to determine how effectively the program was operating. To do so, the GAO examined records and visited the sites of several waste treatment plants.

One objective of the GAO audit was to ensure that the effluent (treated sewage) at the plants met certain standards. Among other things, the audits reviewed sample data on the oxygen content, the pH level, and the amount of suspended solids in the effluent. A requirement of the program was that a variety of tests be taken daily at each plant and that the collected data be sent periodically to the state engineering department. The GAO's investigation of the data showed whether various characteristics of the effluent were within acceptable limits.

For example, the mean or average pH level of the effluent was examined carefully. In addition, the variance in the reported pH levels was reviewed. The following hypothesis test was conducted about the variance in pH level for the population of effluent.

$$H_0: \sigma^2 = \sigma_0^2$$

$$H_a: \sigma^2 \neq \sigma_0^2$$

In this test, σ_0^2 is the population variance in pH level expected at a properly functioning plant. In one particular

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Effluent at this facility must fall within a statistically determined pH range. © John B. Boykin.

plant, the null hypothesis was rejected. Further analysis showed that this plant had a variance in pH level that was significantly less than normal.

The auditors visited the plant to examine the measuring equipment and to discuss their statistical findings with the plant manager. The auditors found that the measuring equipment was not being used because the operator did not know how to work it. Instead, the operator had been told by an engineer what level of pH was acceptable and had simply recorded similar values without actually conducting the test. The unusually low variance in this plant's data resulted in rejection of H_0 . The GAO suspected that other plants might have similar problems and recommended an operator training program to improve the data collection aspect of the pollution control program.

In this chapter you will learn how to conduct statistical inferences about the variances of one and two populations. Two new distributions, the chi-square distribution and the F distribution, will be introduced and used to make interval estimates and hypothesis tests about population variances.

In the preceding four chapters we examined methods of statistical inference involving population means and population proportions. In this chapter we expand the discussion to situations involving inferences about population variances. As an example of a case in which a variance can provide important decision-making information, consider the production process of filling containers with a liquid detergent product. The filling mechanism for the process is adjusted so that the mean filling weight is 16 ounces per container. Although a mean of 16 ounces is desired, the variance of the filling weights is also critical.

In many manufacturing applications, controlling the process variance is extremely important in maintaining quality.

That is, even with the filling mechanism properly adjusted for the mean of 16 ounces, we cannot expect every container to have exactly 16 ounces. By selecting a sample of containers, we can compute a sample variance for the number of ounces placed in a container. This value will serve as an estimate of the variance for the population of containers being filled by the production process. If the sample variance is modest, the production process will be continued. However, if the sample variance is excessive, overfilling and underfilling may be occurring even though the mean is correct at 16 ounces. In this case, the filling mechanism will be readjusted in an attempt to reduce the filling variance for the containers.

In the first section we consider inferences about the variance of a single population. Subsequently, we will discuss procedures that can be used to make inferences about the variances of two populations.

11.1

Inferences About a Population Variance

The sample variance

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n - 1} \quad (11.1)$$

is the point estimator of the population variance σ^2 . In using the sample variance as a basis for making inferences about a population variance, the sampling distribution of the quantity $(n - 1)s^2/\sigma^2$ is helpful. This sampling distribution is described as follows.

SAMPLING DISTRIBUTION OF $(n - 1)s^2/\sigma^2$

Whenever a simple random sample of size n is selected from a normal population, the sampling distribution of

$$\frac{(n - 1)s^2}{\sigma^2} \quad (11.2)$$

has a chi-square distribution with $n - 1$ degrees of freedom.

The chi-square distribution is based on sampling from a normal population.

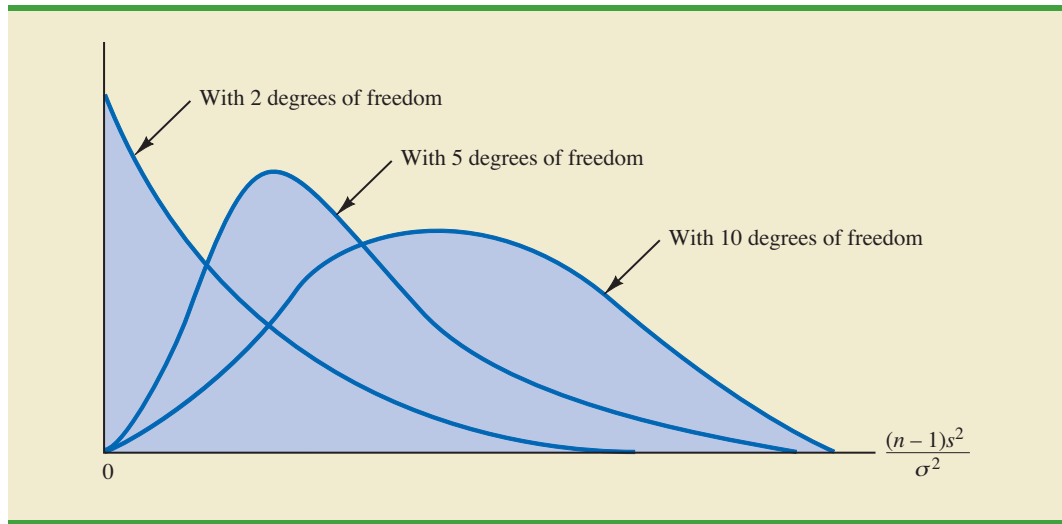
Figure 11.1 shows some possible forms of the sampling distribution of $(n - 1)s^2/\sigma^2$.

Since the sampling distribution of $(n - 1)s^2/\sigma^2$ is known to have a chi-square distribution whenever a simple random sample of size n is selected from a normal population, we can use the chi-square distribution to develop interval estimates and conduct hypothesis tests about a population variance.

Interval Estimation

To show how the chi-square distribution can be used to develop a confidence interval estimate of a population variance σ^2 , suppose that we are interested in estimating the population variance for the production filling process mentioned at the beginning of this chapter. A sample of 20 containers is taken, and the sample variance for the filling quantities is found to be $s^2 = .0025$. However, we know we cannot expect the variance of a sample of 20 containers to provide the exact value of the variance for the population of containers filled by the production process. Hence, our interest will be in developing an interval estimate for the population variance.

FIGURE 11.1 EXAMPLES OF THE SAMPLING DISTRIBUTION OF $(n - 1)s^2/\sigma^2$ (A CHI-SQUARE DISTRIBUTION)



We will use the notation χ^2_{α} to denote the value for the chi-square distribution that provides an area or probability of α to the *right* of the χ^2_{α} value. For example, in Figure 11.2 the chi-square distribution with 19 degrees of freedom is shown with $\chi^2_{.025} = 32.852$ indicating that 2.5% of the chi-square values are to the right of 32.852, and $\chi^2_{.975} = 8.907$ indicating that 97.5% of the chi-square values are to the right of 8.907. Tables of areas or probabilities are readily available for the chi-square distribution. Refer to Table 11.1 and verify that these chi-square values with 19 degrees of freedom (19th row of the table) are correct. Table 3 of Appendix B provides a more extensive table of chi-square values.

