

RISK

Portfolio Theory and the Capital Asset Pricing Model

► In Chapter 7 we began to come to grips with the problem of measuring risk. Here is the story so far.

The stock market is risky because there is a spread of possible outcomes. The usual measure of this spread is the standard deviation or variance. The risk of any stock can be broken down into two parts. There is the *specific* or *diversifiable risk* that is peculiar to that stock, and there is the *market risk* that is associated with marketwide variations. Investors can eliminate specific risk by holding a well-diversified portfolio, but they cannot eliminate market risk. *All* the risk of a fully diversified portfolio is market risk.

A stock's contribution to the risk of a fully diversified portfolio depends on its sensitivity to market changes. This sensitivity is generally known as *beta*. A security with a beta of 1.0 has average market risk—a well-diversified portfolio of such securities has the same

standard deviation as the market index. A security with a beta of .5 has below-average market risk—a well-diversified portfolio of these securities tends to move half as far as the market moves and has half the market's standard deviation.

In this chapter we build on this newfound knowledge. We present leading theories linking risk and return in a competitive economy, and we show how these theories can be used to estimate the returns required by investors in different stock-market investments. We start with the most widely used theory, the capital asset pricing model, which builds directly on the ideas developed in the last chapter. We will also look at another class of models, known as arbitrage pricing or factor models. Then in Chapter 9 we show how these ideas can help the financial manager cope with risk in practical capital budgeting situations.



8-1 Harry Markowitz and the Birth of Portfolio Theory

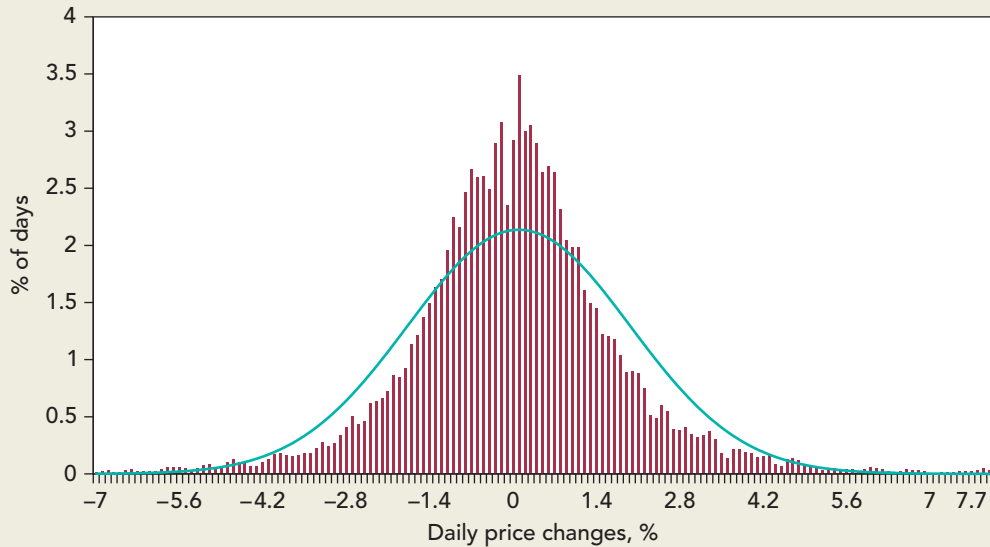
Most of the ideas in Chapter 7 date back to an article written in 1952 by Harry Markowitz.¹ Markowitz drew attention to the common practice of portfolio diversification and showed exactly how an investor can reduce the standard deviation of portfolio returns by choosing stocks that do not move exactly together. But Markowitz did not stop there; he went on to work out the basic principles of portfolio construction. These principles are the foundation for much of what has been written about the relationship between risk and return.

We begin with Figure 8.1, which shows a histogram of the daily returns on IBM stock from 1988 to 2008. On this histogram we have superimposed a bell-shaped normal

¹ H. M. Markowitz, "Portfolio Selection," *Journal of Finance* 7 (March 1952), pp. 77–91.

FIGURE 8.1

Daily price changes for IBM are approximately normally distributed. This plot spans 1988 to 2008.



distribution. The result is typical: When measured over a short interval, the past rates of return on any stock conform fairly closely to a normal distribution.²

Normal distributions can be completely defined by two numbers. One is the average or expected return; the other is the variance or standard deviation. Now you can see why in Chapter 7 we discussed the calculation of expected return and standard deviation. They are not just arbitrary measures: if returns are normally distributed, expected return and standard deviation are the *only* two measures that an investor need consider.

Figure 8.2 pictures the distribution of possible returns from three investments. A and B offer an expected return of 10%, but A has the much wider spread of possible outcomes. Its standard deviation is 15%; the standard deviation of B is 7.5%. Most investors dislike uncertainty and would therefore prefer B to A.

Now compare investments B and C. This time both have the *same* standard deviation, but the expected return is 20% from stock C and only 10% from stock B. Most investors like high expected return and would therefore prefer C to B.

Combining Stocks into Portfolios

Suppose that you are wondering whether to invest in the shares of Campbell Soup or Boeing. You decide that Campbell offers an expected return of 3.1% and Boeing offers an expected return of 9.5%. After looking back at the past variability of the two stocks, you also decide that the standard deviation of returns is 15.8% for Campbell Soup and 23.7% for Boeing. Boeing offers the higher expected return, but it is more risky.

Now there is no reason to restrict yourself to holding only one stock. For example, in Section 7-3 we analyzed what would happen if you invested 60% of your money in Campbell Soup and 40% in Boeing. The expected return on this portfolio is about 5.7%, simply a weighted average of the expected returns on the two holdings. What about the risk of such a portfolio? We know that thanks to diversification the portfolio risk is less than the average

² If you were to measure returns over *long* intervals, the distribution would be skewed. For example, you would encounter returns greater than 100% but none less than -100%. The distribution of returns over periods of, say, one year would be better approximated by a *lognormal* distribution. The lognormal distribution, like the normal, is completely specified by its mean and standard deviation.

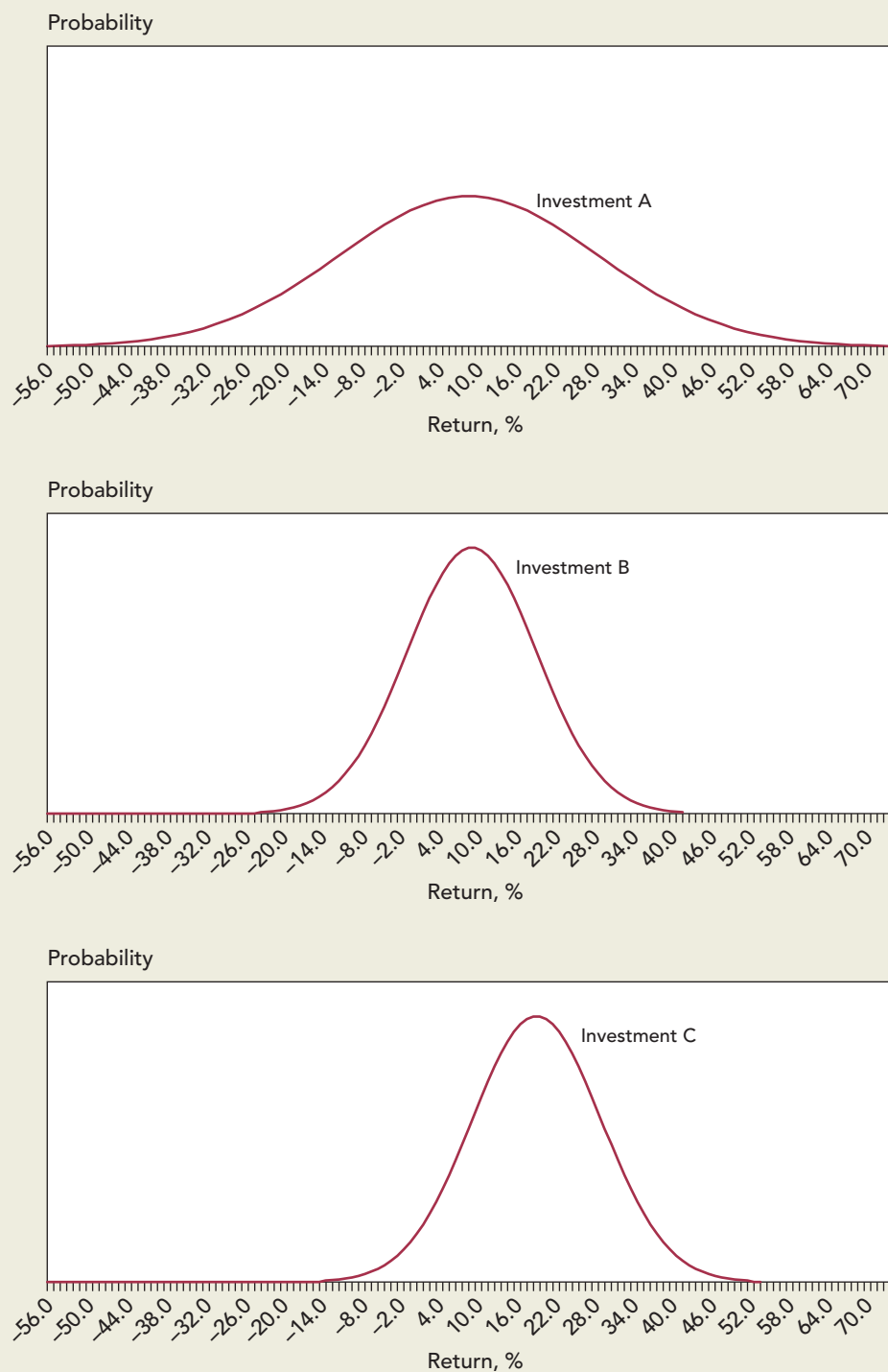


FIGURE 8.2

Investments A and B both have an *expected* return of 10%, but because investment A has the greater spread of *possible* returns, it is more risky than B. We can measure this spread by the standard deviation. Investment A has a standard deviation of 15%; B, 7.5%. Most investors would prefer B to A. Investments B and C both have the same standard deviation, but C offers a higher expected return. Most investors would prefer C to B.

of the risks of the separate stocks. In fact, on the basis of past experience the standard deviation of this portfolio is 14.6%.³

The curved blue line in Figure 8.3 shows the expected return and risk that you could achieve by different combinations of the two stocks. Which of these combinations is best depends on your stomach. If you want to stake all on getting rich quickly, you should put all your money in Boeing. If you want a more peaceful life, you should invest most of your money in Campbell Soup, but you should keep at least a small investment in Boeing.⁴

