

VALUE

Valuing Bonds

► **Investment in new** plant and equipment requires money—often a lot of money. Sometimes firms can retain and accumulate earnings to cover the cost of investment, but often they need to raise extra cash from investors. If they choose not to sell additional shares of common stock, the cash has to come from borrowing. If cash is needed for only a short while, firms may borrow from a bank. If they need cash for long-term investments, they generally issue bonds, which are simply long-term loans.

Companies are not the only bond issuers. Municipalities also raise money by selling bonds. So do national governments. There is always some risk that a company or municipality will not be able to come up with the cash to repay its bonds, but investors in government bonds can generally be confident that the promised payments will be made in full and on time.

We start our analysis of the bond market by looking at the valuation of government bonds and at the interest rate that the government pays when it borrows. Do not confuse this interest rate with the cost of capital for a corporation. The projects that companies undertake are almost invariably risky and investors demand higher prospective returns from these projects than from safe government bonds. (In Chapter 7 we start to look at the additional returns that investors demand from risky assets.)

The markets for government bonds are huge. At the end of February 2009, investors held \$6.6 trillion of U.S. government securities, and U.S. government agencies held \$4.3 trillion more. The bond markets are also sophisticated. Bond traders make massive trades motivated by tiny price discrepancies. This book is not for professional bond traders, but if you are

to be involved in managing the company's debt, you will have to get beyond the simple mechanics of bond valuation. Financial managers need to understand the bond pages in the financial press and know what bond dealers mean when they quote spot rates or yields to maturity. They realize why short-term rates are usually lower (but sometimes higher) than long-term rates and why the longest-term bond prices are most sensitive to fluctuations in interest rates. They can distinguish real (inflation-adjusted) interest rates and nominal (money) rates and anticipate how future inflation can affect interest rates. We cover all these topics in this chapter.

Companies can't borrow at the same low interest rates as governments. The interest rates on government bonds are benchmarks for all interest rates, however. When government interest rates go up or down, corporate rates follow more or less proportionally. Therefore, financial managers had better understand how the government rates are determined and what happens when they change.

Corporate bonds are more complex securities than government bonds. A corporation may not be able to come up with the money to pay its debts, so investors have to worry about default risk. Corporate bonds are also less liquid than government bonds: they are not as easy to buy or sell, particularly in large quantities or on short notice. Some corporate bonds give the borrower an option to repay early; others can be exchanged for the company's common stock. All of these complications affect the "spread" of corporate bond rates over interest rates on government bonds of similar maturities.

This chapter only introduces corporate debt. We take a more detailed look in Chapters 23 and 24.



3-1 Using the Present Value Formula to Value Bonds

If you own a bond, you are entitled to a fixed set of cash payoffs. Every year until the bond matures, you collect regular interest payments. At maturity, when you get the final interest payment, you also get back the **face value** of the bond, which is called the bond's **principal**.

A Short Trip to Paris to Value a Government Bond

Why are we going to Paris, apart from the cafés, restaurants, and sophisticated nightlife? Because we want to start with the simplest type of bond, one that makes payments just once a year.

French government bonds, known as OATs (short for Obligations Assimilables du Trésor), pay interest and principal in euros (€). Suppose that in December 2008 you decide to buy €100 face value of the 8.5% OAT maturing in December 2012. Each December until the bond matures you are entitled to an interest payment of $.085 \times 100 = €8.50$. This amount is the bond's **coupon**.¹ When the bond matures in 2012, the government pays you the final €8.50 interest, plus the principal payment of the €100 face value. Your first coupon payment is in one year's time, in December 2009. So the cash payments from the bond are as follows:

Cash Payments (€)			
2009	2010	2011	2012
€8.50	€8.50	€8.50	€108.50

What is the present value of these payments? It depends on the opportunity cost of capital, which in this case equals the rate of return offered by other government debt issues denominated in euros. In December 2008, other medium-term French government bonds offered a return of about 3.0%. That is what you were giving up when you bought the 8.5% OATs. Therefore, to value the 8.5% OATs, you must discount the cash flows at 3.0%:

$$PV = \frac{8.50}{1.03} + \frac{8.50}{1.03^2} + \frac{8.50}{1.03^3} + \frac{108.50}{1.03^4} = €120.44$$

Bond prices are usually expressed as a percentage of face value. Thus the price of your 8.5% OAT was quoted as 120.44%.

You may have noticed a shortcut way to value this bond. Your OAT amounts to a package of two investments. The first investment gets the four annual coupon payments of €8.50 each. The second gets the €100 face value at maturity. You can use the annuity formula from Chapter 2 to value the coupon payments and then add on the present value of the final payment.

$$\begin{aligned} PV(\text{bond}) &= PV(\text{annuity of coupon payments}) + PV(\text{final payment of principal}) \\ &= (\text{coupon} \times 4\text{-year annuity factor}) + (\text{final payment} \times \text{discount factor}) \\ &= 8.50 \left[\frac{1}{.03} - \frac{1}{.03(1.03)^4} \right] + \frac{100}{(1.03)^4} = 31.59 + 88.85 = €120.44 \end{aligned}$$

¹Bonds used to come with coupons attached, which had to be clipped off and presented to the issuer to obtain the interest payments. This is still the case with *bearer bonds*, where the only evidence of indebtedness is the bond itself. In many parts of the world bearer bonds are still issued and are popular with investors who would rather remain anonymous. The alternative is *registered bonds*, where the identity of the bond's owner is recorded and the coupon payments are sent automatically. OATs are registered bonds.

Thus the bond can be valued as a package of an annuity (the coupon payments) and a single, final payment (the repayment of principal).²

We just used the 3% interest rate to calculate the present value of the OAT. Now we turn the valuation around: If the price of the OAT is 120.44%, what is the interest rate? What return do investors get if they buy the bond? To answer this question, you need to find the value of the variable y that solves the following equation:

$$120.44 = \frac{8.50}{1+y} + \frac{8.50}{(1+y)^2} + \frac{8.50}{(1+y)^3} + \frac{108.50}{(1+y)^4}$$

The rate of return y is called the bond's **yield to maturity**. In this case, we already know that the present value of the bond is €120.44 at a 3% discount rate, so the yield to maturity must be 3.0%. If you buy the bond at 120.44% and hold it to maturity, you will earn a return of 3.0% per year.

Why is the yield to maturity less than the 8.5% coupon payment? Because you are paying €120.44 for a bond with a face value of only €100. You lose the difference of €20.44 if you hold the bond to maturity. On the other hand, you get four annual cash payments of €8.50. (The immediate, *current yield* on your investment is $8.50/120.44 = .071$, or 7.1%.) The yield to maturity blends the return from the coupon payments with the declining value of the bond over its remaining life.

The only general procedure for calculating the yield to maturity is trial and error. You guess at an interest rate and calculate the present value of the bond's payments. If the present value is greater than the actual price, your discount rate must have been too low, and you need to try a higher rate. The more practical solution is to use a spreadsheet program or a specially programmed calculator to calculate the yield. At the end of this chapter, you will find a box which lists the Excel function for calculating yield to maturity plus several other useful functions for bond analysts.

Back to the United States: Semiannual Coupons and Bond Prices

Just like the French government, the U.S. Treasury raises money by regular auctions of new bond issues. Some of these issues do not mature for 20 or 30 years; others, known as *notes*, mature in 10 years or less. The Treasury also issues short-term debt maturing in a year or less. These short-term securities are known as *Treasury bills*. Treasury bonds, notes, and bills are traded in the *fixed-income market*.

Let's look at an example of a U.S. government note. In 2007 the Treasury issued 4.875% notes maturing in 2012. These notes are called "the 4.875s of 2012." Treasury bonds and notes have face values of \$1,000, so if you own the 4.875s of 2012, the Treasury will give you back \$1,000 at maturity. You can also look forward to a regular coupon but, in contrast to our French bond, coupons on Treasury bonds and notes are paid *semiannually*.³ Thus, the 4.875s of 2012 provide a coupon payment of $4.875/2 = 2.4375\%$ of face value every six months.

You can't buy Treasury bonds, notes, or bills on the stock exchange. They are traded by a network of bond dealers, who quote prices at which they are prepared to buy and sell. For example, suppose that in 2009 you decide to buy the 4.875s of 2012. You phone a broker who checks the current price on her screen. If you are happy to go ahead with the purchase, your broker contacts a bond dealer and the trade is done.

The prices at which you can buy or sell Treasury notes and bonds are shown each day in the financial press and on the Web. Figure 3.1 is taken from the *The Wall Street Journal's*

² You could also value a three-year annuity of €8.50 plus a final payment of €108.50.

³ The frequency of interest payments varies from country to country. For example, most euro bonds pay interest annually, while most bonds in the U.K., Canada, and Japan pay interest semiannually.

FIGURE 3.1

Sample Treasury bond quotes from *The Wall Street Journal*, February 2009.

Source: *The Wall Street Journal* Web site, www.wsj.com.

Maturity	Coupon	Bid	Asked	Chg	Asked yield
2010 Feb 15	4.750	104:00	104:01	unch.	0.6651
2011 Feb 15	5.000	108:16	108:18	+5	0.6727
2012 Feb 15	4.875	110:24	110:25	+14	1.2006
2013 Feb 15	3.875	109:27	109:29	+24	1.3229
2014 Feb 15	4.000	111:19	111:21	+30	1.5664
2015 Feb 15	4.000	111:20	111:23	+34	1.9227
2016 Feb 15	4.500	115:03	115:04	+59	1.9856
2017 Feb 15	4.625	115:20	115:21	+65	2.4555
2018 Feb 15	3.500	107:14	107:15	+64	2.5646
2019 Feb 15	2.750	100:23	100:25	+63	2.6622
2020 Feb 15	8.500	147:19	147:21	+83	3.2976

Web page and shows the prices of a small sample of Treasury bonds. Look at the entry for our 4.875s of February 2012. The **asked price** of 110:25 is the price you need to pay to buy the note from a dealer. This price is quoted in 32nds rather than decimals. Thus a price of 110:25 means that each bond costs $110 + 25/32$, or 110.78125% of face value. The face value of the note is \$1,000, so each note costs \$1,107.8125.⁴

The **bid price** is the price investors receive if they sell to a dealer. The dealer earns her living by charging a *spread* between the bid and the asked price. Notice that the spread for the 4.875s of 2012 is only 1/32, or about .03% of the note's value.

The next column in Figure 3.1 shows the change in price since the previous day. The price of the 4.875% notes has risen by 14/32, an unusually large move for a single day. Finally, the column "Asked Yield" shows the **asked yield to maturity**. Because interest is semiannual, yields on U.S. bonds are usually quoted as semiannually compounded yields. Thus, if you buy the 4.875% note at the asked price and hold it to maturity, you earn a semiannually compounded return of 1.2006%. This means that every six months you earn a return of $1.2006/2 = .6003\%$.

You can now repeat the present value calculations that we did for the French government bond. You just need to recognize that bonds in the U.S. have a face value of \$1,000, that their coupons are paid semiannually, and that the quoted yield is a semiannually compounded rate.