From the graph in Figure 11.2 we see that .95, or 95%, of the chi-square values are between $\chi^2_{.975}$ and $\chi^2_{.025}$. That is, there is a .95 probability of obtaining a χ^2 value such that

$$\chi^2_{.975} \leq \chi^2 \leq \chi^2_{.025}$$

FIGURE 11.2 A CHI-SQUARE χ^2 DISTRIBUTION WITH 19 DEGREES OF FREEDOM

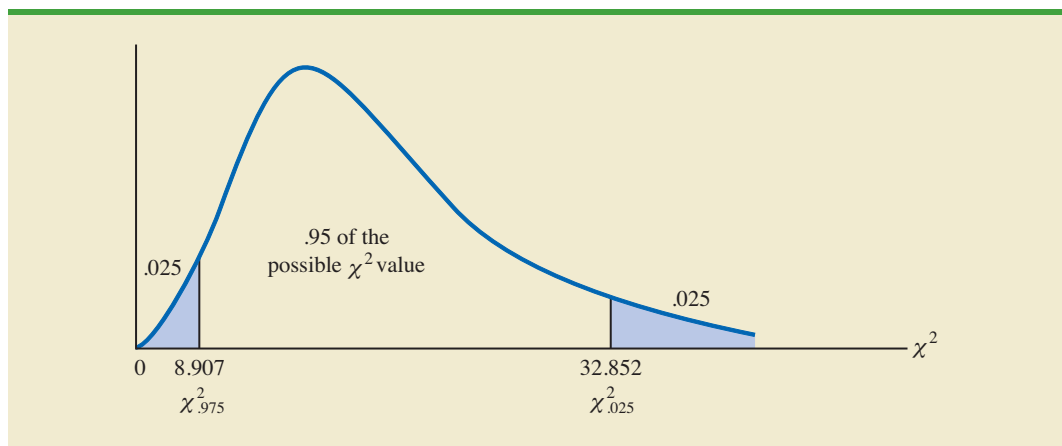
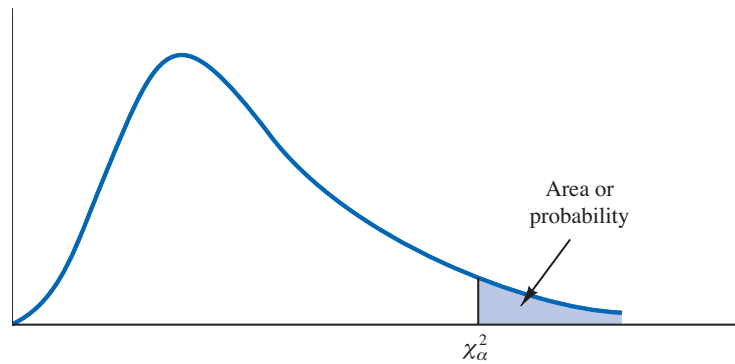


TABLE 11.1 SELECTED VALUES FROM THE CHI-SQUARE DISTRIBUTION TABLE*



Degrees of Freedom	Area in Upper Tail							
	.99	.975	.95	.90	.10	.05	.025	.01
1	.000	.001	.004	.016	2.706	3.841	5.024	6.635
2	.020	.051	.103	.211	4.605	5.991	7.378	9.210
3	.115	.216	.352	.584	6.251	7.815	9.348	11.345
4	.297	.484	.711	1.064	7.779	9.488	11.143	13.277
5	.554	.831	1.145	1.610	9.236	11.070	12.832	15.086
6	.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.041	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.878	14.573	16.151	18.114	36.741	40.113	43.195	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892
40	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691
60	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379
80	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329
100	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807

*Note: A more extensive table is provided as Table 3 of Appendix B.

We stated in expression (11.2) that $(n - 1)s^2/\sigma^2$ follows a chi-square distribution; therefore we can substitute $(n - 1)s^2/\sigma^2$ for χ^2 and write

$$\chi_{.975}^2 \leq \frac{(n - 1)s^2}{\sigma^2} \leq \chi_{.025}^2 \quad (11.3)$$

In effect, expression (11.3) provides an interval estimate in that .95, or 95%, of all possible values for $(n - 1)s^2/\sigma^2$ will be in the interval $\chi_{.975}^2$ to $\chi_{.025}^2$. We now need to do some algebraic manipulations with expression (11.3) to develop an interval estimate for the population variance σ^2 . Working with the leftmost inequality in expression (11.3), we have

$$\chi_{.975}^2 \leq \frac{(n - 1)s^2}{\sigma^2}$$

Thus

$$\sigma^2 \chi_{.975}^2 \leq (n - 1)s^2$$

or

$$\sigma^2 \leq \frac{(n - 1)s^2}{\chi_{.975}^2} \quad (11.4)$$

Performing similar algebraic manipulations with the rightmost inequality in expression (11.3) gives

$$\frac{(n - 1)s^2}{\chi_{.025}^2} \leq \sigma^2 \quad (11.5)$$

The results of expressions (11.4) and (11.5) can be combined to provide

$$\frac{(n - 1)s^2}{\chi_{.025}^2} \leq \sigma^2 \leq \frac{(n - 1)s^2}{\chi_{.975}^2} \quad (11.6)$$

Because expression (11.3) is true for 95% of the $(n - 1)s^2/\sigma^2$ values, expression (11.6) provides a 95% confidence interval estimate for the population variance σ^2 .

Let us return to the problem of providing an interval estimate for the population variance of filling quantities. Recall that the sample of 20 containers provided a sample variance of $s^2 = .0025$. With a sample size of 20, we have 19 degrees of freedom. As shown in Figure 11.2, we have already determined that $\chi_{.975}^2 = 8.907$ and $\chi_{.025}^2 = 32.852$. Using these values in expression (11.6) provides the following interval estimate for the population variance.

$$\frac{(19)(.0025)}{32.852} \leq \sigma^2 \leq \frac{(19)(.0025)}{8.907}$$

or

$$.0014 \leq \sigma^2 \leq .0053$$

Taking the square root of these values provides the following 95% confidence interval for the population standard deviation.

$$.0380 \leq \sigma \leq .0730$$

A confidence interval for a population standard deviation can be found by computing the square roots of the lower limit and upper limit of the confidence interval for the population variance.

Thus, we illustrated the process of using the chi-square distribution to establish interval estimates of a population variance and a population standard deviation. Note specifically that because $\chi^2_{.975}$ and $\chi^2_{.025}$ were used, the interval estimate has a .95 confidence coefficient. Extending expression (11.6) to the general case of any confidence coefficient, we have the following interval estimate of a population variance.

INTERVAL ESTIMATE OF A POPULATION VARIANCE

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}} \quad (11.7)$$

where the χ^2 values are based on a chi-square distribution with $n - 1$ degrees of freedom and where $1 - \alpha$ is the confidence coefficient.

Hypothesis Testing

Using σ_0^2 to denote the hypothesized value for the population variance, the three forms for a hypothesis test about a population variance are as follows:

$$\begin{array}{lll} H_0: \sigma^2 \geq \sigma_0^2 & H_0: \sigma^2 \leq \sigma_0^2 & H_0: \sigma^2 = \sigma_0^2 \\ H_a: \sigma^2 < \sigma_0^2 & H_a: \sigma^2 > \sigma_0^2 & H_a: \sigma^2 \neq \sigma_0^2 \end{array}$$

These three forms are similar to the three forms that we used to conduct one-tailed and two-tailed hypothesis tests about population means and proportions in Chapters 9 and 10.

The procedure for conducting a hypothesis test about a population variance uses the hypothesized value for the population variance σ_0^2 and the sample variance s^2 to compute the value of a χ^2 test statistic. Assuming that the population has a normal distribution, the test statistic is as follows:

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION VARIANCE

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \quad (11.8)$$

where χ^2 has a chi-square distribution with $n - 1$ degrees of freedom.

After computing the value of the χ^2 test statistic, either the p -value approach or the critical value approach may be used to determine whether the null hypothesis can be rejected.

Let us consider the following example. The St. Louis Metro Bus Company wants to promote an image of reliability by encouraging its drivers to maintain consistent schedules. As a standard policy the company would like arrival times at bus stops to have low variability. In terms of the variance of arrival times, the company standard specifies an arrival time variance of 4 or less when arrival times are measured in minutes. The following hypothesis test is formulated to help the company determine whether the arrival time population variance is excessive.

$$\begin{array}{l} H_0: \sigma^2 \leq 4 \\ H_a: \sigma^2 > 4 \end{array}$$

In tentatively assuming H_0 is true, we are assuming that the population variance of arrival times is within the company guideline. We reject H_0 if the sample evidence indicates that the population variance exceeds the guideline. In this case, follow-up steps should be taken to reduce the population variance. We conduct the hypothesis test using a level of significance of $\alpha = .05$.

