CHAPTER 18

Credit Indices and Their Tranches

1. Introduction

In this chapter we introduce the modeling of collateralized debt obligations and their tranches. The chapter also deals with credit indices and their implications in creating *tranched* securities. In particular we discuss the *standard* tranches.

Tranching a basket of credit-risky instruments makes the trading and pricing of credit correlation possible. Once we learn how to strip correlation from tranched products we can price it and then trade it. The issues are discussed within credit default context, but the techniques themselves are very general and buying and selling correlation is routinely applied in creating equity and FX-based structured products as well. Yet, in this chapter we focus on *default* correlation and discuss the issue using the credit names included in the *iTraxx* or, alternatively, *CDX* indices. The chapter also discusses more recent credit indices ABX and LCDX.

2. Credit Indices

Markets prefer to trade credit risk through standardized and transparent credit indices. As discussed in the previous chapter, credit indices are liquid benchmarks that are based on portfolios of reference names. The corresponding default risks are then repackaged by slicing the portfolio into various *standard tranches* bearing different risk and return characteristics.[1]

The indices allow investors to take a position on the broad corporate credit market with a single trade and provide liquid hedging instruments for institutions to hedge their complex credit exposures. Within the investment grade (IG) corporate credit names, there exist two major CDS index categories in the world: in the United States they are called the *Dow Jones CDX*; in Europe, Asia, and Australia they are called iTraxx *Europe* or iTraxx *Asia* respectively. During the discussion in this chapter we focus mostly on *iTraxx* indices.

[1] A custom made tranche is known as a *bespoke tranche*. Note that a bank or corporation trying to hedge a basket of loans would in general need or sell bespoke tranches.

The credit indices are constructed by the International Index Company (IIC) after a dealer liquidity poll. Market makers submit a list of names to the IIC based on the following criteria.

1. The entities have to be incorporated in Europe and have to have the highest credit default swap (CDS) trading volume, as measured over the previous 6 months. Traded volumes for entities that fall under the same ticker, but trade separately in the CDS market, are summed to arrive at an overall volume for each ticker. The most liquid entity under the ticker qualifies for index membership.
2. The list of entities in the index is ranked according to trading volumes. IIC removes any entities rated below BBB− (by S&P) and those on negative outlook.
3. The final portfolio is created using 125 names.[2]

The markets prefer to trade mostly the subinvestment grade indices called *iTraxx* Crossover (XO) in Europe and CDX High Yield (HY) in the United States. These indices see a large majority of the trading, they contain fewer names, and the spreads are significantly higher.[3]

3. Introduction to ABS and CDO

This book cannot deal with the technical issues concerning asset backed securities (ABS) and collateralized debt obligations (CDO). Still, credit indices should be put in context so that they can be compared to ABSs and CDOs. This is also a good opportunity to introduce the basic definitions of and the differences between the ABS and the CDO type securities. The credit crisis of the years 2007–2008 is one example of the important role they play in world financial markets.

Imagine three different classes of defaultable securities. The first class is defaultable bonds. Except for U.S. Treasury bonds, all existing bonds are defaultable and fall into this category.[4] The second class can be defined as "loans." There are several types of loans, but for our purposes here, we consider just the secured loans extended to businesses. Finally, there are loans extended to households, the most important of which are mortgages.

ABS securities can be defined by either using other assets such as loans, bonds, or mortgages, or more commonly using the stream of cash flows generated by various assets such as credit cards or student or equity loans. In this section we consider the first kind. Figure 18-1 displays the way we can structure an ABS security. A basket of debt securities is divided into subclasses with different ratings, then the subclasses are placed "behind" different *classes* of the ABS security. This means that any cash flows or the corresponding shortfalls from the original debt instruments would be passed on to the investor who buys that particular class of ABS security. In contrast, if there are defaults, the investors receive an accordingly lower coupon or may even lose their principal. Note that according to Figure 18-1, classes of ABS securities have different ratings because they are backed by *different* debt instruments. Often in an ABS, the credit pool is made of loans such as credit cards, auto loans, home equity loans, and other similar consumer-related borrowing. When the underlying instrument is a mortgage, the ABS is called a mortgage backed

[2] It is assembled according to the following classification: 10 Autos, 30 Consumers, 20 Energy, 20 Industrials, 20 TMT, and 25 Financials. Each name is weighted equally in the overall and subindices.

[3] For more information on the indices the reader should visit www.iboxx.com and www.markit.com. The last site belongs to the company that actually collects and processes the quotes for the indices.

[4] During the 2007–2008 credit crisis, even U.S. Treasuries had a default risk in August–September 2008. Spread on U.S. Treasuries was 15–20 bp.

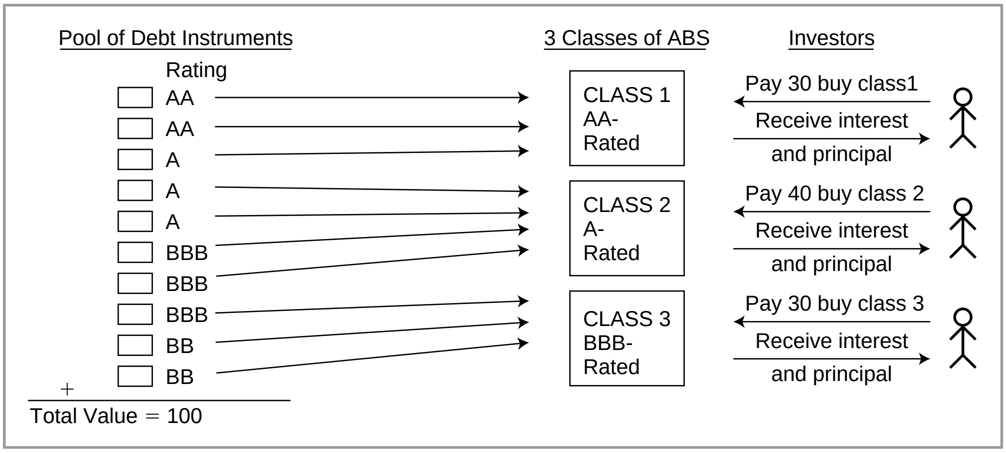

FIGURE 18-1. ABS Structure.

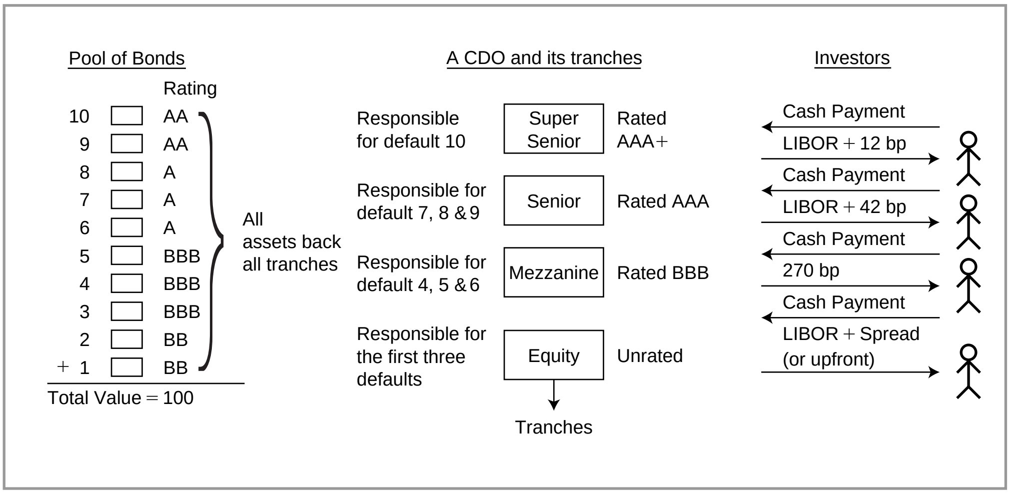

FIGURE 18-2. A CAS# CDO (Note that these are cash bonds backing the CDO).

security (MBS). In each case the rating of the ABS is determined by the rating of the loans that back it.[5]

Figure 18-2 shows a "cash" CDO, also called a *funded* CDO. Again a pool of credit instruments are selected. But they are classified in a very different way. The CDO classes called *tranches* are formed, not by classifying the underlying securities, but the risk in them. In fact, all CDO tranches will be backed by the *same* pool of securities. What distinguishes the tranches is the *subordination* of the default risk. The ABS categorizes the securities *themselves*. A CDO categorizes the *priority* of payments during defaults. The first few defaults will be the first tranche, then if defaults continue the next tranche will suffer and so on.

If the default risk comes from a pool of bonds, the CDO is called a collateralized bond obligation (CBO). If the underlying securities are loans, then it is called a collateralized loan obligation (CLO). The term CDO is more general and the securities it represents may be a mixture

[5] There are other ways one can define the classes of ABS securities.