Here are the cash payments from the 4.875s of 2012:

Cash Payments (\$)					
Aug. 2009	Feb. 2010	Aug. 2010	Feb. 2011	Aug. 2011	Feb. 2012
\$24.375	\$24.375	\$24.375	\$24.375	\$24.375	\$1,024.375

⁴ The quoted bond price is known as the *flat* (or *clean*) price. The price that the bond buyer actually pays (sometimes called the *full* or *dirty price*) is equal to the flat price *plus* the interest that the seller has already earned on the bond since the last interest payment. The precise method for calculating this *accrued interest* varies from one type of bond to another. We use the flat price to calculate the yield.

If investors demand a semiannual return of .6003%, then the present value of these cash flows is

$$PV = \frac{24.375}{1.006003} + \frac{24.375}{1.006003^2} + \frac{24.375}{1.006003^3} + \frac{24.375}{1.006003^4} + \frac{24.375}{1.006003^5} + \frac{1024.375}{1.006003^6} = \$1107.95$$

Each note is worth \$1,107.95, or 110.795% of face value.

Again we could turn the valuation around: given the price, what's the yield to maturity? Try it, and you'll find (no surprise) that the yield to maturity is $y = .006003$. This is the semiannual rate of return that you can earn over the six remaining half-year periods until the note matures. Take care to remember that the yield is *reported* as an annual rate, calculated as $2 \times .006003 = .012006$, or 1.2006%. If you see a reported yield to maturity of $R\%$, you have to remember to use $y = R/2\%$ as the semiannual rate for discounting cash flows received every six months.

3-2 How Bond Prices Vary with Interest Rates

Figure 3.2 plots the yield to maturity on 10-year U.S. Treasury bonds⁵ from 1900 to 2008. Notice how much the rate fluctuates. For example, interest rates climbed sharply after 1979 when Paul Volcker, the new chairman of the Fed, instituted a policy of tight money to rein in inflation. Within two years the interest rate on 10-year government bonds rose from 9% to a midyear peak of 15.8%. Contrast this with 2008, when investors fled to the safety of U.S. government bonds. By the end of that year long-term Treasury bonds offered a measly 2.2% rate of interest.

As interest rates change, so do bond prices. For example, suppose that investors demanded a semiannual return of 4% on the 4.875s of 2012, rather than the .6003% return we used above. In that case the price would be

$$PV = \frac{24.375}{1.04} + \frac{24.375}{1.04^2} + \frac{24.375}{1.04^3} + \frac{24.375}{1.04^4} + \frac{24.375}{1.04^5} + \frac{1024.375}{1.04^6} = \$918.09$$

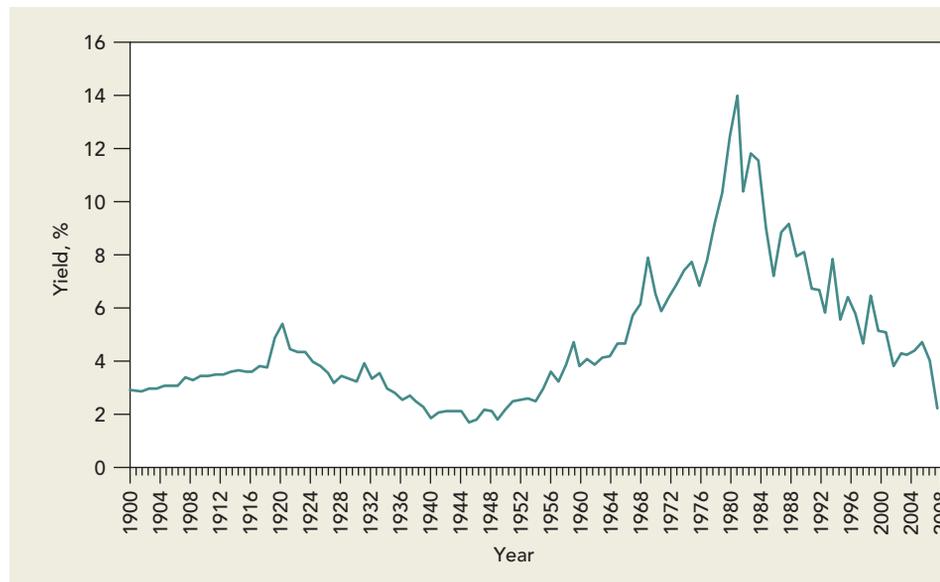


FIGURE 3.2
The interest rate on 10-year U.S. Treasury bonds.

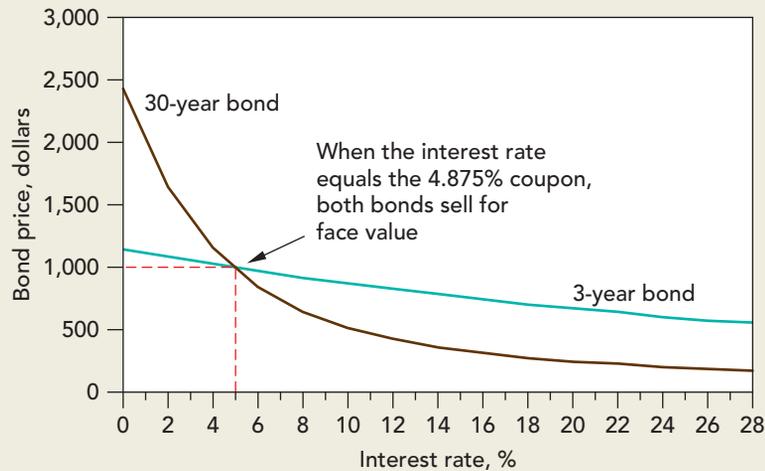
⁵ From this point forward, we will just say “bonds,” and not distinguish notes from bonds unless we are referring to a specific security. Note also that bonds with long maturities end up with short maturities when they approach the final payment date. Thus you will encounter 30-year bonds trading 20 years later at the same prices as new 10-year notes (assuming equal coupons).

FIGURE 3.3

Plot of bond prices as a function of the interest rate. The price of long-term bonds is more sensitive to changes in the interest rate than is the price of short-term bonds.

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The higher interest rate results in a lower price.

Bond prices and interest rates *must* move in opposite directions. The yield to maturity, our measure of the interest rate on a bond, is *defined* as the discount rate that explains the bond price. When bond prices fall, interest rates (that is, yields to maturity) must rise. When interest rates rise, bond prices must fall. We recall a hapless TV pundit who intoned, “The recent decline in long-term interest rates suggests that long-term bond prices may rise over the next week or two.” Of course the bond prices had already gone up. We are confident that you won’t make the pundit’s mistake.

The solid green line in Figure 3.3 shows the value of our 4.875% note for different interest rates. As the yield to maturity falls, the bond price increases. When the annual yield is equal to the note’s annual coupon rate (4.875%), the note sells for exactly its face value. When the yield is higher than 4.875%, the note sells at a discount to face value. When the yield is lower than 4.875%, the note sells at a premium.

Bond investors cross their fingers that market interest rates will fall, so that the price of their securities will rise. If they are unlucky and interest rates jump up, the value of their investment declines.

A change in interest rates has only a modest impact on the value of near-term cash flows but a much greater impact on the value of distant cash flows. Thus the price of long-term bonds is affected more by changing interest rates than the price of short-term bonds. For example, compare the two curves in Figure 3.3. The green line shows how the price of the three-year 4.875% note varies with the interest rate. The brown line shows how the price of a 30-year 4.875% bond varies. You can see that the 30-year bond is much more sensitive to interest rate fluctuations than the three-year note.

Duration and Volatility

Changes in interest rates have a greater impact on the prices of long-term bonds than on those of short-term bonds. But what do we mean by “long-term” and “short-term”? A coupon bond that matures in year 30 makes payments in *each* of years 1 through 30. It’s misleading to describe the bond as a 30-year bond; the average time to each cash payment is less than 30 years.

EXAMPLE 3.1 • Which Is the Longest-Term Bond?

A strip is a special type of Treasury bond that repays principal at maturity, but makes no coupon payments along the way. Strips are also called *zero-coupon* bonds. (We cover strips in more detail in the next section.)

Consider a strip maturing in February 2015 and two coupon bonds maturing on the same date. Table 3.1 calculates the prices of these three Treasuries, assuming a yield to maturity of 2% per year. Take a look at the time pattern of each bond's cash payments and review how the prices are calculated.

Which of the three Treasuries is the longest-term investment? They have the same final maturity, of course, February 2015. But the *timing* of the bonds' cash payments is not the same. The two coupon bonds deliver cash payments earlier than the strip, so the strip has the longest effective maturity. The *average* maturity of the 4s is in turn longer than that of the 11 1/4s, because the 4s deliver relatively more of their cash flows at maturity, when the face value is paid off. The 11 1/4s have the shortest average maturity, because a greater fraction of this bond's cash payments comes as coupons rather than the final payment of face value.

Bond	Price (%)		Cash payments %		
	Feb. 2009	Aug. 2009	Feb. 2010. Aug. 2014	Feb. 2015
Strip for Feb. 2015	88.74	0	0 0	100.00
4s of Feb. 2015	111.26	2.00	2.00 2.00	102.00
11 1/4s of Feb. 2015	152.05	5.625	5.625 5.625	105.625

TABLE 3.1 A comparison of the cash flows and prices of three Treasuries in February 2009, assuming a yield to maturity of 2%.

Note: All three securities mature in February 2015.

Investors and financial managers calculate a bond's average maturity by its **duration**. They keep track of duration because it measures the exposure of the bond's price to fluctuations in interest rates. Duration is often called *Macaulay duration* after its inventor.

Duration is the weighted average of the times when the bond's cash payments are received. The times are the future years 1, 2, 3, etc., extending to the final maturity date, which we call T . The weight for each year is the present value of the cash flow received at that time divided by the total present value of the bond.

$$\text{Duration} = \frac{1 \times \text{PV}(C_1)}{\text{PV}} + \frac{2 \times \text{PV}(C_2)}{\text{PV}} + \frac{3 \times \text{PV}(C_3)}{\text{PV}} + \dots + \frac{T \times \text{PV}(C_T)}{\text{PV}}$$

Table 3.2 shows how to compute duration for the French OATs maturing in 2012. First, we value each of the three annual coupon payments of €8.50 and the final payment of coupon plus face value of €108.50. Of course the present values of these payments add up to the bond price of €120.44. Then we calculate the fraction of the price accounted for by each cash flow and multiply each fraction by the year of the cash flow. The results sum across to a duration of 3.60 years.

Year (t)	1	2	3	4	
Cash payment (C_t)	€8.50	€8.50	€8.50	€108.50	
PV(C_t) at 3%	€8.25	€8.01	€7.78	€96.40	PV = €120.44
Fraction of total value [PV(C_t)/PV]	0.069	0.067	0.065	0.800	
Year \times Fraction of total value [$t \times$ PV(C_t)/PV]	0.069	0.133	0.194	3.202	Total = duration = 3.60

TABLE 3.2 Calculating duration for the French OATs maturing in 2012. The yield to maturity is 3% per year.