Suppose that a random sample of 24 bus arrivals taken at a downtown intersection provides a sample variance of $s^2 = 4.9$. Assuming that the population distribution of arrival times is approximately normal, the value of the test statistic is as follows.

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(24-1)(4.9)}{4} = 28.18$$

The chi-square distribution with $n-1 = 24-1 = 23$ degrees of freedom is shown in Figure 11.3. Because this is an upper tail test, the area under the curve to the right of the test statistic $\chi^2 = 28.18$ is the p -value for the test.

Like the t distribution table, the chi-square distribution table does not contain sufficient detail to enable us to determine the p -value exactly. However, we can use the chi-square distribution table to obtain a range for the p -value. For example, using Table 11.1, we find the following information for a chi-square distribution with 23 degrees of freedom.

Area in Upper Tail	.10	.05	.025	.01
χ^2 Value (23 df)	32.007	35.172	38.076	41.638

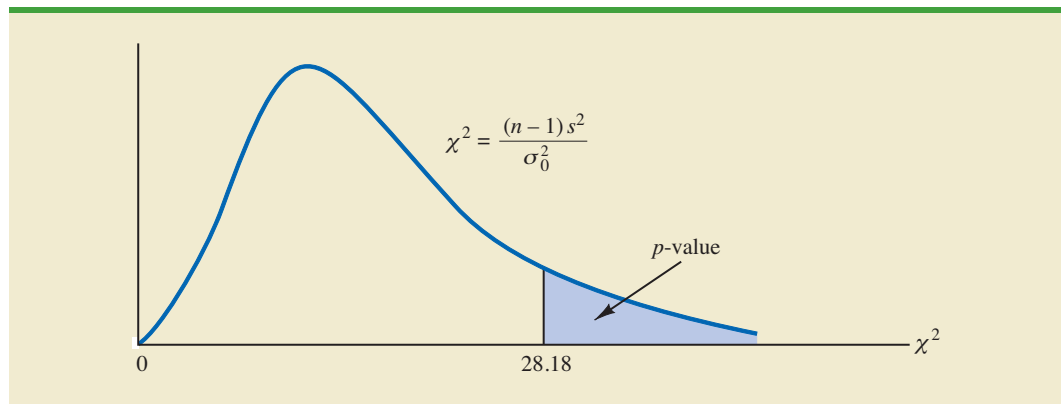
\uparrow
 $\chi^2 = 28.18$

Because $\chi^2 = 28.18$ is less than 32.007, the area in upper tail (the p -value) is greater than .10. With the p -value $> \alpha = .05$, we cannot reject the null hypothesis. The sample does not support the conclusion that the population variance of the arrival times is excessive.

Because of the difficulty of determining the exact p -value directly from the chi-square distribution table, a computer software package such as Minitab or Excel is helpful. Appendix F, at the back of the book, describes how to compute p -values. In the appendix, we show that the exact p -value corresponding to $\chi^2 = 28.18$ is .2091.

As with other hypothesis testing procedures, the critical value approach can also be used to draw the hypothesis testing conclusion. With $\alpha = .05$, $\chi_{.05}^2$ provides the critical value for

FIGURE 11.3 CHI-SQUARE DISTRIBUTION FOR THE ST. LOUIS METRO BUS EXAMPLE



the upper tail hypothesis test. Using Table 11.1 and 23 degrees of freedom, $\chi^2_{.05} = 35.172$. Thus, the rejection rule for the bus arrival time example is as follows:

$$\text{Reject } H_0 \text{ if } \chi^2 \geq 35.172$$

Because the value of the test statistic is $\chi^2 = 28.18$, we cannot reject the null hypothesis.

In practice, upper tail tests as presented here are the most frequently encountered tests about a population variance. In situations involving arrival times, production times, filling weights, part dimensions, and so on, low variances are desirable, whereas large variances are unacceptable. With a statement about the maximum allowable population variance, we can test the null hypothesis that the population variance is less than or equal to the maximum allowable value against the alternative hypothesis that the population variance is greater than the maximum allowable value. With this test structure, corrective action will be taken whenever rejection of the null hypothesis indicates the presence of an excessive population variance.

As we saw with population means and proportions, other forms of hypothesis tests can be developed. Let us demonstrate a two-tailed test about a population variance by considering a situation faced by a bureau of motor vehicles. Historically, the variance in test scores for individuals applying for driver's licenses has been $\sigma^2 = 100$. A new examination with new test questions has been developed. Administrators of the bureau of motor vehicles would like the variance in the test scores for the new examination to remain at the historical level. To evaluate the variance in the new examination test scores, the following two-tailed hypothesis test has been proposed.

$$\begin{aligned} H_0: \sigma^2 &= 100 \\ H_a: \sigma^2 &\neq 100 \end{aligned}$$

Rejection of H_0 will indicate that a change in the variance has occurred and suggest that some questions in the new examination may need revision to make the variance of the new test scores similar to the variance of the old test scores. A sample of 30 applicants for driver's licenses will be given the new version of the examination. We will use a level of significance $\alpha = .05$ to conduct the hypothesis test.

The sample of 30 examination scores provided a sample variance $s^2 = 162$. The value of the chi-square test statistic is as follows:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(30-1)(162)}{100} = 46.98$$

Now, let us compute the p -value. Using Table 11.1 and $n-1 = 30-1 = 29$ degrees of freedom, we find the following.

Area in Upper Tail	.10	.05	.025	.01
χ^2 Value (29 df)	39.087	42.557	45.722	49.588

$\chi^2 = 46.98$

Thus, the value of the test statistic $\chi^2 = 46.98$ provides an area between .025 and .01 in the upper tail of the chi-square distribution. Doubling these values shows that the two-tailed

TABLE 11.2 SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION VARIANCE

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \sigma^2 \geq \sigma_0^2$ $H_a: \sigma^2 < \sigma_0^2$	$H_0: \sigma^2 \leq \sigma_0^2$ $H_a: \sigma^2 > \sigma_0^2$	$H_0: \sigma^2 = \sigma_0^2$ $H_a: \sigma^2 \neq \sigma_0^2$
Test Statistic	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$
Rejection Rule: p-value Approach	Reject H_0 if $p\text{-value} \leq \alpha$	Reject H_0 if $p\text{-value} \leq \alpha$	Reject H_0 if $p\text{-value} \leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $\chi^2 \leq \chi_{(1-\alpha)}^2$	Reject H_0 if $\chi^2 \geq \chi_{\alpha}^2$	Reject H_0 if $\chi^2 \leq \chi_{(1-\alpha/2)}^2$ or if $\chi^2 \geq \chi_{\alpha/2}^2$

p -value is between .05 and .02. Excel or Minitab can be used to show the exact p -value = .0374. With $p\text{-value} \leq \alpha = .05$, we reject H_0 and conclude that the new examination test scores have a population variance different from the historical variance of $\sigma^2 = 100$. A summary of the hypothesis testing procedures for a population variance is shown in Table 11.2.