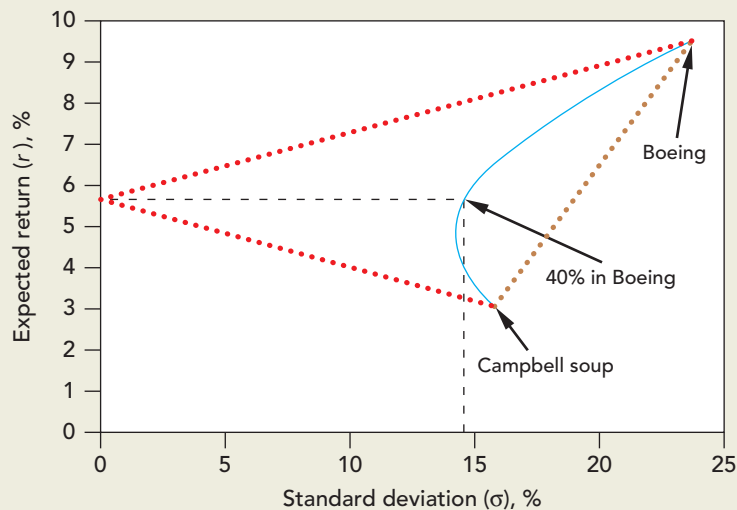
We saw in Chapter 7 that the gain from diversification depends on how highly the stocks are correlated. Fortunately, on past experience there is only a small positive correlation between the returns of Campbell Soup and Boeing ($\rho = +.18$). If their stocks moved in exact lockstep ($\rho = +1$), there would be no gains at all from diversification. You can see this by the brown dotted line in Figure 8.3. The red dotted line in the figure shows a second extreme (and equally unrealistic) case in which the returns on the two stocks are perfectly *negatively* correlated ($\rho = -1$). If this were so, your portfolio would have no risk.

In practice, you are not limited to investing in just two stocks. For example, you could decide to choose a portfolio from the 10 stocks listed in the first column of Table 8.1. After analyzing the prospects for each firm, you come up with forecasts of their returns. You are most optimistic about the outlook for Amazon, and forecast that it will provide a return of 22.8%. At the other extreme, you are cautious about the prospects for Johnson & Johnson and predict a return of 3.8%. You use data for the past five years to estimate the risk of each stock and the correlation between the returns on each pair of stocks.⁵

Now look at Figure 8.4. Each diamond marks the combination of risk and return offered by a different individual security. For example, Amazon has both the highest standard deviation and the highest expected return. It is represented by the upper-right diamond in the figure.

FIGURE 8.3

The curved line illustrates how expected return and standard deviation change as you hold different combinations of two stocks. For example, if you invest 40% of your money in Boeing and the remainder in Campbell Soup, your expected return is 12%, which is 40% of the way between the expected returns on the two stocks. The standard deviation is 14.6%, which is less than 40% of the way between the standard deviations of the two stocks. This is because diversification reduces risk.



³ We pointed out in Section 7-3 that the correlation between the returns of Campbell Soup and Boeing has been about .18. The variance of a portfolio which is invested 60% in Campbell and 40% in Boeing is

$$\begin{aligned}\text{Variance} &= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2 \\ &= [(.6)^2 \times (15.8)^2] + [(.4)^2 \times (23.7)^2] + 2(.6 \times .4 \times .18 \times 15.8 \times 23.7) \\ &= 212.1\end{aligned}$$

The portfolio standard deviation is $\sqrt{212.1} = 14.6\%$.

⁴ The portfolio with the minimum risk has 73.1% in Campbell Soup. We assume in Figure 8.3 that you may not take negative positions in either stock, i.e., we rule out short sales.

⁵ There are 45 different correlation coefficients, so we have not listed them in Table 8.1.

Stock	Expected Return	Standard Deviation	Efficient Portfolios—Percentages Allocated to Each Stock		
			A	B	C
Amazon	22.8%	50.9%	100	10.9	
Ford	19.0	47.2		11.0	
Dell	13.4	30.9		10.3	
Starbucks	9.0	30.3		10.7	3.6
Boeing	9.5	23.7		10.5	
Disney	7.7	19.6		11.2	
Newmont	7.0	36.1		9.9	10.2
Exxon Mobil	4.7	19.1		9.7	18.4
Johnson & Johnson	3.8	12.5		7.4	33.9
Campbell Soup	3.1	15.8		8.4	33.9
Expected portfolio return			22.8	10.5	4.2
Portfolio standard deviation			50.9	16.0	8.8

TABLE 8.1 Examples of efficient portfolios chosen from 10 stocks.

Note: Standard deviations and the correlations between stock returns were estimated from monthly returns, January 2004–December 2008. Efficient portfolios are calculated assuming that short sales are prohibited.

By holding different proportions of the 10 securities, you can obtain an even wider selection of risk and return: in fact, *anywhere* in the shaded area in Figure 8.4. But where in the shaded area is best? Well, what is your goal? Which direction do you want to go? The answer should be obvious: you want to go up (to increase expected return) and to the left (to reduce risk). Go as far as you can, and you will end up with one of the portfolios that lies along the heavy solid line. Markowitz called them **efficient portfolios**. They offer the highest expected return for any level of risk.

We will not calculate this set of efficient portfolios here, but you may be interested in how to do it. Think back to the capital rationing problem in Section 5-4. There we wanted to deploy a limited amount of capital investment in a mixture of projects to give the highest NPV. Here we want to deploy an investor's funds to give the highest expected return for a given standard deviation. In principle, both problems can be solved by hunting and pecking—but only in principle. To solve the capital rationing problem, we can employ linear programming; to solve the portfolio problem, we would turn to a variant of linear programming known as *quadratic programming*. Given the expected return and standard deviation for each stock, as well as the correlation between each pair of stocks, we could use a standard quadratic computer program to calculate the set of efficient portfolios.

Three of these efficient portfolios are marked in Figure 8.4. Their compositions are summarized in Table 8.1. Portfolio B offers the highest expected return: it is invested entirely in one stock, Amazon. Portfolio C offers the minimum risk; you can see from Table 8.1 that it has large holdings in Johnson & Johnson and Campbell Soup, which have the lowest standard deviations. However, the portfolio also has a sizable holding in Newmont even though it is individually very risky. The reason? On past evidence the fortunes of gold-mining shares, such as Newmont, are almost uncorrelated with those of other stocks and so provide additional diversification.

FIGURE 8.4

Each diamond shows the expected return and standard deviation of 1 of the 10 stocks in Table 8.1. The shaded area shows the possible combinations of expected return and standard deviation from investing in a *mixture* of these stocks. If you like high expected returns and dislike high standard deviations, you will prefer portfolios along the heavy line. These are *efficient* portfolios. We have marked the three efficient portfolios described in Table 8.1 (A, B, and C).

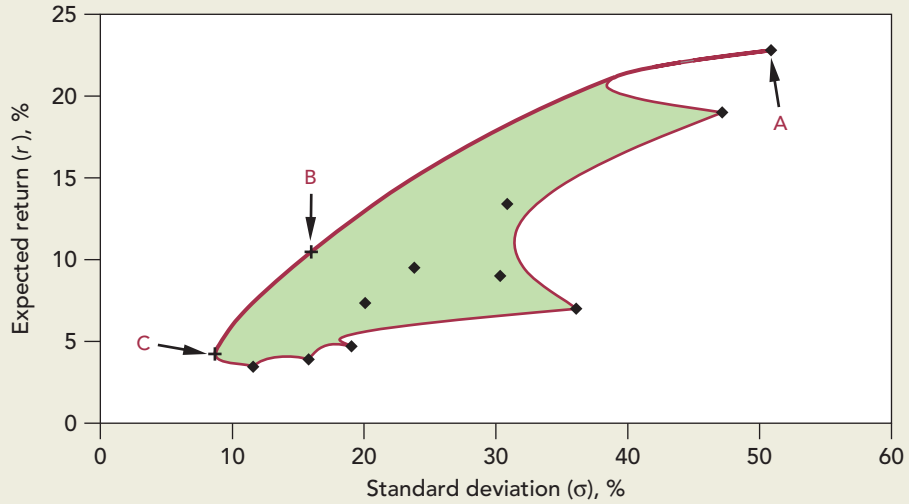


FIGURE 8.5

Lending and borrowing extend the range of investment possibilities. If you invest in portfolio S and lend or borrow at the risk-free interest rate, r_f , you can achieve any point along the straight line from r_f through S. This gives you a higher expected return for any level of risk than if you just invest in common stocks.

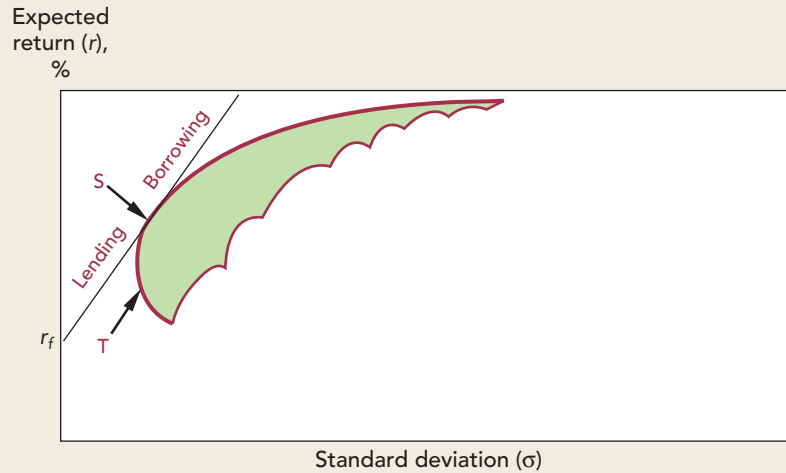


Table 8.1 also shows the compositions of a third efficient portfolio with intermediate levels of risk and expected return.

Of course, large investment funds can choose from thousands of stocks and thereby achieve a wider choice of risk and return. This choice is represented in Figure 8.5 by the shaded, broken-egg-shaped area. The set of efficient portfolios is again marked by the heavy curved line.

We Introduce Borrowing and Lending

Now we introduce yet another possibility. Suppose that you can also lend or borrow money at some risk-free rate of interest r_f . If you invest some of your money in Treasury bills (i.e., lend money) and place the remainder in common stock portfolio S, you can obtain any combination of expected return and risk along the straight line joining r_f and

S in Figure 8.5. Since borrowing is merely negative lending, you can extend the range of possibilities to the right of S by borrowing funds at an interest rate of r_f and investing them as well as your own money in portfolio S.

