MODELLING THE US SWAP SPREAD

Hon-Lun Chung, Wai-Sum Chan and
Jonathan A. Batten

ABSTRACT

The dynamics between five-year US Treasury bonds and interest rate swaps are examined using bivariate threshold autoregressive (BTAR) models to determine the drivers of spread changes and the nature of the lead–lag relation between the two instruments. This model is able to identify the economic – or threshold – value that market participants consider significant before realigning their portfolios. Specifically, three different regimes are identified: when the swap spread in the previous week is either high or low, the Treasury bond market leads the swap market. However, when the swap spread is low, none of the markets leads each other. Thus, yield movements are shown to be governed by the direction and magnitude of the change in the swap spread, which in turn provides an economic insight into the rebalancing between swap and bond portfolios.

1. INTRODUCTION

Trading in Treasury bonds and interest rate swaps comprise two key activities in the global financial marketplace. Treasury bonds are regarded as risk-free securities and carry the highest ratings in local markets by rating

Research in Finance, Volume 26, 155–181

ISSN: 0196-3821/doi:10.1108/S0196-3821(2010)0000026010

agencies,[1] whereas interest rate swaps are derivative contracts traded in over-the-counter markets. Importantly, counterparties through the use of collateral support and netting arrangements can eliminate almost, if not entirely, the credit risk associated with these instruments. In fact recent academic research (Collin-Dufresne & Solnik, 2001; Feldhutter & Lando, 2008 among others) assumes that the swap contract is free of default risk. The swap comprises counterparties with two offsetting sets of underlying cash-flows that generally contain a fixed and floating rate component.[2] Financial market participants can use the swap to hedge existing interest rate exposure, or for speculative interest rate risk taking. For example, if interest rates are expected to decline, investors 'buy' or invest in the fixed rate, whereas if they expect interest rates to increase, they buy or invest in the floating rate. The reverse is true of those who wish to borrow.

Nonetheless, the swap spread, representing the yield difference between a bond and a swap of equivalent maturity, is affected by macroeconomic sentiment – such as inflation expectations, or business cycle effects (Cortes, 2006; Ito, 2007). This effect is most apparent during periods of economic downturn when spreads typically widen due to portfolio rebalancing into Treasury bonds and away from riskier instruments; a result consistent with an increase in risk-aversion. During boom periods, when the probability of default in corporate bond markets declines, sentiment concerning interest rate direction becomes the primary concern, although the effect is known to vary based on swap maturity (Huang, Chen, & Camacho, 2008) and underlying interest rate volatility (Malhotra, Bhargava, & Chaudhry, 2005). Overall, the interplay between default and interest rate expectations results in time-varying spreads, which have critical impacts for the financial decision making by corporations, traders and portfolio managers

The objective of this chapter is to determine the exact nature of the relation between swaps and risk-free bonds through the application of nonlinear threshold models, which previously have been widely used for investigating the dynamics within currency and stock markets (Chappell, Mistry, & Ellis, 1996; Tsay, 1998). These techniques are specifically applied to an investigation of the lead–lag dynamics between the US Treasury bond and US$-denominated swap markets where the change in the swap spread is the dependent variable. US Treasury bonds comprise the largest government bond market in the world, whereas US-denominated interest rate swaps comprise daily turnover in excess of US$81.3 trillion (BIS, 2008). We build upon earlier investigations in interest rate and swap markets (e.g. Malhotra et al., 2005; Ito, 2007) by utilising a new class of bivariate threshold autoregressive (BTAR) models (Chan & Cheung, 2005) to capture the regime-switching, lead–lag dynamics that exist between the US bond and swap markets.

The BTAR model is chosen for a number of reasons. First, this model provides an exact measure of the economic incentive for a portfolio investor to shift funds, in this case between two financial instruments – the swap and a fixed rate bond of equivalent maturity. This measure, termed a 'threshold' or 'critical' value in the BTAR model, may also be interpreted as the hidden cost necessary for financial market participants to shift between these two asset classes. Second, if indeed these thresholds values can be identified, then they can be used to anticipate the change in the yield curve dynamics. This will allow traders to be more cautious in managing risk and help policymakers and central banks fine tune monetary policies. Third, the threshold value can be expressed in terms of interest rate percentages or 'basis points'; a number that can be easily understood and interpreted by financial markets. This is quite different from the information provided by other models, such as Markov Switching Models (Hamilton, 1996).

The study uses weekly five-year US Treasury bond yield and interest rate swap rates from January 1995 to December 2004. This period is a representative period for both markets with the swap market having fully matured since its inception in the early 1980s. This period also provides a novel setting for investigation of the impact of regime change since it includes the longest period of economic expansion in the United States, the Russian bond default, the near failure of long-term capital management (LTCM) and historically low Fed Funds and Treasury bond yields.

The chapter is set out as follows: in Section 2, a brief review of the recent literature on spread trading and modelling is provided; then in Section 3, the data used in the study and the six possible scenarios of bond and swap price movements are explained. Section 4 provides details on the lead–lag modelling techniques utilised as well as the BTAR model. The results are presented in Section 5, which also allows for some concluding remarks.

2. LITERATURE REVIEW

A number of studies have examined the lead–lag relationship between different markets, and the majority use the intraday price data from stock indices and stock index futures. For example, Kawaller, Koch, and Koch (1987) examine the intraday price relationship between the Standard & Poor's (S&P) 500 futures and the S&P 500 Index, whereas Harris (1989) studies the five-minute changes in the S&P 500 Index and futures contracts over the 10-day period surrounding the October 1987 stock market crash. Stoll and Whaley (1990) explore the time-series properties of the 5-min intraday returns of the stock index and stock index futures contracts, and Chan (1992) examines the asymmetric lead–lag relationship between futures and component stocks. Thus, the current study provides valuable new insights into the interest rate and swap markets, which are generally overlooked by academic investigators.

A common question raised by researchers' concerns is whether the lead–lag relationship between different markets changes over time as a result of changing exogenous or endogenous factors. That is, the way in which different instruments interact may be a regime-dependent phenomenon that varies if the internal or external environment changes. To help address this question, Tsay (1998) studies the relationship between three-month and three-year interest rates and uses the difference between the logarithms of the two interest rates as the threshold variable. He identifies three different regimes that represent economic expansion, a stable economy and economic slowdown. Although Ito (2007) investigates interest rate spreads on Japanese-denominated swaps and Malhotra et al. (2005) does so in the US setting, neither consider regime dependent threshold values or utilise BTAR modelling techniques.

More recently, Huang et al. (2008) investigate the determinants of variations in the yield spreads between Japanese yen interest rate swaps and Japan government bonds for a similar period to this study (from 1997 to 2005), although they use a smooth transition vector autoregressive model to analyse the impact of various economic shocks on swap spreads. They find that GARCH volatility is useful for identifying regime change. More specifically they identify the end of the Japanese banking crisis as a significant control variable and that the impact of economic shocks on swap spreads varies across maturity and regimes. This finding is also consistent with Ito (2007) who finds the effect of Treasury interest rates and the term structure yield difference between long and short rates (the slope of the yield curve) on swap spreads also varies by spread maturity. However, the economic implications of these results are moderated by the failure to include threshold values.

The class of threshold autoregressive (TAR) models (Tong, 1978, 1983) has now been widely employed in the literature to explain the various empirical nonlinear phenomena that are observed in many financial and economic time series. Yadav, Pope, and Paudyal (1994) suggest that TAR models are potentially of interest whenever financial decisions are triggered by the threshold values of a control variable, such as arbitrage in the presence of transaction costs and market interventions by regulators. Brooks and Garrett (2002) use self-exciting threshold autoregressive models (SETAR) to explain the daily dynamics of the FTSE 100 index basis. To date this is the first study to apply these techniques to understand the dynamics of the relation between interest rate swaps and the underlying fixed rate bond.