Exercises

Methods

- Find the following chi-square distribution values from Table 11.1 or Table 3 of Appendix B.
 - $\chi_{.05}^2$ with $df = 5$
 - $\chi_{.025}^2$ with $df = 15$
 - $\chi_{.975}^2$ with $df = 20$
 - $\chi_{.01}^2$ with $df = 10$
 - $\chi_{.95}^2$ with $df = 18$
- A sample of 20 items provides a sample standard deviation of 5.
 - Compute the 90% confidence interval estimate of the population variance.
 - Compute the 95% confidence interval estimate of the population variance.
 - Compute the 95% confidence interval estimate of the population standard deviation.
- A sample of 16 items provides a sample standard deviation of 9.5. Test the following hypotheses using $\alpha = .05$. What is your conclusion? Use both the p -value approach and the critical value approach.

$$H_0: \sigma^2 \leq 50$$

$$H_a: \sigma^2 > 50$$

Applications

- The variance in drug weights is critical in the pharmaceutical industry. For a specific drug, with weights measured in grams, a sample of 18 units provided a sample variance of $s^2 = .36$.
 - Construct a 90% confidence interval estimate of the population variance for the weight of this drug.
 - Construct a 90% confidence interval estimate of the population standard deviation.

SELF test

5. The daily car rental rates for a sample of eight cities follow.

City	Daily Car Rental Rate (\$)
Atlanta	47
Chicago	50
Dallas	53
New Orleans	45
Phoenix	40
Pittsburgh	43
San Francisco	39
Seattle	37

- Compute the variance and the standard deviation for these data.
 - What is the 95% confidence interval estimate of the variance of car rental rates for the population?
 - What is the 95% confidence interval estimate of the standard deviation for the population?
6. The Fidelity Growth & Income mutual fund received a three-star, or neutral, rating from Morningstar. Shown here are the quarterly percentage returns for the five-year period from 2001 to 2005 (*Morningstar Funds 500*, 2006).

WEB file
Return

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
2001	-10.91	5.80	-9.64	6.45
2002	0.83	-10.48	-14.03	5.58
2003	-2.27	10.43	0.85	9.33
2004	1.34	1.11	-0.77	8.03
2005	-2.46	0.89	2.55	1.78

- Compute the mean, variance, and standard deviation for the quarterly returns.
 - Financial analysts often use standard deviation as a measure of risk for stocks and mutual funds. Develop a 95% confidence interval for the population standard deviation of quarterly returns for the Fidelity Growth & Income mutual fund.
7. To analyze the risk, or volatility, associated with investing in Chevron Corporation common stock, a sample of the monthly total percentage return for 12 months was taken. The returns for the 12 months of 2005 are shown here (*Compustat*, February 24, 2006). Total return is price appreciation plus any dividend paid.

Month	Return (%)	Month	Return (%)
January	3.60	July	3.74
February	14.86	August	6.62
March	-6.07	September	5.42
April	-10.82	October	-11.83
May	4.29	November	1.21
June	3.98	December	-0.94

- Compute the sample variance and sample standard deviation as a measure of volatility of monthly total return for Chevron.
 - Construct a 95% confidence interval for the population variance.
 - Construct a 95% confidence interval for the population standard deviation.
8. March 4, 2009, was one of the few good days for the stock market in early 2009. The Dow Jones Industrial Average went up 149.82 points (*The Wall Street Journal*, March 5, 2009). The following table shows the stock price changes for a sample of 12 companies on that day.

WEB file
PriceChange

Price Change		Price Change	
Company	(\$)	Company	(\$)
Aflac	0.81	John.&John.	1.46
Bank of Am.	-0.05	Loews Cp	0.92
Cablevision	0.41	Nokia	0.21
Diageo	1.32	SmpraEngy	0.97
Flour Cp	2.37	Sunoco	0.52
Goodrich	0.3	Tyson Food	0.12

- Compute the sample variance for the daily price change.
 - Compute the sample standard deviation for the price change.
 - Provide 95% confidence interval estimates of the population variance and the population standard deviation.
- An automotive part must be machined to close tolerances to be acceptable to customers. Production specifications call for a maximum variance in the lengths of the parts of .0004. Suppose the sample variance for 30 parts turns out to be $s^2 = .0005$. Use $\alpha = .05$ to test whether the population variance specification is being violated.
 - The average standard deviation for the annual return of large cap stock mutual funds is 18.2% (*The Top Mutual Funds*, AAIL, 2004). The sample standard deviation based on a sample of size 36 for the Vanguard PRIMECAP mutual fund is 22.2%. Construct a hypothesis test that can be used to determine whether the standard deviation for the Vanguard fund is greater than the average standard deviation for large cap mutual funds. With a .05 level of significance, what is your conclusion?
 - At the end of 2008, the variance in the semiannual yields of overseas government bond was $\sigma^2 = .70$. A group of bond investors met at that time to discuss future trends in overseas bond yields. Some expected the variability in overseas bond yields to increase and others took the opposite view. The following table shows the semiannual yields for 12 overseas countries as of March 6, 2009 (*Barron's*, March 9, 2009).

WEB file
Yields

Country	Yield (%)	Country	Yield (%)
Australia	3.98	Italy	4.51
Belgium	3.78	Japan	1.32
Canada	2.95	Netherlands	3.53
Denmark	3.55	Spain	3.90
France	3.44	Sweden	2.48
Germany	3.08	U.K.	3.76

- Compute the mean, variance, and standard deviation of the overseas bond yields as of March 6, 2009.
 - Develop hypotheses to test whether the sample data indicate that the variance in bond yields has changed from that at the end of 2008.
 - Use $\alpha = .05$ to conduct the hypothesis test formulated in part (b). What is your conclusion?
- A *Fortune* study found that the variance in the number of vehicles owned or leased by subscribers to *Fortune* magazine is .94. Assume a sample of 12 subscribers to another magazine provided the following data on the number of vehicles owned or leased: 2, 1, 2, 0, 3, 2, 2, 1, 2, 1, 0, and 1.
 - Compute the sample variance in the number of vehicles owned or leased by the 12 subscribers.
 - Test the hypothesis $H_0: \sigma^2 = .94$ to determine whether the variance in the number of vehicles owned or leased by subscribers of the other magazine differs from $\sigma^2 = .94$ for *Fortune*. At a .05 level of significance, what is your conclusion?

11.2

Inferences About Two Population Variances

In some statistical applications we may want to compare the variances in product quality resulting from two different production processes, the variances in assembly times for two assembly methods, or the variances in temperatures for two heating devices. In making comparisons about the two population variances, we will be using data collected from two independent random samples, one from population 1 and another from population 2. The two sample variances s_1^2 and s_2^2 will be the basis for making inferences about the two population variances σ_1^2 and σ_2^2 . Whenever the variances of two normal populations are equal ($\sigma_1^2 = \sigma_2^2$), the sampling distribution of the ratio of the two sample variances s_1^2/s_2^2 is as follows.

SAMPLING DISTRIBUTION OF s_1^2/s_2^2 WHEN $\sigma_1^2 = \sigma_2^2$

Whenever independent simple random samples of sizes n_1 and n_2 are selected from two normal populations with equal variances, the sampling distribution of

$$\frac{s_1^2}{s_2^2} \quad (11.9)$$

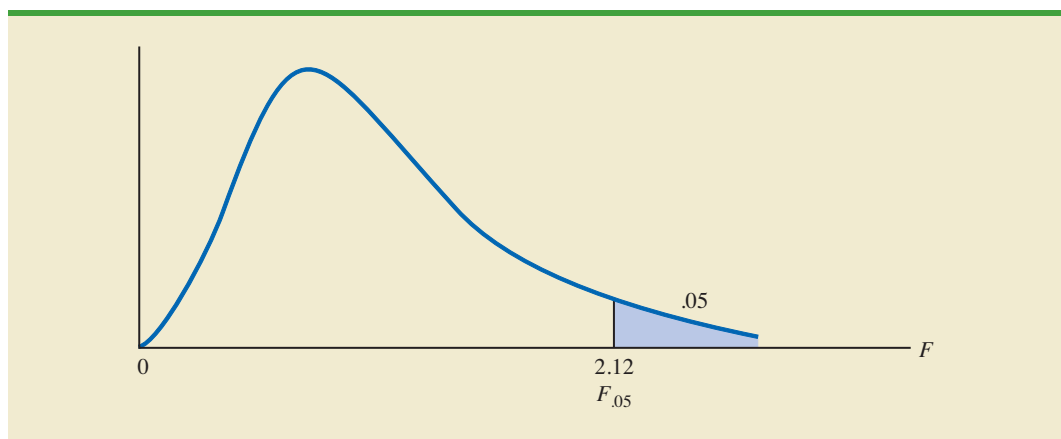
has an F distribution with $n_1 - 1$ degrees of freedom for the numerator and $n_2 - 1$ degrees of freedom for the denominator; s_1^2 is the sample variance for the random sample of n_1 items from population 1, and s_2^2 is the sample variance for the random sample of n_2 items from population 2.