Table 3.3 shows the same calculation for the 11 $\frac{1}{4}$ % U.S. Treasury bond maturing in February 2015. The present value of each cash payment is calculated using a 2% yield to maturity. Again we calculate the fraction of the price accounted for by each cash flow and multiply each fraction by the year. The calculations look more formidable than in Table 3.2, but only because the final maturity date is 2016 rather than 2012 and coupons are paid semiannually. Thus in Table 3.3 we have to track 12 dates rather than 4. The duration of the 11 $\frac{1}{4}$ s equals 4.83 years.

We leave it to you to calculate durations for the other two bonds in Table 3.1. You will find that duration increases to 5.43 years for the 4s of 2015. The duration of the strip is six years exactly, the same as its maturity. Because there are no coupons, 100% of the strip's value comes from payment of principal in year 6.

We mentioned that investors and financial managers track duration because it measures how bond prices change when interest rates change. For this purpose it's best to use *modified duration* or *volatility*, which is just duration divided by one plus the yield to maturity:

$$\text{Modified duration} = \text{volatility (\%)} = \frac{\text{duration}}{1 + \text{yield}}$$

Date	Aug. 2009	Feb. 2010	Aug. 2010	Feb. 2011	...	Aug. 2013	Feb. 2014	Aug. 2014	Feb. 2015	
Year (t)	0.5	1.0	1.5	2.0	...	4.5	5.0	5.5	6.0	
Cash payment (C_t)	5.63	5.63	5.63	5.63	...	5.63	5.63	5.63	105.625	
PV(C_t) at 2%	5.57	5.51	5.46	5.41	...	5.14	5.09	5.04	93.74	PV = 152.05
fraction of total value [PV(C_t)/PV]	0.0366	0.0363	0.0359	0.0355	...	0.0338	0.0335	0.0332	0.6165	
Year \times fraction of total value [$t \times$ PV(C_t)/PV]	0.0183	0.0363	0.0539	0.0711	...	0.1522	0.1674	0.1824	3.6988	Total = duration = 4.83
Duration (years) = 4.83										

TABLE 3.3 Calculating the duration of the 11 $\frac{1}{4}$ % Treasuries of 2015. The yield to maturity is 2%.

Modified duration measures the percentage change in bond price for a 1 percentage-point change in yield.⁶ Let's try out this formula for the OAT bond in Table 3.2. The bond's modified duration is $\text{duration}/(1 + \text{yield}) = 3.60/1.03 = 3.49\%$. This means that a 1% change in the yield to maturity should change the bond price by 3.49%.

Let's check that prediction. Suppose the yield to maturity either increases or declines by .5%:

Yield to Maturity	Price	Change (%)
3.5%	118.37	- 1.767
3.0	120.44	—
2.5	122.57	+ 1.726

The total difference between price at 2.5% and 3.5% is $1.767 + 1.726 = 3.49\%$. Thus a 1% change in interest rates means a 3.49% change in bond price, just as predicted.

The modified duration for the 11¼% U.S. Treasury in Table 3.3 is $4.83/1.02 = 4.74\%$. In other words, a 1% change in yield to maturity results in a 4.74% change in the bond's price. Modified durations for the other bonds in Table 3.1 are larger, which means more exposure of price to changes in interest rates. For example, the modified duration of the strip is $6/1.02 = 5.88\%$.

You can see why duration (or modified duration) is a handy measure of interest-rate risk. For example, investment managers regularly monitor the duration of their bond portfolios to ensure that they are not running undue risk.⁷

3-3 The Term Structure of Interest Rates

When we explained in Chapter 2 how to calculate present values, we used the same discount rate to calculate the value of each period's cash flow. A single yield to maturity y can also be used to discount all future cash payments from a bond. For many purposes, using a single discount rate is a perfectly acceptable approximation, but there are also occasions when you need to recognize that short-term interest rates are different from long-term rates.

The relationship between short- and long-term interest rates is called the **term structure of interest rates**. Look for example at Figure 3.4, which shows the term structure in two different years. Notice that in the later year the term structure sloped downward; long-term interest rates were lower than short-term rates. In the earlier year the pattern was reversed and long-term bonds offered a much higher interest rate than short-term bonds. You now need to learn how to measure the term structure and understand why long- and short-term rates differ.

Consider a simple loan that pays \$1 at the end of one year. To find the present value of this loan you need to discount the cash flow by the one-year rate of interest rate, r_1 :

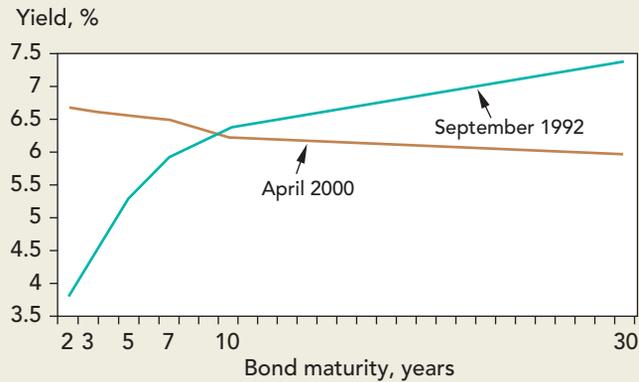
$$PV = 1/(1 + r_1)$$

⁶ In other words, the derivative of the bond price with respect to a change in yield to maturity is $dPV/dy = -\text{duration}/(1 + y) = -\text{modified duration}$.

⁷ The portfolio duration is a weighted average of the durations of the bonds in the portfolio. The weight for each bond is the fraction of the portfolio invested in that bond. Note that as time passes and interest rates change, the portfolio manager needs to recalculate duration.

FIGURE 3.4

Short- and long-term interest rates do not always move in parallel. Between September 1992 and April 2000 U.S. short-term rates rose sharply while long-term rates declined.



This rate, r_1 is called the one-year **spot rate**. To find the present value of a loan that pays \$1 at the end of two years, you need to discount by the two-year spot rate, r_2 :

$$PV = 1/(1 + r_2)^2$$

The first year's cash flow is discounted at today's one-year spot rate and the second year's flow is discounted at today's two-year spot rate. The series of spot rates $r_1, r_2, \dots, r_b, \dots$ traces out the term structure of interest rates.

Now suppose you have to value \$1 paid at the end of years 1 *and* 2. If the spot rates are different, say $r_1 = 3\%$ and $r_2 = 4\%$, then we need two discount rates to calculate present value:

$$PV = \frac{1}{1.03} + \frac{1}{1.04^2} = 1.895$$

Once we know that $PV = 1.895$, we can go on to calculate a single discount rate that would give the right answer. That is, we could calculate the yield to maturity by solving for y in the following equation:

$$PV = 1.895 = \frac{1}{(1 + y)} + \frac{1}{(1 + y)^2}$$

This gives a yield to maturity of 3.66%. Once we have the yield, we could use it to value other two-year annuities. But we can't get the yield to maturity until we know the price. The price is determined by the spot interest rates for dates 1 and 2. Spot rates come first. Yields to maturity come later, after bond prices are set. That is why professionals identify spot interest rates and discount each cash flow at the spot rate for the date when the cash flow is received.

Spot Rates, Bond Prices, and the Law of One Price

The *law of one price* states that the same commodity must sell at the same price in a well-functioning market. Therefore, all safe cash payments delivered on the same date must be discounted at the same spot rate.

Table 3.4 illustrates how the law of one price applies to government bonds. It lists four government bonds, which we assume make annual coupon payments. We have put at the top the shortest-duration bond, the 8% coupon bond maturing in year 2, and we've put at the bottom the longest-duration bond, the 4-year strip. Of course the strip pays off only at maturity.

	Year (t)				Bond Price (PV)	Yield to Maturity (y , %)
	1	2	3	4		
Spot rates	.035	.04	.042	.044		
Discount factors	.9662	.9246	.8839	.8418		
Bond A (8% coupon):						
Payment (C_t)	\$80	1,080				
PV(C_t)	\$77.29	998.52			\$1,075.82	3.98
Bond B (11% coupon):						
Payment (C_t)	\$110	110	1,110			
PV(C_t)	\$106.28	101.70	981.11		\$1,189.10	4.16
Bond C (6% coupon):						
Payment (C_t)	\$60	60	60	1,060		
PV(C_t)	\$57.97	55.47	53.03	892.29	\$1,058.76	4.37
Bond D (strip):						
Payment (C_t)				\$1,000		
PV(C_t)				\$841.78	\$841.78	4.40

TABLE 3.4 The law of one price applied to government bonds.

Excel
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Spot rates and discount factors are given at the top of each column. The law of one price says that investors place the same value on a risk-free dollar regardless of whether it is provided by bond A, B, C, or D. You can check that the law holds in the table.

Each bond is priced by adding the present values of each of its cash flows. Once total PV is calculated, we have the bond price. Only then can the yield to maturity be calculated.

Notice how the yield to maturity increases as bond maturity increases. The yields increase with maturity because the term structure of spot rates is upward-sloping. Yields to maturity are complex averages of spot rates.

Financial managers who want a quick, summary measure of interest rates bypass spot interest rates and look in the financial press at yields to maturity. They may refer to the *yield curve*, which plots yields to maturity, instead of referring to the term structure, which plots spot rates. They may use the yield to maturity on one bond to value another bond with roughly the same coupon and maturity. They may speak with a broad brush and say, “Ampersand Bank will charge us 6% on a three-year loan,” referring to a 6% yield to maturity.

Throughout this book, we too use the yield to maturity to summarize the return required by bond investors. But you also need to understand the measure’s limitations when spot rates are not equal.

Measuring the Term Structure

You can think of the spot rate, r_t , as the rate of interest on a bond that makes a single payment at time t . Such simple bonds do exist—we have already seen examples. They are known as **stripped bonds**, or **strips**. On request the U.S. Treasury will split a normal coupon bond into a package of mini-bonds, each of which makes just one cash payment. Our 4.875% notes of 2012 could be exchanged for six semiannual coupon strips, each paying

\$24.375, and a principal strip paying \$1,000. In February 2009 this package of coupon strips would have cost \$143.83 and the principal strip would have cost \$964.42, making a total cost of \$1,108.25, just a few cents more than it cost to buy one 4.875% note. That should be no surprise. Because the two investments provide identical cash payments, they must sell for very close to the same price.

We can use the prices of strips to measure the term structure of interest rates. For example, in February 2009 a 10-year strip cost \$714.18. In return, investors could look forward to a single payment of \$1,000 in February 2019. Thus investors were prepared to pay \$.71418 for the promise of \$1 at the end of 10 years. The 10-year discount factor was $DF_{10} = 1/(1 + r_{10})^{10} = .71418$, and the 10-year spot rate was $r_{10} = (1/.71418)^{10} - 1 = .0342$, or 3.42%. In Figure 3.5 we use the prices of strips with different maturities to plot the term structure of spot rates from 1 to 10 years. You can see that in 2009 investors required a somewhat higher interest rate for lending for 10 years rather than for 1.