Let us put some numbers on this. Suppose that portfolio S has an expected return of 15% and a standard deviation of 16%. Treasury bills offer an interest rate (r_f) of 5% and are risk-free (i.e., their standard deviation is zero). If you invest half your money in portfolio S and lend the remainder at 5%, the expected return on your investment is likewise halfway between the expected return on S and the interest rate on Treasury bills:

$$\begin{aligned} r &= \left(\frac{1}{2} \times \text{expected return on S}\right) + \left(\frac{1}{2} \times \text{interest rate}\right) \\ &= 10\% \end{aligned}$$

And the standard deviation is halfway between the standard deviation of S and the standard deviation of Treasury bills:⁶

$$\begin{aligned} \sigma &= \left(\frac{1}{2} \times \text{standard deviation of S}\right) + \left(\frac{1}{2} \times \text{standard deviation of bills}\right) \\ &= 8\% \end{aligned}$$

Or suppose that you decide to go for the big time: You borrow at the Treasury bill rate an amount equal to your initial wealth, and you invest everything in portfolio S. You have twice your own money invested in S, but you have to *pay* interest on the loan. Therefore your expected return is

$$\begin{aligned} r &= (2 \times \text{expected return on S}) - (1 \times \text{interest rate}) \\ &= 25\% \end{aligned}$$

And the standard deviation of your investment is

$$\begin{aligned} \sigma &= (2 \times \text{standard deviation of S}) - (1 \times \text{standard deviation of bills}) \\ &= 32\% \end{aligned}$$

You can see from Figure 8.5 that when you lend a portion of your money, you end up part-way between r_f and S; if you can borrow money at the risk-free rate, you can extend your possibilities beyond S. You can also see that regardless of the level of risk you choose, you can get the highest expected return by a mixture of portfolio S and borrowing or lending. S is the *best* efficient portfolio. There is no reason ever to hold, say, portfolio T.

If you have a graph of efficient portfolios, as in Figure 8.5, finding this best efficient portfolio is easy. Start on the vertical axis at r_f and draw the steepest line you can to the curved heavy line of efficient portfolios. That line will be tangent to the heavy line. The efficient portfolio at the tangency point is better than all the others. Notice that it offers the highest *ratio* of risk premium to standard deviation. This ratio of the risk premium to the standard deviation is called the *Sharpe ratio*:

$$\text{Sharpe ratio} = \frac{\text{Risk premium}}{\text{Standard deviation}} = \frac{r - r_f}{\sigma}$$

Investors track Sharpe ratios to measure the risk-adjusted performance of investment managers. (Take a look at the mini-case at the end of this chapter.)

We can now separate the investor's job into two stages. First, the best portfolio of common stocks must be selected—S in our example. Second, this portfolio must be blended with borrowing or lending to obtain an exposure to risk that suits the particular investor's taste. Each investor, therefore, should put money into just two benchmark investments—a risky portfolio S and a risk-free loan (borrowing or lending).

⁶ If you want to check this, write down the formula for the standard deviation of a two-stock portfolio:

$$\text{Standard deviation} = \sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2}$$

Now see what happens when security 2 is riskless, i.e., when $\sigma_2 = 0$.

What does portfolio S look like? If you have better information than your rivals, you will want the portfolio to include relatively large investments in the stocks you think are undervalued. But in a competitive market you are unlikely to have a monopoly of good ideas. In that case there is no reason to hold a different portfolio of common stocks from anybody else. In other words, you might just as well hold the market portfolio. That is why many professional investors invest in a market-index portfolio and why most others hold well-diversified portfolios.

8-2 The Relationship Between Risk and Return

In Chapter 7 we looked at the returns on selected investments. The least risky investment was U.S. Treasury bills. Since the return on Treasury bills is fixed, it is unaffected by what happens to the market. In other words, Treasury bills have a beta of 0. We also considered a much riskier investment, the market portfolio of common stocks. This has average market risk: its beta is 1.0.

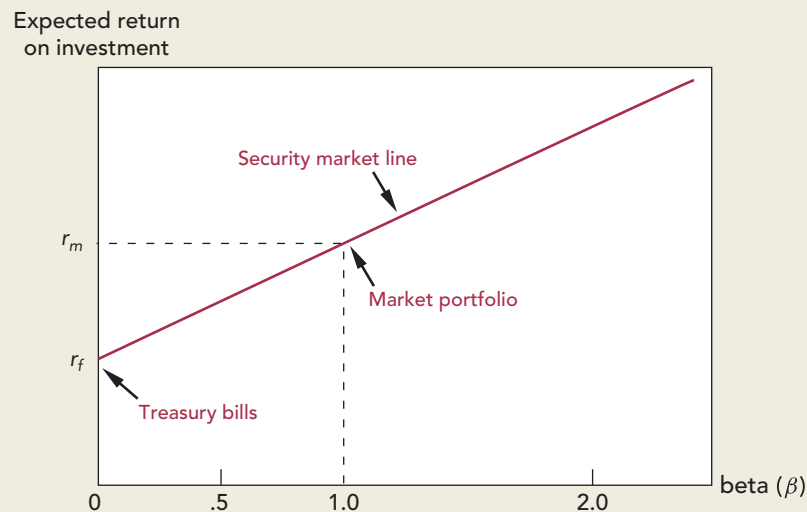
Wise investors don't take risks just for fun. They are playing with real money. Therefore, they require a higher return from the market portfolio than from Treasury bills. The difference between the return on the market and the interest rate is termed the *market risk premium*. Since 1900 the market risk premium ($r_m - r_f$) has averaged 7.1% a year.

In Figure 8.6 we have plotted the risk and expected return from Treasury bills and the market portfolio. You can see that Treasury bills have a beta of 0 and a risk premium of 0.⁷ The market portfolio has a beta of 1 and a risk premium of $r_m - r_f$. This gives us two benchmarks for the expected risk premium. But what is the expected risk premium when beta is not 0 or 1?

In the mid-1960s three economists—William Sharpe, John Lintner, and Jack Treynor—produced an answer to this question.⁸ Their answer is known as the **capital asset pricing**

FIGURE 8.6

The capital asset pricing model states that the expected risk premium on each investment is proportional to its beta. This means that each investment should lie on the sloping security market line connecting Treasury bills and the market portfolio.



⁷ Remember that the risk premium is the difference between the investment's expected return and the risk-free rate. For Treasury bills, the difference is zero.

⁸ W. F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* 19 (September 1964), pp. 425–442; and J. Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* 47 (February 1965), pp. 13–37. Treynor's article has not been published.

model, or CAPM. The model's message is both startling and simple. In a competitive market, the expected risk premium varies in direct proportion to beta. This means that in Figure 8.6 all investments must plot along the sloping line, known as the **security market line**. The expected risk premium on an investment with a beta of .5 is, therefore, *half* the expected risk premium on the market; the expected risk premium on an investment with a beta of 2 is *twice* the expected risk premium on the market. We can write this relationship as

$$\begin{aligned} \text{Expected risk premium on stock} &= \text{beta} \times \text{expected risk premium on market} \\ r - r_f &= \beta(r_m - r_f) \end{aligned}$$

Some Estimates of Expected Returns

Before we tell you where the formula comes from, let us use it to figure out what returns investors are looking for from particular stocks. To do this, we need three numbers: β , r_f , and $r_m - r_f$. We gave you estimates of the betas of 10 stocks in Table 7.5. In February 2009 the interest rate on Treasury bills was about .2%.

How about the market risk premium? As we pointed out in the last chapter, we can't measure $r_m - r_f$ with precision. From past evidence it appears to be 7.1%, although many economists and financial managers would forecast a slightly lower figure. Let us use 7% in this example.

Table 8.2 puts these numbers together to give an estimate of the expected return on each stock. The stock with the highest beta in our sample is Amazon. Our estimate of the expected return from Amazon is 15.4%. The stock with the lowest beta is Campbell Soup. Our estimate of its expected return is 2.4%, 2.2% more than the interest rate on Treasury bills. Notice that these expected returns are not the same as the hypothetical forecasts of return that we assumed in Table 8.1 to generate the efficient frontier.

You can also use the capital asset pricing model to find the discount rate for a new capital investment. For example, suppose that you are analyzing a proposal by Dell to expand its capacity. At what rate should you discount the forecasted cash flows? According to Table 8.2, investors are looking for a return of 10.2% from businesses with the risk of Dell. So the cost of capital for a further investment in the same business is 10.2%.⁹

Stock	Beta (β)	Expected Return [$r_f + \beta(r_m - r_f)$]
Amazon	2.16	15.4
Ford	1.75	12.6
Dell	1.41	10.2
Starbucks	1.16	8.4
Boeing	1.14	8.3
Disney	.96	7.0
Newmont	.63	4.7
Exxon Mobil	.55	4.2
Johnson & Johnson	.50	3.8
Campbell Soup	.30	2.4

TABLE 8.2 These estimates of the returns expected by investors in February 2009 were based on the capital asset pricing model. We assumed .2% for the interest rate r_f and 7% for the expected risk premium $r_m - r_f$.

⁹ Remember that instead of investing in plant and machinery, the firm could return the money to the shareholders. The opportunity cost of investing is the return that shareholders could expect to earn by buying financial assets. This expected return depends on the market risk of the assets.

In practice, choosing a discount rate is seldom so easy. (After all, you can't expect to be paid a fat salary just for plugging numbers into a formula.) For example, you must learn how to adjust the expected return for the extra risk caused by company borrowing. Also you need to consider the difference between short- and long-term interest rates. In early 2009 short-term interest rates were at record lows and well below long-term rates. It is possible that investors were content with the prospect of quite modest equity returns in the short run, but they almost certainly required higher long-run returns than the figures shown in Table 8.2.¹⁰ If that is so, a cost of capital based on short-term rates may be inappropriate for long-term capital investments. But these refinements can wait until later.

Review of the Capital Asset Pricing Model

Let us review the basic principles of portfolio selection:

1. Investors like high expected return and low standard deviation. Common stock portfolios that offer the highest expected return for a given standard deviation are known as *efficient portfolios*.
2. If the investor can lend or borrow at the risk-free rate of interest, one efficient portfolio is better than all the others: the portfolio that offers the highest ratio of risk premium to standard deviation (that is, portfolio S in Figure 8.5). A risk-averse investor will put part of his money in this efficient portfolio and part in the risk-free asset. A risk-tolerant investor may put all her money in this portfolio or she may borrow and put in even more.
3. The composition of this best efficient portfolio depends on the investor's assessments of expected returns, standard deviations, and correlations. But suppose everybody has the same information and the same assessments. If there is no superior information, each investor should hold the same portfolio as everybody else; in other words, everyone should hold the market portfolio.

Now let us go back to the risk of individual stocks:

4. Do not look at the risk of a stock in isolation but at its contribution to portfolio risk. This contribution depends on the stock's sensitivity to changes in the value of the portfolio.
5. A stock's sensitivity to changes in the value of the *market* portfolio is known as *beta*. Beta, therefore, measures the marginal contribution of a stock to the risk of the market portfolio.