3. DATA

The weekly closing rates of US Treasury bond yields and US$ interest rate swap rates from January 1995 to December 2004 are used in this study. In line with the market practice for end of week portfolio realignment, we use weekly data. Doing so, also overcomes stickiness that is otherwise evident in daily data. Thus, significant changes in the swap spread is clearer in weekly data, whereas monthly observations lose information, and the number of observations is fewer. The data were downloaded from the Bloomberg Fixed Income Database, on which the Treasury bond yields and swap rates are monitored closely by thousands of traders worldwide. A sample start date in the mid-1990s is more appropriate due to the tremendous growth in the interest rate derivative markets from its commencement in the early 1980s and the more recent structural change in the pricing and trading of swaps, such as the introduction of master agreements from the International Swaps and Derivatives Association, Inc. (ISDA) and the netting agreements for credit risk reduction that were developed in the early 1990s. As noted earlier, the price data from 1995 to 2004 covers a diverse range of economic experiences, including the later period covering the longest episode of economic expansion in US history, the Asian financial crisis and the near-failure of LTCM.

Depending on the expected direction of interest rate movement, market participants will have a preference for using different instruments when they expect the spread between the two markets (the swap spread) to narrow or widen. There are basically six different scenarios. In the first four of the scenarios, the interest rates in the Treasury and swap markets are moving in the same direction, which means changes in the two markets are positively correlated. In the last two scenarios, the interest rates in the Treasury market and the swaps market are moving in different directions, which means changes in the two markets are negatively correlated. These different scenarios are detailed below:

- Scenario I – Bond and swap rates rise (positive correlation) with a widened swap spread. In this scenario, market participants prefer to pay fixed in a swap than to short sell government bonds, assuming that the transaction cost and liquidity of the two markets are similar. This scenario usually occurs when the market expects interest rates to increase and corporations pay fixed to hedge their interest rate exposure, which is known as 'macro hedging'.

- Scenario II – Bond and swap rates decline (positive correlation) with a widened swap spread. In this scenario, market participants prefer to purchase government bonds than receive fixed in a swap, all other things being equal. This usually occurs when there is uncertainty in the financial markets, such as resulted from the Russian default in 1998. The rush to high-quality and liquid assets is termed the 'flight to quality' and the 'flight to liquidity' by Longstaff (2004) and occurred once again during the subprime crisis of 2007/2008.
- Scenario III – Bond and swap rates increasing (positive correlation) with a reducing swap spread. In this scenario, market participants prefer to short sell government bonds than pay fixed in a swap, all other things being equal. This usually happens after a financial crisis, when investors switch from quality debt securities to risky debt securities.
- Scenario IV – Bond and swap rates declining (positive correlation) with a reducing swap spread. In this scenario, market participants prefer to receive fixed in a swap than purchase government bonds, all other things being equal. This occurs when investors use interest rate swaps as a hedge for their floating rate assets and mortgage backed securities. Interest rate swaps can be used to lock in the interest rate that is received for floating rate securities and hedge mortgage backed securities that possess a negative convexity. The use of receive fixed in interest rate swaps to hedge the capital loss of mortgage backed securities became very common after the US Treasury cut down the volume of long-dated US government bonds that it issued. This is commonly known as 'mortgage hedging' in the financial markets.
- Scenario V – Narrowing swap spread with swap and bond rates converging (negative correlation). In this scenario, market participants sell bonds and receive fixed in the swaps market simultaneously, which causes a narrowing of the swap spread.
- Scenario VI – Widening swap spread with swap and bond rates converging (negative correlation). In this scenario, market participants buy bonds and pay fixed in the swaps market simultaneously, which causes a widening of the swap spread. For scenarios V and VI, the interest rates in the Treasury bond and swaps markets are moving in opposite directions. Given that the expected direction of movement of the swap spread may have an impact on the preference for the usage of an

instrument, we use the change in the swap spread as the threshold variable to test whether it can cause regime switching.

The summary statistics for the yields and swap spreads are recorded in Table 1. Note that over the sample period the average yield for the five-year Treasury bond is 5.01%, and the average five-year swap rate is 5.54%, which gives an average swap spread of 52 basis points over the period. The volatilities are similar, at 1.31% and 1.32%, respectively. Both the bond yield and the swap rate exhibit negative skewness, which means that yields tend to stay at the high end of the range. This is confirmed by the median bond yield and the median swap rate of 5.39% and 5.86%, respectively, which are higher than the average over the sample period. Both the bond yield and the swap rate have a bimodal distribution, which means that they tend to remain at either the high end or at the low end of the range. This was in line with the monetary policy of the Federal Reserve over the period, during which the Fed Fund target rate was usually lifted or lowered in consecutive Federal Open Market Committee (FOMC) meetings so that interest rates would either be kept below 3.0% to avoid a liquidity crunch, or maintained above 5.0% to curtail inflationary pressures in the broader economy.

The swap spread has positive skewness, and the median swap spread is 45 basis points, which is 8 basis points less than the average. The swap rate is

Table 1. Summary Statistics of the Government Bond Yield, Swap Rate and Swap Spread.

	Government Bond Yield (GOV_t)	Interest Rate Swap Rate (IRS_t)	Swap Spread (SS_t)
Mean	5.013	5.537	0.524
Standard deviation	1.313	1.318	0.231
Skewness	−0.338	−0.464	0.536
Kurtosis	1.959	2.120	2.174
Maximum	7.866	8.210	1.055
Median	5.390	5.858	0.452
Minimum	2.030	2.371	−0.016
ADF	−1.812	−1.710	−1.567
Correlation			
GOV_t	1.000		
IRS_t	0.985	1.000	
SS_t	−0.066	0.109	1.000

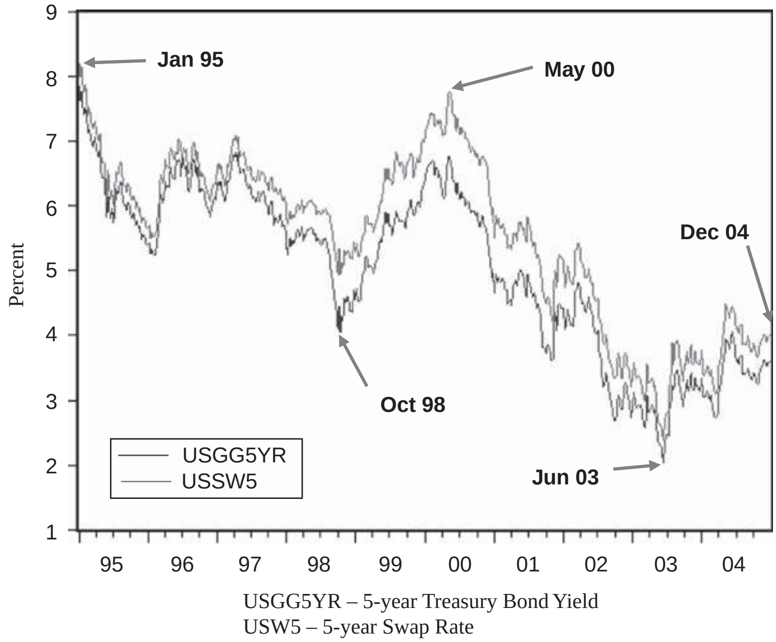

Fig. 1. US Five-Year Treasury Bond Yield and Swap Rate.

highly correlated with the Treasury bond yield (correlation coefficient of 0.985), but the swap spread is not correlated with either the bond yield or the swap rate. All three of the yield time-series failed to pass the Augmented Dickey–Fuller (ADF) test, and thus cannot be considered to be weakly stationary.