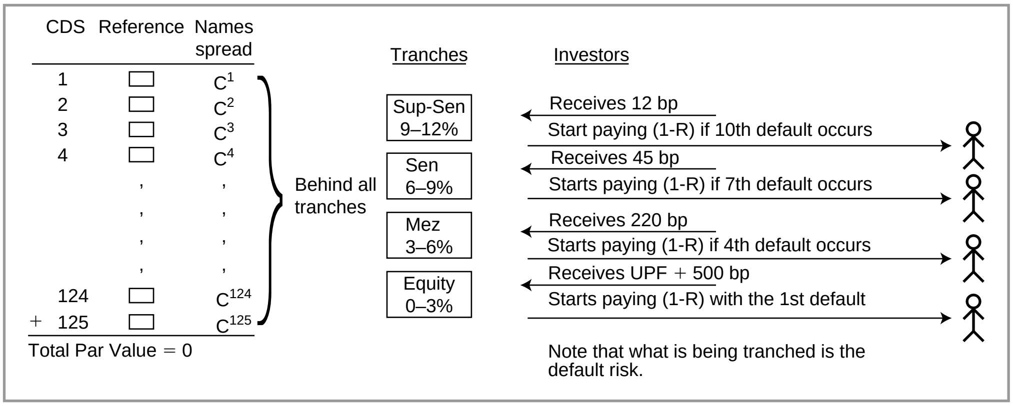

FIGURE 18-3. A credit index and the implied synthetic CDO (Note that there is no initial cash investment).

of all these. In fact, these underlying securities may include MBS or other ABS securities. Even tranches of other CDOs are sometimes included in a CDO.[6]

The investor pays 100 in cash to buy the ABS and the CDO structures shown in Figures 18-1 and 18-2. By buying the ABS or the CDO the investor is, in a sense, buying the underlying securities as a pool. The underlying pool can be arranged so that the investor can choose among classes of ABS securities with different ratings. In the CDO, the *priority* of interest and principal payments determine the rating of the tranche. Thus different classes of ABS securities (at least as defined here) represent different underlying assets grouped according to their credit rating, whereas the tranches of CDOs are actually backed by the same underlying assets, while having different ratings due to how quickly they will be hit during successive defaults.

A credit index and the associated tranches is a synthetic version of a CDO. The underlying assets are related to the bonds of the reference portfolio names but they are *not* purchased! Hence, they are *unfunded*. Thus, with an index and most of the tranches, there is no initial cash payment or receipt involved.[7] While a cash CDO pays Libor plus some spread, the synthetic (unfunded) CDO pays just the spread because it involves no initial cash payments. In this sense, the relationship between an index (unfunded CDO) and a funded CDO is similar to the relationship between cash bonds versus the CDS written on that name.

Figure 18-3 shows how the index can be interpreted as a synthetic unfunded CDO. In this figure the tranches are selected so that they conform to the standard *iTraxx* tranches.

4. A Setup for Credit Indices

Consider in Figure 18-4 a single name CDS. The maturity T is assumed to be *five years*, the notional amount N is 100. The CDS is written on a reference name. The CDS premium is C_{t_0}. The recovery rate is denoted by R.

A credit index is obtained by selecting n such reference entities indexed by $j = 1, \ldots, n$. This pool is called the reference portfolio. The associated CDS rates at time t, denoted by $\{c_t^j\}$ are assumed to be arbitrage-free. A tradeable CDS index is formed by putting these names in a single contract, where if N dollars insurance is sold on the index, then this would correspond to a sale of insurance on *each* name by an amount $\frac{1}{n}N$.

[6] When the reference pool is made of other CDO tranches the CDO is referred to as CDO-squared.

[7] Only the most risky tranche is traded with *upfront*.

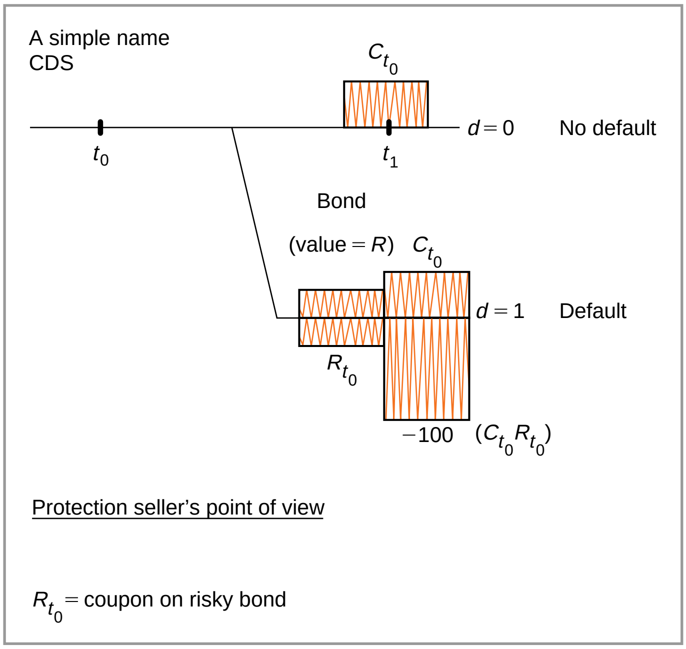

FIGURE 18-4

Let I_t represent the spread on the credit index for the preselected n names. The "index" would then trade as a *separate* security from the underlying single name CDSs. It should be regarded as a standalone security with a known maturity, coupon, and standardized documentation. Trading the index is equivalent to buying or selling protection on the reference portfolio names with *equal weights*.[8] The spread of this portfolio, i.e., the I_t, is quoted separately from the underlying CDSs.[9]

At the outset, one may think that the I_t would (approximately) equal the simple average of the underlying CDSs. This turns out not to be the case, except in exceptional circumstances when the all the CDS rates and their volatilities are the same. In general, we will have

$$I_t \leq \frac{1}{n} \sum_{j=1} c_t^j \tag{1}$$

In fact, why should a traded credit index trade as if all credits are weighted equally? It is more reasonable that the pricing of a reference portfolio would weigh the underlying names using their survival probabilities as well as the level of the corresponding CDS rates. In other words, the index *spread* would be *DV01-weighted* even though the composition of the index is weighted equally.

[8] Trading the series 9 *iTraxx* Europe index in the five-year maturity corresponds to buying or selling a security that pays a fixed coupon of 1656 for five years with quarterly settlements. When a default occurs, the protection seller compensates the protection buyer by an amount equal to $\frac{1}{n}N$, with $n = 125$.

[9] Actual indices trade somewhat differently than the underlying CDSs. For example the single name CDSs trade in first short coupon, whereas the indices trade in accrued. There are other differences concerning the recovery, restructuring, settlement, and other aspects, one of which is the constant contract spread (coupon) in the indices. See www.iboxx.com.

EXAMPLE:

The index has two names:

$$c^1 = 20\,bp \tag{2}$$

$$c^2 = 4500\,bp \tag{3}$$

reminiscent of the spreads during the credit crisis. The average spread will be:

$$\frac{4500 + 20}{2} = 2260 \tag{4}$$

According to these spreads it is much less likely that the first name defaults in the near future compared to the second name. This means that if an investor sold protection with notional $N = 50$ on each of the individual CDSs, the average receipt during the next five years is unlikely to be 2260. This is the case since the default of the second name appears to be imminent. Assuming that default occurs immediately after the transaction, the average return of the remaining 50 invested in the first credit would be 20 bp for the next five years.

On the other hand, if the index traded at 2260 bp, the index protection seller will continue to receive this spread on the remaining $50.

According to this, the index spread will deviate more from the simple average of the underlying CDS spreads, the more dispersed the latter are. This is due to the DV01 weighing mentioned above.

Thus, the credit indices are fundamentally different from the better-known equity indices such as S&P500 or Dow. The latter are supposed to equal some average of the price of the underlying stocks, otherwise there would be an (index) arbitrage opportunity. In the credit sector this difference is far from zero and the traders trade this difference if it deviates from a calculated fair value.

For the sake of presentation, the discussion will continue using the iTraxx investment grade (IG) index as representing the I_t. Some information about the recent history of the indices is in the appendix at the end of this chapter. The next example will help to understand the cash flow structure of such an index.

EXAMPLE:

Suppose n =3. The cash flow diagram for the index is shown in Figure 18-5. We consider a one-year maturity with settlement in arrears at the end of the year.

Suppose $N = 3$ is invested in this index. All names are equally weighted and all probabilities of default p^i are assumed to be the same. This makes the position similar to putting $1 on each name in a reference portfolio of three single name CDSs. However, as mentioned above, the spread on the portfolio as a whole may deviate significantly from the average of the three independent single name CDSs.