Now if everyone holds the market portfolio, and if beta measures each security's contribution to the market portfolio risk, then it is no surprise that the risk premium demanded by investors is proportional to beta. That is what the CAPM says.

What If a Stock Did Not Lie on the Security Market Line?

Imagine that you encounter stock A in Figure 8.7. Would you buy it? We hope not¹¹—if you want an investment with a beta of .5, you could get a higher expected return by investing half your money in Treasury bills and half in the market portfolio. If everybody shares your view of the stock's prospects, the price of A will have to fall until the expected return matches what you could get elsewhere.

¹⁰ The estimates in Table 8.2 may also be too low for the *short term* if investors required a higher risk premium in the short term to compensate for the unusual market volatility in 2009.

¹¹ Unless, of course, we were trying to sell it.

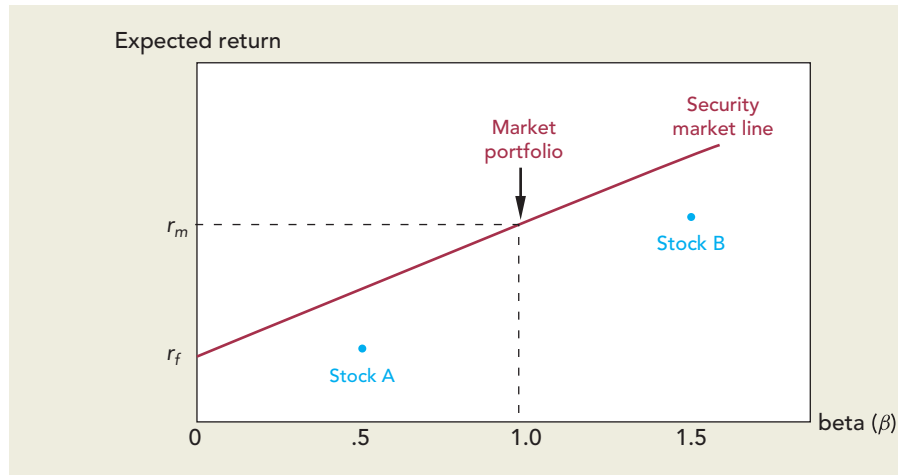


FIGURE 8.7

In equilibrium no stock can lie below the security market line. For example, instead of buying stock A, investors would prefer to lend part of their money and put the balance in the market portfolio. And instead of buying stock B, they would prefer to borrow and invest in the market portfolio.

What about stock B in Figure 8.7? Would you be tempted by its high return? You wouldn't if you were smart. You could get a higher expected return for the same beta by borrowing 50 cents for every dollar of your own money and investing in the market portfolio. Again, if everybody agrees with your assessment, the price of stock B cannot hold. It will have to fall until the expected return on B is equal to the expected return on the combination of borrowing and investment in the market portfolio.¹²

We have made our point. An investor can always obtain an expected risk premium of $\beta(r_m - r_f)$ by holding a mixture of the market portfolio and a risk-free loan. So in well-functioning markets nobody will hold a stock that offers an expected risk premium of *less* than $\beta(r_m - r_f)$. But what about the other possibility? Are there stocks that offer a higher expected risk premium? In other words, are there any that lie above the security market line in Figure 8.7? If we take all stocks together, we have the market portfolio. Therefore, we know that stocks *on average* lie on the line. Since none lies *below* the line, then there also can't be any that lie *above* the line. Thus each and every stock must lie on the security market line and offer an expected risk premium of

$$r - r_f = \beta(r_m - r_f)$$

8-3 Validity and Role of the Capital Asset Pricing Model

Any economic model is a simplified statement of reality. We need to simplify in order to interpret what is going on around us. But we also need to know how much faith we can place in our model.

Let us begin with some matters about which there is broad agreement. First, few people quarrel with the idea that investors require some extra return for taking on risk. That is why common stocks have given on average a higher return than U.S. Treasury bills. Who would want to invest in risky common stocks if they offered only the *same* expected return as bills? We would not, and we suspect you would not either.

Second, investors do appear to be concerned principally with those risks that they cannot eliminate by diversification. If this were not so, we should find that stock prices increase whenever two companies merge to spread their risks. And we should find that investment

¹² Investing in A or B only would be stupid; you would hold an undiversified portfolio.

companies which invest in the shares of other firms are more highly valued than the shares they hold. But we do not observe either phenomenon. Mergers undertaken just to spread risk do not increase stock prices, and investment companies are no more highly valued than the stocks they hold.

The capital asset pricing model captures these ideas in a simple way. That is why financial managers find it a convenient tool for coming to grips with the slippery notion of risk and why nearly three-quarters of them use it to estimate the cost of capital.¹³ It is also why economists often use the capital asset pricing model to demonstrate important ideas in finance even when there are other ways to prove these ideas. But that does not mean that the capital asset pricing model is ultimate truth. We will see later that it has several unsatisfactory features, and we will look at some alternative theories. Nobody knows whether one of these alternative theories is eventually going to come out on top or whether there are other, better models of risk and return that have not yet seen the light of day.

Tests of the Capital Asset Pricing Model

Imagine that in 1931 ten investors gathered together in a Wall Street bar and agreed to establish investment trust funds for their children. Each investor decided to follow a different strategy. Investor 1 opted to buy the 10% of the New York Stock Exchange stocks with the lowest estimated betas; investor 2 chose the 10% with the next-lowest betas; and so on, up to investor 10, who proposed to buy the stocks with the highest betas. They also planned that at the end of each year they would reestimate the betas of all NYSE stocks and reconstitute their portfolios.¹⁴ And so they parted with much cordiality and good wishes.

In time the 10 investors all passed away, but their children agreed to meet in early 2009 in the same bar to compare the performance of their portfolios. Figure 8.8 shows how they had fared. Investor 1's portfolio turned out to be much less risky than the market; its beta was only .49. However, investor 1 also realized the lowest return, 8.0% above the risk-free rate of interest. At the other extreme, the beta of investor 10's portfolio was 1.53, about three times that of investor 1's portfolio. But investor 10 was rewarded with the highest return, averaging 14.3% a year above the interest rate. So over this 77-year period returns did indeed increase with beta.

As you can see from Figure 8.8, the market portfolio over the same 77-year period provided an average return of 11.8% above the interest rate¹⁵ and (of course) had a beta of 1.0. The CAPM predicts that the risk premium should increase in proportion to beta, so that the returns of each portfolio should lie on the upward-sloping security market line in Figure 8.8. Since the market provided a risk premium of 11.8%, investor 1's portfolio, with a beta of .49, should have provided a risk premium of 5.8% and investor 10's portfolio, with a beta of 1.53, should have given a premium of 18.1%. You can see that, while high-beta stocks performed better than low-beta stocks, the difference was not as great as the CAPM predicts.

Although Figure 8.8 provides broad support for the CAPM, critics have pointed out that the slope of the line has been particularly flat in recent years. For example, Figure 8.9 shows how our 10 investors fared between 1966 and 2008. Now it is less clear who is buying the drinks: returns are pretty much in line with the CAPM with the important exception of the

¹³ See J. R. Graham and C. R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 61 (2001), pp. 187–243. A number of the managers surveyed reported using more than one method to estimate the cost of capital. Seventy-three percent used the capital asset pricing model, while 39% stated they used the average historical stock return and 34% used the capital asset pricing model with some extra risk factors.

¹⁴ Betas were estimated using returns over the previous 60 months.

¹⁵ In Figure 8.8 the stocks in the "market portfolio" are weighted equally. Since the stocks of small firms have provided higher average returns than those of large firms, the risk premium on an equally weighted index is higher than on a value-weighted index. This is one reason for the difference between the 11.8% market risk premium in Figure 8.8 and the 7.1% premium reported in Table 7.1.

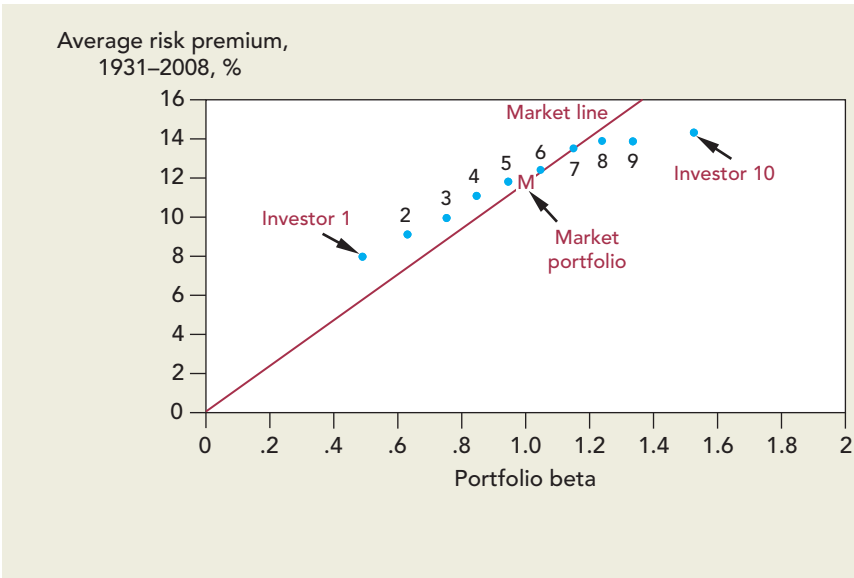


FIGURE 8.8

The capital asset pricing model states that the expected risk premium from any investment should lie on the security market line. The dots show the actual average risk premiums from portfolios with different betas. The high-beta portfolios generated higher average returns, just as predicted by the CAPM. But the high-beta portfolios plotted below the market line, and the low-beta portfolios plotted above. A line fitted to the 10 portfolio returns would be “flatter” than the market line.

Source: F. Black, “Beta and Return,” *Journal of Portfolio Management* 20 (Fall 1993), pp. 8–18. © 1993 Institutional Investor. Used with permission. We are grateful to Adam Kolasinski for updating the calculations.