The trend in five-year US Treasury and swap yields is clear in Fig. 1, which shows that the Treasury bond yield was at its high of 7.87% in January 1995 and declined to a low of 4.94% in October 1998. This was due to the 'Flight to Quality' phenomenon that was triggered by the Russian default in August 1998. The Federal Reserve tightened interest rates after Y2K, and the Treasury bond yield rose to a high of 6.77% in May 2000. Following the technology stocks crash, the Federal Reserve eased the Fed Fund target rate to an all-time low of 1.0%, and the Treasury bond yield declined to 2.03% in June 2003. With concerns over a 'bubble' developing in the property market in the United States, the Fed Fund rate was raised again in May 2004, and in December 2004 the Treasury bond yield was 3.61%.

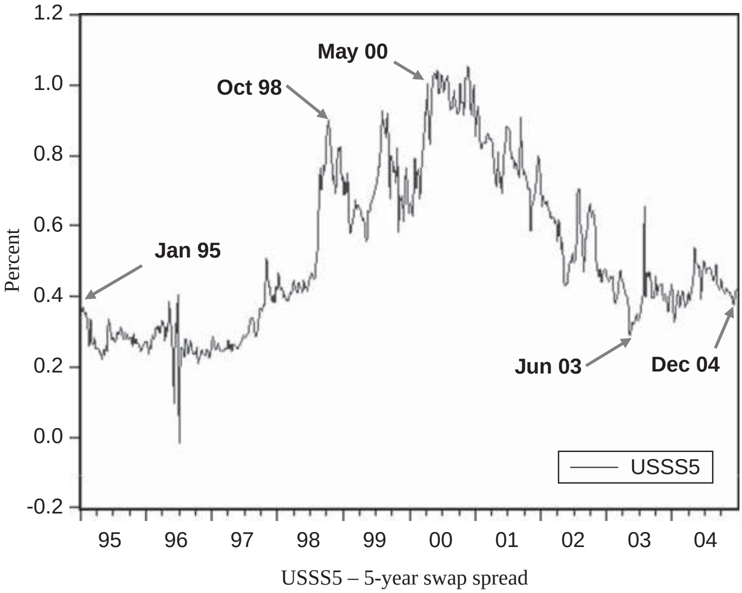

Fig. 2. US Five-Year Swap Spread.

Fig. 2 plots the US five-year spread over the sample period and highlights the volatility that existed in the swap spread. Before 1998, the swap spread seldom moved above 40 basis points, but it increased sharply following the Russian default and the near failure of LTCM. By October 1998, the five-year swap spread had increased to 90 basis points (from its mean of 52.4 basis points), which contributed to the record loss of US$1.6 billion for the hedge fund. After the crisis triggered by the collapse of LTCM, the swap spread also broke through the 90 basis point level twice, in 1999 and 2000, before declining to 34 basis points in June 2003, then rising slightly to 42 basis points in December 2004.

It is interesting to note that the direction of interest rates and the swap spread in terms of the previously discussed in Scenarios (I to VI). First, note the two different settings for the two periods 1995–2000 and 2000–2004. From January 1995 to May 2000, interest rates and the swap spread overall moved in opposite directions, that is, interest rates tended to decline, whereas the swap spread increased (the negative correlation in Scenario VI). However, from June 2000, the swap spread began to move in the same direction as interest rates: initially, the swap spread narrowed from June 2000 to May 2003 with the decline in interest rates (the positive correlation in Scenario II),

Table 2. Analysis of the Change in the Swap Spread from 1994 to 2004.

Period	Date	IRS_t	GOV_t	SS_t	Change in IRS_t	Change in GOV_t	Change in SS_t
–	06/01/1995	7.99	7.87	0.34	–	–	–
1	02/01/1998	6.03	5.61	0.42	−1.97	−2.26	0.08
2	16/10/1998	4.95	4.04	0.90	−1.08	−1.56	0.48
3	05/02/1999	5.54	4.96	0.58	0.60	0.92	−0.33
4	06/08/1999	6.84	5.91	0.92	1.30	0.95	0.35
5	03/12/1999	6.69	6.07	0.61	−0.16	0.16	−0.31
6	09/06/2000	7.40	6.36	1.04	0.72	0.29	0.43
7	17/05/2002	5.03	4.59	0.43	−2.38	−1.77	−0.61
8	02/08/2002	3.92	3.21	0.71	−1.11	−1.39	0.28
9	16/05/2003	2.68	2.38	0.29	−1.24	−0.82	−0.42
10	07/05/2004	4.48	3.94	0.54	1.81	1.56	0.25
11	31/12/2004	4.03	3.61	0.42	−0.46	−0.34	−0.12

Period	Date	Swap Spread	Scenario	Duration (months)	Observation	Explanation
–	06/01/1995	–	–	–	–	
1	02/01/1998	Widen	Scenario II	36	Buy bond	Flight to quality
2	16/10/1998	Widen	Scenario II	9	Buy bond	Flight to quality
3	05/02/1999	Narrowed	Scenario III	4	Sell bond	Unwind position
4	06/08/1999	Widen	Scenario I	6	Pay fixed in swap	Macro hedging
5	03/12/1999	Narrowed	Scenario V	4	Curve depart	Unknown
6	09/06/2000	Widen	Scenario I	6	Pay fixed in swap	Macro hedging
7	17/05/2002	Narrowed	Scenario IV	23	Received fixed in swap	Mortgage hedging
8	02/08/2002	Widen	Scenario II	3	Buy bond	Flight to quality
9	16/05/2003	Narrowed	Scenario IV	9	Received fixed in swap	Mortgage hedging
10	07/05/2004	Widen	Scenario I	12	Pay fixed in swap	Macro hedging
11	31/12/2004	Narrowed	Scenario IV	8	Received fixed in swap	Mortgage hedging

and both increased after June 2003 (the positive correlation in Scenario I). These observations highlight the importance of monitoring regime-switching effects when modelling the time-varying properties of the spreads.

Table 2 provides more detail on these general trends by dividing the sample period into 11 sub-periods with the first panel showing the levels in the swap (IRS) and bond (GOV) and spread (SS) rates and their respective changes. The bottom panel in Table 2 explains whether the swap spread narrowed or widened, its duration and the Scenario (from I to VI). The explanation for the observed behaviour is explained in the last column.

Thus, if we observe the highs and lows in the swap spread and compare the relative movement of the change in the Treasury bond yield relative to the change in the interest rate swap rate, one can see that the widening of the swap spread from January 1998 to October 1998 and from May 2002 to August 2002 was mainly due to a flight to quality (Scenario II). However, the more recent widening of the swap spread from 2003 to 2004 was due to macro hedging (Scenario I). As interest rates declined sharply from June 2000, institutions tended to receive fixed in swaps to hedge their investment in mortgage backed securities that possess negative convexity (Mortgage Hedging Scenario IV). The narrowing of the swap spread in the first part of 2000 was therefore closely related to the sharp decline in the swap rate.

Given that the interest rate and swap spread levels are not weakly stationary, the first difference in the three time series is used in the subsequent analysis. The summary statistics, for the first difference (or change) in the Government bond and interest rate swap yields, are provided in Table 3. Descriptive statistics for the averages of the weekly change in the Treasury bond yield and the swap rate are both negative, at 0.8 basis points, whereas the standard deviations are quite high, at 14.9 basis

Table 3. Summary Statistics of the Change in the Government Bond Yield, Swap Rate and Swap Spread.