Why the Discount Factor Declines As Futurity Increases—and a Digression on Money Machines

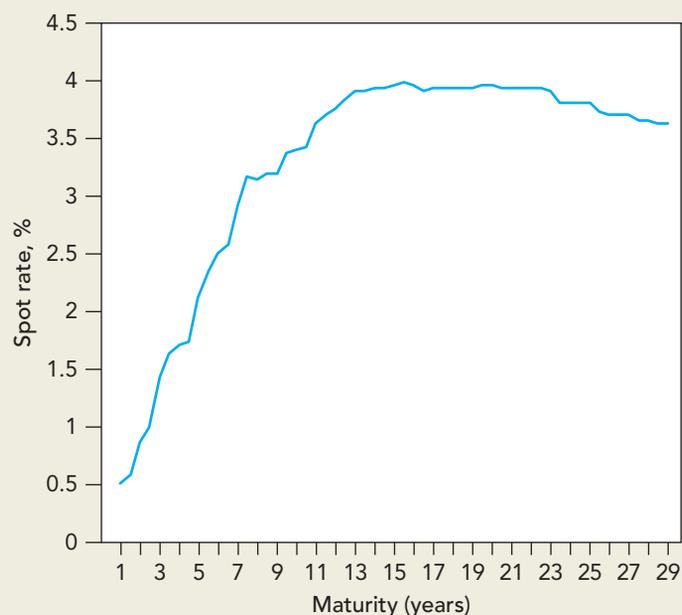
In Chapter 2 we saw that the longer you have to wait for your money, the less is its present value. In other words, the two-year discount factor $DF_2 = 1/(1 + r_2)^2$ is less than the one-year discount factor $DF_1 = 1/(1 + r_1)$. But is this *necessarily* the case when there can be a different spot interest rate for each period?

Suppose that the one-year spot rate of interest is $r_1 = 20\%$, and the two-year spot rate is $r_2 = 7\%$. In this case the one-year discount factor is $DF_1 = 1/1.20 = .833$ and the two-year discount factor is $DF_2 = 1/1.07^2 = .873$. Apparently a dollar received the day after tomorrow is *not* necessarily worth less than a dollar received tomorrow.

But there is something wrong with this example. Anyone who could borrow and invest at these interest rates could become a millionaire overnight. Let us see how such a “money machine” would work. Suppose the first person to spot the opportunity is

FIGURE 3.5

Spot rates on U.S. Treasury strips, February 2009.



Hermione Kraft. Ms. Kraft first buys a one-year Treasury strip for $.833 \times \$1,000 = \833 . Now she notices that there is a way to earn an *immediate* surefire profit on this investment. She reasons as follows. Next year the strip will pay off \$1,000 that can be reinvested for a further year. Although she does not know what interest rates will be at that time, she does know that she can always put the money in a checking account and be certain of having \$1,000 at the end of year 2. Her next step, therefore, is to go to her bank and borrow the present value of this \$1,000. At 7% interest the present value is $PV = 1000/(1.07)^2 = \$873$.

So Ms. Kraft borrows \$873, invests \$830, and walks away with a profit of \$43. If that does not sound like very much, notice that by borrowing more and investing more she can make much larger profits. For example, if she borrows \$21,778,584 and invests \$20,778,584, she would become a millionaire.⁸

Of course this story is completely fanciful. Such an opportunity would not last long in well-functioning capital markets. Any bank that allowed you to borrow for two years at 7% when the one-year interest rate was 20% would soon be wiped out by a rush of small investors hoping to become millionaires and a rush of millionaires hoping to become billionaires. There are, however, two lessons to our story. The first is that a dollar tomorrow *cannot* be worth less than a dollar the day after tomorrow. In other words, the value of a dollar received at the end of one year (DF_1) cannot be less than the value of a dollar received at the end of two years (DF_2). There must be some extra gain from lending for two periods rather than one: $(1 + r_2)^2$ cannot be less than $1 + r_1$.

Our second lesson is a more general one and can be summed up by this precept: “There is no such thing as a surefire money machine.” The technical term for money machine is **arbitrage**. In well-functioning markets, where the costs of buying and selling are low, arbitrage opportunities are eliminated almost instantaneously by investors who try to take advantage of them.

Later in the book we invoke the *absence* of arbitrage opportunities to prove several useful properties about security prices. That is, we make statements like, “The prices of securities X and Y must be in the following relationship—otherwise there would be potential arbitrage profits and capital markets would not be in equilibrium.”

3-4 Explaining the Term Structure

The term structure that we showed in Figure 3.5 was upward-sloping. Long-term rates of interest in February 2009 were more than 3.5%; short-term rates were 1% or less. Why then didn't everyone rush to buy long-term bonds? Who were the (foolish?) investors who put their money into the short end of the term structure?

Suppose that you held a portfolio of one-year U.S. Treasuries in February 2009. Here are three possible reasons why you might decide to hold on to them, despite their low rate of return:

1. You believe that short-term interest rates will be higher in the future.
2. You worry about the greater exposure of long-term bonds to changes in interest rates.
3. You worry about the risk of higher future inflation.

We review each of these reasons now.

⁸ We exaggerate Ms. Kraft's profits. There are always costs to financial transactions, though they may be very small. For example, Ms. Kraft could use her investment in the one-year strip as security for the bank loan, but the bank would need to charge more than 7% on the loan to cover its costs.

Expectations Theory of the Term Structure

Recall that you own a portfolio of one-year Treasuries. A year from now, when the Treasuries mature, you can reinvest the proceeds for another one-year period and enjoy whatever interest rate the bond market offers then. The interest rate for the second year may be high enough to offset the first year's low return. You often see an upward-sloping term structure when future interest rates are expected to rise.

EXAMPLE 3.2 ● Expectations and the Term Structure

Suppose that the one-year interest rate, r_1 , is 5%, and the two-year rate, r_2 , is 7%. If you invest \$100 for one year, your investment grows to $100 \times 1.05 = \$105$; if you invest for two years, it grows to $100 \times 1.07^2 = \$114.49$. The *extra* return that you earn for that second year is $1.07^2/1.05 - 1 = .090$, or 9.0%.⁹

Would you be happy to earn that extra 9% for investing for two years rather than one? The answer depends on how you expect interest rates to change over the coming year. If you are confident that in 12 months' time one-year bonds will yield more than 9.0%, you would do better to invest in a one-year bond and, when that matured, reinvest the cash for the next year at the higher rate. If you forecast that the future one-year rate is exactly 9.0%, then you will be indifferent between buying a two-year bond or investing for one year and then rolling the investment forward at next year's short-term interest rate.

If everyone is thinking as you just did, then the two-year interest rate has to adjust so that everyone is equally happy to invest for one year or two. Thus the two-year rate will incorporate both today's one-year rate and the consensus forecast of next year's one-year rate.

We have just illustrated (in Example 3.2) the **expectations theory** of the term structure. It states that in equilibrium investment in a series of short-maturity bonds must offer the same expected return as an investment in a single long-maturity bond. Only if that is the case would investors be prepared to hold both short- and long-maturity bonds.

The expectations theory implies that the *only* reason for an upward-sloping term structure is that investors expect short-term interest rates to rise; the *only* reason for a declining term structure is that investors expect short-term rates to fall.

If short-term interest rates are significantly lower than long-term rates, it is tempting to borrow short-term rather than long-term. The expectations theory implies that such naïve strategies won't work. If short-term rates are lower than long-term rates, then investors must be expecting interest rates to rise. When the term structure is upward-sloping, you are likely to make money by borrowing short only if investors are *overestimating* future increases in interest rates.

Even at a casual glance the expectations theory does not seem to be the complete explanation of term structure. For example, if we look back over the period 1900–2008, we find that the return on long-term U.S. Treasury bonds was on average 1.5 percentage points higher than the return on short-term Treasury bills. Perhaps short-term interest rates stayed lower than investors expected, but it seems more likely that investors wanted some extra return for holding long bonds and that on average they got it. If so, the expectations theory is only a first step.

These days the expectations theory has few strict adherents. Nevertheless, most economists believe that expectations about future interest rates have an important effect on the term structure. For example, you will hear market commentators remark that the six-month interest rate is higher than the three-month rate and conclude that the market is expecting the

⁹ The extra return for lending for one more year is termed the *forward rate of interest*. In our example the forward rate is 9.0%. In Ms. Kraft's arbitrage example, the forward interest rate was negative. In real life, forward interest rates can't be negative. At the lowest they are zero.

Federal Reserve Board to raise interest rates. There is good evidence for this type of reasoning. Suppose that every month from 1950 to 2008 you used the extra return from lending for six months rather than three to predict the likely change in interest rates. You would have found on average that the steeper the term structure, the more that interest rates subsequently rose. So it looks as if the expectations theory has some truth to it even if it is not the whole truth.

Introducing Risk

What does the expectations theory leave out? The most obvious answer is “risk.” If you are confident about the future level of interest rates, you will simply choose the strategy that offers the highest return. But, if you are not sure of your forecasts, you may well opt for a less risky strategy even if it means giving up some return.

Remember that the prices of long-duration bonds are more volatile than prices of short-duration bonds. A sharp increase in interest rates can knock 30% or 40% off the price of long-term bonds.

For some investors, this extra volatility of long-duration bonds may not be a concern. For example, pension funds and life insurance companies have fixed long-term liabilities, and may prefer to lock in future returns by investing in long-term bonds. However, the volatility of long-term bonds *does* create extra risk for investors who do not have such long-term obligations. These investors will be prepared to hold long bonds only if they offer the compensation of a higher return. In this case the term structure will be upward-sloping more often than not. Of course, if interest rates are expected to fall, the term structure could be downward-sloping and still reward investors for lending long. But the additional reward for risk offered by long bonds would result in a less dramatic downward slope.

Inflation and Term Structure

Suppose you are saving for your retirement 20 years from now. Which of the following strategies is more risky? Invest in a succession of one-year Treasuries, rolled over annually, or invest once in 20-year strips? The answer depends on how confident you are about future inflation.

If you buy the 20-year strips, you know exactly how much money you will have at year 20, but you don’t know what that money will buy. Inflation may seem benign now, but who knows what it will be in 10 or 15 years? This uncertainty about inflation may make it uncomfortably risky for you to lock in one 20-year interest rate by buying the strips. This was a problem facing investors in 2009, when no one could be sure whether the country was facing the prospect of prolonged deflation or whether the high levels of government borrowing would prompt rapid inflation.

You can reduce exposure to inflation risk by investing short-term and rolling over the investment. You do not know future short-term interest rates, but you do know that future interest rates will adapt to inflation. If inflation takes off, you will probably be able to roll over your investment at higher interest rates.

If inflation is an important source of risk for long-term investors, borrowers must offer some extra incentive if they want investors to lend long. That is why we often see a steeply upward-sloping term structure when inflation is particularly uncertain.