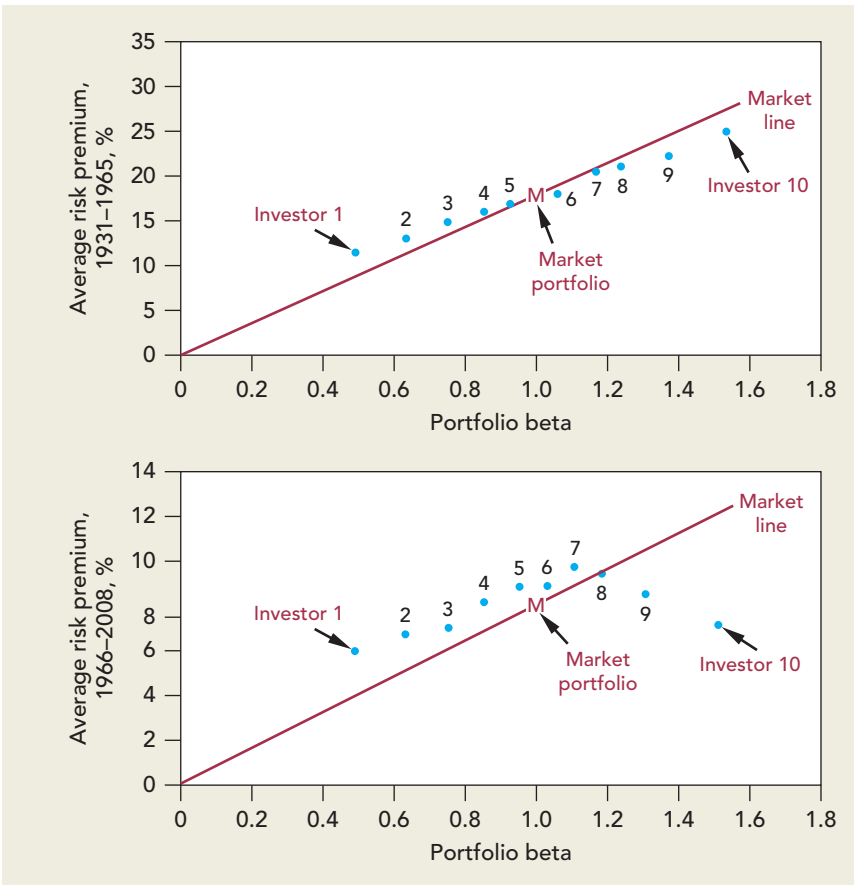


FIGURE 8.9

The relationship between beta and actual average return has been weaker since the mid-1960s. Stocks with the highest betas have provided poor returns.

Source: F. Black, “Beta and Return,” *Journal of Portfolio Management* 20 (Fall 1993), pp. 8–18. © 1993 Institutional Investor. Used with permission. We are grateful to Adam Kolasinski for updating the calculations.

two highest-risk portfolios. Investor 10, who rode the roller coaster of a high-beta portfolio, earned a return that was below that of the market. Of course, before 1966 the line was correspondingly steeper. This is also shown in Figure 8.9.

What is going on here? It is hard to say. Defenders of the capital asset pricing model emphasize that it is concerned with *expected* returns, whereas we can observe only *actual* returns. Actual stock returns reflect expectations, but they also embody lots of “noise”—the steady flow of surprises that conceal whether on average investors have received the returns they expected. This noise may make it impossible to judge whether the model holds better in one period than another.¹⁶ Perhaps the best that we can do is to focus on the longest period for which there is reasonable data. This would take us back to Figure 8.8, which suggests that expected returns do indeed increase with beta, though less rapidly than the simple version of the CAPM predicts.¹⁷

The CAPM has also come under fire on a second front: although return has not risen with beta in recent years, it has been related to other measures. For example, the red line in Figure 8.10 shows the cumulative difference between the returns on small-firm stocks and large-firm stocks. If you had bought the shares with the smallest market capitalizations and sold those with the largest capitalizations, this is how your wealth would have changed. You can see that small-cap stocks did not always do well, but over the long haul their owners have made substantially higher returns. Since the end of 1926 the average annual difference between the returns on the two groups of stocks has been 3.6%.

Now look at the green line in Figure 8.10, which shows the cumulative difference between the returns on value stocks and growth stocks. Value stocks here are defined as those with high ratios of book value to market value. Growth stocks are those with low ratios of book to market. Notice that value stocks have provided a higher long-run return than growth stocks.¹⁸ Since 1926 the average annual difference between the returns on value and growth stocks has been 5.2%.

Figure 8.10 does not fit well with the CAPM, which predicts that beta is the *only* reason that expected returns differ. It seems that investors saw risks in “small-cap” stocks and value stocks that were not captured by beta.¹⁹ Take value stocks, for example. Many of these stocks may have sold below book value because the firms were in serious trouble; if the economy slowed unexpectedly, the firms might have collapsed altogether. Therefore, investors, whose jobs could also be on the line in a recession, may have regarded these stocks as particularly risky and demanded compensation in the form of higher expected returns. If that were the case, the simple version of the CAPM cannot be the whole truth.

Again, it is hard to judge how seriously the CAPM is damaged by this finding. The relationship among stock returns and firm size and book-to-market ratio has been well documented. However, if you look long and hard at past returns, you are bound to find some strategy that just by chance would have worked in the past. This practice is known as “data-mining” or “data snooping.” Maybe the size and book-to-market effects are simply chance results that stem from data snooping. If so, they should have vanished once they were discovered. There is some evidence that this is the case. For example, if you look again at Figure 8.10, you will see that in the past 25 years small-firm stocks have underperformed just about as often as they have overperformed.

¹⁶ A second problem with testing the model is that the market portfolio should contain all risky investments, including stocks, bonds, commodities, real estate—even human capital. Most market indexes contain only a sample of common stocks.

¹⁷ We say “simple version” because Fischer Black has shown that if there are borrowing restrictions, there should still exist a positive relationship between expected return and beta, but the security market line would be less steep as a result. See F. Black, “Capital Market Equilibrium with Restricted Borrowing,” *Journal of Business* 45 (July 1972), pp. 444–455.

¹⁸ Fama and French calculated the returns on portfolios designed to take advantage of the size effect and the book-to-market effect. See E. F. Fama and K. R. French, “The Cross-Section of Expected Stock Returns,” *Journal of Financial Economics* 47 (June 1992), pp. 427–465. When calculating the returns on these portfolios, Fama and French control for differences in firm size when comparing stocks with low and high book-to-market ratios. Similarly, they control for differences in the book-to-market ratio when comparing small- and large-firm stocks. For details of the methodology and updated returns on the size and book-to-market factors see Kenneth French’s Web site (mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

¹⁹ An investor who bought small-company stocks and sold large-company stocks would have incurred some risk. Her portfolio would have had a beta of .28. This is not nearly large enough to explain the difference in returns. There is no simple relationship between the return on the value- and growth-stock portfolios and beta.

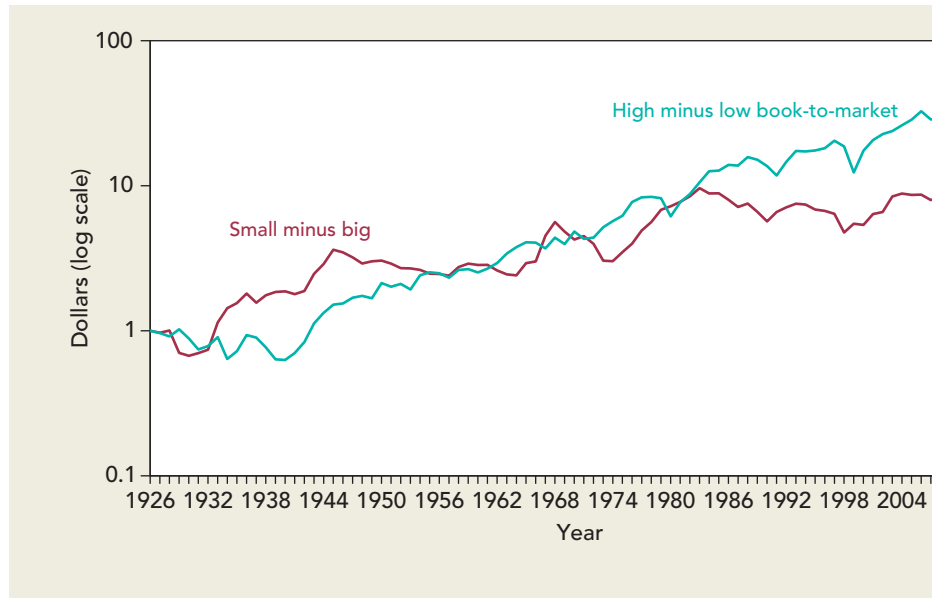


FIGURE 8.10

The red line shows the cumulative difference between the returns on small-firm and large-firm stocks. The green line shows the cumulative difference between the returns on high book-to-market-value stocks (i.e., value stocks) and low book-to-market-value stocks (i.e., growth stocks).

Source: Kenneth French's Web site, mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Used with permission.

There is no doubt that the evidence on the CAPM is less convincing than scholars once thought. But it will be hard to reject the CAPM beyond all reasonable doubt. Since data and statistics are unlikely to give final answers, the plausibility of the CAPM *theory* will have to be weighed along with the empirical “facts.”

Assumptions behind the Capital Asset Pricing Model

The capital asset pricing model rests on several assumptions that we did not fully spell out. For example, we assumed that investment in U.S. Treasury bills is risk-free. It is true that there is little chance of default, but bills do not guarantee a *real* return. There is still some uncertainty about inflation. Another assumption was that investors can *borrow* money at the same rate of interest at which they can lend. Generally borrowing rates are higher than lending rates.

It turns out that many of these assumptions are not crucial, and with a little pushing and pulling it is possible to modify the capital asset pricing model to handle them. The really important idea is that investors are content to invest their money in a limited number of benchmark portfolios. (In the basic CAPM these benchmarks are Treasury bills and the market portfolio.)

In these modified CAPMs expected return still depends on market risk, but the definition of market risk depends on the nature of the benchmark portfolios. In practice, none of these alternative capital asset pricing models is as widely used as the standard version.

8-4 Some Alternative Theories

The capital asset pricing model pictures investors as solely concerned with the level and uncertainty of their future wealth. But this could be too simplistic. For example, investors may become accustomed to a particular standard of living, so that poverty tomorrow may be particularly difficult to bear if you were wealthy yesterday. Behavioral psychologists have also observed that investors do not focus solely on the *current* value of their holdings, but look back at whether their investments are showing a profit. A gain, however small, may be

an additional source of satisfaction. The capital asset pricing model does not allow for the possibility that investors may take account of the price at which they purchased stock and feel elated when their investment is in the black and depressed when it is in the red.²⁰

Arbitrage Pricing Theory

The capital asset pricing theory begins with an analysis of how investors construct efficient portfolios. Stephen Ross's **arbitrage pricing theory**, or **APT**, comes from a different family entirely. It does not ask which portfolios are efficient. Instead, it starts by *assuming* that each stock's return depends partly on pervasive macroeconomic influences or "factors" and partly on "noise"—events that are unique to that company. Moreover, the return is assumed to obey the following simple relationship:

$$\text{Return} = a + b_1(r_{\text{factor 1}}) + b_2(r_{\text{factor 2}}) + b_3(r_{\text{factor 3}}) + \dots + \text{noise}$$

The theory does not say what the factors are: there could be an oil price factor, an interest-rate factor, and so on. The return on the market portfolio *might* serve as one factor, but then again it might not.