	Change in Government Bond Yield (z_{1t})	Change in Interest Rate Swap Rate (z_{2t})	Threshold Variable Change in Swap Spread (y_t)
Mean	−0.008	−0.008	0.000
Standard deviation	0.149	0.153	0.052
Skewness	0.323	0.419	−0.795
Kurtosis	3.804	4.291	15.906
Maximum	0.584	0.655	0.267
Median	−0.009	−0.010	0.001
Minimum	−0.488	−0.464	−0.424
ADF	−9.802	−9.552	−11.825
Correlation			
z_{1t}	1.000		
z_{2t}	−0.099	1.000	
y_t	0.941	0.244	1.000

points and 15.3 basis points, respectively. The average of the change in the swap spread is zero, and the standard deviation is 5.2 basis points. Both the change in the Treasury bond yield and the change in the swap rate are positively skewed, whereas the change in the swap spread is negatively skewed. The three time series are proved to be weakly stationary after the ADF test.

It is interesting to note that the correlations between the yields and spread (bottom panel of Table 3) shows that the change in the swap spread is highly correlated with the change in the Treasury bond yield (correlation coefficient of 0.941), but only weakly correlated with the change in the swap rate (correlation coefficient of 0.244). The very weak correlation between the change in the Treasury bond yield and the change in the swap rate (correlation coefficient of −0.099) is bad news for institutions that use Treasury bonds to hedge their swap positions.

4. METHODOLOGY

4.1. Modelling Techniques

On the basis of Chan's (1992) study of stock indices and index futures, the lead–lag behaviour of the change (Δ) in the US Treasury bond (GOV_t) and the interest rate swap (IRS_t) at time (t) can be examined using the following regression, Eq. (1), where ε is a random variable:

$$\Delta\text{IRS}_t = a + \sum_{k=-4}^{4} b_k \Delta\text{GOV}_{t+k} + \varepsilon_t \tag{1}$$

Given that the change in the Treasury bond yield and the change in the swap rate are stationary time series, the Granger causality test (up to lag L) can be performed using the following sets of regressions (Eq. (2)):

$$\begin{aligned} \Delta\text{IRS}_t &= c + \sum_{i=1}^{L} d_i \Delta\text{IRS}_{t-i} + \sum_{j=4}^{L} e_j \Delta\text{GOV}_{t-j} + \varepsilon_t \\ \Delta\text{GOV}_t &= f + \sum_{i=1}^{L} g_i \Delta\text{GOV}_{t-i} + \sum_{j=1}^{L} h_j \Delta\text{IRS}_{t-j} + \varepsilon_t \end{aligned} \tag{2}$$

BTAR modelling techniques are then applied to the examination of the dynamic relationship between the change in the five-year US Treasury yield (ΔGOV_t) and the change in the five-year interest rate swap rate (ΔIRS_t), where $\mathbf{Z}_t = (z_{1t},\ z_{2t})'$ with $z_{1t} = \Delta\text{GOV}_t = \text{GOV}_t - \text{GOV}_{t-1}$ and $z_{2t} = \Delta\text{IRS}_t = \text{IRS}_t - \text{IRS}_{t-1}$. The series under study is the weekly closing prices over the period January 1995 to December 2004, which gives 522 observations. For the threshold variable, defined as $y_t = \Delta\text{SS}_t = \text{SS}_t - \text{SS}_{t-1}$, the weekly change in the swap spread (ΔSS_t) is used. The threshold variable series is plotted in Fig. 3, which displays y_t from January 1995 to December 2004. The volatility spikes in the series are clear, especially during 1996 and 2003.

Tiao and Box (1981) suggest summarising the cross-correlation relationship of the data series $\mathbf{Z}_t = (z_{1t},\ z_{2t})'$, using indicator matrices where the indicator symbols (+), (−) and (·), where (+) denotes a value that is greater than twice the estimated standard error, (−) denotes a value that is less than twice the estimated standard error and (·) denotes an insignificant value that is based on the aforementioned criteria. The $(i,\ j)$ element of the indicator matrix at lag l summarises the significance of the lag-l cross-correlation when the component series z_{jt} leads the component series z_{it}. Furthermore, the diagonal elements summarise the significance of the sample autocorrelations for each series. Analogous to the Tsay (1989) procedure for univariate TAR modelling, Tsay (1998) extends the univariate

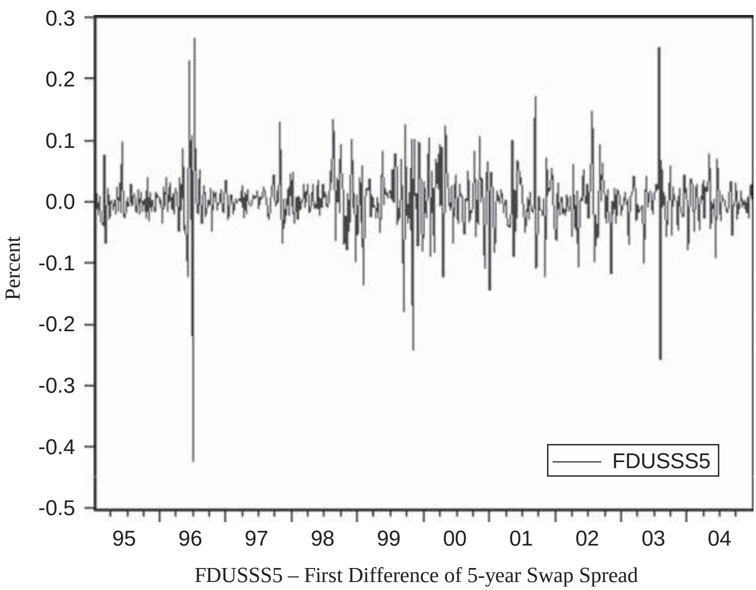

Fig. 3. Change in Five-Year Swap Spread.

method to the multivariate situation. In this section, we consider a bivariate time series $\mathbf{Z}_t = (z_{1t}, z_{2t})'$. A k-regime BTAR (d; $p_1,\ldots, p_k$) model defined as

$$\mathbf{Z}_t = \begin{cases} w_0^{(1)} + \sum_{j=1}^{p_1} \mathbf{\Phi}_j^{(1)} \mathbf{Z}_{t-j} + \boldsymbol{a}_t^{(1)}, & \text{if } y_{t-d} \leq r \\ w_0^{(2)} + \sum_{j=1}^{p_2} \mathbf{\Phi}_j^{(2)} \mathbf{Z}_{t-j} + \boldsymbol{a}_t^{(2)}, & \text{if } r_1 < y_{t-d} \leq r_2 \\ \vdots \quad \vdots \quad \vdots & \vdots \\ w_0^{(k)} + \sum_{j=1}^{p_k} \mathbf{\Phi}_j^{(k)} \mathbf{Z}_{t-j} + \boldsymbol{a}_t^{(k)}, & \text{if } r_{k-1} < y_{t-d} \end{cases} \tag{3}$$

where k is the number of regimes in the model, d is the delay parameter, p_i is the autoregressive order in the ith regime of the model, $w_0^{(i)}$ are (2×1)-dimensional constant vectors, and $\mathbf{\Phi}_j^{(i)}$ are (2×2)-dimensional matrix parameters for $i = 1, \ldots, k$. The threshold parameters satisfy the constraint $-\infty = r_0 < r_1 < r_2 < \ldots < r_{k-1} < r_k = \infty$. The innovational vectors in the ith regime satisfy $\boldsymbol{a}_t^{(i)} = \sum_i^{1/2} \boldsymbol{e}_t$, where $\sum_i^{1/2}$ are symmetric positive definite matrices, and $\{\boldsymbol{e}_t\}$ is a sequence of serially uncorrelated normal random vectors with a mean of 0 and a covariance matrix $\mathbf{I}$, the (2×2)-dimensional identity matrix. The threshold variable y_{t-d} is assumed to be stationary and depends on the observable past history of $\mathbf{Z}_{t-d}$. For example, we can set

$$y_{t-d} = \boldsymbol{\eta}' \mathbf{Z}_{t-d} \tag{4}$$

where $\boldsymbol{\eta}'$ is a pre-specified (2×1) dimensional vector. When $\boldsymbol{\eta} = (1, 0)'$, the threshold variable is simply $y_{t-d} = z_{1,t-d}$. When $\boldsymbol{\eta} = (1/2, 1/2)'$, the threshold variable is the average of the two elements in $\mathbf{Z}_{t-d}$.