The F distribution is based on sampling from two normal populations.

Figure 11.4 is a graph of the F distribution with 20 degrees of freedom for both the numerator and denominator. As indicated by this graph, the F distribution is not symmetric, and the F values can never be negative. The shape of any particular F distribution depends on its numerator and denominator degrees of freedom.

We will use F_α to denote the value of F that provides an area or probability of α in the upper tail of the distribution. For example, as noted in Figure 11.4, $F_{.05}$ denotes the upper tail area of .05 for an F distribution with 20 degrees of freedom for the numerator and 20 degrees of freedom for the denominator. The specific value of $F_{.05}$ can be found by

FIGURE 11.4 F DISTRIBUTION WITH 20 DEGREES OF FREEDOM FOR THE NUMERATOR AND 20 DEGREES OF FREEDOM FOR THE DENOMINATOR



referring to the F distribution table, a portion of which is shown in Table 11.3. Using 20 degrees of freedom for the numerator, 20 degrees of freedom for the denominator, and the row corresponding to an area of .05 in the upper tail, we find $F_{.05} = 2.12$. Note that the table can be used to find F values for upper tail areas of .10, .05, .025, and .01. See Table 4 of Appendix B for a more extensive table for the F distribution.

Let us show how the F distribution can be used to conduct a hypothesis test about the variances of two populations. We begin with a test of the equality of two population variances. The hypotheses are stated as follows.

$$\begin{aligned}H_0: \sigma_1^2 &= \sigma_2^2 \\H_a: \sigma_1^2 &\neq \sigma_2^2\end{aligned}$$

We make the tentative assumption that the population variances are equal. If H_0 is rejected, we will draw the conclusion that the population variances are not equal.

The procedure used to conduct the hypothesis test requires two independent random samples, one from each population. The two sample variances are then computed. We refer to the population providing the *larger* sample variance as population 1. Thus, a sample size of n_1 and a sample variance of s_1^2 correspond to population 1, and a sample size of n_2 and a sample variance of s_2^2 correspond to population 2. Based on the assumption that both populations have a normal distribution, the ratio of sample variances provides the following F test statistic.

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT POPULATION VARIANCES
WITH $\sigma_1^2 = \sigma_2^2$

$$F = \frac{s_1^2}{s_2^2} \quad (11.10)$$

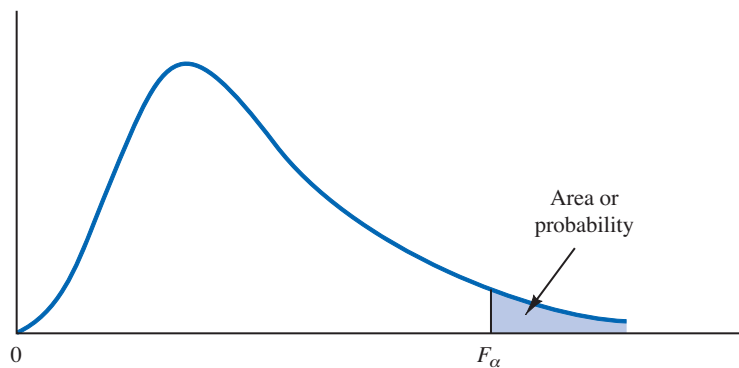
Denoting the population with the larger sample variance as population 1, the test statistic has an F distribution with $n_1 - 1$ degrees of freedom for the numerator and $n_2 - 1$ degrees of freedom for the denominator.

Because the F test statistic is constructed with the larger sample variance s_1^2 in the numerator, the value of the test statistic will be in the upper tail of the F distribution. Therefore, the F distribution table as shown in Table 11.3 and in Table 4 of Appendix B need only provide upper tail areas or probabilities. If we did not construct the test statistic in this manner, lower tail areas or probabilities would be needed. In this case, additional calculations or more extensive F distribution tables would be required. Let us now consider an example of a hypothesis test about the equality of two population variances.

Dullus County Schools is renewing its school bus service contract for the coming year and must select one of two bus companies, the Milbank Company or the Gulf Park Company. We will use the variance of the arrival or pickup/delivery times as a primary measure of the quality of the bus service. Low variance values indicate the more consistent and higher-quality service. If the variances of arrival times associated with the two services are equal, Dullus School administrators will select the company offering the better financial terms. However, if the sample data on bus arrival times for the two companies indicate a significant difference between the variances, the administrators may want to give special consideration to the company with the better or lower variance service. The appropriate hypotheses follow.

$$\begin{aligned}H_0: \sigma_1^2 &= \sigma_2^2 \\H_a: \sigma_1^2 &\neq \sigma_2^2\end{aligned}$$

TABLE 11.3 SELECTED VALUES FROM THE *F* DISTRIBUTION TABLE*



Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom				
		10	15	20	25	30
10	.10	2.32	2.24	2.20	2.17	2.16
	.05	2.98	2.85	2.77	2.73	2.70
	.025	3.72	3.52	3.42	3.35	3.31
	.01	4.85	4.56	4.41	4.31	4.25
15	.10	2.06	1.97	1.92	1.89	1.87
	.05	2.54	2.40	2.33	2.28	2.25
	.025	3.06	2.86	2.76	2.69	2.64
	.01	3.80	3.52	3.37	3.28	3.21
20	.10	1.94	1.84	1.79	1.76	1.74
	.05	2.35	2.20	2.12	2.07	2.04
	.025	2.77	2.57	2.46	2.40	2.35
	.01	3.37	3.09	2.94	2.84	2.78
25	.10	1.87	1.77	1.72	1.68	1.66
	.05	2.24	2.09	2.01	1.96	1.92
	.025	2.61	2.41	2.30	2.23	2.18
	.01	3.13	2.85	2.70	2.60	2.54
30	.10	1.82	1.72	1.67	1.63	1.61
	.05	2.16	2.01	1.93	1.88	1.84
	.025	2.51	2.31	2.20	2.12	2.07
	.01	2.98	2.70	2.55	2.45	2.39

**Note:* A more extensive table is provided as Table 4 of Appendix B.

If H_0 can be rejected, the conclusion of unequal service quality is appropriate. We will use a level of significance of $\alpha = .10$ to conduct the hypothesis test.

A sample of 26 arrival times for the Milbank service provides a sample variance of 48 and a sample of 16 arrival times for the Gulf Park service provides a sample variance of 20. Because the Milbank sample provided the larger sample variance, we will denote Milbank as population 1. Using equation (11.10), we find the value of the test statistic:


$$F = \frac{s_1^2}{s_2^2} = \frac{48}{20} = 2.40$$



The corresponding F distribution has $n_1 - 1 = 26 - 1 = 25$ numerator degrees of freedom and $n_2 - 1 = 16 - 1 = 15$ denominator degrees of freedom.

As with other hypothesis testing procedures, we can use the p -value approach or the critical value approach to obtain the hypothesis testing conclusion. Table 11.3 shows the following areas in the upper tail and corresponding F values for an F distribution with 25 numerator degrees of freedom and 15 denominator degrees of freedom.