Some stocks will be more sensitive to a particular factor than other stocks. Exxon Mobil would be more sensitive to an oil factor than, say, Coca-Cola. If factor 1 picks up unexpected changes in oil prices, b_1 will be higher for Exxon Mobil.

For any individual stock there are two sources of risk. First is the risk that stems from the pervasive macroeconomic factors. This cannot be eliminated by diversification. Second is the risk arising from possible events that are specific to the company. Diversification eliminates specific risk, and diversified investors can therefore ignore it when deciding whether to buy or sell a stock. The expected risk premium on a stock is affected by factor or macroeconomic risk; it is *not* affected by specific risk.

Arbitrage pricing theory states that the expected risk premium on a stock should depend on the expected risk premium associated with each factor and the stock's sensitivity to each of the factors (b_1, b_2, b_3 , etc.). Thus the formula is²¹

$$\begin{aligned} \text{Expected risk premium} &= r - r_f \\ &= b_1(r_{\text{factor 1}} - r_f) + b_2(r_{\text{factor 2}} - r_f) + \dots \end{aligned}$$

Notice that this formula makes two statements:

1. If you plug in a value of zero for each of the b 's in the formula, the expected risk premium is zero. A diversified portfolio that is constructed to have zero sensitivity to each macroeconomic factor is essentially risk-free and therefore must be priced to offer the risk-free rate of interest. If the portfolio offered a higher return, investors could make a risk-free (or "arbitrage") profit by borrowing to buy the portfolio. If it offered a lower return, you could make an arbitrage profit by running the strategy in reverse; in other words, you would *sell* the diversified zero-sensitivity portfolio and *invest* the proceeds in U.S. Treasury bills.
2. A diversified portfolio that is constructed to have exposure to, say, factor 1, will offer a risk premium, which will vary in direct proportion to the portfolio's sensitivity to that factor. For example, imagine that you construct two portfolios, A and B, that are affected only by factor 1. If portfolio A is twice as sensitive as portfolio B to factor 1,

²⁰ We discuss aversion to loss again in Chapter 13. The implications for asset pricing are explored in S. Benartzi and R. Thaler, "Myopic Loss Aversion and the Equity Premium Puzzle," *Quarterly Journal of Economics* 110 (1995), pp. 75–92; and in N. Barberis, M. Huang, and T. Santos, "Prospect Theory and Asset Prices," *Quarterly Journal of Economics* 116 (2001), pp. 1–53.

²¹ There may be some macroeconomic factors that investors are simply not worried about. For example, some macroeconomists believe that money supply doesn't matter and therefore investors are not worried about inflation. Such factors would not command a risk premium. They would drop out of the APT formula for expected return.

portfolio A must offer twice the risk premium. Therefore, if you divided your money equally between U.S. Treasury bills and portfolio A, your combined portfolio would have exactly the same sensitivity to factor 1 as portfolio B and would offer the same risk premium.

Suppose that the arbitrage pricing formula did *not* hold. For example, suppose that the combination of Treasury bills and portfolio A offered a higher return. In that case investors could make an arbitrage profit by selling portfolio B and investing the proceeds in the mixture of bills and portfolio A.

The arbitrage that we have described applies to well-diversified portfolios, where the specific risk has been diversified away. But if the arbitrage pricing relationship holds for all diversified portfolios, it must generally hold for the individual stocks. Each stock must offer an expected return commensurate with its contribution to portfolio risk. In the APT, this contribution depends on the sensitivity of the stock's return to unexpected changes in the macroeconomic factors.

A Comparison of the Capital Asset Pricing Model and Arbitrage Pricing Theory

Like the capital asset pricing model, arbitrage pricing theory stresses that expected return depends on the risk stemming from economywide influences and is not affected by specific risk. You can think of the factors in arbitrage pricing as representing special portfolios of stocks that tend to be subject to a common influence. If the expected risk premium on each of these portfolios is proportional to the portfolio's market beta, then the arbitrage pricing theory and the capital asset pricing model will give the same answer. In any other case they will not.

How do the two theories stack up? Arbitrage pricing has some attractive features. For example, the market portfolio that plays such a central role in the capital asset pricing model does not feature in arbitrage pricing theory.²² So we do not have to worry about the problem of measuring the market portfolio, and in principle we can test the arbitrage pricing theory even if we have data on only a sample of risky assets.

Unfortunately you win some and lose some. Arbitrage pricing theory does not tell us what the underlying factors are—unlike the capital asset pricing model, which collapses *all* macroeconomic risks into a well-defined *single* factor, the return on the market portfolio.

The Three-Factor Model

Look back at the equation for APT. To estimate expected returns, you first need to follow three steps:

Step 1: Identify a reasonably short list of macroeconomic factors that could affect stock returns.

Step 2: Estimate the expected risk premium on each of these factors ($r_{\text{factor } 1} - r_f$, etc.).

Step 3: Measure the sensitivity of each stock to the factors (b_1, b_2 , etc.).

One way to shortcut this process is to take advantage of the research by Fama and French, which showed that stocks of small firms and those with a high book-to-market ratio have provided above-average returns. This could simply be a coincidence. But there is also some evidence that these factors are related to company profitability and therefore may be picking up risk factors that are left out of the simple CAPM.²³

²² Of course, the market portfolio *may* turn out to be one of the factors, but that is not a necessary implication of arbitrage pricing theory.

²³ E. F. Fama and K. R. French, "Size and Book-to-Market Factors in Earnings and Returns," *Journal of Finance* 50 (1995), pp. 131-155.

If investors do demand an extra return for taking on exposure to these factors, then we have a measure of the expected return that looks very much like arbitrage pricing theory:

$$r - r_f = b_{\text{market}}(r_{\text{market factor}}) + b_{\text{size}}(r_{\text{size factor}}) + b_{\text{book-to-market}}(r_{\text{book-to-market factor}})$$

This is commonly known as the Fama–French three-factor model. Using it to estimate expected returns is the same as applying the arbitrage pricing theory. Here is an example.²⁴

Step 1: Identify the Factors Fama and French have already identified the three factors that appear to determine expected returns. The returns on each of these factors are

Factor	Measured by
Market factor	Return on market index <i>minus</i> risk-free interest rate
Size factor	Return on small-firm stocks <i>less</i> return on large-firm stocks
Book-to-market factor	Return on high book-to-market-ratio stocks <i>less</i> return on low book-to-market-ratio stocks

Step 2: Estimate the Risk Premium for Each Factor We will keep to our figure of 7% for the market risk premium. History may provide a guide to the risk premium for the other two factors. As we saw earlier, between 1926 and 2008 the difference between the annual returns on small and large capitalization stocks averaged 3.6% a year, while the difference between the returns on stocks with high and low book-to-market ratios averaged 5.2%.

Step 3: Estimate the Factor Sensitivities Some stocks are more sensitive than others to fluctuations in the returns on the three factors. You can see this from the first three columns of numbers in Table 8.3, which show some estimates of the factor sensitivities of 10 industry groups for the 60 months ending in December 2008. For example, an increase of 1% in the return on the book-to-market factor *reduces* the return on computer stocks by .87% but *increases* the return on utility stocks by .77%. In other words, when value stocks (high book-to-market) outperform growth stocks (low book-to-market), computer stocks tend to perform relatively badly and utility stocks do relatively well.

Once you have estimated the factor sensitivities, it is a simple matter to multiply each of them by the expected factor return and add up the results. For example, the expected risk premium on computer stocks is $r - r_f = (1.43 \times 7) + (.22 \times 3.6) - (.87 \times 5.2) = 6.3\%$. To calculate the return that investors expected in 2008 we need to add on the risk-free interest rate of about .2%. Thus the three-factor model suggests that expected return on computer stocks in 2008 was $.2 + 6.3 = 6.5\%$.

Compare this figure with the expected return estimate using the capital asset pricing model (the final column of Table 8.3). The three-factor model provides a substantially lower estimate of the expected return for computer stocks. Why? Largely because computer stocks are growth stocks with a low exposure (−.87) to the book-to-market factor. The three-factor model produces a lower expected return for growth stocks, but it produces a higher figure for value stocks such as those of auto and construction companies which have a high book-to-market ratio.

²⁴ The three-factor model was first used to estimate the cost of capital for different industry groups by Fama and French. See E. F. Fama and K. R. French, “Industry Costs of Equity,” *Journal of Financial Economics* 43 (1997), pp. 153–193. Fama and French emphasize the imprecision in using either the CAPM or an APT-style model to estimate the returns that investors expect.

	Three-Factor Model			CAPM	
	Factor Sensitivities			Expected Return*	Expected Return**
	b_{market}	b_{size}	$b_{\text{book-to-market}}$		
Autos	1.51	.07	.91	15.7	7.9
Banks	1.16	-.25	.72	11.1	6.2
Chemicals	1.02	-.07	.61	10.2	5.5
Computers	1.43	.22	-.87	6.5	12.8
Construction	1.40	.46	.98	16.6	7.6
Food	.53	-.15	.47	5.8	2.7
Oil and gas	.85	-.13	.54	8.5	4.3
Pharmaceuticals	.50	-.32	-.13	1.9	4.3
Telecoms	1.05	-.29	-.16	5.7	7.3
Utilities	.61	-.01	.77	8.4	2.4

TABLE 8.3 Estimates of expected equity returns for selected industries using the Fama–French three-factor model and the CAPM.

* The expected return equals the risk-free interest rate plus the factor sensitivities multiplied by the factor risk premiums, that is, $r_f + (b_{\text{market}} \times 7) + (b_{\text{size}} \times 3.6) + (b_{\text{book-to-market}} \times 5.2)$.

** Estimated as $r_f + \beta(r_m - r_f)$, that is, $r_f + \beta \times 7$. Note that we used *simple* regression to estimate β in the CAPM formula. This beta may, therefore, be different from b_{market} that we estimated from a *multiple* regression of stock returns on the three factors.

The basic principles of portfolio selection boil down to a commonsense statement that investors try to increase the expected return on their portfolios and to reduce the standard deviation of that return. A portfolio that gives the highest expected return for a given standard deviation, or the lowest standard deviation for a given expected return, is known as an *efficient portfolio*. To work out which portfolios are efficient, an investor must be able to state the expected return and standard deviation of each stock and the degree of correlation between each pair of stocks.

Investors who are restricted to holding common stocks should choose efficient portfolios that suit their attitudes to risk. But investors who can also borrow and lend at the risk-free rate of interest should choose the *best* common stock portfolio *regardless* of their attitudes to risk. Having done that, they can then set the risk of their overall portfolio by deciding what proportion of their money they are willing to invest in stocks. The best efficient portfolio offers the highest ratio of forecasted risk premium to portfolio standard deviation.