4.2. Nonlinearity Testing

Given $p = \max\{p_1, \ldots, p_k\}$ and $d \leq p$, we can observe the bivariate vector time series $\{\mathbf{Z}_1, \ldots, \mathbf{Z}_n\}$. It should be noted that the threshold variable y_{t-d} in Eq. (1) can only assume values in $Y = \{y_{p+1-d}, \ldots, y_{n-d}\}$. Let (i) be the time index of the ith smallest observation in Y. Tsay (1998) considers the multivariate generalisation of the ordered regression arrangement. Rolling ordered bivariate autoregressions in the form

$$\begin{pmatrix} \mathbf{Z}'_{(1)+d} \\ \mathbf{Z}'_{(2)+d} \\ \vdots \\ \mathbf{Z}'_{(j)+d} \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{Z}'_{(1)+d-1} & \cdots & \mathbf{Z}'_{(1)+d-p} \\ 1 & \mathbf{Z}'_{(2)+d-1} & \cdots & \mathbf{Z}'_{(2)+d-p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mathbf{Z}'_{(j)+d-1} & \cdots & \mathbf{Z}'_{(j)+d-p} \end{pmatrix} \times \begin{pmatrix} \omega'_0 \\ \mathbf{\Phi}'_1 \\ \vdots \\ \mathbf{\Phi}'_p \end{pmatrix} + \begin{pmatrix} \boldsymbol{a}'_{(1)+d} \\ \boldsymbol{a}'_{(2)+d} \\ \vdots \\ \boldsymbol{a}'_{(j)+d} \end{pmatrix} \quad (5)$$

can be arranged successively, where $j = m, m+1, \ldots, n-p$, and m is the number of start-up observations in the ordered autoregression. Tsay (1998) suggests a range of m (between $3\sqrt{n}$ and $5\sqrt{n}$). Different values of m can be used to investigate the sensitivity of the modelling results with respect to the choice. It should be noted that the ordered autoregressions are sorted by the variable y_{t-d}, which is the regime indicator in the BTAR model.

Let $\hat{\varepsilon}_{(m+1)+d}$ denote the one-step-ahead standardised predictive residual from the least-squares fitted multivariate regression for $j = m$. Tsay (1998) provides the direct computational formula for $\hat{\varepsilon}_{(m+1)+d}$, but it can easily be obtained from many commonly used statistical software packages (Timm & Mieczkowski, 1997). Analogous to the univariate case, if the underlying model is a linear autoregressive process, then the predictive residuals are white noise and are uncorrelated with the regressor $\mathbf{X}'_t = \{1, \mathbf{Z}'_{t-1}, \mathbf{Z}'_{t-2}, \ldots, \mathbf{Z}'_{t-p}\}$. However, if $\mathbf{Z}_t$ follows a threshold process, then the predictive residuals are correlated with the regressor. Tsay (1998) utilises this property and considers the multivariate regression

$$\hat{\varepsilon}'_{(l)+d} = \mathbf{X}'_{(l)+d}\mathbf{B} + \mathbf{w}'_{(l)+d} \quad (6)$$

for $l = m+1, \ldots, n-p$, where $\mathbf{B}$ is the matrix regression parameter, and $\mathbf{w}'_{(l)+d}$ is the matrix of the residuals. The problem of testing nonlinearity is then transformed into the testing of the hypothesis H_0: $\mathbf{B} = 0$ in this regression. Tsay (1998) employs the test statistic

$$C(d) = (n - p - m - kp - 1) \times \{\ln|S_0| - \ln|S_1|\} \quad (7)$$

where $|S|$ denotes the determinant of the matrix $\mathbf{S}$, and

$$\begin{aligned} S_0 &= \tfrac{1}{n-p-m} \sum_{l=m-1}^{n-p} \hat{\varepsilon}_{(l)+d}\hat{\varepsilon}'_{(l)+d} \\ S_1 &= \tfrac{1}{n-p-m} \sum_{l=m-1}^{n-p} \hat{w}_{(l)+d}\hat{w}'_{(l)+d} \end{aligned} \quad (8)$$

where $\hat{w}$ is the least-squares residual of regression (6). Under the null hypothesis that $\mathbf{Z}_t$ is linear, Tsay (1998) shows that $C(d)$ is asymptotically a chi-squared random variable with (pk^2+k) degrees of freedom.

4.3. Model Specification, Estimation and Diagnostic Checking

To perform the $C(d)$ test for nonlinearity in Eq. (7), the values of both p and d must be given. In practice, we can select p from the partial autoregression matrix (PAM) of $\mathbf{Z}_t$. Tiao and Box (1981) define the PAM at lag l, which is denoted by $\mathbf{P}(l)$, to be the last matrix coefficient when the data are fitted to a vector autoregressive process of order l. This is a direct extension of the definition of Box and Jenkins (1976) of the partial autocorrelation function for a univariate time series. The PAM $\mathbf{P}(l)$ of a linear vector AR(p) process are zero for $l > p$. This 'cut-off' property provides useful information for the identification of the order p. Once p is selected, d is chosen, such that it provides the most significant $C(d)$ statistic.

In univariate TAR modelling, various scatterplots are used to specify the number of regimes k and the threshold parameters (i.e. the r values). Unfortunately, these plots are not applicable to high-dimensional multivariate TAR analysis. Following Tong (1983), Akaike's information (AIC) is used to search for these parameters. Given p, d, k and $R_k = \{r_1, \ldots, r_{k-1}\}$, the full-length ordered bivariate autoregression can be divided into different regimes. For the jth regime of the data, allow a general model of the form $\mathbf{Z}_j = \mathbf{A}_j\mathbf{\Phi}^{(j)}\mathbf{a}_j$, where

$$\begin{aligned}
\mathbf{Z}_j &= (\mathbf{Z}'_{(\pi_{j-1}+1)+d}, \mathbf{Z}'_{(\pi_{j-1}+2)+d}, \ldots, \mathbf{Z}'_{(\pi_j)+d})' \\
\mathbf{\Phi}^{(j)} &= (\omega'_0, \mathbf{\Phi}'^{(j)}_1, \ldots, \mathbf{\Phi}'^{(j)}_p)' \\
\mathbf{a}_j &= (\mathbf{a}'_{(\pi_{j-1}+1)+d}, \mathbf{a}'_{(\pi_{j-1}+2)+d}, \ldots, \mathbf{a}'_{(\pi_j)+d})' \\
\mathbf{A}_j &= \begin{pmatrix} 1 & \mathbf{Z}'_{(\pi_{j-1}+1)+d-1} & \cdots & \mathbf{Z}'_{(\pi_{j-1}+1)+d-p} \\ 1 & \mathbf{Z}'_{(\pi_{j-1}+2)+d-1} & \cdots & \mathbf{Z}'_{(\pi_{j-1}+2)+d-p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mathbf{Z}'_{(\pi_j)+d-1} & \cdots & \mathbf{Z}'_{(\pi_j)+d-p} \end{pmatrix}
\end{aligned} \tag{9}$$

and π_j is the largest value of (j) such that $\{r_{j-1} < z_{(j)} \leq r_j\}$ for $j = 1, \ldots, k-1$. We define $\pi_0 = 0$ and $\pi_k = n-p$. The number of observations in the jth regime is $n_j = \pi_j - \pi_{j-1}$. The least-squares estimate of $\mathbf{\Phi}_j$ can be obtained by the ordinary multivariate least-squares method:

$$\hat{\Phi}^{(j)} = (\mathbf{A}_j'\mathbf{A}_j)^{-1}(\mathbf{A}_j'\mathbf{Z}_j) \tag{10}$$