Area in Upper Tail	.10	.05	.025	.01
F Value ($df_1 = 25, df_2 = 15$)	1.89	2.28	2.69	3.28



 $F = 2.40$

Because $F = 2.40$ is between 2.28 and 2.69, the area in the upper tail of the distribution is between .05 and .025. For this two-tailed test, we double the upper tail area, which results in a p -value between .10 and .05. Because we selected $\alpha = .10$ as the level of significance, the p -value $< \alpha = .10$. Thus, the null hypothesis is rejected. This finding leads to the conclusion that the two bus services differ in terms of pickup/delivery time variances. The recommendation is that the Dullus County School administrators give special consideration to the better or lower variance service offered by the Gulf Park Company.

We can use Excel or Minitab to show that the test statistic $F = 2.40$ provides a two-tailed p -value = .0811. With $.0811 < \alpha = .10$, the null hypothesis of equal population variances is rejected.

To use the critical value approach to conduct the two-tailed hypothesis test at the $\alpha = .10$ level of significance, we would select critical values with an area of $\alpha/2 = .10/2 = .05$ in each tail of the distribution. Because the value of the test statistic computed using equation (11.10) will always be in the upper tail, we only need to determine the upper tail critical value. From Table 11.3, we see that $F_{.05} = 2.28$. Thus, even though we use a two-tailed test, the rejection rule is stated as follows.

$$\text{Reject } H_0 \text{ if } F \geq 2.28$$

Because the test statistic $F = 2.40$ is greater than 2.28, we reject H_0 and conclude that the two bus services differ in terms of pickup/delivery time variances.

One-tailed tests involving two population variances are also possible. In this case, we use the F distribution to determine whether one population variance is significantly greater than the other. A one-tailed hypothesis test about two population variances will always be formulated as an *upper tail* test:

A one-tailed hypothesis test about two population variances can always be formulated as an upper tail test. This approach eliminates the need for lower tail F values.

$$H_0: \sigma_1^2 \leq \sigma_2^2$$

$$H_a: \sigma_1^2 > \sigma_2^2$$

This form of the hypothesis test always places the p -value and the critical value in the upper tail of the F distribution. As a result, only upper tail F values will be needed, simplifying both the computations and the table for the F distribution.

Let us demonstrate the use of the F distribution to conduct a one-tailed test about the variances of two populations by considering a public opinion survey. Samples of 31 men and 41 women will be used to study attitudes about current political issues. The researcher conducting the study wants to test to see whether the sample data indicate that women show a greater variation in attitude on political issues than men. In the form of the one-tailed

TABLE 11.4 SUMMARY OF HYPOTHESIS TESTS ABOUT TWO POPULATION VARIANCES

	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \sigma_1^2 \leq \sigma_2^2$ $H_a: \sigma_1^2 > \sigma_2^2$	$H_0: \sigma_1^2 = \sigma_2^2$ $H_a: \sigma_1^2 \neq \sigma_2^2$ Note: Population 1 has the larger sample variance
Test Statistic	$F = \frac{s_1^2}{s_2^2}$	$F = \frac{s_1^2}{s_2^2}$
Rejection Rule: <i>p</i>-value	Reject H_0 if $p\text{-value} \leq \alpha$	Reject H_0 if $p\text{-value} \leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $F \geq F_\alpha$	Reject H_0 if $F \geq F_{\alpha/2}$

hypothesis test given previously, women will be denoted as population 1 and men will be denoted as population 2. The hypothesis test will be stated as follows.

$$H_0: \sigma_{\text{women}}^2 \leq \sigma_{\text{men}}^2$$

$$H_a: \sigma_{\text{women}}^2 > \sigma_{\text{men}}^2$$

A rejection of H_0 gives the researcher the statistical support necessary to conclude that women show a greater variation in attitude on political issues.

With the sample variance for women in the numerator and the sample variance for men in the denominator, the F distribution will have $n_1 - 1 = 41 - 1 = 40$ numerator degrees of freedom and $n_2 - 1 = 31 - 1 = 30$ denominator degrees of freedom. We will use a level of significance $\alpha = .05$ to conduct the hypothesis test. The survey results provide a sample variance of $s_1^2 = 120$ for women and a sample variance of $s_2^2 = 80$ for men. The test statistic is as follows.

$$F = \frac{s_1^2}{s_2^2} = \frac{120}{80} = 1.50$$

Referring to Table 4 in Appendix B, we find that an F distribution with 40 numerator degrees of freedom and 30 denominator degrees of freedom has $F_{.10} = 1.57$. Because the test statistic $F = 1.50$ is less than 1.57, the area in the upper tail must be greater than .10. Thus, we can conclude that the p -value is greater than .10. Using Excel or Minitab provides a p -value = .1256. Because the p -value $> \alpha = .05$, H_0 cannot be rejected. Hence, the sample results do not support the conclusion that women show greater variation in attitude on political issues than men. Table 11.4 provides a summary of hypothesis tests about two population variances.

NOTES AND COMMENTS

Research confirms the fact that the F distribution is sensitive to the assumption of normal populations. The F distribution should not be used unless it is

reasonable to assume that both populations are at least approximately normally distributed.

Exercises

Methods

13. Find the following F distribution values from Table 4 of Appendix B.
- $F_{.05}$ with degrees of freedom 5 and 10
 - $F_{.025}$ with degrees of freedom 20 and 15
 - $F_{.01}$ with degrees of freedom 8 and 12
 - $F_{.10}$ with degrees of freedom 10 and 20
14. A sample of 16 items from population 1 has a sample variance $s_1^2 = 5.8$ and a sample of 21 items from population 2 has a sample variance $s_2^2 = 2.4$. Test the following hypotheses at the .05 level of significance.

$$H_0: \sigma_1^2 \leq \sigma_2^2$$

$$H_a: \sigma_1^2 > \sigma_2^2$$

- What is your conclusion using the p -value approach?
- Repeat the test using the critical value approach.

15. Consider the following hypothesis test.

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_a: \sigma_1^2 \neq \sigma_2^2$$

- What is your conclusion if $n_1 = 21$, $s_1^2 = 8.2$, $n_2 = 26$, and $s_2^2 = 4.0$? Use $\alpha = .05$ and the p -value approach.
- Repeat the test using the critical value approach.

SELF test

Applications

16. Investors commonly use the standard deviation of the monthly percentage return for a mutual fund as a measure of the risk for the fund; in such cases, a fund that has a larger standard deviation is considered more risky than a fund with a lower standard deviation. The standard deviation for the American Century Equity Growth fund and the standard deviation for the Fidelity Growth Discovery fund were recently reported to be 15.0% and 18.9%, respectively (*The Top Mutual Funds*, AAI, 2009). Assume that each of these standard deviations is based on a sample of 60 months of returns. Do the sample results support the conclusion that the Fidelity fund has a larger population variance than the American Century fund? Which fund is more risky?
17. Most individuals are aware of the fact that the average annual repair cost for an automobile depends on the age of the automobile. A researcher is interested in finding out whether the variance of the annual repair costs also increases with the age of the automobile. A sample of 26 automobiles 4 years old showed a sample standard deviation for annual repair costs of \$170 and a sample of 25 automobiles 2 years old showed a sample standard deviation for annual repair costs of \$100.
- State the null and alternative versions of the research hypothesis that the variance in annual repair costs is larger for the older automobiles.
 - At a .01 level of significance, what is your conclusion? What is the p -value? Discuss the reasonableness of your findings.
18. Data were collected on the top 1000 financial advisers by *Barron's* (*Barron's*, February 9, 2009). Merrill Lynch had 239 people on the list and Morgan Stanley had 121 people on the list. A sample of 16 of the Merrill Lynch advisers and 10 of the Morgan Stanley advisers showed that the advisers managed many very large accounts with a large variance in the total amount of funds managed. The standard deviation of the amount managed by the Merrill Lynch advisers was $s_1 = \$587$ million. The standard deviation of the amount managed by the Morgan Stanley advisers was $s_2 = \$489$ million. Conduct a hypothesis test

SELF test

at $\alpha = .10$ to determine if there is a significant difference in the population variances for the amounts managed by the two companies. What is your conclusion about the variability in the amount of funds managed by advisers from the two firms?