For an investor who has only the same opportunities and information as everybody else, the best stock portfolio is the same as the best stock portfolio for other investors. In other words, he or she should invest in a mixture of the market portfolio and a risk-free loan (i.e., borrowing or lending).

A stock's marginal contribution to portfolio risk is measured by its sensitivity to changes in the value of the portfolio. The marginal contribution of a stock to the risk of the *market portfolio* is measured by *beta*. That is the fundamental idea behind the capital asset pricing model (CAPM), which concludes that each security's expected risk premium should increase in proportion to its beta:

$$\begin{aligned} \text{Expected risk premium} &= \text{beta} \times \text{market risk premium} \\ r - r_f &= \beta(r_m - r_f) \end{aligned}$$

The capital asset pricing theory is the best-known model of risk and return. It is plausible and widely used but far from perfect. Actual returns are related to beta over the long run, but the relationship is not as strong as the CAPM predicts, and other factors seem to explain returns better since the mid-1960s. Stocks of small companies, and stocks with high book values relative to market prices, appear to have risks not captured by the CAPM.



SUMMARY

The arbitrage pricing theory offers an alternative theory of risk and return. It states that the expected risk premium on a stock should depend on the stock's exposure to several pervasive macroeconomic factors that affect stock returns:

$$\text{Expected risk premium} = b_1(r_{\text{factor 1}} - r_f) + b_2(r_{\text{factor 2}} - r_f) + \dots$$

Here b 's represent the individual security's sensitivities to the factors, and $r_{\text{factor}} - r_f$ is the risk premium demanded by investors who are exposed to this factor.

Arbitrage pricing theory does not say what these factors are. It asks for economists to hunt for unknown game with their statistical toolkits. Fama and French have suggested three factors:

- The return on the market portfolio less the risk-free rate of interest.
- The difference between the return on small- and large-firm stocks.
- The difference between the return on stocks with high book-to-market ratios and stocks with low book-to-market ratios.

In the Fama–French three-factor model, the expected return on each stock depends on its exposure to these three factors.

Each of these different models of risk and return has its fan club. However, all financial economists agree on two basic ideas: (1) Investors require extra expected return for taking on risk, and (2) they appear to be concerned predominantly with the risk that they cannot eliminate by diversification.

Near the end of Chapter 9 we list some Excel Functions that are useful for measuring the risk of stocks and portfolios.



FURTHER READING

A number of textbooks on portfolio selection explain both Markowitz's original theory and some ingenious simplified versions. See, for example:

E. J. Elton, M. J. Gruber, S. J. Brown, and W. N. Goetzmann: *Modern Portfolio Theory and Investment Analysis*, 7th ed. (New York: John Wiley & Sons, 2007).

The literature on the capital asset pricing model is enormous. There are dozens of published tests of the capital asset pricing model. Fisher Black's paper is a very readable example. Discussions of the theory tend to be more uncompromising. Two excellent but advanced examples are Campbell's survey paper and Cochrane's book.

F. Black, "Beta and Return," *Journal of Portfolio Management* 20 (Fall 1993), pp. 8–18.

J. Y. Campbell, "Asset Pricing at the Millennium," *Journal of Finance* 55 (August 2000), pp. 1515–1567.

J. H. Cochrane, *Asset Pricing*, revised ed. (Princeton, NJ: Princeton University Press, 2004).



Select problems are available in McGraw-Hill Connect. Please see the preface for more information.

PROBLEM SETS

BASIC

1. Here are returns and standard deviations for four investments.

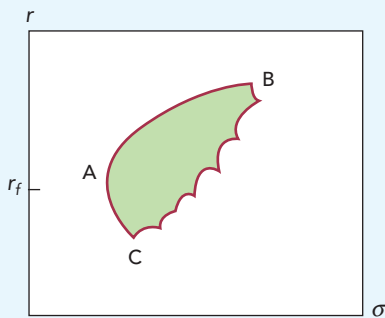
	Return	Standard Deviation
Treasury bills	6 %	0%
Stock P	10	14
Stock Q	14.5	28
Stock R	21	26

Calculate the standard deviations of the following portfolios.

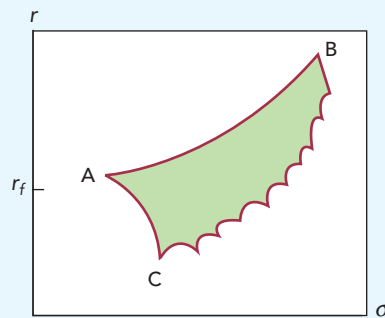
- a. 50% in Treasury bills, 50% in stock P.
 - b. 50% each in Q and R, assuming the shares have
 - perfect positive correlation
 - perfect negative correlation
 - no correlation
 - c. Plot a figure like Figure 8.3 for Q and R, assuming a correlation coefficient of .5.
 - d. Stock Q has a lower return than R but a higher standard deviation. Does that mean that Q's price is too high or that R's price is too low?
2. For each of the following pairs of investments, state which would always be preferred by a rational investor (assuming that these are the *only* investments available to the investor):
 - a. Portfolio A $r = 18\%$ $\sigma = 20\%$
Portfolio B $r = 14\%$ $\sigma = 20\%$
 - b. Portfolio C $r = 15\%$ $\sigma = 18\%$
Portfolio D $r = 13\%$ $\sigma = 8\%$
 - c. Portfolio E $r = 14\%$ $\sigma = 16\%$
Portfolio F $r = 14\%$ $\sigma = 10\%$
 3. Use the long-term data on security returns in Sections 7-1 and 7-2 to calculate the historical level of the Sharpe ratio of the market portfolio.
 4. Figure 8.11 below purports to show the range of attainable combinations of expected return and standard deviation.
 - a. Which diagram is incorrectly drawn and why?
 - b. Which is the efficient set of portfolios?
 - c. If r_f is the rate of interest, mark with an X the optimal stock portfolio.
 5. a. Plot the following risky portfolios on a graph:

	Portfolio							
	A	B	C	D	E	F	G	H
Expected return (r), %	10	12.5	15	16	17	18	18	20
Standard deviation (σ), %	23	21	25	29	29	32	35	45

- b. Five of these portfolios are efficient, and three are not. Which are *inefficient* ones?
- c. Suppose you can also borrow and lend at an interest rate of 12%. Which of the above portfolios has the highest Sharpe ratio?



(a)



(b)

FIGURE 8.11

See Problem 4.

- d. Suppose you are prepared to tolerate a standard deviation of 25%. What is the maximum expected return that you can achieve if you cannot borrow or lend?
 - e. What is your optimal strategy if you can borrow or lend at 12% and are prepared to tolerate a standard deviation of 25%? What is the maximum expected return that you can achieve with this risk?
6. Suppose that the Treasury bill rate were 6% rather than 4%. Assume that the expected return on the market stays at 10%. Use the betas in Table 8.2.
 - a. Calculate the expected return from Dell.
 - b. Find the highest expected return that is offered by one of these stocks.
 - c. Find the lowest expected return that is offered by one of these stocks.
 - d. Would Ford offer a higher or lower expected return if the interest rate were 6% rather than 4%? Assume that the expected market return stays at 10%.
 - e. Would Exxon Mobil offer a higher or lower expected return if the interest rate were 8%?
 7. True or false?
 - a. The CAPM implies that if you could find an investment with a negative beta, its expected return would be less than the interest rate.
 - b. The expected return on an investment with a beta of 2.0 is twice as high as the expected return on the market.
 - c. If a stock lies below the security market line, it is undervalued.
 8. Consider a three-factor APT model. The factors and associated risk premiums are

Factor	Risk Premium
Change in GNP	5%
Change in energy prices	-1
Change in long-term interest rates	+2

Calculate expected rates of return on the following stocks. The risk-free interest rate is 7%.

- a. A stock whose return is uncorrelated with all three factors.
- b. A stock with average exposure to each factor (i.e., with $b = 1$ for each).
- c. A pure-play energy stock with high exposure to the energy factor ($b = 2$) but zero exposure to the other two factors.
- d. An aluminum company stock with average sensitivity to changes in interest rates and GNP, but negative exposure of $b = -1.5$ to the energy factor. (The aluminum company is energy-intensive and suffers when energy prices rise.)

INTERMEDIATE

9. True or false? Explain or qualify as necessary.
 - a. Investors demand higher expected rates of return on stocks with more variable rates of return.
 - b. The CAPM predicts that a security with a beta of 0 will offer a zero expected return.
 - c. An investor who puts \$10,000 in Treasury bills and \$20,000 in the market portfolio will have a beta of 2.0.
 - d. Investors demand higher expected rates of return from stocks with returns that are highly exposed to macroeconomic risks.
 - e. Investors demand higher expected rates of return from stocks with returns that are very sensitive to fluctuations in the stock market.

10. Look back at the calculation for Campbell Soup and Boeing in Section 8.1. Recalculate the expected portfolio return and standard deviation for different values of x_1 and x_2 , assuming the correlation coefficient $\rho_{12} = 0$. Plot the range of possible combinations of expected return and standard deviation as in Figure 8.3. Repeat the problem for $\rho_{12} = +.5$.
11. Mark Harrywitz proposes to invest in two shares, X and Y. He expects a return of 12% from X and 8% from Y. The standard deviation of returns is 8% for X and 5% for Y. The correlation coefficient between the returns is .2.
- Compute the expected return and standard deviation of the following portfolios:

Portfolio	Percentage in X	Percentage in Y
1	50	50
2	25	75
3	75	25

- Sketch the set of portfolios composed of X and Y.
 - Suppose that Mr. Harrywitz can also borrow or lend at an interest rate of 5%. Show on your sketch how this alters his opportunities. Given that he can borrow or lend, what proportions of the common stock portfolio should be invested in X and Y?
12. Ebenezer Scrooge has invested 60% of his money in share A and the remainder in share B. He assesses their prospects as follows:

	A	B
Expected return (%)	15	20
Standard deviation (%)	20	22
Correlation between returns	.5	

- What are the expected return and standard deviation of returns on his portfolio?
 - How would your answer change if the correlation coefficient were 0 or $-.5$?
 - Is Mr. Scrooge's portfolio better or worse than one invested entirely in share A, or is it not possible to say?
13. Look back at Problem 3 in Chapter 7. The risk-free interest rate in each of these years was as follows:

	2003	2004	2005	2006	2007
Interest rate%	1.01	1.37	3.15	4.73	4.36

- Calculate the average return and standard deviation of returns for Ms. Sauros's portfolio and for the market. Use these figures to calculate the Sharpe ratio for the portfolio and the market. On this measure did Ms. Sauros perform better or worse than the market?
 - Now calculate the average return that you could have earned over this period if you had held a combination of the market and a risk-free loan. Make sure that the combination has the same beta as Ms. Sauros's portfolio. Would your average return on this portfolio have been higher or lower?
- Explain your results.
14. Look back at Table 7.5 on page 174.
- What is the beta of a portfolio that has 40% invested in Disney and 60% in Exxon Mobil?

