

CHAPTER 22

Engineering of Equity Instruments: Pricing and Replication

1. Introduction

Fixed-income instruments involve *payoffs* that are, in general, known and “fixed.” They also have set *maturity* dates. Putting aside the credit quality of the instrument, fixed-income assets have relatively simple cash flows that depend on a known, small set of variables and, hence, risk factors. There are also well-established and quite accurate ways to calculate the relevant term structure. Finally, there are several liquid and efficient fixed-income derivatives markets such as swaps, forward rate agreements (FRAs), and futures, which simplify the replication and pricing problems existing in this sector.

There is no such luxury in equity analysis. The underlying asset, which is often a stock or a stock index, does not have a set maturity date. It depends on a nontransparent, idiosyncratic set of risks, and the resulting cash flows are complex. The timing and the size of future cash flows may not be known. There are also complex issues of growth, investment, and management decisions that further complicate the replication and pricing of equity instruments. Finally, relatively few related derivatives markets are liquid and usable for a replication exercise.

Yet, the general principles of pricing, replicating, and risk-managing equity cannot be that different. Whatever is possible in the fixed-income sector should, *in principle*, be possible in equity as well. Of course, the approximation process, and the resulting modeling of these instruments may become more difficult and the success rate of the potential methods may drop.¹

In this chapter, we extend the methods introduced earlier to equity and equity-linked products. We also discuss the engineering applications of some products that are representative of this sector.

Our intention is to show how the methods used in fixed income can, in principle, be used in the equity sector as well. In doing this, we will analyze the major differences and some similarities

¹ Of course, some replication is possible using options or other derivatives directly written on the stock, but this is quite different from, say, splitting a coupon bond into constituent discount bonds.

between the two sectors. There are two additional difficulties with equity. First, equity analysis may require a modeling effort to project the underlying earnings. This is because the implied cash flows of a stock are never known exactly and are difficult to predict.² Financial engineering methods that use the fundamental theorem of asset pricing avoid this issue by replacing true “expected returns” with risk-free return. Yet, this cannot always be done. For some exercises, future cash flows implied by stocks need to be projected using real-world probabilities.

This chapter also introduces financial engineering applications that relate to asset-backed securities (ABS) and securitization. It turns out that securitization and hybrid asset creation are similar procedures with different objectives. From the *issuer's* point of view, one is a solution to balance-sheet problems and it helps to reduce funding costs. From an *investor's* point of view, securitization gives access to payoffs the investor had no access to before and provides opportunities for better diversification. Hybrid assets, on the other hand, can be regarded as complex, ready-made portfolios.

A financial engineer needs to know how to construct an ABS. In fact, engineering is implicit in this asset class. The remaining tasks of pricing and risk managing are straightforward. A similar statement can be made about hybrid assets. We begin the chapter by reviewing the basics of equity instruments and by adapting the tools we have seen thus far to this sector.

2. What Is Equity?

Bonds are contracts that promise the delivery of known cash flows, at known dates. Sometimes these cash flows are floating, but the dates are almost always known, and with floating-rate instruments, pricing and risk management is less of an issue. Finally, the owner of a bond is a *lender* to the institution that issues the bond. This means a certain set of covenants would exist.

Stocks, on the other hand, entitle the holder to some *ownership* of the company that issues the instrument.³ Thus, the position of the equity holder is similar to that of a partner of the company, benefiting directly from increasing profits and getting hurt by losses. In principle, the corporation is managed by the people selected by stock holders. The equity should then be regarded as a tradeable security where the underlying cash flows are future earnings of the corporation.

2.1. A Comparison of Approaches

The best way to begin discussing the engineering of equity-based instruments is to review the valuation problem of a simple, fixed-income instrument and *simultaneously* try to duplicate the same steps for equity. The resulting comparison clarifies the differences and indicates how new methods can be put together for use in the equity sector.

Consider first the cash flows and the parameters associated with a three-period coupon bond $P(t_0, T)$, shown in Figure 22-1. The bond is to be sold at time t_0 and pays coupon c three times during $\{t_1, t_2, t_3\}$. The date t_3 is also the maturity date denoted by T . The par value of the bond is \$100, and there is no default risk.

² For example, what is the value of the earnings of a company? Analyses that depart from the same generally accepted accounting principles often disagree on the exact number.

³ Not all stocks are like this. There is Euro-equity, where the asset belongs to the bearer of the security and is not registered. In this case, the owner is anonymous, and, hence, it is difficult to speak of an owner. Yet, the owner still has access to the cash flows earned by the company, although he or she has no voting rights and, hence, cannot influence how the company should be run. This justifies the claim that the Euro-stock owner is not a “real” owner of the company.

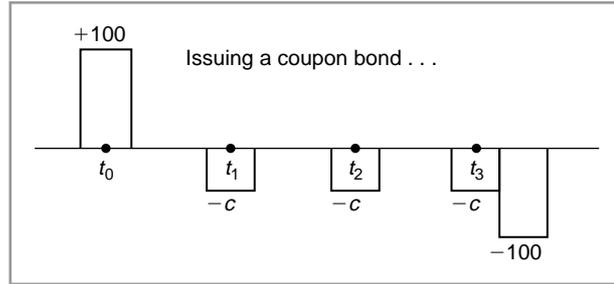


FIGURE 22-1

Next, consider the stock of a publicly traded company denoted by S_t . Let Z_t be a process that represents the relevant *index* for the market where S_t trades. The corporation has future earnings *per share* denoted by e_t . We will now try to synthetically recreate these two instruments, one fixed income, the other equity.

Suppose there are $P(t_0, T)$ dollars to invest. Consider first a savings deposit. Investing this sum in the short-term spot-rate, L_{t_i} , instead of the coupon bond, will yield the sum:

$$P(t_0, T)(1 + \delta L_{t_0})(1 + \delta L_{t_1})(1 + \delta L_{t_2}) \quad (1)$$

in three periods, at time t_3 . Here, δ is the usual adjustment for the day count and $\{L_{t_0}, L_{t_1}, L_{t_2}\}$ are the short-term rates that will be observed at times t_0, t_1 , and t_2 , respectively.

A second possibility is the purchase of the default-free bond $P(t_0, T)$. This will result in the receipt of three coupon payments and the payment of the principal. Finally, we can buy k units of the stock S_t .

The simplest approach to price or risk manage the bond portfolio would be to proceed along a line such as the following. The coupon bond that pays c three times is equivalent to a properly chosen portfolio of zero-coupon bonds:

$$\text{Portfolio} = \{c \text{ units } B(t_0, t_1), c \text{ units } B(t_0, t_2), (c + 100) \text{ units } B(t_0, t_3)\} \quad (2)$$

where $B(t_0, t_i)$ are default-free, zero-coupon bonds that mature at dates t_i . Clearly, this portfolio results in the same cash flow as the original coupon bond $P(t_0, T)$. Given that the two investments are assumed to have no credit risk or any other cash flows, their value must be the same:

$$P(t_0, T) = cB(t_0, t_1) + cB(t_0, t_2) + (c + 100)B(t_0, t_3) \quad (3)$$

But, we know that the arbitrage-free prices of the zero-coupon bonds are given by

$$B(t_0, t_1) = \frac{1.00}{(1 + \delta L_{t_0})} \quad (4)$$

$$B(t_0, t_2) = E_{t_0}^{\tilde{P}} \frac{1.00}{(1 + \delta L_{t_0})(1 + \delta L_{t_1})} \quad (5)$$

$$B(t_0, t_3) = E_{t_0}^{\tilde{P}} \frac{1.00}{(1 + \delta L_{t_0})(1 + \delta L_{t_1})(1 + \delta L_{t_2})} \quad (6)$$

We obtain the valuation equation that uses risk-neutral probability \tilde{P} , with random L_{t_1} and L_{t_2} at time t_0 :

$$P(t_0, T) = E_{t_0}^{\tilde{P}} \left[\frac{c}{(1 + \delta L_{t_0})} + \frac{c}{(1 + \delta L_{t_0})(1 + \delta L_{t_1})} + \frac{c + 100}{(1 + \delta L_{t_0})(1 + \delta L_{t_1})(1 + \delta L_{t_2})} \right] \quad (7)$$

Here, L_{t_1} and L_{t_2} are random variables distributed with probability \tilde{P} .