3-5 Real and Nominal Rates of Interest

It is now time to review more carefully the relation between inflation and interest rates. Suppose you invest \$1,000 in a one-year bond that makes a single payment of \$1,100 at the end of the year. Your cash flow is certain, but the government makes no promises about what that money will buy. If the prices of goods and services increase by more than 10%, you will lose ground in terms of purchasing power.

Several indexes are used to track the general level of prices. The best known is the Consumer Price Index (CPI), which measures the number of dollars that it takes to pay for a typical family's purchases. The change in the CPI from one year to the next measures the rate of inflation.

Figure 3.6 shows the rate of inflation in the U.S. since 1900. Inflation touched a peak at the end of World War I, when it reached 21%. However, this figure pales into insignificance compared with the hyperinflation in Zimbabwe in 2008. Prices there rose so fast that a Z\$50 trillion bill was barely enough to buy a loaf of bread.

Prices can fall as well as rise. The U.S. experienced severe *deflation* in the Great Depression, when prices fell by 24% in three years. More recently, consumer prices in Hong Kong fell by nearly 15% in the six years from 1999 to 2004.

The *average* U.S. inflation rate from 1900 to 2008 was 3.1%. As you can see from Figure 3.7, among major economies the U.S. has been almost top of the class in holding inflation in check. Countries torn by war have generally experienced much higher

FIGURE 3.6

Annual rates of inflation in the United States from 1900–2008.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

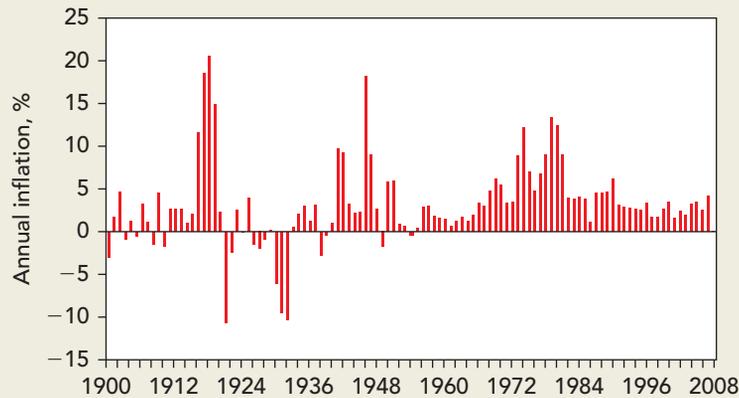
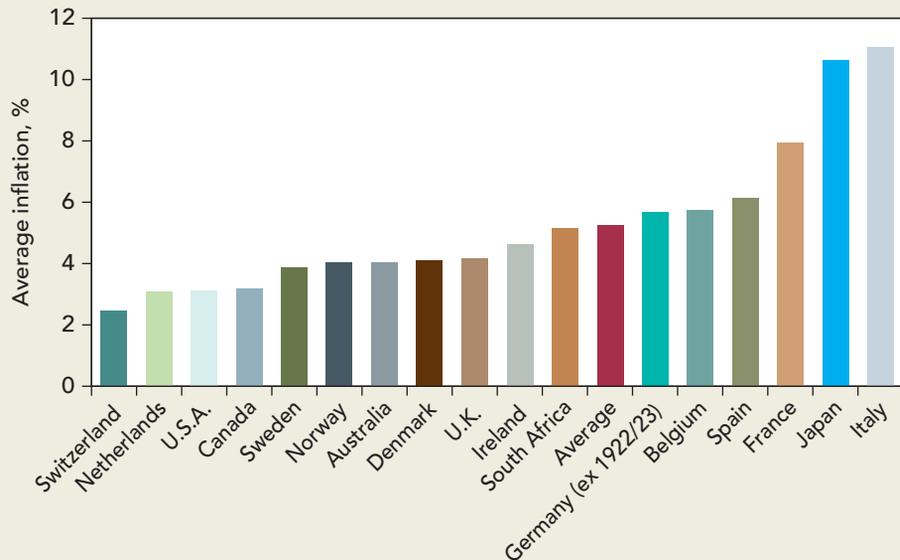


FIGURE 3.7

Average rates of inflation in 17 countries from 1900–2008.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.



inflation. For example, in Italy and Japan, inflation since 1900 has averaged about 11% a year.

Economists and financial managers refer to current, or *nominal*, dollars versus constant, or *real*, dollars. For example, the nominal cash flow from your one-year bond is \$1,100. But if prices rise over the year by 6%, then each dollar will buy you 6% less next year than it does today. So at the end of the year \$1,100 has the same purchasing power as $1,100/1.06 = \$1,037.74$ today. The nominal payoff on the bond is \$1,100, but the real payoff is only \$1,037.74.

The formula for converting nominal cash flows in a future period t to real cash flows today is

$$\text{Real cash flow at date } t = \frac{\text{Nominal cash flow at date } t}{(1 + \text{inflation rate})^t}$$

For example, suppose you invest in a 20-year Treasury strip, but inflation over the 20 years averages 6% per year. The strip pays \$1,000 in year 20, but the real value of that payoff is only $1,000/1.06^{20} = \$311.80$. In this example, the purchasing power of \$1 today declines to just over \$.31 after 20 years.

These examples show you how to get from nominal to real cash flows. The journey from nominal to real interest rates is similar. When a bond dealer says that your bond yields 10%, she is quoting a nominal interest rate. That rate tells you how rapidly your money will grow, say over one year:

Invest Current Dollars	Receive Dollars in Year 1	Result
\$1,000 →	\$1,100	10% <i>nominal</i> rate of return

However, with an expected inflation rate of 6%, you are only 3.774% better off at the end of the year than at the start:

Invest Current Dollars	Expected Real Value of Dollars in Year 1	Result
\$1,000 →	\$1,037.74 (= 1,100/1.06)	3.774% expected <i>real</i> rate of return

Thus, we could say, “The bond offers a 10% nominal rate of return,” or “It offers a 3.774% expected real rate of return.”

The formula for calculating the real rate of return is:

$$1 + r_{\text{real}} = (1 + r_{\text{nominal}})/(1 + \text{inflation rate})$$

In our example, $1 + r_{\text{real}} = 1.10/1.06 = 1.03774$.¹⁰

Indexed Bonds and the Real Rate of Interest

Most bonds are like our U.S. Treasury bonds; they promise you a fixed *nominal* rate of interest. The *real* interest rate that you receive is uncertain and depends on inflation. If the inflation rate turns out to be higher than you expected, the real return on your bonds will be lower.

¹⁰ A common rule of thumb states that $r_{\text{real}} = r_{\text{nominal}} - \text{inflation rate}$. In our example this gives $r_{\text{real}} = .10 - .06 = .04$, or 4%. This is not a bad approximation to the true real interest rate of 3.774%. But when inflation is high, it pays to use the full formula.

You *can* nail down a real return, however. You do so by buying an *indexed bond* that makes cash payments linked to inflation. Indexed bonds have been around in many other countries for decades, but they were almost unknown in the United States until 1997, when the U.S. Treasury began to issue inflation-indexed bonds known as TIPS (Treasury Inflation-Protected Securities).¹¹

The real cash flows on TIPS are fixed, but the nominal cash flows (interest and principal) increase as the CPI increases.¹² For example, suppose that the U.S. Treasury issues 3% 20-year TIPS at a price equal to its face value of \$1,000. If during the first year the CPI rises by 10%, then the coupon payment on the bond increases by 10% from \$30 to $30 \times 1.10 = \$33$. The amount that you will be paid at maturity also increases to $\$1,000 \times 1.10 = \$1,100$. The purchasing power of the coupon and face value remain constant at $\$33/1.10 = \30 and $\$1,100/1.10 = \$1,000$. Thus, an investor who buys the bond at the issue price earns a real interest rate of 3%.

Long-term TIPS offered a yield of about 1.7% in February 2009. This is a *real* yield to maturity. It measures the extra goods and services your investment will allow you to buy. The 1.7% yield on TIPS was about 1.0% less than the nominal yield on ordinary Treasury bonds. If the annual inflation rate turns out to be higher than 1.0%, investors will earn a higher return by holding long-term TIPS; if the inflation rate turns out to be less than 1.0%, they would have been better off with nominal bonds.

What Determines the Real Rate of Interest?

The real rate of interest depends on people's willingness to save (the supply of capital)¹³ and the opportunities for productive investment by governments and businesses (the demand for capital). For example, suppose that investment opportunities generally improve. Firms have more good projects, so they are willing to invest more than previously at the current real interest rate. Therefore, the rate has to rise to induce individuals to save the additional amount that firms want to invest.¹⁴ Conversely, if investment opportunities deteriorate, there will be a fall in the real interest rate.

Thus the real rate of interest depends on the balance of saving and investment in the overall economy.¹⁵ A high aggregate willingness to save may be associated with high aggregate wealth (because wealthy people usually save more), an uneven distribution of wealth (an even distribution would mean fewer rich people who do most of the saving), and a high proportion of middle-aged people (the young don't need to save and the old don't want to—"You can't take it with you"). A high propensity to invest may be associated with a high level of industrial activity or major technological advances.

Real interest rates do change but they do so gradually. We can see this by looking at the U.K., where the government has issued indexed bonds since 1982. The green line in

¹¹ Indexed bonds were not completely unknown in the United States before 1997. For example, in 1780 American Revolutionary soldiers were compensated with indexed bonds that paid the value of "five bushels of corn, 68 pounds and four-seventh parts of a pound of beef, ten pounds of sheep's wool, and sixteen pounds of sole leather."

¹² The reverse happens if there is deflation. In this case the coupon payment and principal amount are adjusted downward. However, the U.S. government guarantees that when the bond matures it will not pay less than its original face value.

¹³ Some of this saving is done indirectly. For example, if you hold 100 shares of IBM stock, and IBM retains and reinvests earnings of \$1.00 a share, IBM is saving \$100 on your behalf. The government may also oblige you to save by raising taxes to invest in roads, hospitals, etc.

¹⁴ We assume that investors save more as interest rates rise. It doesn't have to be that way. Suppose that 20 years hence you will need \$50,000 in today's dollars for your children's college tuition. How much will you have to set aside today to cover this obligation? The answer is the present value of a real expenditure of \$50,000 after 20 years, or $50,000/(1 + \text{real interest rate})^{20}$. The higher the real interest rate, the lower the present value and the less you have to set aside.

¹⁵ Short- and medium-term real interest rates are also affected by the monetary policy of central banks. For example, sometimes central banks keep short-term nominal interest rates low despite significant inflation. The resulting real rates can be negative. Nominal interest rates cannot be negative, however, because investors can simply hold cash. Cash always pays zero interest, which is better than negative interest.