The residual variance–covariance matrix of the *j*th regime can then be obtained by

$$\hat{\sum}_j = \frac{1}{n_j}\sum_{t=1}^{n_j}\{\hat{a}_{(\pi_{j-1}+t)+d}\hat{a}'_{(\pi_{j-1}+t)+d}\} \tag{11}$$

Finally, the AIC of the bivariate fitted TAR model in Eq. (1) is defined as

$$\text{AIC}(p, d, k, R_k) = \sum_{j=1}^{k}\{n_j \ln\left|\hat{\sum}_j\right| + 2k(kp+1)\} \tag{12}$$

Given p and d, we can search for the parameters k and R_k by minimising the AIC. Owing to the computational complexity and possible interpretations of the final model, k is usually restricted to be a small number, such as 2 or 3. For the threshold parameters R_k, the data may be divided into subgroups according to the empirical percentiles of y_{t-d} and use the AIC to select the r values. Finally, the AIC is used to refine the AR order ($p_k \leq p$) in each regime. To guard against incorrect specification of the model, a detailed diagnostic analysis of the residuals is required. This includes an examination of the plots of the standardised residuals and the sample cross-correlation (SCC) matrices of the residuals (Tiao & Box, 1981).

5. RESULTS

5.1. Lead–Lag and Causality Testing

Table 4 reports the results if the simple lead–lag analysis (Eq. (1)). In this analysis the only coefficient that is statistically significant is b_0. Neither the lead variables (b_1 to b_4) nor the lag variable (b_{-1} to b_{-4}) are statistically significant. The results indicate that there is no lead–lag relationship between the change in the swap rate with respect to the change in the Treasury bond yield for the lead variables, or the lag variable, over the entire sample period.

The next analysis involves pairwise testing for Granger causality (Eqs. (2) and (3)). The results for this testing are reported in Tables 5–7, where

Table 4. Results of the Simple Lead–Lag Study.

	Coefficient	t-Statistics
b_{-4}	0.017	1.098
b_{-3}	−0.002	−0.110
b_{-2}	−0.010	−0.626
b_{-1}	−0.011	−0.699
b_0	0.964*	61.444
b_1	0.020	1.300
b_2	0.000	0.018
b_3	0.020	1.248
b_4	0.004	0.263
R^2	0.885	
F-Statistics	431.782	

*Significance at the 5% level.

Table 5 reports the results for the complete sample and Tables 6 and 7 for two sub-periods. For the entire sample period (January 1995 to December 2004), the null hypothesis that the change in the Treasury bond yield does not Granger-cause a change in the swap rate is rejected (F-statistic = 5.029) at least at the 95% level. The null hypothesis is also rejected for periods from one lag up to five lags (F-statistic = 3.697 to 2.677) at least at the 95% level. However, the null hypothesis that a change in the swap rate does not Granger-cause a change in the Treasury bond yield cannot be rejected, which shows that the change in the Treasury bond yield has only a unilateral causality on the change in the swap rate.

If the sample period is divided into two sub-periods, the first from January 1995 to May 2000 (Table 6), then the result is similar to that for the whole sample period (reported in Table 5): there is unilateral causality with the change in the Treasury bond yield Granger-causing the change in the swap rate. However, for the first sub-period, the F-statistics (3.853 to 2.352) are only significant (at least at the 95% level) from three lags to five lags.

For the second sub-period, from June 2000 to December 2004, the results are reported in Table 7. These findings are also consistent with the first sub-period and the overall sample. However, the F-statistics (7.031 and 3.349, respectively) are only significant (at least the 95% level) for the one lag period and two lag period only. An interesting result is that the change in the swap rate is marginally shown to Granger-cause a change in the Treasury bond yield for one lag period (at the 10% level with F-statistic = 3.855).

Table 5. Results of the Granger Causality Tests (January 1995 to December 2004).

Pairwise Granger Causality Tests			
Null Hypothesis	Observations	*F*-Statistics	*p*-value
Lag = 1			
Change in GOV_t does not Granger-cause a change in IRS_t	520	5.029*	0.025
Change in IRS_t does not Granger-cause a change in GOV_t	520	0.243	0.622
Lag = 2			
Change in GOV_t does not Granger-cause a change in IRS_t	519	3.697*	0.025
Change in IRS_t does not Granger-cause a change in GOV_t	519	0.487	0.615
Lag = 3			
Change in GOV_t does not Granger-cause a change in IRS_t	518	3.852*	0.010
Change in IRS_t does not Granger-cause a change in GOV_t	518	0.387	0.763
Lag = 4			
Change in GOV_t does not Granger-cause a change in IRS_t	517	2.850*	0.023
Change in IRS_t does not Granger-cause a change in GOV_t	517	0.559	0.692
Lag = 5			
Change in GOV_t does not Granger-cause a change in IRS_t	516	2.677*	0.021
Change in IRS_t does not Granger-cause a change in GOV_t	516	0.708	0.617

*Significance at the 5% level.

5.2. Bivariate Threshold Autoregressive Models

We first examine the SCC matrices using indicator symbols and conclude that there are no moving average elements in the BTAR models. Then the PAM of the observed bivariate vector time series are observed, with the PAM matrices summarised using indicator symbols in Table 8.

The likelihood ratio statistic $M(l)$ can be used to test the null hypothesis that the PAM are zero matrices. Originally, Bartlett (1938) shows that the

Table 6. Results of the Granger Causality Tests (January 1995 to May 2000).

Pairwise Granger Causality Tests (January 1995 to May 2000)			
Null Hypothesis	Observations	*F*-Statistics	*p*-value
Lag = 1			
Change in GOV_t does not Granger-cause a change in IRS_t	278	0.792	0.374
Change in IRS_t does not Granger-cause a change in GOV_t	278	0.312	0.577
Lag = 2			
Change in GOV_t does not Granger-cause a change in IRS_t	277	2.485	0.085
Change in IRS_t does not Granger-cause a change in GOV_t	277	0.963	0.383
Lag = 3			
Change in GOV_t does not Granger-cause a change in IRS_t	276	3.852*	0.010
Change in IRS_t does not Granger-cause a change in GOV_t	276	0.940	0.422
Lag = 4			
Change in GOV_t does not Granger-cause a change in IRS_t	275	2.692*	0.032
Change in IRS_t does not Granger-cause a change in GOV_t	275	0.962	0.429
Lag = 5			
Change in GOV_t does not Granger-cause a change in IRS_t	516	2.352*	0.041
Change in IRS_t does not Granger-cause a change in GOV_t	516	0.726	0.604

*Significance at the 5% level.

$M(l)$ statistic is asymptotically χ^2 distributed with four degrees of freedom if the null hypothesis is true. From Table 8, one can observe that the $M(l)$ statistics drop significantly after $l = 3$ (from 17.81 at lag 3 to 3.76 at lag 4). Therefore, it is possible to tentatively specify $p = 3$ for the $C(d)$ test for nonlinearity. This allows the $C(d)$ test with $p = 3$, $d \leq p$ and $m = 150$. These results for nonlinearity are then reported in Table 9. Since the critical value for this test is 23.68. The results clearly reject the linear hypothesis, which indicates that BTAR-type nonlinearity is detected in the data. The test statistics also suggest using the delay parameter $d = 1$.