Essentially, there will be 4 possibilities at the end of the year. There may be no defaults, one default, two defaults or three defaults at time t_1. The structure will be as shown in Figure 18-6.

On the other hand, if the position was for more than one year the default possibilities would be more complicated for the second year. This is discussed later in this chapter.

What happens when an entity that belongs to the underlying reference names defaults? Consider the case of the *iTraxx* index with $n = 125$ names. The resulting process for this default

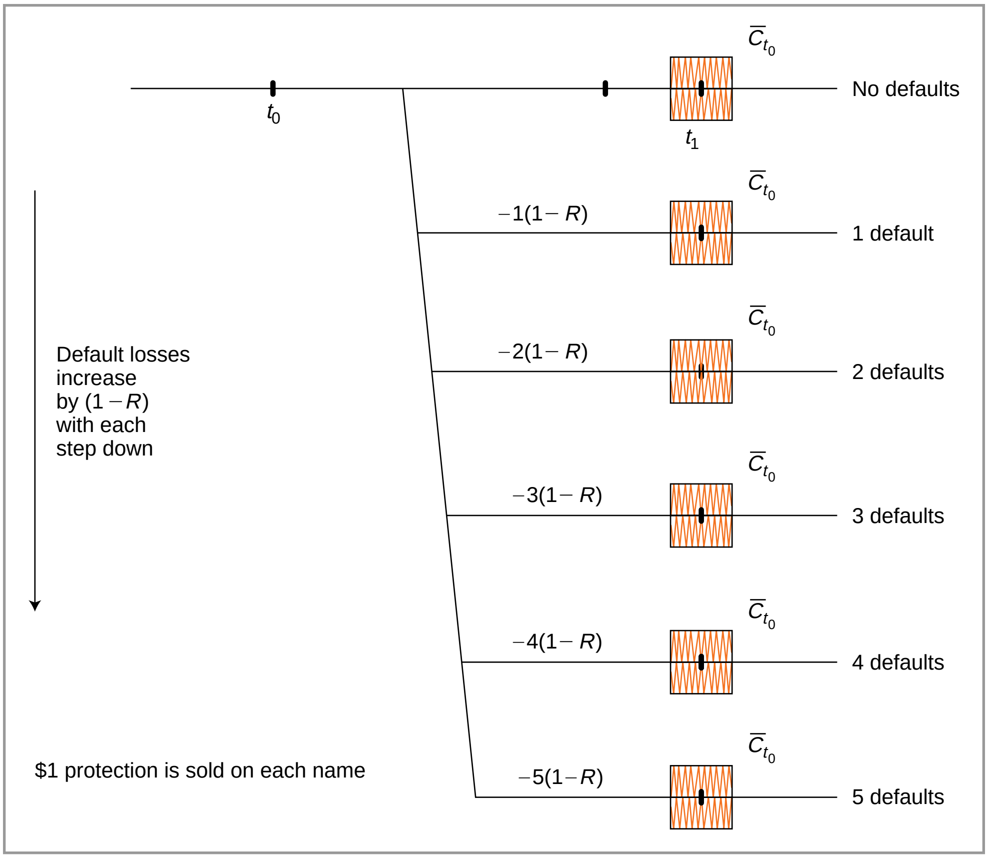

FIGURE 18-5

is similar to that for a single name CDS. The protection seller pays to the protection buyer a compensation equal to $\frac{1}{125}N$. In return, the protection buyer delivers deliverable bonds with the same face value.

After the default, trading will continue in the case of a credit index, albeit with the notional amount reduced by $\frac{1}{n}N$. The index will then have $n-1$ names and any contracts written on it will continue as if the notional amount has become:

$$N - \frac{1}{n}N = \frac{n-1}{n}N \tag{5}$$

Further defaults would lower this notional in a similar way.

5. Index Arbitrage

Index arbitrage is well known in equity markets. A stock index is made of a given and known number of stocks. The Index itself can be traded in the futures markets. The carry strategy applies and by buying (short selling) the underlying stocks, one can create synthetic futures contract which can be used to take arbitrage positions on the stock index.[10]

[10] Specifically, this is done by selling the index and buying the underlying stocks to carry to the expiration date if the index is too high relative to the reference portfolio, or vice versa.

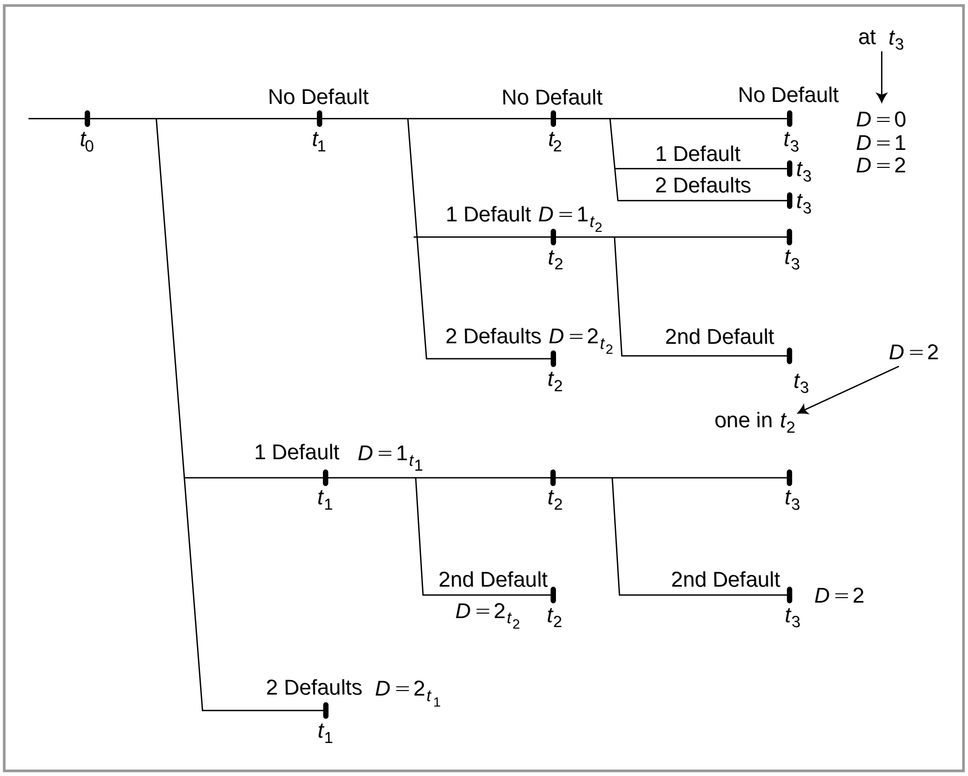

FIGURE 18-6

By analogy one can define index-arbitrage in credit as positioning in the index itself versus its underlying reference names using the single name CDSs. In equity markets this strategy is reasonable. In credit markets it faces several complications even though theoretically it sounds similar.[11]

There are two major issues that lead to different valuations in the index versus its constituents. The first is the differential treatment of *restructuring*. The second is a technical issue and deals with the *convexity* of the index versus the underlying CDSs. Credit index arbitrage is possible, but only after taking into account such divergences explicitly.

First note that CDX indices trade with no restructuring. Yet, the CDSs for most of the investment grade credits treat restructuring as a credit event.[12] To correct for this valuation difference, the value of the restructuring risk must be subtracted from the individual CDSs. This value, however, is not observed separately and has to be "estimated."

The convexity issue is more technical. Fixed income instruments have convexity as discussed previously, whereas equity does not. This also differentiates index arbitrage in credit from the index arbitrage in equity. Consider the 5-year CDS with a CDS rate c_t^j bps for the jth name in the reference portfolio. As the underlying CDS rate changes, the market value of the future CDS coupon payments will change nonlinearly since the fixed coupons will be discounted

[11] One issue is the liquidity of single-name CDSs. In an index consisting of 125 names, not all underlying CDSs may be liquid. The bid-ask spreads for these individual CDSs may be too wide in many cases and trading the underlying against the index may become too costly.

[12] A restructuring event may trigger a payment associated with the credit name on an individual CDS, yet this will not affect the corresponding index.

using discount factors that will be a function of *survival probabilities*.[13] According to this, the CDS value would be a nonlinear function of the c_t^j, the CDS rate, which leads to convexity gains.

On the other hand, an investor to the credit index receives a single coupon. The convexity of this cash flow will be different from the convexity of the portfolio of underlying CDSs, since the convexity of the average is different from the simple average of convexities. The effects of convexity and of restructuring should be taken out explicitly in order to come up with a *fair value* for the index.