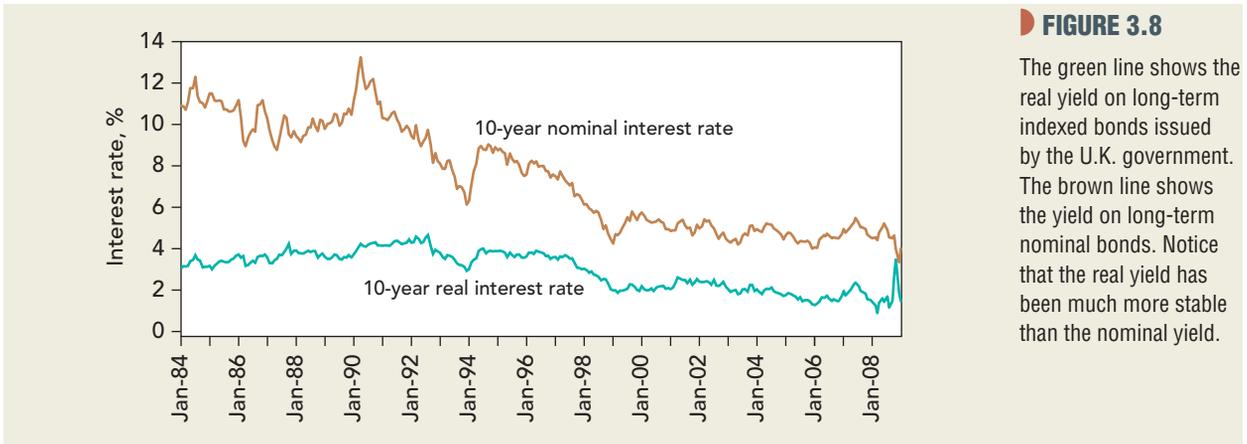


FIGURE 3.8

The green line shows the real yield on long-term indexed bonds issued by the U.K. government. The brown line shows the yield on long-term nominal bonds. Notice that the real yield has been much more stable than the nominal yield.

Figure 3.8 shows that the real yield to maturity on these bonds has fluctuated within a relatively narrow range, while the yield on nominal government bonds (the brown line) has declined dramatically.

Inflation and Nominal Interest Rates

How does the inflation outlook affect the nominal rate of interest? Here is how economist Irving Fisher answered the question. Suppose that consumers are equally happy with 100 apples today or 103 apples in a year's time. In this case the real or "apple" interest rate is 3%. If the price of apples is constant at (say) \$1 each, then we will be equally happy to receive \$100 today or \$103 at the end of the year. That extra \$3 will allow us to buy 3% more apples at the end of the year than we could buy today.

But suppose now that the apple price is expected to increase by 5% to \$1.05 each. In that case we would *not* be happy to give up \$100 today for the promise of \$103 next year. To buy 103 apples in a year's time, we will need to receive $1.05 \times \$103 = \108.15 . In other words, the nominal rate of interest must increase by the expected rate of inflation to 8.15%.

This is Fisher's theory: A change in the expected inflation rate causes the same proportionate change in the *nominal* interest rate; it has no effect on the required real interest rate. The formula relating the nominal interest rate and expected inflation is

$$1 + r_{\text{nominal}} = (1 + r_{\text{real}})(1 + i)$$

where r_{real} is the real interest rate that consumers require and i is the expected inflation rate. In our example, the prospect of inflation causes $1 + r_{\text{nominal}}$ to rise to $1.03 \times 1.05 = 1.0815$.

Not all economists would agree with Fisher that the real rate of interest is unaffected by the inflation rate. For example, if changes in prices are associated with changes in the level of industrial activity, then in inflationary conditions I might want more or less than 103 apples in a year's time to compensate me for the loss of 100 today.

We wish we could show you the past behavior of interest rates and *expected* inflation. Instead we have done the next best thing and plotted in Figure 3.9 the return on Treasury bills (short-term government debt) against *actual* inflation for the U.S., U.K., and Germany. Notice that since 1953 the return on Treasury bills has generally been a little above the rate of inflation. Investors in each country earned an average real return of between 1% and 2% during this period.

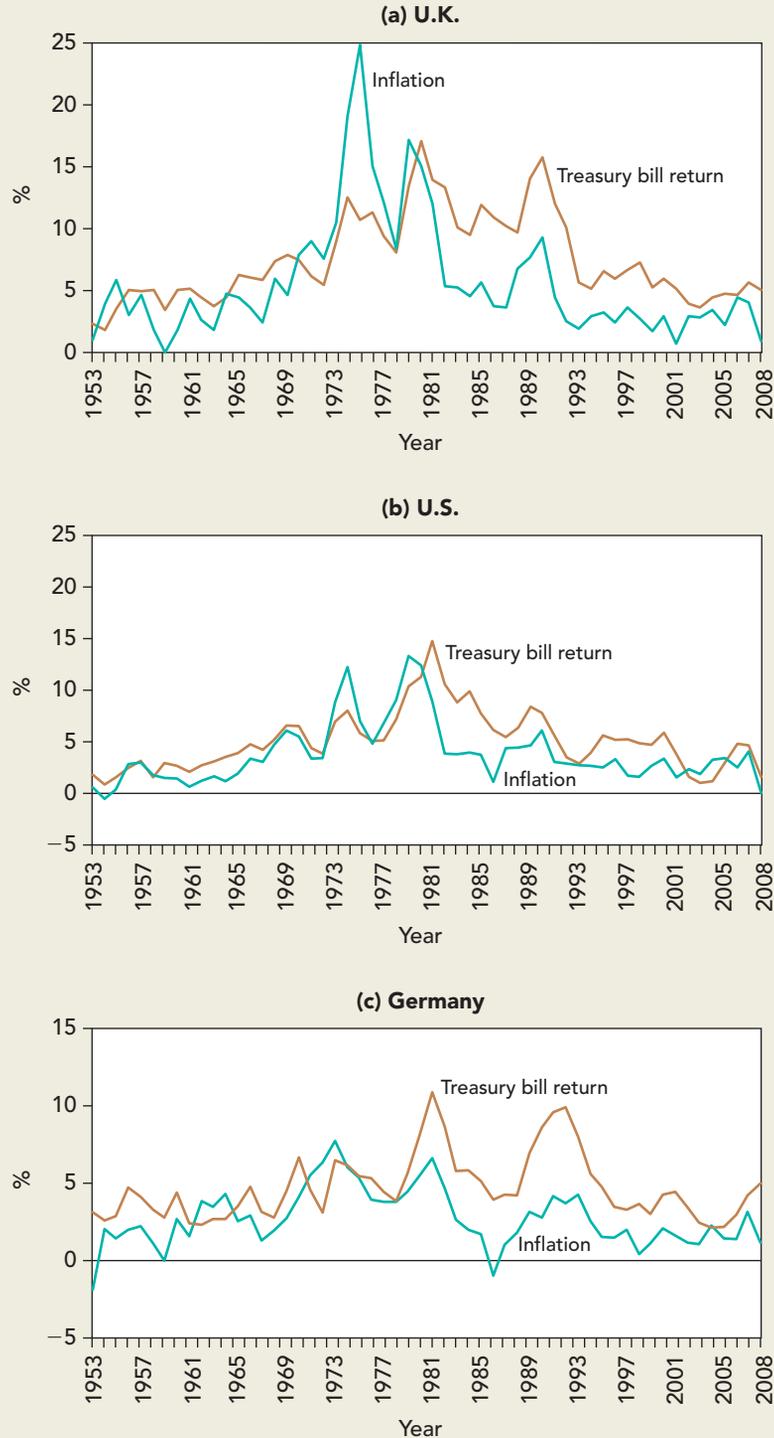
Look now at the relationship between the rate of inflation and the Treasury bill rate. Figure 3.9 shows that investors have for the most part demanded a higher rate of interest

when inflation was high. So it looks as if Fisher's theory provides a useful rule of thumb for financial managers. If the expected inflation rate changes, it is a good bet that there will be a corresponding change in the interest rate. In other words, a strategy of rolling over short-term investments affords some protection against uncertain inflation.

FIGURE 3.9

The return on Treasury bills and the rate of inflation in the U.K., U.S., and Germany, 1953–2008.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.



3-6 Corporate Bonds and the Risk of Default

Our focus so far has been on U.S. Treasury bonds. But the federal government is not the only issuer of bonds. State and local governments borrow by selling bonds.¹⁶ So do corporations. Many foreign governments and corporations also borrow in the U.S. At the same time U.S. corporations may borrow dollars or other currencies by issuing their bonds in other countries. For example, they may issue dollar bonds in London that are then sold to investors throughout the world.

National governments don't go bankrupt—they just print more money. But they can't print money of other countries.¹⁷ Therefore, when a foreign government borrows dollars, investors worry that in some future crisis the government may not be able to come up with enough dollars to repay the debt. This worry shows up in bond prices and yields to maturity. For example, in 2008 a collapse in the Ukrainian exchange rate raised the cost of servicing the country's foreign debts. Despite a bailout from the International Monetary Fund, bondholders worried that the Ukrainian government would not be able to service the dollar bonds that it had issued. By early 2009, the promised yield on Ukrainian government debt had climbed to 25 percentage points above the yield on U.S. Treasuries.

Corporations that get into financial distress may also be forced to default on their bonds. Thus the payments promised to corporate bondholders represent a best-case scenario: The firm will never pay more than the promised cash flows, but in hard times it may pay less.

The safety of most corporate bonds can be judged from bond ratings provided by Moody's, Standard & Poor's (S&P), and Fitch. Table 3.5 lists the possible bond ratings in declining order of quality. For example, the bonds that receive the highest Moody's rating are known as Aaa (or "triple A") bonds. Then come Aa (double A), A, Baa bonds, and so on. Bonds rated Baa and above are called **investment grade**, while those with a rating of Ba or below are referred to as *speculative grade*, *high-yield*, or **junk bonds**.

It is rare for highly rated bonds to default. However, when an investment-grade bond does go under, the shock waves are felt in all major financial centers. For example, in May 2001 WorldCom sold \$11.8 billion of bonds with an investment-grade rating. About one year later, WorldCom filed for bankruptcy, and its bondholders lost more than 80% of their investment. For those bondholders, the agencies that had assigned investment-grade ratings were not the flavor of the month.

Moody's	Standard & Poor's and Fitch
Aaa	AAA
Aa	AA
A	A
Baa	BBB
Ba	BB
B	B
Caa	CCC
Ca	CC
C	C

TABLE 3.5 Key to bond ratings. The highest-quality bonds rated Baa/BBB or above are investment grade. Lower-rated bonds are called high-yield, or junk, bonds.

¹⁶ These *municipal bonds* enjoy a special tax advantage, because investors are exempt from federal income tax on the coupon payments on state and local government bonds. As a result, investors accept lower pretax yields on "munis."

¹⁷ The U.S. government can print dollars and the Japanese government can print yen. But governments in the Eurozone don't even have the luxury of being able to print their own currency. For example, during the 2008 credit crisis, investors worried that the Greek government would not be able to service its euro debts. At one point Greek bonds yielded 3% more than equivalent German government bonds.