We are not yet done with this equation since it involves an expectation operator and is therefore only a representation and not an operational formula. But, we should stop here and consider how the derivation up to this point would be different in the case of equity.

2.2. The Case of Stocks

In the following, we try to apply the same methodology to price a stock. We assume the following:

- The stock does not pay dividends.
- There are no other corporate actions such as stock splits, capital injections, or secondary issues.
- There exists a market stock *index* calculated using all the traded stocks in this market.

We can buy one unit of S_t to get the title for future earnings $\{e_{t_i}\}$. Following the same steps, we need to do two things. We find a synthetic for the stock using other liquid and possibly elementary securities, and then equate their price. Suppose we put together the following portfolio:

$$\{e_{t_1} \text{ units of } B(t_0, t_1), e_{t_2} \text{ units of } B(t_0, t_2), e_{t_3} \text{ units of } B(t_0, t_3), \dots\} \quad (8)$$

and then we proceed similarly to pricing the bond. There are at least *two* potential problems with this method. First, the dollars that the company promises to pay through future earnings e_{t_i} , and the dollars promised by the maturing zero-coupon bonds $B(t_0, t_i)$, may not have the same *credit-risk* characteristics. Therefore $B(t_0, t_i)$ may not be an appropriate present value for e_{t_i} . Of course, assuming (unrealistically) that there is no credit risk eliminates this problem. But a second problem remains. Unlike a coupon bond where the coupon payments c were constant and gave *constant* weights in the replicating portfolio, the future earnings e_{t_i} are random. So the weights of the portfolio in equation (8) are *not* known and, thus, the portfolio itself *cannot* be a replicating portfolio. This means that in the case of equity the pricing logic is not the same.

One way to look at it is to ask the following question: Can we *modify* the approach used for the fixed instrument a little and employ a method that is similar? In fact, by imposing some further (restrictive) assumptions, we can get a meaningful answer. The one-factor version of this approach is equivalent to the application of the so-called *CAPM theory*.

This book is not the place to discuss the *capital asset pricing model* (CAPM), but a fairly simple description illustrating the parallels of derivatives and fixed income will still be given. The idea goes as follows. Suppose Z_t is a correct stock index for the market where S_t trades. Assume that we have the following (discretized) risk-neutral dynamics for the pair S_t, Z_t :

$$\Delta S_t = rS_t\Delta + \sigma_s S_t \Delta W_{st} + \sigma_m S_t \Delta W_{mt} \quad (9)$$

$$\Delta Z_t = rZ_t\Delta + \sigma Z_t \Delta W_{mt} \quad (10)$$

where $\Delta Z_t, \Delta S_t$ are increments in the Z_t, S_t variables and r is the constant risk-free rate. ΔW_{st} and ΔW_{mt} are two independent increments. We assume that ΔW_{st} is a risk that is *diversifiable* and specific to the single stock S_t only. The market index is affected only by ΔW_{mt} . This represents a risk that is nondiversifiable. It has to be borne by stockholders. Thus, this is a model with *two* factors, but one of the factors is not a true risk, although it is a true source of fluctuation in the stock S_t .

To obtain a formula similar to the bond pricing representation, we postulate that expected future earnings properly discounted should equal the current price S_t . We then use the *real-world probability* P and the real-world discount rate d_t that apply to the dollars earned by this company to write an equation that is similar to the representation for the coupon bond price:⁴

$$S_t = E_t^P \left[\sum_{i=1}^{\infty} \frac{e_{t+i}}{\prod_{j=1}^i (1 + d_{t+j})} \right] \quad (11)$$

It is worth re-emphasizing that, in this expression, we are using the real-world probability. Thus, the relevant discount rate will differ from the risk-free rate:

$$d_t \neq r \quad (12)$$

We need to discuss how such a d_t can be obtained.

To do this, we need to use the following economic equilibrium condition: If a risk is diversifiable, then in equilibrium it has a zero price. The market does not have to compensate an investor who holds a diversifiable risk by offering a positive risk premium as we will see in the section that follows.

2.2.1. Beta

The only source of risk that the investor needs to be compensated for is W_{mt} . But if this is the case, and if W_{st} risk can be considered as having zero price, then we *can* use Z_t as a hedge to eliminate the movements in S_t caused by W_{mt} only. There are two ways we can look at this.

The first would be to use the equation for Z_t to get

$$\Delta W_{mt} = \frac{\Delta Z_t - rZ_t\Delta}{\sigma Z_t} \quad (13)$$

and then substitute the right-hand side in

$$\Delta S_t = rS_t\Delta + \sigma_s S_t \Delta W_{st} + \sigma_m S_t \left(\frac{\Delta Z_t - rZ_t\Delta}{\sigma Z_t} \right) \quad (14)$$

Divide by $\sigma_m S_t$ and rearrange:

$$\frac{\Delta S_t - rS_t\Delta}{\sigma_m S_t} = \frac{\sigma_s}{\sigma_m} \Delta W_{st} + \left(\frac{\Delta Z_t - rZ_t\Delta}{\sigma Z_t} \right) \quad (15)$$

Since the first term on the right is diversifiable by taking expectations with respect to the real-world probability, we can write this using the corresponding expected (annual) returns, R_t^s , and R_t^m

$$\frac{R_t^s \Delta - r\Delta}{\sigma_m} = \frac{R_t^m \Delta - r\Delta}{\sigma} \quad (16)$$

⁴ In the following formula, t is in years.

Now, from a pricing perspective, the market price of a diversifiable risk is zero. This implies that there is a single factor that matters. Accordingly, we posit the following relationship involving σ_m :

$$\sigma_m = \beta\sigma \quad (17)$$

Then, we can substitute this in equation (16) to obtain a formula that gives a discount factor for the equity earnings⁵

$$R_t^s = r + \beta(R_t^m - r) \quad (18)$$

If we are given the right-hand side values, we can calculate the R_t^s and use it as a discount factor in

$$S_t = E_t^P \left[\sum_{i=1}^{\infty} \frac{e_{t+i}}{\prod_{j=1}^i (1 + d_{t+j})} \right] \quad (19)$$

Again, this is a representation only and not a usable formula yet. Next, we show how to get usable formulas for the two cases.

2.3. Analytical Formulas

How do we get operational formulas from the representations in equations (7) and (19), respectively? For fixed income, the answer is relatively easy, but for equity, further work is needed.

To convert the bond representation into an operational formula, we can use two liquid FRA contracts as shown in Figure 22-2. These contracts show that market participants are willing to pay the known cash flow $F(t_0, t_1)$ against the unknown (at time t_0) cash flow L_{t_1} and that they are willing to pay the known $F(t_0, t_2)$ against the random L_{t_2} . Thus, any risk premia or other calculations concerning the random payments L_{t_1} and L_{t_2} are already included in $F(t_0, t_1)$ and $F(t_0, t_2)$. This means that, at time t_0 , the unknowns L_{t_1} and L_{t_2} can be “replaced” by $F(t_0, t_1)$ and $F(t_0, t_2)$, since the latter are equivalent in value as shown by the FRA contracts.