6. Tranches: Standard and Bespoke

The popularity of the indices is mostly due to the existence of *standard* index tranches that permit trading credit risk at different levels of subordination. At the present time *default correlation* can be traded only by using index tranches. The index itself is used to hedge the sensitivity of tranches to changes in the probability of default.[14] We discuss the formal aspects of pricing default correlation in the next chapter. In this section we introduce standard index tranches and then introduce their relationship to default correlation.

The *equity tranche* is the first loss piece. By convention equity tranche bears the highest default risk.[15] A protection seller on the standard equity tranche bears the risk of the first 0–3% of defaults on the reference portfolio. The equity tranche spread is quoted in two components. The first, which is quoted by the market maker, is paid *upfront*. It is for the investor to "keep." The second is the 500 bps *running* fee which is paid depending on how much time passes between relevant events.[16]

The *mezzanine tranche* bears the second highest risk. A protection seller on the tranche is responsible, by convention, for 3–6% of the defaults.[17]

The *senior* and *super senior iTraxx* tranches bear the default risk of 6–9% and 9–12% of names respectively.

The numbers such as 0–3% are called the lower and upper *attachment points*. The ones introduced above are the attachment points for *standard tranches*. If attachment points are different from those and negotiated individually with the market maker they become *bespoke tranches*.[18] A bespoke trance will not naturally have the same liquidity as a standard tranche.

The value of the tranches depends on *two* important factors: The first is the risk of a change in the average probability of default; the second is the change in *default correlation*. This is a complex and important idea and leads to the market known as correlation trading. It turns out that from a typical bank's point of view, one of the biggest risks that may lead to bankruptcy is the event of defaults of its clients *at the same time.* Banks normally make provisions for "expected" defaults. It is part of the business. If individual defaults occur now and then it will

[13] Remember that a default event means the coupons expected by the protection seller would stop. Hence, the cash flow characteristic of the CDS changes with the default event and the discounting should take this into account.

[14] Hence, the more popular correlation trading becomes, the higher will be the liquidity of the indices.

[15] Also, the *delta* of the equity tranche is the highest with respect to the underlying index. It has a higher sensitivity to index spread changes. This is another risk.

[16] For example, suppose all 3% of the defaults occur exactly in 6 months, then the running fee will be paid by the protection buyer only for 6 months. The upfront fee will not be affected by such timing issues.

[17] For the CDX index the attachment points of the Mezzanine tranche are different and they equal 3–7%.

[18] However, a more important characteristic of most bespoke tranches is that the selection of the reference portfolio may be different than the reference portfolio used in tradeable credit indices.

not be very harmful. Yet, joint defaults of the borrowers can be fatal. Hence, the importance of default correlation for the banking sector.

In fact, the reference names in a credit index are affected by the same macroeconomic and financial conditions that prevail in an economy (sector) and hence are likely to be quite highly correlated at times, and the level of the correlation would change depending on the prevailing conditions. In an environment where credit conditions are benign and liquidity is ample, default correlation is likely to be low and any defaults occur mostly due to *idiosnycratic* reasons, that is to say, effects that relate to the defaulting company only, rather than the overall negative economic and financial conditions. During periods of stress this changes and defaults occur in bunches due to the underlying adverse macroeconomic conditions.

Thus default correlation is a stochastic process itself. During the last few years, market professionals have learned how to strip, price, hedge, and trade the default correlation. We will study this more formally in the next chapter. Here we note that the value of the equity tranche depends *positively* on the level of default correlation. The higher the default correlation in the reference portfolio, the higher the value of an investment in the equity tranche, which means that the spread associated with it will be lower.[19] On the other hand the value of the senior and supersenior tranches depend *negatively* on the default correlation. As default correlation increases, the investment in a super senior tranche will become less valuable and its spread will increase. This is called the *correlation smile* and is discussed in the next chapter.

7. Tranche Modeling and Pricing

Markets trade the indices and the index tranches actively. As a result, the spreads associated with these instruments should be considered to be arbitrage-free. Still, we would like to understand the price formation, and this requires a modeling effort. The market has over the past few years developed a market standard for this purpose. The specifics of this market standard model are discussed in the next chapter. Here we discuss the heuristics of CDO tranche valuation.

What determines the tranche values is of course the receipts due to spreads and the potential payoffs due to defaults. The general idea is the same as in any swap. The expected value of the properly discounted cash inflows should equal the expected value of the properly discounted cash outflows. The arbitrage-free spread is that number which makes the expected value of the two streams equal. Clearly, in order to accomplish this we need to find a proper probability distribution to work with. We discuss this issue using tranche pricing. Tranche values depend on the probabilities that are associated with the payoffs the tranche protection seller will have to make. These probabilities are the ones associated with the number of defaulting companies during a particular time period and their correlation.

We limit ourselves to one period tranche contracts on an index with $n = 3$ names.

7.1. *A Mechanical View of the Tranches*

Consider a reference portfolio of $n = 3$ names. Call them A, B, C respectively. Limit the time frame to 1-year maturities. Let D represent, as usual, the total number of defaults in a year. The first step in discussing tranche valuation is to obtain the distribution of D. How many possible values can D have? It is clear that with $n = 3$, there are only *four* possibilities as shown in

[19] This is similar to the relationship between the value of a bond and its return.

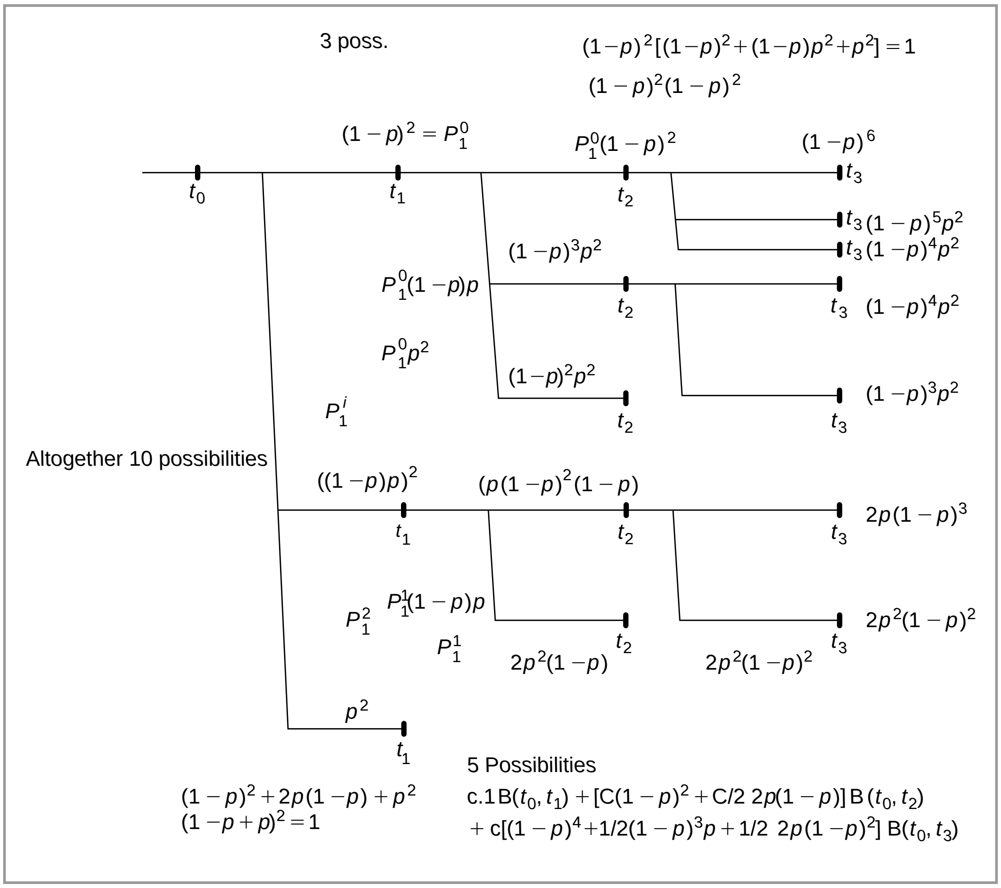

FIGURE 18-7

Figure 18-7. According to this figure the probability is quite high that there will be no defaults at all, i.e., $D = 0$. The probability associated with more defaults D then gets smaller.

At this moment ignore how such probabilities can be obtained and take them as given. Assume that the probability of default is the same for each name and that the recovery is R. Thus,

$$p^A = p^B = p^C = p \tag{6}$$

We now show how tranche values depend on this probability and on the correlation between the defaults of the three names.