19. The variance in a production process is an important measure of the quality of the process. A large variance often signals an opportunity for improvement in the process by finding ways to reduce the process variance. Conduct a statistical test to determine whether there is a significant difference between the variances in the bag weights for two machines. Use a .05 level of significance. What is your conclusion? Which machine, if either, provides the greater opportunity for quality improvements?

WEB file
Bags

Machine 1	2.95	3.45	3.50	3.75	3.48	3.26	3.33	3.20
	3.16	3.20	3.22	3.38	3.90	3.36	3.25	3.28
	3.20	3.22	2.98	3.45	3.70	3.34	3.18	3.35
	3.12							
Machine 2	3.22	3.30	3.34	3.28	3.29	3.25	3.30	3.27
	3.38	3.34	3.35	3.19	3.35	3.05	3.36	3.28
	3.30	3.28	3.30	3.20	3.16	3.33		

20. On the basis of data provided by a Romac salary survey, the variance in annual salaries for seniors in public accounting firms is approximately 2.1 and the variance in annual salaries for managers in public accounting firms is approximately 11.1. The salary data were provided in thousands of dollars. Assuming that the salary data were based on samples of 25 seniors and 26 managers, test the hypothesis that the population variances in the salaries are equal. At a .05 level of significance, what is your conclusion?
21. Fidelity Magellan is a large cap growth mutual fund and Fidelity Small Cap Stock is a small cap growth mutual fund (*Morningstar Funds 500*, 2006). The standard deviation for both funds was computed based on a sample of size 26. For Fidelity Magellan, the sample standard deviation is 8.89%; for Fidelity Small Cap Stock, the sample standard deviation is 13.03%. Financial analysts often use the standard deviation as a measure of risk. Conduct a hypothesis test to determine whether the small cap growth fund is riskier than the large cap growth fund. Use $\alpha = .05$ as the level of significance.
22. A research hypothesis is that the variance of stopping distances of automobiles on wet pavement is substantially greater than the variance of stopping distances of automobiles on dry pavement. In the research study, 16 automobiles traveling at the same speeds are tested for stopping distances on wet pavement and then tested for stopping distances on dry pavement. On wet pavement, the standard deviation of stopping distances is 32 feet. On dry pavement, the standard deviation is 16 feet.
- At a .05 level of significance, do the sample data justify the conclusion that the variance in stopping distances on wet pavement is greater than the variance in stopping distances on dry pavement? What is the p -value?
 - What are the implications of your statistical conclusions in terms of driving safety recommendations?

Summary

In this chapter we presented statistical procedures that can be used to make inferences about population variances. In the process we introduced two new probability distributions: the chi-square distribution and the F distribution. The chi-square distribution can be used as the basis for interval estimation and hypothesis tests about the variance of a normal population.

We illustrated the use of the F distribution in hypothesis tests about the variances of two normal populations. In particular, we showed that with independent simple random

samples of sizes n_1 and n_2 selected from two normal populations with equal variances $\sigma_1^2 = \sigma_2^2$, the sampling distribution of the ratio of the two sample variances s_1^2/s_2^2 has an F distribution with $n_1 - 1$ degrees of freedom for the numerator and $n_2 - 1$ degrees of freedom for the denominator.

Key Formulas

Interval Estimate of a Population Variance

$$\frac{(n-1)s^2}{\chi_{\alpha/2}^2} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi_{(1-\alpha/2)}^2} \quad (11.7)$$

Test Statistic for Hypothesis Tests About a Population Variance

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \quad (11.8)$$

Test Statistic for Hypothesis Tests About Population Variances with $\sigma_1^2 = \sigma_2^2$

$$F = \frac{s_1^2}{s_2^2} \quad (11.10)$$

Supplementary Exercises

23. Because of staffing decisions, managers of the Gibson-Marimont Hotel are interested in the variability in the number of rooms occupied per day during a particular season of the year. A sample of 20 days of operation shows a sample mean of 290 rooms occupied per day and a sample standard deviation of 30 rooms.
 - a. What is the point estimate of the population variance?
 - b. Provide a 90% confidence interval estimate of the population variance.
 - c. Provide a 90% confidence interval estimate of the population standard deviation.
24. Initial public offerings (IPOs) of stocks are on average underpriced. The standard deviation measures the dispersion, or variation, in the underpricing-overpricing indicator. A sample of 13 Canadian IPOs that were subsequently traded on the Toronto Stock Exchange had a standard deviation of 14.95. Develop a 95% confidence interval estimate of the population standard deviation for the underpricing-overpricing indicator.
25. The estimated daily living costs for an executive traveling to various major cities follow. The estimates include a single room at a four-star hotel, beverages, breakfast, taxi fares, and incidental costs.

City	Daily Living Cost (\$)	City	Daily Living Cost (\$)
Bangkok	242.87	Mexico City	212.00
Bogotá	260.93	Milan	284.08
Cairo	194.19	Mumbai	139.16
Dublin	260.76	Paris	436.72
Frankfurt	355.36	Rio de Janeiro	240.87
Hong Kong	346.32	Seoul	310.41
Johannesburg	165.37	Tel Aviv	223.73
Lima	250.08	Toronto	181.25
London	326.76	Warsaw	238.20
Madrid	283.56	Washington, D.C.	250.61

WEB file
Travel

- a. Compute the sample mean.
 - b. Compute the sample standard deviation.
 - c. Compute a 95% confidence interval for the population standard deviation.
26. Part variability is critical in the manufacturing of ball bearings. Large variances in the size of the ball bearings cause bearing failure and rapid wearout. Production standards call for a maximum variance of .0001 when the bearing sizes are measured in inches. A sample of 15 bearings shows a sample standard deviation of .014 inches.
- a. Use $\alpha = .10$ to determine whether the sample indicates that the maximum acceptable variance is being exceeded.
 - b. Compute the 90% confidence interval estimate of the variance of the ball bearings in the population.
27. The filling variance for boxes of cereal is designed to be .02 or less. A sample of 41 boxes of cereal shows a sample standard deviation of .16 ounces. Use $\alpha = .05$ to determine whether the variance in the cereal box fillings is exceeding the design specification.
28. City Trucking, Inc., claims consistent delivery times for its routine customer deliveries. A sample of 22 truck deliveries shows a sample variance of 1.5. Test to determine whether $H_0: \sigma^2 \leq 1$ can be rejected. Use $\alpha = .10$.
29. A sample of 9 days over the past six months showed that a dentist treated the following numbers of patients: 22, 25, 20, 18, 15, 22, 24, 19, and 26. If the number of patients seen per day is normally distributed, would an analysis of these sample data reject the hypothesis that the variance in the number of patients seen per day is equal to 10? Use a .10 level of significance. What is your conclusion?
30. A sample standard deviation for the number of passengers taking a particular airline flight is 8. A 95% confidence interval estimate of the population standard deviation is 5.86 passengers to 12.62 passengers.
- a. Was a sample size of 10 or 15 used in the statistical analysis?
 - b. Suppose the sample standard deviation of $s = 8$ was based on a sample of 25 flights. What change would you expect in the confidence interval for the population standard deviation? Compute a 95% confidence interval estimate of σ with a sample size of 25.
31. Is there any difference in the variability in golf scores for players on the LPGA Tour (the women's professional golf tour) and players on the PGA Tour (the men's professional golf tour)? A sample of 20 tournament scores from LPGA events showed a standard deviation of 2.4623 strokes, and a sample of 30 tournament scores from PGA events showed a standard deviation of 2.2118 (*Golfweek*, February 7, 2009, and March 7, 2009). Conduct a hypothesis test for equal population variances to determine if there is any statistically significant difference in the variability of golf scores for male and female professional golfers. Use $\alpha = .10$. What is your conclusion?
32. The grade point averages of 352 students who completed a college course in financial accounting have a standard deviation of .940. The grade point averages of 73 students who dropped out of the same course have a standard deviation of .797. Do the data indicate a difference between the variances of grade point averages for students who completed a financial accounting course and students who dropped out? Use a .05 level of significance. *Note:* $F_{.025}$ with 351 and 72 degrees of freedom is 1.466.
33. The accounting department analyzes the variance of the weekly unit costs reported by two production departments. A sample of 16 cost reports for each of the two departments shows cost variances of 2.3 and 5.4, respectively. Is this sample sufficient to conclude that the two production departments differ in terms of unit cost variance? Use $\alpha = .10$.
34. Two new assembly methods are tested and the variances in assembly times are reported. Use $\alpha = .10$ and test for equality of the two population variances.