This implies that, in the formula

$$P(t_0, T) = E_t^{\bar{P}} \left[\frac{c}{(1 + \delta L_{t_0})} + \frac{c}{(1 + \delta L_{t_0})(1 + \delta L_{t_1})} + \frac{c + 100}{(1 + \delta L_{t_0})(1 + \delta L_{t_1})(1 + \delta L_{t_2})} \right] \quad (20)$$

a no-arbitrage condition will permit us to “replace” the random L_{t_1} and L_{t_2} by the known $F(t_0, t_1)$ and $F(t_0, t_2)$. We then have

$$P(t_0, T) = \frac{c}{(1 + \delta L_{t_0})} + \frac{c}{(1 + \delta L_{t_0})(1 + \delta F(t_0, t_1))} + \frac{c + 100}{(1 + \delta L_{t_0})(1 + \delta F(t_0, t_1))(1 + \delta F(t_0, t_2))} \quad (21)$$

This is the bond-pricing equation obtained through the risk-neutral pricing approach. Note that to use this formula, all we need is to get the latest spot and forward Libor rates L_{t_0} , $F(t_0, t_1)$, $F(t_0, t_2)$ from the markets and then substitute.

⁵ The β in CAPM equals the covariance between stock and index returns, divided by the variance of the index return.

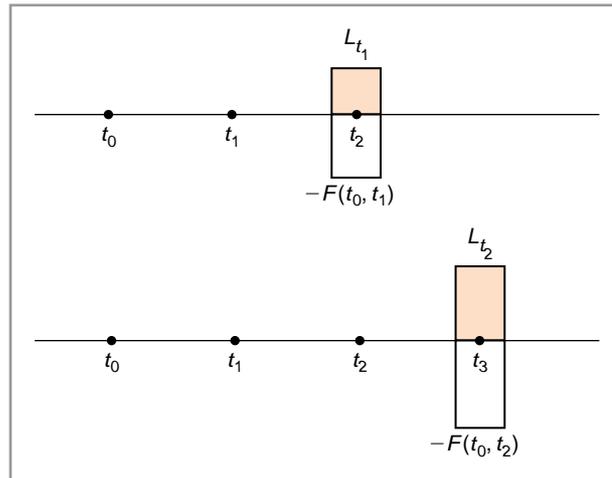


FIGURE 22-2

Obtaining an analytical formula in the case of equity is not as easy and requires further assumptions beyond the ones already made. Thus, starting with the original representation:

$$S_t = E_t^P \left[\sum_{i=1}^{\infty} \frac{e_{t+i}}{\prod_{j=1}^i [1 + (r + \beta(R_{t+j}^m - r))]} \right] \quad (22)$$

To convert this into a usable formula, the following set of assumptions is needed.

There are an infinite number of e_{t+i} in the numerator. First, we need to truncate this at some large but finite number n . Then, assume that the company earnings will grow at an estimated future rate of g , so that we can write for all i ,

$$e_{t+i} = e_t(1 + g)^i \quad (23)$$

Finally, using some econometric or judgmental method, we need to estimate the earnings per share, e_t . After estimating e_t , β , R_{t+i}^m , we can let

$$S_t = \sum_{i=1}^n \frac{e_t(1 + g)^i}{\prod_{j=1}^i [1 + (r + \beta(R_{t+j}^m - r))]} \quad (24)$$

This equation can be used to value S_t . It turns out that most equity analysts use some version of this logic to value stocks. The number of underlying assumptions is more than those of fixed income, and they are stronger.

2.3.1. Summary

The valuation of the fixed-income instrument is simple for the following reasons:

1. Given that the coupon rate c is known, we can easily find a replicating portfolio using appropriate zero-coupon bonds where the weights depend on the coupon.
2. The maturity of the bond is known and is finite so that we have a known, finite number of instruments with which to replicate the bond.

3. The existence of FRA contracts permits “replacing” the unknown random variables with market-equivalent dollar quantities that are known and exact.

The valuation of equity requires further restrictions.

1. A model for the market return or something similar needs to be adopted. This is the modeling component.
2. The number of factors needs to be specified explicitly in this model.
3. Economic equilibrium needs to be invoked to claim that diversifiable risks won’t be rewarded by the markets, and that the only volatility that “matters” is the volatility of nondiversifiable risks.

After this brief conceptual review, we can now consider some examples of equity products.

3. Engineering Equity Products

The second purpose of this chapter is to discuss the engineering of some popular equity instruments.

A large class of synthetic securities has been created using equity products, and the popularity of such instruments keeps increasing. This is not the place to discuss the details of these large asset classes; yet, they provide convenient examples of how financial engineering can be used to meet various objectives and to structure *hybrid equity products*. The discussion here is not comprehensive, but at the end of the chapter, we provide some additional references.

The plan of this section is as follows. We begin by considering the earliest and best-known equity-linked instruments. Namely, we discuss *convertible bonds* and their relative, warrant-linked bonds. These instruments are well studied and a brief discussion should clarify almost all the financial engineering issues in the equity-linked sector.

Then we move to *index-linked products*, which are a more recent variant. Here, even though the general structures are not much different, the synthetics are constructed for different purposes, using equity indices instead of individual stocks which is the case in convertibles and warrant-linked securities.

The third group is composed of the more recent *hybrid securities* that have a wider area of application.

3.1. Purpose

Companies raise capital by issuing debt or equity.⁶ Suppose a corporation or a bank decides to raise funds by issuing *equity*. Are there more advantageous ways of doing this? It turns out that the company can directly sell equity and raise funds. But the company may have specific needs. Financial engineering offers several alternatives.

1. Some strategies may decrease the cost of equity financing.
2. Other strategies may result in modifying the composition of the balance sheet.
3. There are steps directed toward better timing for issuing securities depending on the direction of interest rates, stock markets, and currencies.
4. Finally, there are strategies directed toward broadening the investor base.

⁶ There is also what is called the “mezzanine finance,” which comes close to a combination of these two.

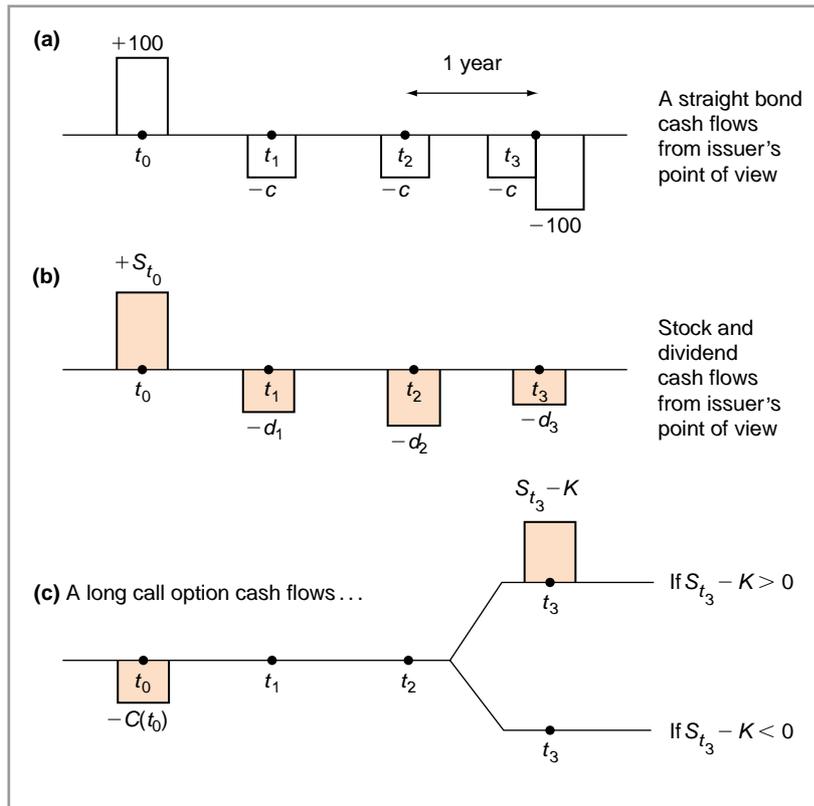


FIGURE 22-3

In discussing these strategies, we consider three basic instruments that the reader is already familiar with. First, we need a *straight coupon bond* issued by the corporation. The cash flows from this instrument are shown in Figure 22-3a.⁷ The bond is assumed to have zero probability of default so that the cash flows are known exactly. The coupon is fixed at c , and the bond is sold at par, so that the initial price is \$100.