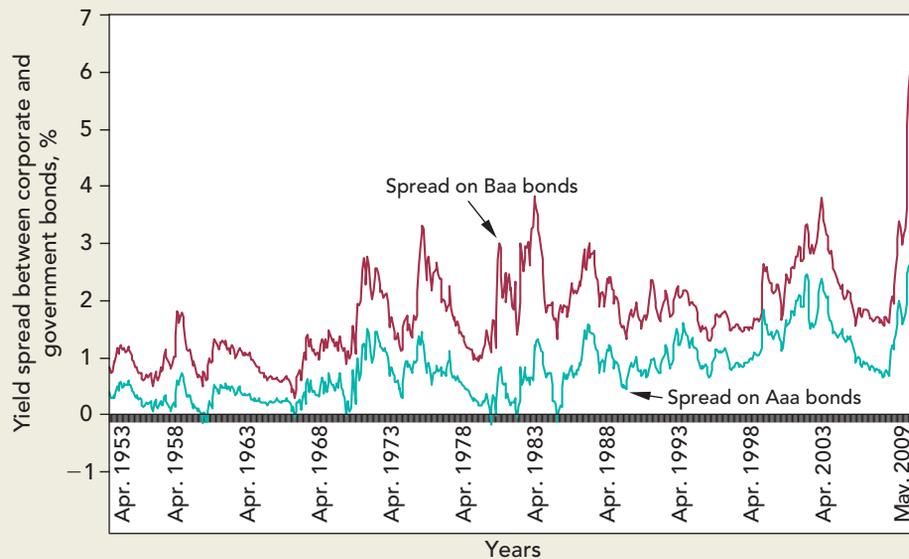
As you would expect, yields on corporate bonds vary with the bond rating. Figure 3.10 shows the yield spread of corporate bonds against U.S. Treasuries. Notice how spreads widen as safety falls off.

The credit crunch that began in 2008 saw a dramatic widening in yield spreads. Look at Table 3.6, which shows the prices and yields in December 2008 for a sample of corporate bonds ranked by Standard & Poor's rating. The yield on General Motors bonds exceeded 50%. That may seem like a mouth-watering rate of return, but investors foresaw that GM was likely to go bankrupt and that bond investors would not get their money back.

The yields to maturity reported in Table 3.6 depend on the probability of default, the amount recovered by the bondholder in the event of default, and also on *liquidity*. Corporate bonds are less liquid than Treasuries: they are more difficult and expensive to trade, particularly in large quantities or on short notice. Many investors value liquidity and will demand a higher interest rate on a less liquid bond. Lack of liquidity accounts for some of the spread between yields on corporate and Treasury bonds.

FIGURE 3.10

Yield spreads between corporate and 10-year Treasury bonds.



Issuer	Coupon	Maturity	S&P Rating	Price, % of Face Value	Yield to Maturity
Pfizer	4.65%	2018	AAA	104.00%	4.12%
Wal-Mart	6.75	2023	AA	111.45	5.60
DuPont	6	2018	A	101.97	5.73
ConAgra	9.75	2021	BBB	111.00	8.30
Woolworth	8.50	2022	BB	93.99	9.30
Eastman Kodak	9.20	2021	B	70.00	14.46
General Motors	8.8	2021	CCC	16.66	56.78

TABLE 3.6 Prices and yields of a sample of corporate bonds, December 2008.

Source: Bond transactions reported on FINRA's TRACE service: <http://cxa.marketwatch.com/finra/BondCenter>.

Valuing Bonds

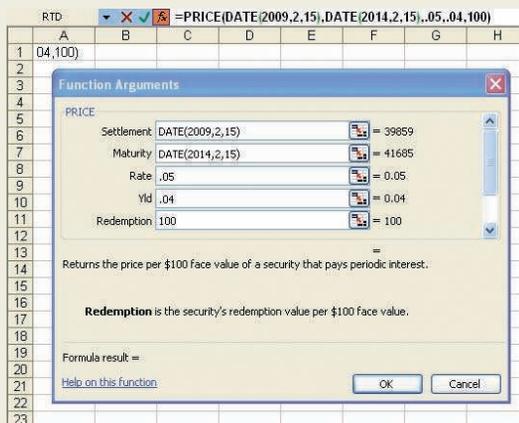
Spreadsheet programs such as Excel provide built-in functions to solve for a variety of bond valuation problems. You can find these functions by pressing *fx* on the Excel toolbar. If you then click on the function that you wish to use, Excel will ask you for the inputs that it needs. At the bottom left of the function box there is a Help facility with an example of how the function is used.

Here is a list of useful functions for valuing bonds, together with some points to remember when entering data:

- **PRICE:** The price of a bond given its yield to maturity.
- **YLD:** The yield to maturity of a bond given its price.
- **DURATION:** The duration of a bond.
- **MDURATION:** The modified duration (or volatility) of a bond.

Note:

- You can enter all the inputs in these functions directly as numbers or as the addresses of cells that contain the numbers.



- You must enter the yield and coupon as decimal values, for example, for 3% you would enter .03.
- Settlement is the date that payment for the security is made. Maturity is the maturity date. You can enter these dates directly using the Excel date function, for example, you would enter 15 Feb 2009 as DATE(2009,02,15). Alternatively, you can enter these dates in a cell and then enter the cell address in the function.
- In the functions for PRICE and YLD you need to scroll down in the function box to enter the frequency of coupon payments. Enter 1 for annual payments or 2 for semiannual.
- The functions for PRICE and YLD ask for an entry for "basis." We suggest you leave this blank. (See the Help facility for an explanation.)

SPREADSHEET QUESTIONS

The following questions provide an opportunity to practice each of these functions.

- 3.1 (PRICE)** In February 2009, Treasury 8.5s of 2020 yielded 3.2976% (see Figure 3.1). What was their price? If the yield rose to 4%, what would happen to the price?
- 3.2 (YLD)** On the same day Treasury 3.5s of 2018 were priced at 107:15 (see Figure 3.1). What was their yield to maturity? Suppose that the price was 110:00. What would happen to the yield?
- 3.3 (DURATION)** What was the duration of the Treasury 8.5s? How would duration change if the yield rose to 4%? Can you explain why?
- 3.4 (MDURATION)** What was the modified duration of the Treasury 8.5s? How would modified duration differ if the coupon were only 7.5%?

Corporate Bonds Come in Many Forms

Most corporate bonds are similar to the government bonds that we have analyzed in this chapter. In other words, they promise to make a fixed nominal coupon payment for each year until maturity, at which point they also promise to repay the face value. However, you will find that corporate bonds offer far greater variety than governments. Here are just two examples.

Floating-Rate Bonds Some corporate bonds are *floating rate*. They make coupon payments that are tied to some measure of current market rates. The rate might be reset once a year to the current short-term Treasury rate plus a spread of 2%, for example. So if the Treasury bill rate at the start of the year is 6%, the bond's coupon rate over the next year is set at 8%. This arrangement means that the bond's coupon rate always approximates current market interest rates.

Convertible Bonds If you buy a convertible bond, you can choose later to exchange it for a specified number of shares of common stock. For example, a convertible bond that is issued at a face value of \$1,000 may be convertible into 50 shares of the firm's stock. Because convertible bonds offer the opportunity to participate in any price appreciation of the company's stock, convertibles can be issued at lower coupon rates than plain-vanilla bonds.

We have only skimmed the differences between government and corporate bonds. More detail follows in several later chapters, including Chapters 23 and 24. But you have a sufficient start for now.

SUMMARY

Bonds are simply long-term loans. If you own a bond, you are entitled to a regular interest (or *coupon*) payment and at maturity you get back the bond's face value (or *principal*). In the U.S., coupons are normally paid every six months, but in other countries they may be paid annually.

The value of any bond is equal to its cash payments discounted at the spot rates of interest. For example, the present value of a 10-year bond with a 5% coupon paid annually equals

$$\text{PV}(\% \text{ of face value}) = \frac{5}{(1+r_1)} + \frac{5}{(1+r_2)^2} + \cdots + \frac{105}{(1+r_{10})^{10}}$$

This calculation uses a different spot rate of interest for each period. A plot of spot rates by maturity shows the term structure of interest rates.

Spot interest rates are most conveniently calculated from the prices of strips, which are bonds that make a single payment of face value at maturity, with zero coupons along the way. The price of a strip maturing in a future date t reveals the discount factor and spot rate for cash flows at that date. All other safe cash payments on that date are valued at that same spot rate.

Investors and financial managers use the yield to maturity on a bond to summarize its prospective return. To calculate the yield to maturity on the 10-year 5s, you need to solve for y in the following equation:

$$\text{PV}(\% \text{ of face value}) = \frac{5}{(1+y)} + \frac{5}{(1+y)^2} + \cdots + \frac{105}{(1+y)^{10}}$$

The yield to maturity discounts all cash payments at the same rate, even if spot rates differ. Notice that the yield to maturity for a bond can't be calculated until you know the bond's price or present value.

A bond's maturity tells you the date of its final payment, but it is also useful to know the *average* time to each payment. This is called the bond's *duration*. Duration is important because there is a direct relationship between the duration of a bond and the exposure of its price to changes in interest rates. A change in interest rates has a greater effect on the price of long-duration bonds.

The term structure of interest rates is upward-sloping more often than not. This means that long-term spot rates are higher than short-term spot rates. But it does *not* mean that investing long is more profitable than investing short. The *expectations theory* of the term structure tells us that bonds are priced so that an investor who holds a succession of short bonds can expect the same return as another investor who holds a long bond. The expectations theory predicts an upward-sloping term structure only when future short-term interest rates are expected to rise.

The expectations theory cannot be a complete explanation of term structure if investors are worried about risk. Long bonds may be a safe haven for investors with long-term fixed liabilities. But other investors may not like the extra volatility of long-term bonds or may be concerned that a sudden burst of inflation may largely wipe out the real value of these bonds. These investors will be prepared to hold long-term bonds only if they offer the compensation of a higher rate of interest.

Bonds promise fixed nominal cash payments, but the *real* interest rate that they provide depends on inflation. The best-known theory about the effect of inflation on interest rates was suggested by Irving Fisher. He argued that the nominal, or money, rate of interest is equal to the required real rate plus the expected rate of inflation. If the expected inflation rate increases by 1%, so too will the money rate of interest. During the past 50 years Fisher's simple theory has not done a bad job of explaining changes in short-term interest rates.

When you buy a U.S. Treasury bond, you can be confident that you will get your money back. When you lend to a company, you face the risk that it will go belly-up and will not be able to repay its bonds. Defaults are rare for companies with investment-grade bond ratings, but investors worry nevertheless. Companies need to compensate investors for default risk by promising to pay higher rates of interest.

A good general text on debt markets is:

S. Sundaresan, *Fixed Income Markets and Their Derivatives*, 3rd ed. (San Diego, CA: Academic Press, 2009).

Schaefer's paper is a good review of duration and how it is used to hedge fixed liabilities:

A.M. Schaefer, "Immunisation and Duration: A Review of Theory, Performance and Application," in *The Revolution in Corporate Finance*, ed. J. M. Stern and D. H. Chew, Jr. (Oxford: Basil Blackwell, 1986).