	Method A	Method B
Sample Size	$n_1 = 31$	$n_2 = 25$
Sample Variation	$s_1^2 = 25$	$s_2^2 = 12$

Case Problem Air Force Training Program

An Air Force introductory course in electronics uses a personalized system of instruction whereby each student views a videotaped lecture and then is given a programmed instruction text. The students work independently with the text until they have completed the training and passed a test. Of concern is the varying pace at which the students complete this portion of their training program. Some students are able to cover the programmed instruction text relatively quickly, whereas other students work much longer with the text and require additional time to complete the course. The fast students wait until the slow students complete the introductory course before the entire group proceeds together with other aspects of their training.

A proposed alternative system involves use of computer-assisted instruction. In this method, all students view the same videotaped lecture and then each is assigned to a computer terminal for further instruction. The computer guides the student, working independently, through the self-training portion of the course.

To compare the proposed and current methods of instruction, an entering class of 122 students was assigned randomly to one of the two methods. One group of 61 students used the current programmed-text method and the other group of 61 students used the proposed computer-assisted method. The time in hours was recorded for each student in the study. The following data are provided in the data set Training.

Course Completion Times (hours) for Current Training Method

76	76	77	74	76	74	74	77	72	78	73
78	75	80	79	72	69	79	72	70	70	81
76	78	72	82	72	73	71	70	77	78	73
79	82	65	77	79	73	76	81	69	75	75
77	79	76	78	76	76	73	77	84	74	74
69	79	66	70	74	72					

WEB file

Training

Course Completion Times (hours) for Proposed Computer-Assisted Method

74	75	77	78	74	80	73	73	78	76	76
74	77	69	76	75	72	75	72	76	72	77
73	77	69	77	75	76	74	77	75	78	72
77	78	78	76	75	76	76	75	76	80	77
76	75	73	77	77	77	79	75	75	72	82
76	76	74	72	78	71					

Managerial Report

1. Use appropriate descriptive statistics to summarize the training time data for each method. What similarities or differences do you observe from the sample data?

2. Use the methods of Chapter 10 to comment on any difference between the population means for the two methods. Discuss your findings.
3. Compute the standard deviation and variance for each training method. Conduct a hypothesis test about the equality of population variances for the two training methods. Discuss your findings.
4. What conclusion can you reach about any differences between the two methods? What is your recommendation? Explain.
5. Can you suggest other data or testing that might be desirable before making a final decision on the training program to be used in the future?

Appendix 11.1 Population Variances with Minitab

Here we describe how to use Minitab to conduct a hypothesis test involving two population variances.



We will use the data for the Dullus County School bus study in Section 11.2. The arrival times for Milbank appear in column C1, and the arrival times for Gulf Park appear in column C2. The following Minitab procedure can be used to conduct the hypothesis test $H_0: \sigma_1^2 = \sigma_2^2$ and $H_a: \sigma_1^2 \neq \sigma_2^2$.

- Step 1.** Select the **Stat** menu
- Step 2.** Choose **Basic Statistics**
- Step 3.** Choose **2-Variances**
- Step 4.** When the 2-Variances dialog box appears:
 - Select **Samples in different columns**
 - Enter C1 in the **First** box
 - Enter C2 in the **Second** box
 - Click **OK**

Information about the test will be displayed in the section entitled F-Test which shows the test statistic $F = 2.40$ and the p -value = .081. This Minitab procedure specifically performs the two-tailed test for the equality of population variances. Thus, if this Minitab routine is used for a one-tailed test, remembering that the area in one tail is one-half of the area for the two-tailed p -value should make it relatively easy to compute the p -value for the one-tailed test.

Appendix 11.2 Population Variances with Excel

Here we describe how to use Excel to conduct a hypothesis test involving two population variances.



We will use the data for the Dullus County School bus study in Section 11.2. The Excel worksheet has the label Milbank in cell A1 and the label Gulf Park in cell B1. The times for the Milbank sample are in cells A2:A27 and the times for the Gulf Park sample are in cells B2:B17. The steps to conduct the hypothesis test $H_0: \sigma_1^2 = \sigma_2^2$ and $H_a: \sigma_1^2 \neq \sigma_2^2$ are as follows:

- Step 1.** Click the **Data** tab on the Ribbon
- Step 2.** In the **Analysis** group, click **Data Analysis**
- Step 3.** When the Data Analysis dialog box appears:
 - Choose **F-Test Two-Sample for Variances**
 - Click **OK**
- Step 4.** When the F-Test Two Sample for Variances dialog box appears:
 - Enter A1:A27 in the **Variable 1 Range** box
 - Enter B1:B17 in the **Variable 2 Range** box

Select **Labels**

Enter .05 in the **Alpha** box

(*Note:* This Excel procedure uses alpha as the area in the upper tail.)

Select **Output Range** and enter C1 in the box

Click **OK**

The output $P(F \leq f)$ one-tail = .0405 is the one-tailed area associated with the test statistic $F = 2.40$. Thus, the two-tailed p -value is $2(.0405) = .081$. If the hypothesis test had been a one-tailed test, the one-tailed area in the cell labeled $P(F \leq f)$ one-tail provides the information necessary to determine the p -value for the test.

Appendix 11.3 Single Population Standard Deviation with StatTools



In this appendix we show how StatTools can be used to conduct hypothesis tests about a population standard deviation. StatTools conducts hypothesis tests on the population standard deviation, not on the population variance directly. We use the example discussed in Section 11.1 involving bus arrival times at a downtown intersection to illustrate.

Begin by using the Data Set Manager to create a StatTools data set for the BusTimes data using the procedure described in the appendix in Chapter 1. The following steps can be used to test the hypothesis $H_0 : \sigma \leq 2$ against $H_a : \sigma > 2$.

Step 1. Click the **StatTools** tab on the Ribbon

Step 2. In the **Analyses** group, click **Statistical Inference**

Step 3. Choose the **Hypothesis Test** option

Step 4. Choose **Mean/Std. Deviation**

Step 5. When the StatTools-Hypothesis Test for Mean/Std. Deviation dialog box appears:

For **Analysis Type**, choose **One-Sample Analysis**

In the variables section, select **Times**

In the **Hypothesis Tests to Perform** section:

Remove the check mark from the **Mean** box

Select the **Standard Deviation** option

Enter 2 in the **Null Hypothesis Value** box

Select **Greater Than Null Value (One-Tailed Test)** in the **Alternative Hypothesis** box

Click **OK**

The results from the hypothesis test will appear. They include the p -value and the value of the χ^2 test statistic.