The second instrument is a dividend-paying *stock*. The initial price is S_t and the dividends are random. The company never goes bankrupt. The cash flows are shown in Figure 22-3b. The third instrument is an *option* written on the stock. The (call) option on the stock is of European style, has expiration date T and strike price K . The call is sold at a premium $C(t_0)$. Its payoff at time T is

$$C(T) = \max[S_T - K, 0] \quad (25)$$

These sets of instruments can be complemented by two additional products. In some equity-linked products, we may want to use a call option on the bond as well. The option will be European. In other special cases, we may want to add a credit default swap to the analysis. Many useful synthetics can be created from these building blocks. We start with the engineering of a *convertible bond* in a simplified setting.

⁷ We assume a maturity of three years for simplicity.

3.2. Convertibles

A *convertible* is a bond that incorporates an option to convert the principal into stocks. The principal can be converted to a predetermined number of stocks of the issuing company. Otherwise, the par value is received. It is clear that the convertible bond is a *hybrid product* that gives the bond holder exposure to the company stock in case the underlying equity appreciates significantly. We discuss the engineering of such a convertible bond under simplified assumptions. In the first case we discuss a bond that has no default risk. This is illustrative, but unrealistic. All corporate bonds have some default risk. Sometimes this risk is significant. Hence, we redo the engineering, after adding a *default risk* in the second example.

3.2.1. Case 1: Convertible with No Default Risk

Suppose a *default-free* bond pays \$100 at maturity, and consider the following portfolio:

$$\text{Portfolio} = \{1 \text{ Bond, long } n \text{ call options on the stock with } nK = 100\} \quad (26)$$

where K is the call strike.

This portfolio of a bond and n call options is shown in Figure 22-4. Consider the top part of the figure. Here, the holder of the portfolio is paying for the bond and receiving three coupon payments. At $T = t_3$, the bond holder also receives the principal. This is the cash flow of a typical default-free coupon bond.

The second cash flow shows what happens if the option ends up in-the-money. n such options are bought, so, initially, the portfolio holder pays $nC(t_0)$ dollars for the options. Given that these options are European, there is no other cash flow until expiration. At expiration, if the option is in-the-money, the bond will convert and the payoff will be

$$n[S_T - K] \quad (27)$$

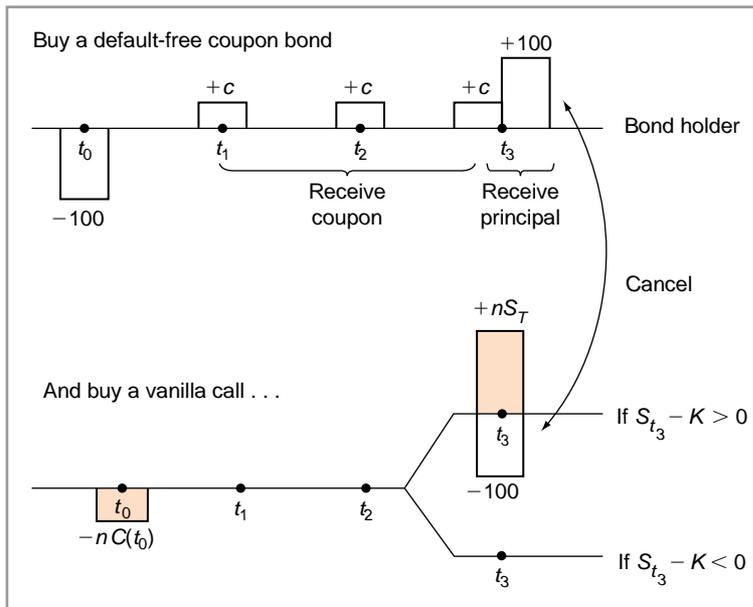


FIGURE 22-4

This can be regarded as an exchange of n stocks, each valued at S_T , against the cash amount nK . But n is selected such that $nK = 100$. Thus, it is as if the portfolio holder is receiving n shares valued at S_T each and paying \$100 for them. This is exactly what a plain vanilla call option will do when it is in-the-money. But, in this case, there is the additional convenience of \$100 being received from the payment of the principal on the bond.

Putting these two cash flows together, we see that the portfolio holder will pay $100 + nC(t_0)$, receive c dollars at every coupon payment date until maturity, and then end up with n shares valued at S_T each, if the option expires in-the-money. Otherwise, the bond holder ends up with the principal of \$100. When options expire out-of-the-money, there will be no additional cash flows originating from option expiration. This case is equivalent to a purchase of a coupon bond. The coupon c is paid by a bond that initially sold at $100 + nC(t_0)$. Because this is *above* the par value 100 on issue date, the *yield to maturity* of this bond will be less than c . This can be seen by using the internal rate of return representation for the *par yield* y :

$$100 + nC(t_0) = \frac{c}{(1+y)} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \frac{100}{(1+y)^3} \quad (28)$$

We need to have $y < c$ as long as $nC(t_0) > 0$.

This discussion shows what a convertible bond is and suggests a way to price it if there is no default risk. A convertible bond is a bond purchased at an “expensive” price if $S_T < K$ —that is to say, if the stock price fails to increase beyond the strike level K . In this case, we say that the bond *fails to convert*. But, if at expiration $S_T > K$, the bond will give its holder n shares valued at S_T with a total value greater than \$100, the principal that a typical bond pays. The bond converts to n shares with a higher value than the principal. In order to price the convertible bonds in this simplistic case, we first price the components separately and then add the values.

3.2.2. Case 2: Adding Default Risk

The decomposition of the convertible bond discussed above is incomplete in one major respect. To simplify the discussion in the previous section, we assumed that the convertible bond is issued by a corporation with no default-risk. This is clearly unrealistic since all corporate bonds have some associated credit risk.

Before we deal with the analytics of this issue, it may be worthwhile to look at market practice. Consider the following reading:

EXAMPLE:

Convertible arb hedge funds in the U.S. are piling into the credit default swaps market. The step-up in demand is in response to the rise in investment-grade convertible bond issuance over the last month, coupled with illiquidity in the U.S. asset swaps market and the increasing credit sensitivity of convertible players’ portfolios, said market officials in New York and Connecticut.

Arb hedge funds are using credit default swaps to strip out the credit risk from convertible bonds, leaving them with only the implicit equity derivative and interest-rate risk. The latter is often hedged through futures or treasuries. Depending on the price of the investment-grade convertible bond, this strategy is often cheaper than buying equity derivatives options outright, said [a trader].