Table 7. Results of the Granger Causality Tests (June 2000 to December 2004).

Pairwise Granger Causality Tests (June 2000 to December 2004)			
Null Hypothesis	Observations	*F*-Statistics	*p*-value
Lag = 1			
Change in GOV_t does not Granger-cause a change in IRS_t	241	7.031*	0.009
Change in IRS_t does not Granger-cause a change in GOV_t	241	3.855*	0.048
Lag = 2			
Change in GOV_t does not Granger-cause a change in IRS_t	240	3.349*	0.037
Change in IRS_t does not Granger-cause a change in GOV_t	240	2.169	0.117
Lag = 3			
Change in GOV_t does not Granger-cause a change in IRS_t	239	2.401	0.069
Change in IRS_t does not Granger-cause a change in GOV_t	239	1.583	0.194
Lag = 4			
Change in GOV_t does not Granger-cause a change in IRS_t	238	1.662	0.160
Change in IRS_t does not Granger-cause a change in GOV_t	238	1.183	0.319
Lag = 5			
Change in GOV_t does not Granger-cause a change in IRS_t	237	1.468	0.201
Change in IRS_t does not Granger-cause a change in GOV_t	237	1.262	0.281

*Significance at the 5% level.

With 522 observations, it is possible to consider the possibilities of BTAR models with two or three regimes, that is, $k = 2$ or 3. Given p, d and k, a grid search method can be used to select the thresholds by minimising the AIC values that are defined in Eq. (12). Let $P_\alpha(y_{t-d})$ be the empirical α-th percentile of y_{t-d}. For the two-regime models assume $r \in [P_{10}(y_{t-d}), P_{90}(y_{t-d})]$. For the three-regime models, assume that $r_1 \in [P_{10}(y_{t-d}), P_{45}(y_{t-d})]$ and $r_2 \in [P_{55}(y_{t-d}), P_{90}(y_{t-d})]$. Table 10 shows the selected threshold values under different combinations of (k, p, d). It is indicated that the overall AIC is −5,182.38 when $k = 3$, $p = 3$, $d = 1$, $\hat{r}_1 = -0.033$ and

Table 8. Indicator Matrices for the Partial Autoregression Matrices.

Lag (l)	1	2	3	4	5	6
	$\begin{pmatrix} - & + \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & + \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$
$M(l)$	50.47	3.77	17.81	3.76	3.13	5.83
Lag (l)	7	8	9	10	11	12
	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$
$M(l)$	8.85	6.41	1.37	1.14	6.11	1.72

Note: The critical value for the $M(l)$ test is $\chi^2_{0.95,4} = 9.49$.

Table 9. Tests for Nonlinearity.

d	1	2	3
$C(d)$	28.63	17.56	19.99

Note: The critical value for the $C(d)$ test is $\chi^2_{0.95,14} = 23.68$.

Table 10. Selection of k, p, d and the Threshold Values..

k	p	d	$\hat{r}_1$	$\hat{r}_2$	AIC
2	3	1	0.049		−5141.69
3	3	1	−0.033	0.017	−5182.38

$\hat{r}_2 = 0.017$. One can further refine the model by allowing different autoregressive orders for different regimes. The AIC selects $(p_1, p_2, p_3) = (2, 3, 1)$ with the least squares estimation results of the specified model provided in Table 11. In this table the results for the first ($k = 1$, $p = 1$), second ($k = 2$, $p = 0$) and third regime ($k = 3$, $p = 3$) are provided. The indicator matrices and the residual PAM are also examined and do not show any model inadequacy [$M(l)$ for lags 1–6 take values 2.84 to $1.70 <$ critical value of 9.49].

Table 11. Model Estimation Results.

The estimated coefficients: $\hat{\Phi}_j^{(k)}$

(A) The first regime ($k = 1$, $p_1 = 1$, $n_1 = 83$)

Lag(j)	0	1
	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -0.74 & 0.78* \\ -0.19 & 0.22 \end{pmatrix}$

(B) The second regime ($k = 2$, $p_2 = 0$, $n_2 = 294$)

Lag(j)	0
	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(C) The third regime ($k = 3$, $p_3 = 3$, $n_3 = 142$)

Lag(j)	0	1	2	3
	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -0.78 & 0.70 \\ -0.29 & 0.19 \end{pmatrix}$	$\begin{pmatrix} -0.51 & 0.62^{*} \\ -0.37 & 0.44 \end{pmatrix}$	$\begin{pmatrix} -0.39 & 0.60^{*} \\ -0.17 & 0.40 \end{pmatrix}$

*The element in the coefficient matrix is statistically significant at the 5% level.

Using the BTAR modelling framework, three regimes can now be constructed for the dynamics of the Treasury and the interest rate swaps markets. The first regime occurs when the weekly change in the swap spread is negative and more than 3.3 basis points ($y_{t-1} \leq r_1 = -0.033$). The second regime occurs when ($r_1 = -0.033 \leq y_{t-1} \leq r_2 = 0.017$). The third regime exists when the weekly change in the swap spread is positive and more than 1.7 basis points ($y_{t-1} > r_2 = 0.017$). The weekly lead–lag relationship between the Treasury and interest rate swaps markets in the k-th regime can be examined using the off-diagonal elements $\hat{\Phi}_j^{(k)}$ reported in Table 11. If any of the upper off-diagonal elements of the estimated matrices are significant, then a change in the Treasury bond yield has a lead effect on the change in the swap rate. If any of the lower off-diagonal elements of the estimated matrices are significant, then a change in the swap rate has a lead effect on the change in the Treasury bond yield. The results are now summarised in Table 12, which shows a with statistically significant lead–lag relationship (with the Government bond leading the swap market) in the first regime at lag 1, no relationship in the second regime, and a statistically significant relationship at lags 2 and 3 in the third regime. The swap market does not lead the bond market in any of the three regimes.

Table 12. Analysis of the Weekly Lead–Lag Relationship.

Government Bond			Interest Rate Swaps Market		
Leads			Leads		
Interest Rate Swaps Market			Government Bond		
Lag	Size	*t*-ratio	Lag	Size	*t*-ratio
(A) The first regime					
1	0.78	2.60*			
(B) The second regime					
No significant lead–lag effects found					
(C) The third regime					
1	0.70	1.94	1	−0.29	−0.83
2	0.62	2.38*	2	−0.37	−1.54
3	0.60	2.07*	3	−0.17	−0.65

*Significance at the 5% level.

5. CONCLUSIONS

This chapter examines the dynamics between US Treasury bond and swap markets using BTAR models. This approach, which may be applied to other interest rate products in other markets, tests whether the lead–lag relationship between the bond market and the swap market is a nonlinear dynamic process, and second whether this relationship is governed by the change in the interest rate differential, or spread, between these two markets.

The findings from the BTAR models may be summarised in Fig. 4 where three regimes are identified. In the first regime, the Treasury market leads the swaps market when the change in the swap spread is negative and more than 3.3 basis points on a weekly basis. In the second regime, there is no significant lead–lag relationship between the two markets when the change in the swap spread falls within a narrow range of negative 3.3 basis points and positive 1.7 basis points. In the third regime, the Treasury bond market leads the swaps market when the change in the swap spread is positive and more than 1.7 basis points.