Now, consider the *equity tranche*. We have three names, and the equity tranche is bearing the risk of *any* first name to default. Mezzanine is the protection for the second name, and senior tranche sells protection on the third name.[20] Suppose a market maker now sells protection on the equity tranche. In other words, the market maker will compensate the counterparty as soon as *any one* of A, B or C defaults. What is the probability of this event? Let D denote the random variable representing the number of defaults. Then the probability that there will be at least one

[20] Hence for a protection seller on the senior tranche to pay, all three names need to default.

default is given by

$$P(D = 1) + P(D = 2) + P(D = 3) = 1 - P(D = 0) \quad (7)$$

Going back to Figure 18-7 we see that this probability is 20% in that particular case. This is much higher than the assumed 5% probability that any name defaults individually. Thus writing insurance on a first to default contract is much riskier than writing insurance on a particular name in the reference portfolio.[21] This is the risk associated with the equity tranche—the tranche that will get hit first in case of a default.

An investor may not be willing to take such a risk. He or she may want to write insurance *only* on the *second* default. This is the investment in the *mezzanine tranche*. The tranche has *subordination*, in the sense that there is a cushion between the defaults and the protection seller's loss. The first default will hit the equity tranche.

We can calculate the probability that the mezzanine tranche will lose money as,

$$P(D = 2) + P(D = 3) \quad (8)$$

One can also write protection for the third default. Here there is even *more* subordination. Before the insurer suffers any losses, two names must default. In this case the investment will represent a *senior tranche*. The probability of making a payoff is simply

$$P(D = 3) \quad (9)$$

This simple case can be generalized easily to *iTraxx* indices.

7.2. Tranche Values and the Default Distribution

We use Figure 18-8 to discuss the important relation between the area under the default density function and the tranche values. First, note that the *iTraxx* attachment points slice the distribution of D into 5 separate pieces. Each tranche is associated with a different area under the density.

Consider the 3–6% mezzanine tranche as an example. The tranche has two *attachment points*, the lower attachment point is 3% and the upper attachment point is 6%. In heuristic terms, the lower attachment point represents the subordination, i.e., the cushion the investor has. Defaults up to this point do not result in payments of default insurance.[22] The upper attachment point represents a threshold of defaults after which the mezzanine protection seller has *exhausted* all the notional amount invested. Any defaults beyond this point do not hurt the mezzanine investor, simply because the investment does not exist anymore. Thus the area to the *right* of the upper attachment point is the probability of losing all the investment for that particular tranche.

Once this point about attachment points is understood, we can now show the relationship between default correlation and tranche values. Consider again Figure 18-8b, giving the distribution of D, the total number of defaults. This distribution depends on the average probability of default p and on the default correlation ρ. Suppose in Figure 18-8 the correlation originally was low at $\rho = .1$. Then, keep the p the *same* and move the correlation up to, say, $\rho = 90\%$. The distribution will shift as shown in Figure 18-8b. Consider the implications.

The first implication is that as correlation goes up, the distribution is being pressed downward from the middle. However, the area needs to equal one. So, as the middle is compressed, the

[21] This is understandable. Consider the analogy. You go to school and you have 50 classmates. It is winter. The probability that tomorrow you come in with a cold is small. But the probability that *someone* in your class will have a cold is much higher. In fact, during a typical winter day this probability is quite close to one.

[22] The spread on mezzanine tranche would go up since the cushion would be getting smaller.

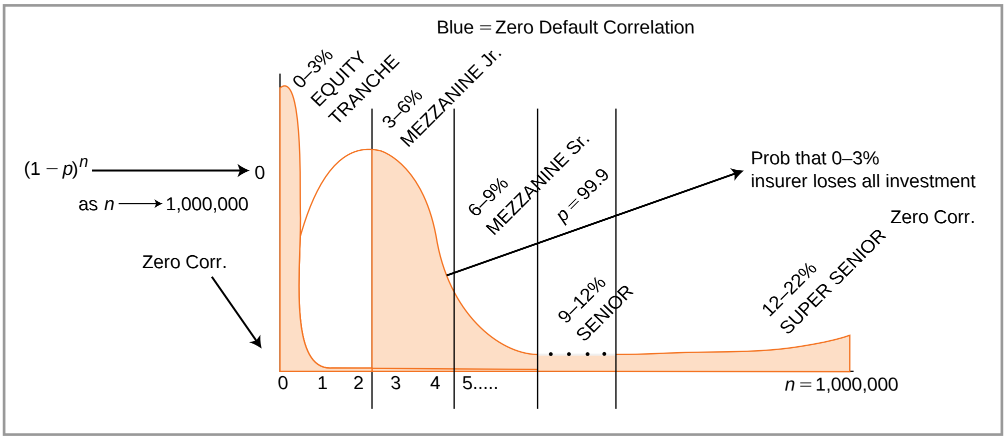

FIGURE 18-8a

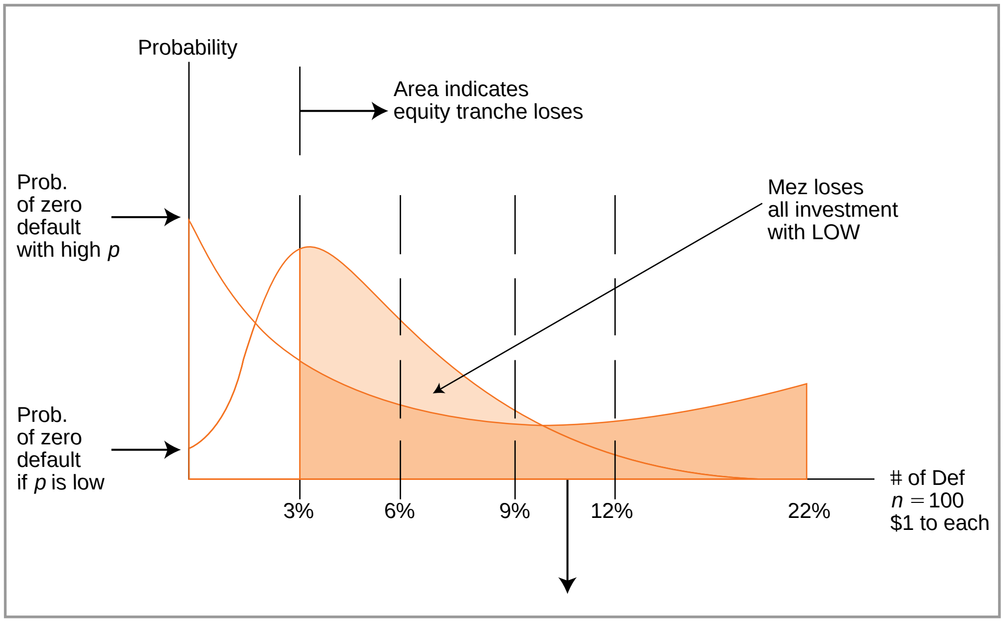

FIGURE 18-8b

weight goes to the two end points. The probability of the area near 0–3% goes up. The same is true for the probability associated with the other tail. The area near the 12–22% tranche also goes up.

But this has a second implication. The area on the right of the equity upper attachment point gets smaller. Implying the probability that the equity investor will lose all his investment has gone *down*. The equity tranche investor will benefit from this change in the distribution function. The equity tranche spread will go down and the investor who sold protection earlier at the higher rate will have mark-to-market gains. Hence the equity tranche protection seller is *long* default correlation.[23]

[23] I.e., the investor will benefit if the correlation increases.

Third, the probability associated with the cushion of the super senior tranche is also getting smaller. This implies that the probability is higher and the super senior protection writer will suffer some losses. The spread on the super senior tranche will go up and the investor who sold protection at a lower spread will have mark-to-market losses. Hence the super senior protection seller is *short* the default correlation.[24]

Finally, note that the effect of these movements are mixed on the mezzanine tranche. The area on the right of the upper attachment point has not changed that much. Hence the probabilities associated with mezzanine tranche losses are approximately the same and the mezzanine investor is more or less neutral toward the correlation changes.

At the extreme as $\rho \uparrow 1$ the distribution that is approximately bell shaped starts looking more like a binomial distribution. As correlation goes to one, the d^j start to behave more and more similarly and at the end, they become the same random variable. Hence the value assumed by the random variable D will be either n or 0, i.e., either every name will default or no name will default.

8. The Roll and the Implications

Every six months, on March and September 20th, the *iTraxx* and CDX indices *roll* and a new *series* starts trading. Some credit names may have defaulted, changed sector, merged, or been downgraded. Other CDSs may have become less liquid. These are considered as no longer eligible to be part of the index. Every such name is replaced by the next most liquid name available in its class. The "old series" continue to trade as long as there are open positions, but they are off-the-run. The new roll will be the *on-the-run* liquid index.