Asset swapping, which involves stripping out the equity derivative from the convertible, is the optimal hedge for these funds, said the [trader] as it allows them to finance the position cheaply, and removes interest-rate risk and credit risk in one fell swoop. But with issuer-credit quality in the U.S. over the last 12 to 18 months declining, finding counterparties

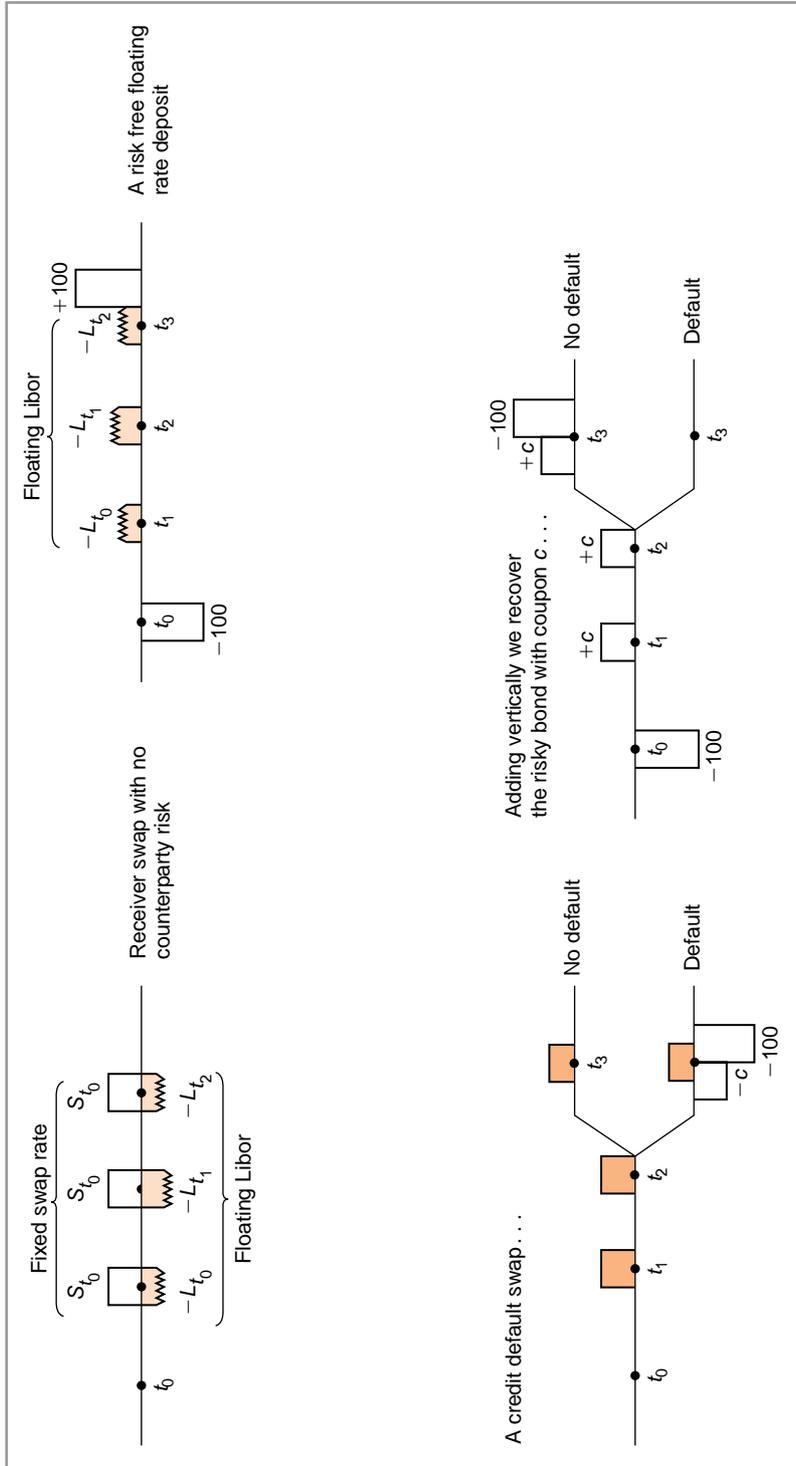


FIGURE 22-5

3.3. Important Variations

This section considers two variations of this basic convertible structure. First of all, the basic convertible can be modified in a way that will make the buyer operate in two *different currencies*. In fact, a dollar-denominated bond may be sold, but the underlying shares may be, say, French shares, denominated in Euros. This amounts, as we will see, to adding a call or put option on a *foreign currency*.

The second alteration is also important. The basic convertible can be made *callable*. This amounts to making the underlying debt issue a callable bond. It leads to adding a call option on the bond. Before we see how these are used, we consider some of the financial engineering issues in each case.

3.3.1. Exchange Rate Exposure

Suppose the convertible bond is structured in *two* currencies. A Thai company secures funding by selling a euro convertible in the Eurodollar market, and the debt component of the structure is denominated in dollars. So, the bonds have a par value of, say, \$100. The conversion is into the shares of the firm, which trade, say, in Bangkok. The shares are baht denominated. We assume, unrealistically, that there is no default risk.

Because Thai shares trade in Thai exchanges and are quoted in Thai baht, the conversion price to be included in the convertible bond needs to specify *something* about the value of the exchange rate to be used during a potential conversion. Otherwise, the conversion rule will not be complete. That is to say, instead of specifying only the number of shares, n , and the conversion price, K , using the equality

$$100\$ = Kn \quad (30)$$

the conversion condition now needs to be

$$e_t 100\$ = Kn \quad (31)$$

where e_t is an exchange rate denoting the price of USD1 in terms of Thai baht at date t . This is needed since the original conversion price, K , will be in Thai baht, yet, the face value of the bond will be in USD. The bond structure can set a value for e_t and include it as a parameter in the contract. Often, this e_t will be the current exchange rate.

Now, suppose a Thai issuer has sold such a Euro convertible at e_t , the current exchange rate. Then, if Thai stocks rise *and* the exchange rate remains stable, the conversion will occur. Here is the important point. With this structure, at maturity, the Thai firm will meet its obligations by using its own *shares* instead of returning the original \$100 to bond holders. Yet, if, in the meantime, e_t rises,⁸ then, in spite of higher stock prices, the value of the original principal \$100, when measured in Thai baht, may still be higher than the nS_T and the conversion may not occur. As a result, the Thai firm may face a significant dollar cash outflow.⁹

This shows that a convertible bond, issued in major currencies but written on domestic stocks, will carry an FX exposure. This point can be seen more clearly if we reconstruct this type of convertible and create its synthetic. This is done in Figure 22-6.

The top part of Figure 22-6 is similar to Figure 22-4. A straight coupon bond with coupon c matures at time t_3 and pays the principal \$100. The difference is in the second part of the figure.

⁸ That is to say, if the Thai baht is devalued, for example.

⁹ This may be something that is occurring at a bad time, since if the currency is devalued, the international markets may not be receptive to rolling over the dollar debt of Thai corporates.

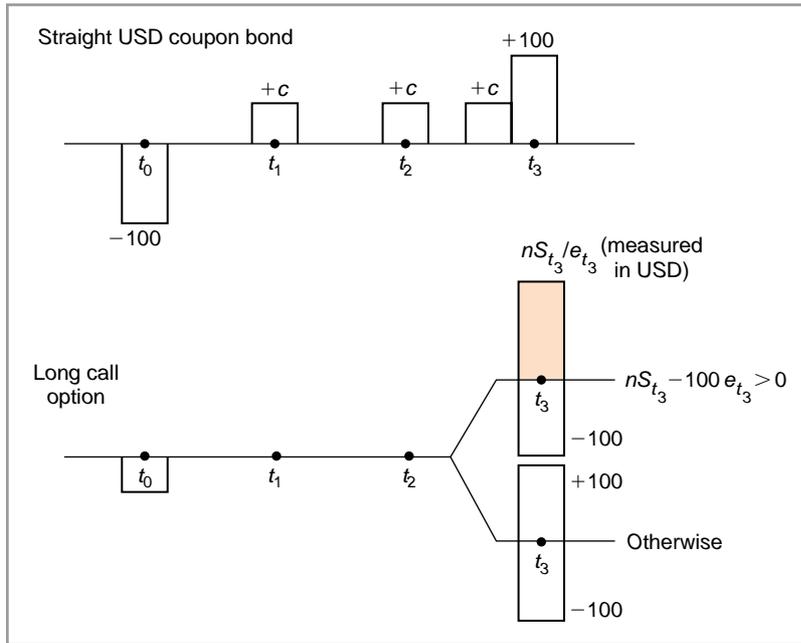


FIGURE 22-6

Here, we have, as usual, the call option on the stock S_t . But S_t is denominated in baht and the call will be in-the-money—that is to say, the conversion will occur only if

$$nS_{t_3} > 100e_{t_3} \tag{32}$$

The idea in Figure 22-6 is the following. We would like to begin with a dollar bond and then convert the new call option into an option as in the case before. But, if the Thai baht collapses,¹⁰ then the \$100 received from the principal at maturity will be much more valuable than $S_{t_3}n/e_{t_0}$.