FURTHER READING



Select problems are available in McGraw-Hill Connect.
Please see the preface for more information.

BASIC

1. A 10-year bond is issued with a face value of \$1,000, paying interest of \$60 a year. If market yields increase shortly after the T-bond is issued, what happens to the bond's
 - a. Coupon rate?
 - b. Price?
 - c. Yield to maturity?
2. The following statements are true. Explain why.
 - a. If a bond's coupon rate is higher than its yield to maturity, then the bond will sell for more than face value.
 - b. If a bond's coupon rate is lower than its yield to maturity, then the bond's price will increase over its remaining maturity.

PROBLEM SETS

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3. In February 2009 Treasury 6s of 2026 offered a semiannually compounded yield of 3.5965%. Recognizing that coupons are paid semiannually, calculate the bond's price.
4. Here are the prices of three bonds with 10-year maturities:

Bond Coupon (%)	Price (%)
2	81.62
4	98.39
8	133.42

If coupons are paid annually, which bond offered the highest yield to maturity? Which had the lowest? Which bonds had the longest and shortest durations?

5. Construct some simple examples to illustrate your answers to the following:
 - a. If interest rates rise, do bond prices rise or fall?
 - b. If the bond yield is greater than the coupon, is the price of the bond greater or less than 100?
 - c. If the price of a bond exceeds 100, is the yield greater or less than the coupon?
 - d. Do high-coupon bonds sell at higher or lower prices than low-coupon bonds?
 - e. If interest rates change, does the price of high-coupon bonds change proportionately more than that of low-coupon bonds?
6. Which comes first in the market for U.S. Treasury bonds:
 - a. Spot interest rates or yields to maturity?
 - b. Bond prices or yields to maturity?
7. Look again at Table 3.4. Suppose that spot interest rates all change to 4%—a “flat” term structure of interest rates.
 - a. What is the new yield to maturity for each bond in the table?
 - b. Recalculate the price of bond A.
8.
 - a. What is the formula for the value of a two-year, 5% bond in terms of spot rates?
 - b. What is the formula for its value in terms of yield to maturity?
 - c. If the two-year spot rate is higher than the one-year rate, is the yield to maturity greater or less than the two-year spot rate?
9. The following table shows the prices of a sample of U.S. Treasury strips in August 2009. Each strip makes a single payment of \$1,000 at maturity.
 - a. Calculate the annually compounded, spot interest rate for each year.
 - b. Is the term structure upward- or downward-sloping, or flat?
 - c. Would you expect the yield on a coupon bond maturing in August 2013 to be higher or lower than the yield on the 2013 strip?

Maturity	Price (%)
August 2010	99.423
August 2011	97.546
August 2012	94.510
August 2013	90.524

10.
 - a. An 8%, five-year bond yields 6%. If the yield remains unchanged, what will be its price one year hence? Assume annual coupon payments.
 - b. What is the total return to an investor who held the bond over this year?
 - c. What can you deduce about the relationship between the bond return over a particular period and the yields to maturity at the start and end of that period?

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11. True or false? Explain.
- Longer-maturity bonds necessarily have longer durations.
 - The longer a bond's duration, the lower its volatility.
 - Other things equal, the lower the bond coupon, the higher its volatility.
 - If interest rates rise, bond durations rise also.
12. Calculate the durations and volatilities of securities A, B, and C. Their cash flows are shown below. The interest rate is 8%.

	Period 1	Period 2	Period 3
A	40	40	40
B	20	20	120
C	10	10	110

13. The one-year spot interest rate is $r_1 = 5\%$ and the two-year rate is $r_2 = 6\%$. If the expectations theory is correct, what is the expected one-year interest rate in one year's time?
14. The two-year interest rate is 10% and the expected annual inflation rate is 5%.
- What is the expected real interest rate?
 - If the expected rate of inflation suddenly rises to 7%, what does Fisher's theory say about how the real interest rate will change? What about the nominal rate?

INTERMEDIATE

15. A 10-year German government bond (bund) has a face value of €100 and a coupon rate of 5% paid annually. Assume that the interest rate (in euros) is equal to 6% per year. What is the bond's PV?
16. A 10-year U.S. Treasury bond with a face value of \$10,000 pays a coupon of 5.5% (2.75% of face value every six months). The semiannually compounded interest rate is 5.2% (a six-month discount rate of $5.2/2 = 2.6\%$).
- What is the present value of the bond?
 - Generate a graph or table showing how the bond's present value changes for semiannually compounded interest rates between 1% and 15%.
17. A six-year government bond makes annual coupon payments of 5% and offers a yield of 3% annually compounded. Suppose that one year later the bond still yields 3%. What return has the bondholder earned over the 12-month period? Now suppose that the bond yields 2% at the end of the year. What return would the bondholder earn in this case?
18. A 6% six-year bond yields 12% and a 10% six-year bond yields 8%. Calculate the six-year spot rate. Assume annual coupon payments. (*Hint*: What would be your cash flows if you bought 1.2 10% bonds?)
19. Is the yield on high-coupon bonds more likely to be higher than that on low-coupon bonds when the term structure is upward-sloping or when it is downward-sloping? Explain.
20. You have estimated spot rates as follows:
 $r_1 = 5.00\%$, $r_2 = 5.40\%$, $r_3 = 5.70\%$, $r_4 = 5.90\%$, $r_5 = 6.00\%$.
- What are the discount factors for each date (that is, the present value of \$1 paid in year t)?
 - Calculate the PV of the following bonds assuming annual coupons: (i) 5%, two-year bond; (ii) 5%, five-year bond; and (iii) 10%, five-year bond.
 - Explain intuitively why the yield to maturity on the 10% bond is less than that on the 5% bond.
 - What should be the yield to maturity on a five-year zero-coupon bond?
 - Show that the correct yield to maturity on a five-year annuity is 5.75%.
 - Explain intuitively why the yield on the five-year bonds described in part (c) must lie between the yield on a five-year zero-coupon bond and a five-year annuity.

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21. Calculate durations and modified durations for the 4% coupon bond and the strip in Table 3.1. The answers for the strip will be easy. For the 4% bond, you can follow the procedure set out in Table 3.3 for the 11¼% coupon bonds. Confirm that modified duration predicts the impact of a 1% change in interest rates on the bond prices.
22. Find the “live” spreadsheet for Table 3.3 on this book’s Web site, www.mhhe.com/bma. Show how duration and volatility change if (a) the bond’s coupon is 8% of face value and (b) the bond’s yield is 6%. Explain your finding.
23. The formula for the duration of a perpetual bond that makes an equal payment each year in perpetuity is $(1 + \text{yield})/\text{yield}$. If each bond yields 5%, which has the longer duration—a perpetual bond or a 15-year zero-coupon bond? What if the yield is 10%?
24. Look up prices of 10 U.S. Treasury bonds with different coupons and different maturities. Calculate how their prices would change if their yields to maturity increased by 1 percentage point. Are long- or short-term bonds most affected by the change in yields? Are high- or low-coupon bonds most affected?
25. Look again at Table 3.4. Suppose the spot interest rates change to the following *downward-sloping* term structure: $r_1 = 4.6\%$, $r_2 = 4.4\%$, $r_3 = 4.2\%$, and $r_4 = 4.0\%$. Recalculate discount factors, bond prices, and yields to maturity for each of the bonds listed in the table.
26. Look at the spot interest rates shown in Problem 25. Suppose that someone told you that the five-year spot interest rate was 2.5%. Why would you not believe him? How could you make money if he was right? What is the minimum sensible value for the five-year spot rate?
27. Look again at the spot interest rates shown in Problem 25. What can you deduce about the one-year spot interest rate in three years if . . .
 - a. The expectations theory of term structure is right?
 - b. Investing in long-term bonds carries additional risks?
28. Suppose that you buy a two-year 8% bond at its face value.
 - a. What will be your nominal return over the two years if inflation is 3% in the first year and 5% in the second? What will be your real return?
 - b. Now suppose that the bond is a TIPS. What will be your real and nominal returns?
29. A bond’s credit rating provides a guide to its price. As we write this in Spring 2009, Aaa bonds yield 5.41% and Baa bonds yield 8.47%. If some bad news causes a 10% five-year bond to be unexpectedly downrated from Aaa to Baa, what would be the effect on the bond price? (Assume annual coupons.)

CHALLENGE

30. Write a spreadsheet program to construct a series of bond tables that show the present value of a bond given the coupon rate, maturity, and yield to maturity. Assume that coupon payments are semiannual and yields are compounded semiannually.
31. Find the arbitrage opportunity (opportunities?). Assume for simplicity that coupons are paid annually. In each case the face value of the bond is \$1,000.

Bond	Maturity (years)	Coupon, \$	Price, \$
A	3	0	751.30
B	4	50	842.30
C	4	120	1,065.28
D	4	100	980.57
E	3	140	1,120.12
F	3	70	1,001.62
G	2	0	834.00

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32. The duration of a bond that makes an equal payment each year in perpetuity is $(1 + \text{yield})/\text{yield}$. Prove it.
33. What is the duration of a common stock whose dividends are expected to grow at a constant rate in perpetuity?
34. a. What spot and forward rates are embedded in the following Treasury bonds? The price of one-year strips is 93.46%. Assume for simplicity that bonds make only annual payments. (*Hint*: Can you devise a mixture of long and short positions in these bonds that gives a cash payoff only in year 2? In year 3?)

Maturity (years)	Coupon (%)	Price (%)
4	2	94.92
8	3	103.64

- b. A three-year bond with a 4% coupon is selling at 95.00%. Is there a profit opportunity here? If so, how would you take advantage of it?
35. Look one more time at Table 3.4.
- a. Suppose you knew the bond prices but not the spot interest rates. Explain how you would calculate the spot rates. (*Hint*: You have four unknown spot rates, so you need four equations.)
- b. Suppose that you could buy bond C in large quantities at \$1,040 rather than at its equilibrium price of \$1,058.76. Show how you could make a zillion dollars without taking on any risk.

1. The Web sites of *The Wall Street Journal* (www.wsj.com) and the *Financial Times* (www.ft.com) are wonderful sources of market data. You should become familiar with them. Use www.wsj.com to answer the following questions:
- Find the prices of coupon strips. Use these prices to plot the term structure. If the expectations theory is correct, what is the expected one-year interest rate three years hence?
 - Find a three- or four-year bond and construct a package of coupon and principal strips that provides the same cash flows. The law of one price predicts that the cost of the package should be very close to that of the bond. Is it?
 - Find a long-term Treasury bond with a low coupon and calculate its duration. Now find another bond with a similar maturity and a higher coupon. Which has the longer duration?
 - Look up the yields on 10-year nominal Treasury bonds and on TIPS. If you are confident that inflation will average 2% a year, which bond will provide the higher real return?
2. Log on to www.smartmoney.com and find the Living Yield Curve, which shows a picture of the yield curve. How does today's yield curve compare with yield curves in the past? Do short-term interest rates move more than long rates?

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