The *Roll* is an important characteristic of the indices from a financial engineering point of view since its presence leads to several strategies and complications that a financial engineer must be aware of. An obvious strategy is to guess the names that will leave the index and the names that will come in. But this happens in index revisions in the equity sector as well. Credit rolls have some novel additional strategies. Note that dropping lower-rated issuers from the index and replacing them with higher-rated ones normally means that the new index should start trading at a narrower spread than the old index, everything else being the same. But, surprisingly, this may not happen, as we will see below.

There is some empirical evidence that because the *buyers* of protection[25] would like to stay with the new, on-the-run index due to its liquidity, right before the new series, they will close their positions. This means they will *sell* protection and this will lower the spreads. Of course, as the new index starts trading, the same names will buy protection and this will widen the spreads.

There are also structured credit products that are partially based on the fact that the index will roll every six months. The constant proportion debt obligation (CPDO) is one good example. During a six-month period, everything else being the same, a 5-year maturity *iTraxx* index will become a 4.5-year maturity index. This means that the index will *roll down* the curve and, if the curve is upward sloping, spreads will tighten automatically. This is due to the shorter maturity and nothing else.

Another classic technical roll is the change in the *basis*. This is the difference between the index spread and the intrinsic value of the underlying credit default swaps of the referenced

[24] The investor will lose if default correlation decreases.

[25] The buyers of protection are not necessarily the ones who desire insurance against default. Indices are mostly used by players who hedge the default risk in tranche positions that they have taken. Hence, often such buyers of protection are hedge funds.

names in the index. The basis exists because normally there are more clients that *buy* protection than *sell* them.[26] In past rolls, the basis between the underlying credit default swaps and the index has narrowed significantly. This happens because more investors are selling protection than buying protection in order to move to the new, more liquid index.

The example below summarizes the mechanics of the roll.

EXAMPLE:

NEW YORK (Dow Jones)—As summer draws to a close, credit derivatives investors are eyeing September's changeover in the credit default swap indexes for trading opportunities. This time around though, trading patterns surrounding the change in the indexes' composition could differ from past ones as more issuers than ever before—a total of eight—will be dropping out of the investment-grade index family.

Credit default swaps allow investors to protect their holdings should issuers default on their debt. Since the inception of the credit default swap indexes—liquid benchmarks that allow investors to take positions on corporate debt issuers without having to buy or sell the underlying, often illiquid cash instruments—there have been [nine rolls until March 2008] which take place every six months.

At each roll date, issuers who have lost their investment-grade ranking by either Moody's Investors Service or Standard & Poor's are dropped from the Dow Jones CDX investment-grade credit default swap index and replaced by other issuers—chosen by a poll of the dealers who belong to the index consortium. At the first index roll, six issuers were replaced, at the second five and at the last roll in March 2006, just three issuers were dropped.

It is important to understand that at each roll the newly introduced series will have a new coupon and will be trading near par initially. The traders who buy and sell index protection are in fact buying and selling this newly introduced standardized contract.

8.1. *Roll and Default Risk*

Default risk in the indices is somewhat different than it looks at the outset due to the existence of the roll. In fact, at reach roll the credit quality of the indices improves. In general, before a company defaults, its credit quality deteriorates. By the time the company is about to default, it is quite likely that it was dropped from the index during some previous roll. This brings up an important distinction. A five-year position that stays with the *same* index will face the default risk of the underlying reference names, and these names will remain the same during five years. Some of these names will deteriorate and some may even default. This is what is meant by *default risk*.

A position that always rolls to the new index faces a somewhat different default risk. The main default risk faced by such a position is when default occurs *all of a sudden*, without any indications. In this case a good corporation may go from a rating of AA to default, without the rating agencies having time to downgrade it, and before the roll date arrives. This is called *jump to default risk*. Note that its probability is significantly lower than staying with the same names during the 5 years and then seeing some of them default.

[26] The market makers will hedge this discrepancy on their books using the underlying CDSs.

9. Credit versus Default Loss Distributions

This section discusses the relationship between tranche pricing and credit risk management. Basel II is the framework associated with current credit risk management practices and indirectly it has a close relationship with tranche pricing.[27]

Suppose we would like to adapt a Value at Risk (VaR)-type risk management approach to portfolios with credit risk. This means that we would like to set enough capital to cover losses $0 < \alpha$ percent of the time. It turns out that there are *two* quite different empirical loss distributions that determine the calculation of possible losses.

The first is the loss distribution due to *defaults*. Consider a portfolio of defaultable assets issued by n different debtors. For simplicity, assume that the recovery rates are the same and are known at R. Also, the exposure to each name is $1, meaning that the total investment in the portfolio is $\$n$. Then, if one name defaults, the investor loses $(1 - R)$ dollars. The total default loss during a horizon of length T will depend on how many names default during this time interval. In other words, default loss depends on the *distribution* of the random variable D defined earlier.[28]

In this setting, in order to calculate possible losses due to default, one first needs to obtain a distribution for this random variable D. This is the *default loss distribution* and is illustrated again in Figure 18-8a. Using this distribution one can calculate the *expected default loss* and the threshold L_α which determines the extreme losses that occur with a probability of (at most) α, during the interval with length T.

The bank would then put aside enough capital to cover default losses up to the point L_α. These have a probability $1 - \alpha$ of occurring. Default losses may be greater than L_α only in α percent of the time. The bank is not obligated to cover these more extreme losses with additional capital.

But this is only *one* way of looking at credit risk. It involves the risk associated with the *default events* only. This way of managing risks is perhaps appropriate if the instruments under consideration are held until maturity and if the *mark-to-market* is not relevant.

On the other hand, if the portfolio under consideration is a trading portfolio where marking-to-market is important or if the instruments may not be held until maturity, then the bank or the hedge fund faces *another* risk.[29] This risk comes from credit quality changes, which incidentally also involves defaults. If credit deteriorates and needs to be sold or marked-to-market, then the bank or the hedge fund will still suffer credit losses although no default has occurred. The *default* loss distribution cannot take such potential losses into account, because it is only directed toward measuring loss due to default. The loss due to credit quality changes requires an *additional* effort. The change in the market value of the portfolio due to credit quality changes can be calculated in a way similar to that of market risk.

The calculation of the *default* loss distribution requires the default probabilities for each name and the default correlations. The calculation of the *credit* return distribution on the other hand requires, in addition, the *conditional transition probabilities* concerning the rating migrations

[27] In this section, we ignore the market risk and concentrate on the *new* aspects of risk management brought about due to the existence of the default event.

[28] Again, assume that when defaults occur they occur at the end of the period, at time $t_0 + T$.

[29] A hedge fund position is a credit instrument and is usually financed by borrowed funds. This will be similar to repo, where the hedge fund buys the instrument and repos it to secure the funds to pay for it. The repo dealer would then mark this instrument to the market. Even when there is no default, the credit spreads may change and may create losses for the hedge fund.

of each name in the portfolio as well as their correlations.[30] Hence the calculation of this second distribution is much more involved.

10. An Important Generalization

Thus far we discussed CDS contracts and credit indices using a one-year maturity. Also we assumed throughout the discussion that default, if it occurs, can occur only at settlement dates. These assumptions were adopted in order to simplify the understanding of these new instruments. Both of these assumptions are unrealistic. The most liquid maturity for CDS contracts is five years and default can occur at any time. The CDS premium should then be pro-rated to the time passed and the contract should be settled immediately.[31]

In this section we modify the assumption that CDS and index contracts are only for one year.[32] Changing this assumption leads to new concepts and changes some of the formulas used thus far. First we begin with CDS contracts then we move to discussing index trading in a similar context.

10.1. *A Five-Year CDS*

The cash flows and the corresponding probability structure of a five-year single-name CDS are shown in Figure 18-9. Note that moving from a 1-year CDS to a five-year CDS makes very little difference in terms of the cash flow geometry. Assume that the annual probability of default is denoted by p,[33] and make the simplifying assumption that defaults can occur *only* at times $\{t_1, . . . , t_5\}$. Then calculate the expected receipts in a straightforward way. Multiply the premium received in a state of the world with the probability that that state is reached. Doing this the expected receipts, *Rec*, of a protection seller will be:

$$E[Rec] = \big[B(t_0, t_1) + B(t_0, t_2)(1-p) + B(t_0, t_3)(1-p)^2 \\ + B(t_0, t_4)(1-p)^3 + B(t_0, t_5)(1-p)^4\big]\, c\delta N \tag{10}$$

where $B(t_0, t_i)$ is the time t_0 value of a *default-free* dollar to be received at time t_i. The δ equals *one* in our case. The N is the agreed-on notional amount. We let

$$\tilde{B}(t_0, t_i) = B(t_0, t_i)(1-p)^{i-1} \tag{11}$$

be the so-called *defaultable* discount factors. The defaultable annuity factor is then

$$A = \left[\tilde{B}(t_0, t_1) + \tilde{B}(t_0, t_2) + \tilde{B}(t_0, t_3) + \tilde{B}(t_0, t_4) + \tilde{B}(t_0, t_5)\right] \delta \tag{12}$$

30 Note that default is only one of the states where credits can migrate. Hence default risk is included in the credit loss distributions due to credit deterioration.