3.4. Making the Convertibles Callable

One can extend the basic convertible structure in a second way, and add a call option on the underlying convertible bond. For example, if the bond maturity is T , then we can add an implicit option that gives the issuer the right to buy the bond back at time, $U, U < T$ at the price

$$\max[\$100, nS_U] \tag{33}$$

This way the company has the right to *force* the conversion and issue new securities at time U . Some corporations may find this a useful strategy.

With this type of convertible, forcing the conversion is the main purpose. Suppose the following two conditions are satisfied:

1. The share is trading at a higher price than the conversion price (i.e., the strike K).
2. The expected future dividends to be paid on the stock are lower than the current coupon of the convertible.

¹⁰ This means that the Thai stock market is also down.

Then, if the convertible is callable, the issuer may force the conversion by calling the bond. This will convert a debt issue in the issuer's balance sheet into equity and affect some important ratios, in case these are relevant. Second, the immediate cash flow of the firm will improve.

3.5. More Complex Structures

The basic convertible-warrant structures can be modified to meet further financial engineering needs. We can consider another example.

Suppose the convertible bond, when it converts, converts into *another* company's security. This may be the case, for example, if company *A* has acquired an interest in company *B*. This way, the company can sell convertible bonds where the conversion is into company *B*'s securities.

From a financial engineering point of view, the structure of this "exchangeable" is the same. Yet, the pricing and risk management are different because now there are *two* credits that affect the price of the bond: the credit of the company that issues the bond and the credit of the company this bond may convert.

Another difference involves the dilution of the shares of the target company. When a convertible is issued and converts at a later date, there may be dilution of the shares, yet, in an exchangeable the shares that are exchanged will come, in general, from the free float.¹¹

3.6. Using Convertibles

A convertible bond has some attractiveness from the point of view of end investors. For example, the investor who buys the convertible will have some exposure to the share price. If S_t increases significantly, the bond becomes a portfolio of shares. On the other hand, if the bond fails to convert, the investor has at least some minimum cash flow to count on as income, and the principal is recovered (when there is no default).

But, our interest in this book is not with the investors, but rather, in the advantages of the product from an issuer's point of view. For what types of purposes can we use a convertible bond?

- The first consequence of issuing convertibles rather than a straight bond is that the convertible carries a lower coupon. Hence, it "seems" like the funds are secured at lower cost.
- More notably for a financial engineer, convertibles have interesting implications for balance sheet management. If an equity-linked capital is regarded as equity, it may have less effect on ratios such as debt to equity. But, in general, rating agencies would consider straight convertibles as debt rather than equity.
- Note that with a convertible, in case conversion occurs, the shares will be sold at a higher price than the original stock price at issue time.
- Finally, convertibles are bonds, and they can be sold in the Euro markets as Euro-convertibles. This way a new investor base can be reached.

We should also point out that convertibles, when combined with other instruments, may have significant and subtle tax advantages. The best way to show this is by looking at an example from the markets.

¹¹ Lyon's is another recent product. The underlying bond is not a coupon-paying bond, but, instead, a discount bond with *long* maturity.

EXAMPLE:

(ABC Capital) has entered into a total return swap on 154,000 shares of Cox Communications preferred stock exchangeable into shares of Sprint PCS, and a total return swap on 225,000 shares of Sprint PCS. In the Cox swap, the hedge fund pays three-month Libor plus 50 basis points and receives the return on the exchangeable preferred shares. In the Sprint swap, ABC pays the return on the stock and receives three-month Libor less 25bps. Both total return swaps mature in about 13 months.

The total return swaps were entered into for tax reasons. ABC's positions are held by a Cayman Islands limited duration company. Because the Cayman Islands do not have a tax treaty with the U.S., income from these securities is withheld at the non-treaty rate of 30%. Entering the total return swaps ensures that ABC does not physically hold the securities, and, hence, is not subject to U.S. withholding.

The underlying position was put on as part of a convertible arb play. ABC bought the exchangeable preferred stock, and is using the cash equity to delta hedge the implicit equity option. The market is undervaluing the exchangeable preferred shares, according to a trader, who noted that although these shares recently traded at USD76.50, the fund's models indicate they should be priced around USD87. The company's model is based in part on the volatility of the underlying stock, the credit quality of the issuer, and the features of the convertible. In this case, the market may be undervaluing the security because it is not pricing in all the features of the complicated preferreds and because of general malaise in the telecom sector. (Derivatives Week, November 2000)

This reading is also an example of how implicit options can be used to form arbitrage portfolios. However, there are many delicate points of doing this as were shown earlier.

3.7. Warrants

Warrants are *detachable* options linked to bonds. In this sense, they are similar to convertibles. But, from a financial engineering point of view, there are important differences.

1. The warrant is detachable and can be sold separately from the bond. Of course, a financial engineer can always detach the implicit option in a convertible bond as well, but still there are differences. The fact that the warrant is detachable means that the principal will always have to be paid at maturity.

The number of warrants will not necessarily be chosen so as to give an exercise cash inflow that equals the cash outflow due to the payment of the principal. Thus, the investor can, in principle, end up with both the debt and the equity arising from the same issue.
2. The exchange rate used in a convertible is fixed. But, because there is no such requirement for a warrant and because the latter is detachable, this is, in general, not the case for a warrant. Hence, there is no implicit option on the exchange rate in the case of warrants. In this sense warrants are said to be relatively more attractive for strong currency borrowers, whereas convertibles are more attractive for weak currency borrowers.
3. Finally, because warrants are detachable, the warrant cannot be forced to convert. The bond can be called, but the conversion is not required.

We now move to another topic and look at securitizing cash flows. This can be regarded as an example of new product structuring.

4. Financial Engineering of Securitization

Every business or financial institution is associated with a “credit” or, more precisely, a credit rating. If this entity issues a debt instrument to secure funding, then the resulting bonds, in general, have the same credit rating as the company. Yet, a company can also be interpreted as the receiver of future cash flows with different credit ratings. The value and the credit rating of the company will be a function of these future cash flows. Not all the receivables will have the same rating. For example, some cash flows may be owed by institutions with a dubious credit record, and these cash flows may not be received in the case of default or delinquency. Other cash flows may be liabilities of highly reputable companies, may carry a low probability of default, and may indeed be received with very high probability.

A debt issue will be backed by an average of these credits, since it is the average receivable cash flows that determine the probability that the bonds will be repaid at maturity. If the receivables of a company carry mostly a relatively high probability of default, then the company may experience difficulties in the future and, hence, may end up defaulting on the loan. Alternatively, the credit spread on the bond will increase and the investors will be subject to mark-to-market losses. All these possibilities reflect on the debt issued by this company, and are factors in the determination of the proper cost of funding.

On the other hand, instead of issuing debt on the back of the average cash flows to be received in the future, the company can issue special types of bonds that are backed only by the *higher-rated* portion of the receivables. Clearly, such receivables have a comparatively lower probability of default, and this makes the bonds carry a lower default probability. The funding cost will decrease significantly. The company has thus securitized a certain portion of the cash flows that are to be received in the future. In other words, securitization can be regarded as a way to issue debt and raise funds that have a higher rating than that of the company. It is also a way of repackaging various cash flows.

What are the critical aspects of such financial engineering? Essentially, various cash flows are to be analyzed and a proper selection is made so as to obtain an optimal basket. This is then sold to investors through special types of bonds.

Yet, besides the financial engineering aspects, securitization involves (1) legal issues, (2) balance sheet considerations, and (3) tax considerations. Securitization is a way of funding an operation. Instead of selling bonds or securing bank lines, the company issues asset-backed securities. The option of securitization helps corporations and banks make decisions among the various funding alternatives.