Simple lead–lag studies do not reveal any particular information about the dynamics between the two markets. However, Granger causality tests are useful for revealing the overall movement of the two markets, because it shows that the change in the Treasury bond yield can Granger-cause a

Regime I – Change in Swap Spread (ΔSS_{t-1}) is more than –3.3 basis point per week
ΔGOV_t leads ΔIRS_t (the coefficient is positive)
Number of observations = 83 (15.99% of the total sample)
Only 1 lag is significant

ΔSS_{t-1} **< -3.3 b.p.**

Regime II – Change in Swap Spread (ΔSS_{t-1}) is between –3.3 basis point and + 1.7 basis point per week
No lead-lag
Number of observation = 294 (56.65% of the total sample)

-3.3 b.p. < ΔSS_{t-1} < 1.7b.p.

Regime III – Change in Swap Spread (ΔSS_{t-1}) is more than 1.7 basis point per week
Mixed lead-lag relation (two conditions)
Number of observations = 142 (27.36% of the total sample)
First condition
ΔGOV_t leads ΔIRS_t (the coefficient is positive)
Longer lags (1, 2, 3) are significant
Second condition
ΔIRS_t lead ΔGOV_t (the coefficient is negative)
Lags 2 marginally significant

ΔSS_{t-1} **> 1.7 b.p.**

Fig. 4. Three Regimes.

change in the swap rate. Subjective judgment can enhance the performance of the Granger causality test. For example, one can identify that lag 3 to lag 5 are significant for the first sub-period from January 1995 to May 2000, whereas lag 1 and lag 2 are significant in the second sub-period from June 2000 to December 2004.

By dividing the data into two sub-periods, we also find that for the second sub-period, the change in the swap rate can marginally Granger-cause a change in the Treasury bond yield for one lag period. However, this information cannot be revealed if a subjective decision is not made. The BTAR model, however, can reveal more information without the use of subjective decision making. The regime switch process can occur in individual observations because the switching is determined by the state of the threshold variable, namely, the magnitude and direction of the change in the swap spread. The dominance of Treasury bonds in leading the swaps market occurs when either a widening or a narrowing of the swap spread occurs. This seems to fall in line with the 'Flight to Quality' and 'Flight to Liquidity' phenomena and is probably due to the nature of Treasury securities. Usually, no credit limit is required for transactions to be carried out for financial institutions, and Treasury securities have a minimal impact on the balance sheets of various institutions.

Overall, the BTAR model provides a deeper insight than simple lead–lag studies and Granger causality tests into the dynamics between the US Treasury bonds and interest rate swaps markets. Importantly, this approach can be applied to other interest rate product in other markets. By identifying the regimes and the conditions for change in the regimes, market participants and regulators can become more informed about the probable changes that will occur in the Treasury bond and interest rate swaps markets. Similar to the findings of Chappell et al. (1996), who identify the bounds within which the French Franc/Deutschmark exchange rate kept to before the launch of the Euro, the movements of the Treasury bond market and the interest rate swaps market are governed by the direction and magnitude of the change in the swap spread. The BTAR model is able to identify the threshold value of the change in the swap spread that bond and swap market participants considered to be significant. Further research can be conducted to explain the existence of the threshold values.

NOTES

1. The main rating agencies are Standard & Poors, Moody's and Fitch Investor Services. For convenience we use the Standard & Poors' notation. The fixed rate side of the swap by convention in financial markets is expressed as a spread over the risk-free fixed rate bond.
2. Bank of International Settlements (BIS, 2008) statistics for June 2007 show that interest rate swaps account for 52.6% of outstandings (US$ 516.4 trillion) in over-the-counter (OTC) derivatives, with plain vanilla or simple fixed-floating swaps accounting for most swap turnover. Of this swap total, about 30% (US$ 81.28 trillion) are denominated in US$.

REFERENCES

Bank for International Settlements (BIS). (2008). Semi annual OTC derivatives turnover at end June 2007, Basle, Switzerland. Available at http://www.bis.org/statistics/derstats.htm. Retrieved on April 21, 2008.

Bartlett, M. S. (1938). Further aspects of the theory of multiple regression. *Proceedings of the Cambridge Philosophical Society*, *34*, 33–40.

Box, G. E. P., & Jenkins, G. M. (1976). *Time series analysis: Forecasting and control* (2nd ed.). San Francisco: Holden-Day.

Brooks, C., & Garrett, I. (2002). Can we explain the dynamics of the UK FTSE 100 stock and stock index futures markets? *Applied Financial Economics*, *12*, 25–31.

Chan, K. (1992). A further analysis of the 1eadlag relationship between the cash market and stock index market. *Review of Financial Studies*, *5*, 123–152.

Chan, W. S., & Cheung, S. H. (2005). A bivariate threshold time series model for analyzing Australian interest rates. *Mathematics and Computers in Simulation 68*, *68*, 429–437.

Chappell, P. J., Mistry, P., & Ellis, C. (1996). A threshold model for the French Franc/Deutschmark exchange rate. *Journal of Forecasting*, *15*, 155–164.

Collin-Dufresne, P., & Solnik, B. (2001). On the term structure of default premia in the swap and LIBOR markets. *Journal of Finance*, *56*, 1095–1114.

Cortes, F. (2006). Understanding the term structure of swap spreads. *Bank of England Quarterly Bulletin 46*, *1*, 45–56.

Feldhutter, P., & Lando, D. (2008). Decomposing swap spreads. *Journal of Financial Economics*, *88*, 375–405.

Hamilton, J. D. (1996). Specification testing in Markov-switching time-series models. *Journal of Econometrics*, *70*, 127–157.

Harris, L. (1989). The October 1987 S&P 500 stock-future basis. *Journal of Finance*, *44*(1), 77–99.

Huang, Y., Chen, C. R., & Camacho, M. (2008). Determinants of Japanese Yen interest rate swap spreads: Evidence from a smooth transition vector autoregressive model. *Journal of Futures Markets*, *28*(1), 82–107.

Ito, T. (2007). The analysis of interest rate swap spreads in Japan. *Applied Financial Economics Letters*, *3*(1-3), 1–4.

Kawaller, I. G., Koch, P. D., & Koch, T. W. (1987). The temporal price relationship between S&P 500 futures and the S&P 500 index. *Journal of Finance*, *42*, 1309–1330.

Longstaff, F. A. (2004). The flight-to-liquidity premium in US Treasury bond prices. *Journal of Business*, *77*(3), 511–526.

Malhotra, D. K., Bhargava, V., & Chaudhry, M. (2005). Determinants of treasury-LIBOR swap spreads. *Review of Pacific Basin Financial Markets and Policies*, *8*(4), 687–705.

Stoll, H. R., & Whaley, R. E. (1990). The dynamics of stock index and stock index futures returns. *Journal of Financial and Quantitative Analysis*, *25*, 444–468.

Tiao, G. C., & Box, G. E. P. (1981). Modelling multiple time series with applications. *Journal of the American Statistical Association*, *76*, 802–816.

Timm, N. H., & Mieczkowski, T. A. (1997). *Univariate and multivariate general linear models: Theory and applications using SAS software*. North Carolina: SAS Publishing.

Tong, H. (1978). On a threshold model. In: C. H. Chen (Ed.), *Pattern recognition and signal processing*. Amsterdam: Sijhoff and Noordoff.

Tong, H. (1983). *Threshold models in non-linear time series analysis*. New York: Springer-Verlag.

Tsay, R. S. (1989). Testing and modelling threshold autoregressive processes. *Journal of the American Statistical Association*, *84*, 231–240.

Tsay, R. S. (1998). Testing and modelling multivariate threshold models. *Journal of the American Statistical Association*, *93*, 1188–1202.

Yadav, P. K., Pope, P. F., & Paudyal, K. (1994). Threshold autoregressive modelling in finance: The price difference of equivalent assets. *Mathematical Finance*, *4*, 205–221.