31 The settlement is two days after the credit event.

32 However, we still assume that defaults occur only at settlement dates. Modifying this assumption would change the numerical calculations slightly, but would not introduce critical new concepts. Interested readers should consult Hull (2008).

33 Normally an annual probability of default in a 5-year contract would be different than the annual probability of default in a, say, 3-year default. This is similar to interest rate swaps. The swap rate will be different in a 5-year swap when compared to a 3-year swap. Credit spreads have a similar structure.

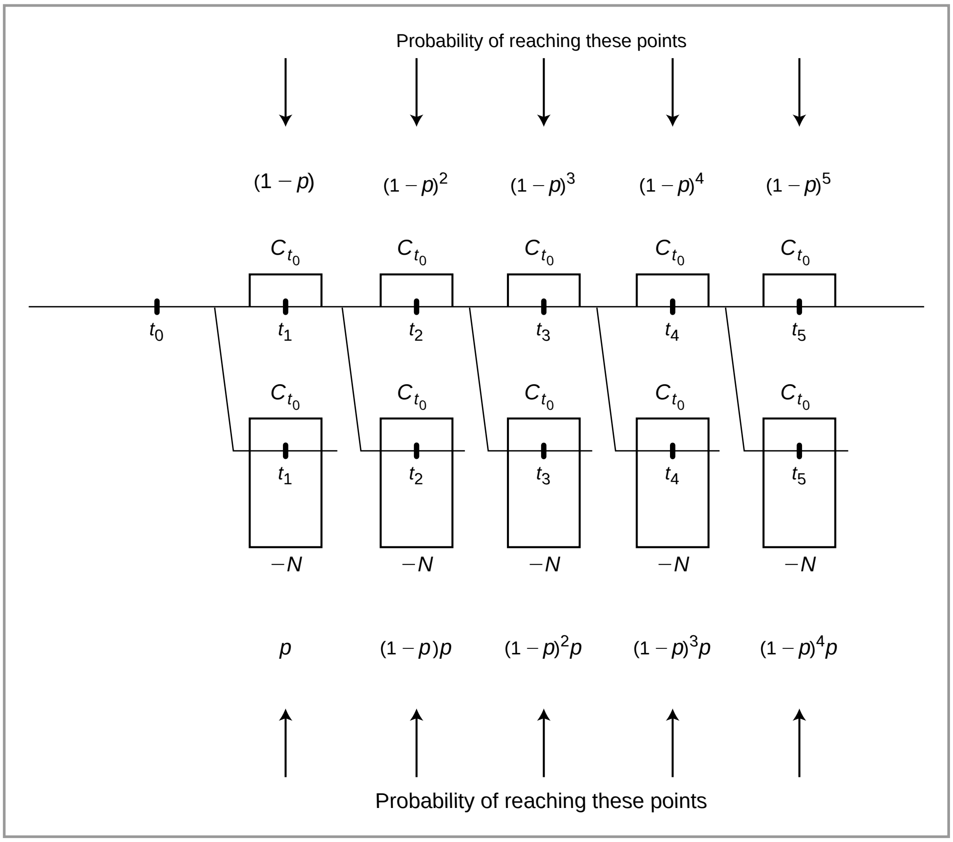

FIGURE 18-9

For small movements in spreads this is close to the *defaultable* DV01. In other words, we have[34]

$$\frac{\partial E[Rec]}{\partial c} = A \tag{13}$$

However, as c changes the p is likely to change as well and we will have

$$\frac{dE[Rec]}{dc} = \text{DV01} \tag{14}$$

In general for small changes in c the annuity factor will be close to the DV01. These defaultable DV01 and annuity factors are important tools to credit traders and play equivalent roles in default-free swap and bond trading. This completes the discussion of the *fixed leg* of the CDS contract.

We can similarly calculate the expected payments, denoted by *Pay* by the protection seller using the same concepts. We obtain

$$\begin{aligned} E[Pay] = \big[&B(t_0,t_1)p + B(t_0,t_2)(1-p)p + B(t_0,t_3)(1-p)^2 p \\ &+B(t_0,t_4)(1-p)^3 p + B(t_0,t_5)(1-p)^4 p\big]\,(1-R)N \end{aligned} \tag{15}$$

A look at the five-year CDS cash flows in Figure 18-9 show that the diagrams are made of five one-year CDS contracts. The first one is a spot contract, the remaining four are similar to forward

[34] Note the use of partial derivative notation.

start CDSs, although, due to five-year maturity they are traded jointly. Thus we will have under the *present* assumptions

$$p = \frac{c}{(1 - R)} \tag{16}$$

in other words, the probability of default will depend on the *five-year* CDS spread.[35]

We should emphasize once again that the formulas given in this section depend on the simplifying assumption that default can occur only at times $\{t_1, . . . , t_5\}$. If default can occur at all $t \in [t_0, t_5]$ then the formulas will change marginally, although the general ideas would be similar.

10.2. A "Multi-Year" Index Contract

Things get a bit more complicated as we move from a one-year index contract to a five-year index contract. In fact, it is next to impossible to show the cash flows of a five-year index contract graphically, in, say, the *iTraxx* index. Still, we would like to look at the geometry of index trading. To achieve an acceptable level of simplicity we assume a two-year index contract and allow only *two* names in the underlying reference portfolio.

We assume that the index is made of two names, $j = 1, 2$ and that the probability of default for both names is p

$$p^1 = p^2 = p \tag{17}$$

All together there are more than two possibilities that matter. As time passes, the number of combinations ("nodes") increases. At time t_1 there are three possibilities. No default, one default, and two defaults. As time t_2 arrives the number of possibilities becomes five. This is one difference in the case of a single-name CDS.

The second difference is in the payment of index spreads. In CDS there are two possibilities. Either the name does not default and a protection seller continues to receive the spreads, or the name defaults and the protection seller will not receive the remaining future spreads. In index trading this changes: the protection seller will continue to receive spreads but the notional amount will decrease by $\frac{k}{n}N$ as k names among n defaults. If one default occurs at time t_1, then the time t_2 spread will go down to $\frac{1}{2}N$.

We can calculate the expected receipts as

$$B(t_0, t_1)c\delta N + B(t_0, t_2)\left[c(1 - p)^2\delta + \frac{c}{2}(2p(1 - p))\right]\delta N$$
$$= [B(t_0, t - 1) + B(t_0, t_2)\Big[(1 - p)^2 + \frac{1}{2}(2p(1 - p))\Big]\delta N \tag{18}$$

Note that we can again define defaultable discount factors $\hat{B}(t_0, t_2)$

$$\hat{B}(t_0, t_2) = B(t_0, t_1)[(1 - p)^2 + 2p(1 - p)] \tag{19}$$

that correspond to the indices, as in CDS contracts, although not identical. The $\hat{B}(t_0, t_2)$ are likely to be quite a bit more complicated to define when compared with $\tilde{B}(t_0, t_2)$. However, the overall idea is the same. We would like to adjust the risk-free discount factors by the probability that we would in fact *get* to that point.

[35] Remember that the assumption that default can occur only at the settlement date plays some role in determining this formula. Otherwise, this would be an approximation.

10.3. *Valuing Tranches*

Using this "multi-year" index contract we can also look at what changes when the maturity moves from a single period to several periods. The previous situation deals with two credit names, so we can only look at two meaningful tranches: the tranche that has exposure to the first default only, and the senior tranche that is exposed to the second default.

Suppose the notional is $N = 1$, and that $\delta = 1$. Then the tranche values can be calculated using the equation

$$c^e\left[B(t_0,t_1)+B(t_0,t_2)\left[c^e(1-p)^2\right]\right]=\left[B(t_0,t-1)(1-(1-p)^2)\right.\\ \left.+B(t_0,t_2)(1-(1-p)^4)\right](1-R) \qquad (20)$$

The main difference between the one-period and two-period cases is similar to the valuation of indexes themselves. Essentially, besides the probabilities being somewhat more complicated, the multiperiod contracts will lead to pricing equations that will involve the defaultable discount factors $\tilde{B}(t_0, t_i)$. The multiperiod case also shows how complicated the trajectories of the index tranches and index valuation can get once we let the underlying reference portfolio start having 100 or 125 names. In fact, it is clear that valuation exercises under such conditions would require heavy calculation efforts.