4.1. Choosing Cash Flows

Consider Figure 22-7. Here, we show a bank that expects three different (random) cash flows in the future. The institutions that are supposed to pay these cash flows have different credit ratings. For example, the first series of cash flows, rated BBB, may represent credit card payments. The third could represent the random cash flows due to mortgages made in the past. Arguably, people pay better attention to the timely payment of mortgages than they do to the timely payments of unsecured credit card proceeds. Credit card defaults are much more common and plausible than mortgage delinquencies. Thus, the mortgage cash flows will be rated higher, say, with an A rating as shown in the figure.

Now, if the company is set to receive these three cash flows only, assuming similar liabilities, the company’s average rating will perhaps be around BBB+. A corporate bond issued by the company will carry a BBB+ credit spread.

Consider two different ways of packaging the same cash flows. If the company “sells” cash flow 3 and backs a bond issue with this cash flow only, the probability of default will be much

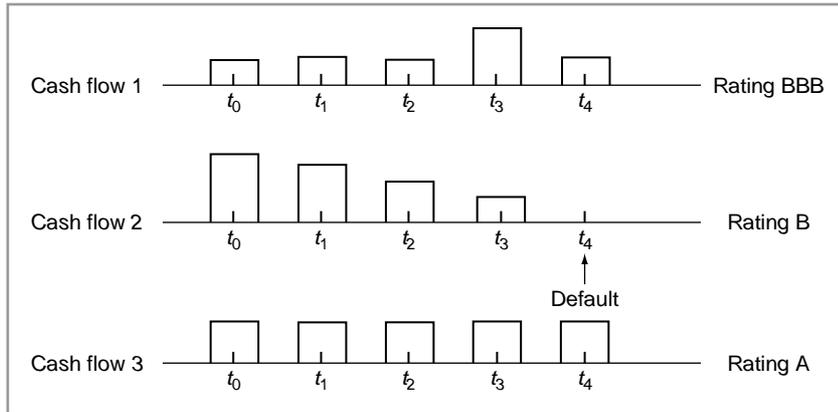


FIGURE 22-7

lower and funding can be secured at a lower rate. A bond backed by cash flows 1 and 3 will have a lower credit rating but still yield a funding cost below that of a general bond issue. The funding cost would be a little higher, but at the same time, more can be borrowed because there is a bigger pool of receivables in this second option.

4.2. The Critical Step: Securing the Cash Flow

The idea of securitization is quite simple. Instead of borrowing against the average quality of the company's receivables, which is what happens if the company sells a straight corporate bond, the entity decides to borrow against a higher quality *subset* of the receivables. In case of default, these receivables have a higher chance of being collected (recovered) and, hence, the cost of these funds will be lower.

But there is a critical step. How can the buyer of an asset-backed security make sure that the receivables that are supposed to back the security are not used by the company for other purposes, and that, in the case of bankruptcy, these receivables will be there to cover losses?

The question is relevant, since after issuing the ABS security, it is still the original company that handles the business of processing new receivables (e.g., by issuing new mortgages), as well as the receipts of cash generated by such cash flows, and then uses them in the daily business of the firm. Clearly, there must be an additional mechanism that guarantees, at least partially, that these cash flows will be there in case of default.

A *bankruptcy remote SPV* is one such mechanism used quite often to resolve such problems in practice. The idea is as follows. (1) The issuing company forms a special-purpose vehicle (SPV), which is a shell company, often independent of the parent company, and whose sole purpose is to act as a *vehicle* in structuring the ABS. (2) Steps are taken to make the SPV *bankruptcy remote*. That is to say, the probability that the SPV itself defaults is zero (since it does not engage in any meaningful economic activity other than that of issuing the paper), and in case the original company goes bankrupt, the underlying cash flows remain in the hands of the SPV. (3) The issuing company draws all the necessary papers so that, at least from a legal point of view, the cash flows are sold to the SPV. This is a *true sale at law*.¹²

¹² This can be confirmed by a letter written by a well-known and respected counsel.

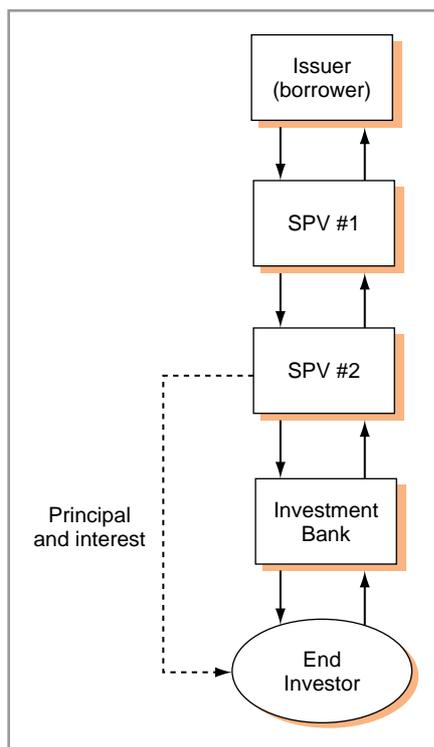


FIGURE 22-8

The idea is to transfer the right to these cash flows and guarantee them under the ABS as much as possible.¹³ In fact, several SPVs with different purposes can be layered to make sure that the ABS has the desired characteristics.

1. An SPV may be needed for tax reasons.
2. Another SPV may be needed for balance sheet reasons.
3. Still another SPV may be needed to comply with other regulations.

Hence, one possible structure can be layered as shown in Figure 22-8. Note that here, the first SPV is a subsidiary of the company, so the company can “buy” the cash flows, and this is the reason for its existence. But if the SPVs keep the cash flows, these will still be on the balance sheet of the original company.

Finally, note the role played by the investment bank. The first three layers make up the ABS structure, and the investment bank still has to handle the original sale of the ABS. The structure clearly shows that the ABS has three important purposes, namely lowering the funding cost, managing the balance sheet, and handling tax and accounting restrictions.

4.3. Some Comparisons

The first use of securitization concerns *funding costs*, as already discussed. Securitization is a form of funding. But we must add that it is also an unconstrained form of funding, and an

¹³ However, the existence of such bankruptcy-remote SPVs is not sufficient to fully guarantee that the cash flows will be there to use. A bankruptcy is sometimes a chaotic event, involving difficult legal issues.

off-balance-sheet form of liquidity for small and medium companies. Finally, it is a diversified funding source. This way it can lower leverage. Securitization also implies less public disclosure.

Securitization is neither secured corporate financing, nor the sale of an asset. It is a hybrid, a combination of both that uses the well-accepted legal, regulatory, tax, and accounting concepts that already exist.

4.3.1. Loan-Sales

We should also compare whole-loan sales versus securitization. Securitization is on a service-retained basis, whereas a loan sale is service released. The buyer of the loan would like to service the loan himself or herself. Another point is the retention of credit and prepayment risk. In a loan sale, 100% of these risks are transferred. With securitization, some of these risks may be retained. A loan sale is also often done at a premium, whereas securitization issues are often around par.

In addition, there is a timing issue. In securitization, cash flows from assets are often invested in short-term investments and then transferred to the bond holders. Thus, the investor receives the payments later than the *servicer*. Finally, securitization sometimes uses credit enhancements, and this makes the paper somewhat more liquid.

4.3.2. Secured Lending

Securitization is similar to secured financing, with one important difference. In an ABS, the issuing company is not liable for its asset-backed securities. It is as if the company has not really “borrowed” the funds. A separate legal entity needs to be established to do the borrowing. Securitization is structured so that this entity becomes the legal owner of the asset. If the company defaults, the cash flows will not belong to the company, but to the SPV. This way the owners of the bonds have an *ownership* interest in the case of securitization, whereas in the case of secured lending, they only have a *security* interest.

5. Conclusions

Financial engineering of equity instruments follows the same principles introduced in earlier chapters. However, valuation may require more restrictive assumptions, as illustrated by this CAPM. This chapter summarized some basic equity and securitization products.

Suggested Reading

This chapter has provided a simple introduction to hybrid equity products. The reader may want to follow up on the discussion in two ways. First, more details about such products may be needed. Several books deal with the current state of hybrid equity instruments. Das (2000) is one example. The reader may also want to learn more about the technical issues related to pricing, hedging, and risk-managing hybrid equity products. Kat (2000) deals with some of the related issues. Other examples of securitization can be found in Fabozzi (1998).

Exercises

1. Read the article below carefully and answer the questions that follow.

Reverse Convertibles in Limbo

Issuance of lira-denominated reverse convertibles ground to a halt in response to growing uncertainty over their tax status under Italian law. The Italian authorities are concerned that investors may buy the instruments in the belief that they are capital-protected fixed-income instruments, when, in fact, they would be exposed to equity downside risk.

According to warrant market participants, about a month ago the Bank of Italy warned potential issuers of lira-denominated reverse convertibles that they might be classified as “abnormal securities.” If classified as such, the coupon on the securities would be taxed at 27% instead of 12.5%—the rate for normal fixed income and derivative structures. Since then, lira reverse convertible issuance has dwindled as structurers await a decision on their status.

Market commentators said the Bank of Italy was concerned about the lack of principal protection in the structure. Reverse convertibles generate a yield considerably higher than that of vanilla bonds by embodying a short equity put position.

The investor receives a high coupon and normal bond redemption as long as a specific equity price is above a particular level at maturity. However, if the equity falls below the specified mark, then the investor is forced to receive the physical equity instead of the normal bond principal.

As a result, the buyer of the reverse convertible could end up with a long stock position at a low level, which would mean an erosion of initial principal. In contrast, the buyer of vanilla bond paper is assured of getting back the initial principal investment. (IFR, May 1998)

- (a) Using a cash-flow diagram, show the decomposition of a reverse convertible into a risky bond and an option.
- (b) What type of option do you need here? Does this explain why the yields on reverse convertibles are higher than usual?
- (c) Why is there no principal protection in reverse convertibles?
- (d) Is there principal protection in risky bonds? So why, and in what sense, is a reverse convertible different?

CASE STUDY: Volatility Trading

With Applications to Convertibles and Reverse-Convertibles

This case study shows another example of volatility trading. We try to accomplish two objectives at the same time. One objective is to define, measure, and price volatility. To do this, it is best to start with some specific products. Hence, in this case study we select *convertible* and *reverse-convertible* bonds, an asset class which is important in its own right. By working with these instruments, we intend to meet our second objective, which is to provide examples as to how a financial engineer may use his or her knowledge toward corporate finance.

The following readings should be taken in their given order. They are all from various issues of IFR and provide the latest examples of the use of volatility trading. The more advanced instruments are, however, not given here.

Questions

1. Let's consider briefly some models of volatility. For example, what is the mean-reverting model for volatility? Are there any other models? Discuss briefly.
2. What is a convertible bond? How would you decompose this instrument? How would a corporate treasurer use convertible bonds?
3. Give an example of a convertible bond issued, recently, in Europe. Discuss the main parameters. What is a *conversion premium*? What is the *dilution effect*?
4. What is a reverse-convertible bond? How would you decompose this instrument? How would a corporate treasurer use reverse-convertible bonds?
5. How are the market professionals using reverse-convertibles? Why is there a "flood"?
6. What is a synthetic convertible?
7. What is the other solution mentioned in the text? What are the possible risks behind this solution?
8. Finally, the regulators. Consider the case of French regulators and reverse-convertible bonds. Why would the regulators have an issue with these products? Do you think this is justified? Discuss briefly.

Reading 1

A resurgence in close-out activity has been triggered by a sharp rise in long-dated implied volatility during the first five months of 1998. Having declined significantly throughout November and December last year, three-year index volatility levels have risen steadily since January.

The trend closely mirrors changes in implied volatility experienced last year. Implied volatility levels first took off in 1997, driven by a surge in the market for guaranteed equity funds. Arrangers of the funds bought longer-dated options from banks to hedge the guarantee embedded in retail products. No natural sellers of long-dated volatility were available, and a short squeeze in volatility quickly developed, pushing up rates.

Market professionals asserted last week that the short squeeze was the result of a vicious circle. Banks closing-out their short options positions had pushed volatility even higher, which in turn has prompted other houses to close out their positions. "It's something of a chain reaction. I think most professionals are fairly concerned," said one head of equity derivatives trading in London.

Although in agreement over the severity of the volatility squeeze, market professionals were divided on its cause or solution. Some professionals alleged that bank risk controllers had exacerbated the volatility squeeze by setting too tight risk limits. “This is a typical example of accountants sticking to their guns, whatever happens. It’s a unique situation which will not last and they should take account of that,” said one.

History is bunk?

Professionals also pointed to factors that they felt would eventually alleviate the demand/supply imbalance. Several market participants said they thought high volatility was a temporary phenomenon; they argued that volatility is mean-reverting and that implied volatility rates would soon descend towards (much lower) historical levels.

The discrepancy between the two views of volatility lies at the heart of conflicting views over the market’s development. Implied volatility rates are calculated by feeding current option prices into an option model, and so are a function of the supply and demand in volatility. In contrast, historical volatility rates are calculated from previous equity market movements. Three-year FTSE 100 historical volatility levels are around 11%.

The yawning gap between historical and implied volatility rates has already created trading opportunities for unconventional suppliers of volatility. While dealers remain naturally short volatility, other trading firms have a more flexible approach to volatility rates. Hedge funds had already been seeking to sell long-dated volatility, with the view that implied levels would descend to historical rates.

Equity derivatives professionals asserted that lower participation rates were the direct result of the rise in implied volatility. Guaranteed products are a mixture of a long position in equities — usually achieved through one or more futures contracts — and an option used to provide a floor on possible losses. If the floor is more expensive, the upside offered to the buyer of the guaranteed product will be reduced to lower the overall cost of the product.

According to several market professionals, the new lower participation rates are a function of the volatility squeeze and are, thus here to stay. “I think participation rates will decline and so reduce demand for volatility. It’s simply the market finding an equilibrium,” said one equity derivatives marketer. (IFR, June 1998)

Reading 2

Reverse convertibles activity soared to new heights last week, driven by sustained high stock index volatility levels across Europe and stock market jitters. Warrant professionals highlighted the increasing number of structuring banks involved in the sector and the broadening range of issuance currencies.

These factors were evidence, they said, of the product’s growing acceptance throughout the industry. Stock index volatilities across Europe have consistently been in the high twenties of percentage points since the beginning of the year.

Explaining the sudden surge in demand, equity derivatives professionals said reverse convertibles were an ideal means of taking advantage of current high volatility rates throughout Europe.

Others said interest in the product was spreading across Europe, whereas it had previously been confined to Switzerland and Germany. “There’s been a flood. It’s all over the place,” enthused one German bank derivatives official.

Reverse convertibles are credit products with an embedded put option referenced 1 to a particular quoted company stock. The face value of the bond is discounted to the equivalent

value of the option premium. If the stock price breaches a certain minimum value threshold, the bond investor receives equity instead of a cash payout.

Credit Lyonnais Equity Derivatives was in the thick of the action, structuring three deals. Volkswagen was chosen as the underlying for an issue paying a 10% coupon; Telecom Italia was selected as the reference stock for an issue paying an 11% coupon; and ABN AMRO was the reference for a third.

If the stock price on the final date is greater than or equal to 95% of the initial price, then the investor receives 100% cash redemption. But if the price is below, then the investor receives one physical. (IFR, June 1998)