11. New Index Markets

The credit sector plays an important role in financial markets for several reasons. One of these is the methodical way new sectors are introduced by the major players. Markets are normally created endogenously with the interaction of thousands of traders, market makers, and risk managers. In the credit sector this effort has been consciously directed by broker-dealers and has come after years of experience in new derivatives markets and trading strategies. As a result, broker-dealers have been quite successful in creating platforms for trading new risks and offering broad numbers of instruments for hedging a range of credit risk exposures.

The ABX and the LCDX indices are the most prominent of the new tradeable indices that permit trading of new risks.[36] They are quite recent in that the first trading began during the years 2006–2007. The market liquidity has not yet reached levels experienced in *iTraxx* and *CDX* indices. However, they already play an important role in financial markets strategies.

The ABX index is a carefully constituted index that permits trading and hedging of mortgage debt exposures. A player who is short mortgage debt will hedge this position by buying the ABX index. The LCDX is similar to the *iTraxx* or CDX, except that the underlying securities are loans instead of being bonds. Hence, this index helps hedging loan exposures. Levx is similar. It is an index of leveraged loans.

A player who is long will hedge the position by selling the index. These indices are traded in form of series, and offer a fixed coupon at each roll.

11.1. *The ABX Index*

The ABX.HE index is made of obligations issued by 20 issuers of *residential* mortgage-backed securities. Altogether there are five sub-indices. These sub-indices are each made of one security from each one of these issuers. These securities and hence the sub-indices have ratings that range

[36] Another index is the Levx. This is the European equivalent of the LCDX. We will not discuss it.

from AAA, to BBB−. Similar to the roll in the *iTraxx* and CDX indices, the new series of ABX indices roll on January 19 and July 19. Each series has a new set of mortgage loans behind it.[37]

The securities included in the index are essentially debt securities entitling the investors to receive cash flows that depend on residential mortgages of one-to-four family residences. Hence variations in these cash flows, defaults, and delinquencies affect the value of the ABX indices. An investor in the ABX indices will receive (pay) a fixed coupon set at the roll date and will make floating payments (receipts) if there is an interest or principal shortfall in the cash flows.

11.2. The LCDS, LCDX

The so-called LCDS contracts and their corresponding index LCDX are relatively new tools in the credit sector. Besides being important tools they provide a good opportunity to summarize the Credit indexing technology from a somewhat different angle. LCDS is similar to the single name CDS that we saw in Chapters 1 and 5. In a CDS the deliverable debts are bonds. With LCDS the deliverable is syndicated *secured* debt in the United States that was originally issued by a syndicate.[38] LCDX is the corresponding tradeable index and it is similar to the *iTraxx* and CDX indices. The underlying for the LCDX are $n = 100$ equally weighted single-name loan-credit default swaps (LCDS). The loans in question trade in the secondary *leveraged loan* market. The index launches with a fixed coupon, paid quarterly. The index trades on a clean price basis. If the price goes down, the corresponding spread goes up.

The protection seller will receive a coupon fixed at the roll date and will make a compensating payment during a credit event. Also during a credit event the protection seller pays the notional amount N and gains possession of the loan.[39]

11.2.1. Cancellability

An issuer can repay the debt and may not issue new debt afterward. If there is no deliverable obligation then the LCDS cannot continue. Thus LCDS contracts are cancelable unlike the standard CDS.[40]

A loan repayment starts a 30 business day period, and during the entity can initiate a new loan. If after 30 days no new loans are made, the name is removed from the index after a dealer vote.

This cancelability affects the valuation. If the LCDS is cancelable upon repayment of debt, then the final maturity for a given LCDS will be unknown. The calculation of the present values should then take into account the probability that the loan will be repaid early. This is similar to the use of the credit-risky DV01.

11.2.2. Quoting Conventions

The iTraxx and the CDS index families are quoted on a spread basis, except for the equity tranche which is quoted with an upfront.[41]

[37] Thus, unlike the *iTraxx* and CDX index, the ABX indices have “vintages.”

[38] Just like the CDS, LCDS transactions are unfunded.

[39] Cash settlement is also permitted. The amount of this cash settlement will be determined in an auction.

[40] An issuer may be upgraded. When this occurs, the issuer can repay the original debt and issue new debt with lower interest cost.

[41] Equity tranche also has a constant 500 bps running fee which is not quoted explicitly since it does not change.

On the other hand the LCDX and the ABX indices are quoted on a price basis and this forms another difference.

12. Conclusions

The so-called credit crisis of 2007–2008 changed the popular perceptions of *funded* CDOs and the market for this important product came to a standstill. Yet, the trading of indexes and the index tranches continued without much structural change. The interest in ABX, LCDX, and LEVX trading increased. This chapter shows why such instruments need to be an important component of any modern financial sector.

Suggested Reading

The material in this chapter is relatively new and there are few written sources on it. Our recommended readings are a pair of handbooks. The first is a two-volume Handbook of Credit Derivatives by Merrill Lynch. The second is by JPMorgan. The latter is the closest approach to the market standard in this sector. For more details on the indices, see www.iboxx.com and www.markit.com.

APPENDIX 18-1: A History of Credit Indices

The following is based on information from Reuters.

JP Morgan and Morgan Stanley set up Trac-X early in 2003 to offer the first indices for the mushrooming CDS market. A few months later iBoxx, a joint venture between Deutsche Borse and seven investment banks to provide bond indices, began offering its own suite of CDS indices.

There was friction immediately, with the Morgan banks resenting iBoxx for stealing their thunder, while the iBoxx banks claimed Trac-X was an in-house product, too closely associated with its two sponsors. Because neither side would trade on the others' indices, the rivalry divided the CDS market, sidelined JP Morgan and Morgan Stanley and left investors confused as to which indices to follow.

In the last few months iBoxx has eclipsed Trac-X almost completely in the United States market, with trading on its indices dwarfing that on Trac-X products. In Europe, iBoxx's dominance was less pronounced, but still convincing. A merger was the obvious solution.

Given the different ownership structures of the indices in Europe and the United States, the merger required two separate agreements, one for London and one for New York.

First, under the terms of a letter of intent, iBoxx's bond indices and European and Asian credit derivative index products will combine forces with the Dow Jones Trac-X European and Asian credit derivative indices. The bond indices will be known as Dow Jones iBoxx and the CDS indices as Dow Jones iTraxx.

The new family of products will be managed by a freshly created company called International Index Co, the shareholders of which are ABN Amro, Barclays Capital, BNP Paribas, Deutsche Bank, Deutsche Borse, Dresdner Kleinwort Wasserstein, JP Morgan, Morgan Stanley, and UBS.

In another agreement, announced the previous night, CDS Indexco, which owns the iBoxx CDX North American indices, merged with Dow Jones Indexes and Trac-X LLC to create a combined family of United States high-grade and junk and emerging markets credit derivative indices. This family of products is known as Dow Jones CDX Indices and is managed by CDS IndexCo.

Exercises

1. Consider the following news from Reuters:

   1008 GMT [Dow Jones] LONDON—SG recommends selling 7-year 0–3% tranche protection versus buying 5-year and 10-year 0–3% protection. 7-year equity correlation tightened versus 5-year and 10-year last year. SG's barbell plays a steepening of the 7-year bucket, as well as offering positive roll down, time decay, and jump to default.

   SG also thinks Alstom's (1022047.FR) 3–5-year curve is too steep, and recommends buying its 6.25% March 2010 bonds versus 3-year CDS.

   (a) What is a barbell? What is positive roll down, time decay?
   (b) What is jump to default?
   (c) Explain the logic behind SG's strategy.

2. Consider the following quote:

   It is only when portfolios are tranched that the relative value of default correlation becomes meaningful.

   So, for subordinate tranches, the risk and spreads decrease as correlation between defaults increases, while for senior tranches the risk and spreads increase as default correlation increases.

   (a) Explain the first sentence carefully.
   (b) Explain the second paragraph.
   (c) Suppose you think that credit correlation would *decrease* in the near future. What type of trade would you put on?

3. Consider the following quote:

   Until last year, this correlation pricing of single-tranche CDOs and first-to-default baskets was dependent on each bank or hedge fund's assessment of correlation. However, in 2003 the banks behind iBoxx and Trac-x started trading tranched versions of the indexes. This standardization in tranches has created a market where bank desks and hedge funds are assessing value and placing prices on the same products rather than on portfolios bespoke single-tranche CDOs and first-to-default baskets. Rather than the price of correlation being based on a model, it is now being set by the market.

   (a) What is the *iTraxx* index?
   (b) What is a *standard* tranche?
   (c) Explain the differences between trading standardized tranches and the tranches of CDOs issued in the market place.