- b. Would you invest in this portfolio if you had no superior information about the prospects for these stocks? Devise an alternative portfolio with the same expected return and less risk.
- c. Now repeat parts (a) and (b) with a portfolio that has 40% invested in Amazon and 60% in Dell.
- 15.** The Treasury bill rate is 4%, and the expected return on the market portfolio is 12%. Using the capital asset pricing model:
- Draw a graph similar to Figure 8.6 showing how the expected return varies with beta.
 - What is the risk premium on the market?
 - What is the required return on an investment with a beta of 1.5?
 - If an investment with a beta of .8 offers an expected return of 9.8%, does it have a positive NPV?
 - If the market expects a return of 11.2% from stock X, what is its beta?
- 16.** Percival Hygiene has \$10 million invested in long-term corporate bonds. This bond portfolio's expected annual rate of return is 9%, and the annual standard deviation is 10%.

Amanda Reckonwith, Percival's financial adviser, recommends that Percival consider investing in an index fund that closely tracks the Standard & Poor's 500 index. The index has an expected return of 14%, and its standard deviation is 16%.

- Suppose Percival puts all his money in a combination of the index fund and Treasury bills. Can he thereby improve his expected rate of return without changing the risk of his portfolio? The Treasury bill yield is 6%.
 - Could Percival do even better by investing equal amounts in the corporate bond portfolio and the index fund? The correlation between the bond portfolio and the index fund is +.1.
- 17.** Epsilon Corp. is evaluating an expansion of its business. The cash-flow forecasts for the project are as follows:

Years	Cash Flow (\$ millions)
0	-100
1-10	+15

The firm's existing assets have a beta of 1.4. The risk-free interest rate is 4% and the expected return on the market portfolio is 12%. What is the project's NPV?

- 18.** Some true or false questions about the APT:
- The APT factors cannot reflect diversifiable risks.
 - The market rate of return cannot be an APT factor.
 - There is no theory that specifically identifies the APT factors.
 - The APT model could be true but not very useful, for example, if the relevant factors change unpredictably.
- 19.** Consider the following simplified APT model:

Factor	Expected Risk Premium
Market	6.4%
Interest rate	-.6
Yield spread	5.1

Calculate the expected return for the following stocks. Assume $r_f = 5\%$.

Factor Risk Exposures			
	Market	Interest Rate	Yield Spread
Stock	(b_1)	(b_2)	(b_3)
P	1.0	-2.0	-.2
P ²	1.2	0	.3
P ³	.3	.5	1.0

20. Look again at Problem 19. Consider a portfolio with equal investments in stocks P, P², and P³.
- What are the factor risk exposures for the portfolio?
 - What is the portfolio's expected return?
21. The following table shows the sensitivity of four stocks to the three Fama–French factors. Estimate the expected return on each stock assuming that the interest rate is .2%, the expected risk premium on the market is 7%, the expected risk premium on the size factor is 3.6%, and the expected risk premium on the book-to-market factor is 5.2%.

	Boeing	Johnson & Johnson	Dow Chemical	Microsoft
Market	0.66	0.54	1.05	0.91
Size	1.19	-0.58	-0.15	-0.04
Book-to-market	-0.76	0.19	0.77	-0.40

CHALLENGE

22. In footnote 4 we noted that the minimum-risk portfolio contained an investment of 73.1% in Campbell Soup and 26.9% in Boeing. Prove it. (*Hint:* You need a little calculus to do so.)
23. Look again at the set of the three efficient portfolios that we calculated in Section 8.1.
- If the interest rate is 10%, which of the four efficient portfolios should you hold?
 - What is the beta of each holding relative to that portfolio? (*Hint:* Note that if a portfolio is efficient, the expected risk premium on each holding must be proportional to the beta of the stock *relative to that portfolio*.)
 - How would your answers to (a) and (b) change if the interest rate were 5%?
24. The following question illustrates the APT. Imagine that there are only two pervasive macroeconomic factors. Investments X, Y, and Z have the following sensitivities to these two factors:

Investment	b_1	b_2
X	1.75	.25
Y	-1.00	2.00
Z	2.00	1.00

We assume that the expected risk premium is 4% on factor 1 and 8% on factor 2. Treasury bills obviously offer zero risk premium.

- According to the APT, what is the risk premium on each of the three stocks?
- Suppose you buy \$200 of X and \$50 of Y and sell \$150 of Z. What is the sensitivity of your portfolio to each of the two factors? What is the expected risk premium?

- c. Suppose you buy \$80 of X and \$60 of Y and sell \$40 of Z. What is the sensitivity of your portfolio to each of the two factors? What is the expected risk premium?
- d. Finally, suppose you buy \$160 of X and \$20 of Y and sell \$80 of Z. What is your portfolio's sensitivity now to each of the two factors? And what is the expected risk premium?
- e. Suggest two possible ways that you could construct a fund that has a sensitivity of .5 to factor 1 only. (*Hint:* One portfolio contains an investment in Treasury bills.) Now compare the risk premiums on each of these two investments.
- f. Suppose that the APT did *not* hold and that X offered a risk premium of 8%, Y offered a premium of 14%, and Z offered a premium of 16%. Devise an investment that has zero sensitivity to each factor and that has a positive risk premium.



REAL-TIME DATA ANALYSIS

STANDARD
& POOR'S

You can download data for the following questions from the Standard & Poor's Market Insight Web site (www.mhhe.com/edumarketinsight)—see the “Monthly Adjusted Prices” spreadsheet—or from finance.yahoo.com.

Note: When we calculated the efficient portfolios in Table 8.1, we assumed that the investor could not hold short positions (i.e., have negative holdings). The book's Web site (www.mhhe.com/bma) contains an Excel program for calculating the efficient frontier with short sales. (We are grateful to Simon Gervais for providing us with a copy of this program.) Excel functions SLOPE, STDEV, and CORREL are especially useful for answering the following questions.

1.
 - a. Look at the efficient portfolios constructed from the 10 stocks in Table 8.1. How does the possibility of short sales improve the choices open to the investor?
 - b. Now download up to 10 years of monthly returns for 10 different stocks and enter them into the Excel program. Enter some plausible figures for the expected return on each stock and find the set of efficient portfolios.
2. Find a low-risk stock—Exxon Mobil or Kellogg would be a good candidate. Use monthly returns for the most recent three years to confirm that the beta is less than 1.0. Now estimate the annual standard deviation for the stock and the S&P index, and the correlation between the returns on the stock and the index. Forecast the expected return for the stock, assuming the CAPM holds, with a market return of 12% and a risk-free rate of 5%.
 - a. Plot a graph like Figure 8.5 showing the combinations of risk and return from a portfolio invested in your low-risk stock and the market. Vary the fraction invested in the stock from 0 to 100%.
 - b. Suppose that you can borrow or lend at 5%. Would you invest in some combination of your low-risk stock and the market, or would you simply invest in the market? Explain.
 - c. Suppose that you forecasted a return on the stock that is 5 percentage points higher than the CAPM return used in part (b). Redo parts (a) and (b) with the higher forecasted return.
 - d. Find a high-risk stock and redo parts (a) and (b).
3. Recalculate the betas for the stocks in Table 8.2 using the latest 60 monthly returns. Recalculate expected rates of return from the CAPM formula, using a current risk-free rate and a market risk premium of 7%. How have the expected returns changed from Table 8.2?

MINI-CASE

John and Marsha on Portfolio Selection

The scene: John and Marsha hold hands in a cozy French restaurant in downtown Manhattan, several years before the mini-case in Chapter 9. Marsha is a futures-market trader. John manages a \$125 million common-stock portfolio for a large pension fund. They have just ordered *tourne-dos financiers* for the main course and *flan financier* for dessert. John reads the financial pages of *The Wall Street Journal* by candlelight.

John: Wow! Potato futures hit their daily limit. Let's add an order of gratin dauphinoise. Did you manage to hedge the forward interest rate on that euro loan?

Marsha: John, please fold up that paper. (*He does so reluctantly.*) John, I love you. Will you marry me?

John: Oh, Marsha, I love you too, but . . . there's something you must know about me—something I've never told anyone.

Marsha (concerned): John, what is it?

John: I think I'm a closet indexer.

Marsha: What? Why?

John: My portfolio returns always seem to track the S&P 500 market index. Sometimes I do a little better, occasionally a little worse. But the correlation between my returns and the market returns is over 90%.

Marsha: What's wrong with that? Your client wants a diversified portfolio of large-cap stocks. Of course your portfolio will follow the market.

John: Why doesn't my client just buy an index fund? Why is he paying *me*? Am I really adding value by active management? I try, but I guess I'm just an . . . indexer.

Marsha: Oh, John, I know you're adding value. You were a star security analyst.

John: It's not easy to find stocks that are truly over- or undervalued. I have firm opinions about a few, of course.

Marsha: You were explaining why Pioneer Gypsum is a good buy. And you're bullish on Global Mining.

John: Right, Pioneer. (*Pulls handwritten notes from his coat pocket.*) Stock price \$87.50. I estimate the expected return as 11% with an annual standard deviation of 32%.

Marsha: Only 11%? You're forecasting a market return of 12.5%.

John: Yes, I'm using a market risk premium of 7.5% and the risk-free interest rate is about 5%. That gives 12.5%. But Pioneer's beta is only .65. I was going to buy 30,000 shares this morning, but I lost my nerve. I've got to stay diversified.

Marsha: Have you tried modern portfolio theory?

John: MPT? Not practical. Looks great in textbooks, where they show efficient frontiers with 5 or 10 stocks. But I choose from hundreds, maybe thousands, of stocks. Where do I get the inputs for 1,000 stocks? That's a million variances and covariances!

Marsha: Actually only about 500,000, dear. The covariances above the diagonal are the same as the covariances below. But you're right, most of the estimates would be out-of-date or just garbage.

John: To say nothing about the expected returns. Garbage in, garbage out.

Marsha: But John, you don't need to solve for 1,000 portfolio weights. You only need a handful. Here's the trick: Take your benchmark, the S&P 500, as security 1. That's what you would

end up with as an indexer. Then consider a few securities you really know something about. Pioneer could be security 2, for example. Global, security 3. And so on. Then you could put your wonderful financial mind to work.

John: I get it: active management means selling off some of the benchmark portfolio and investing the proceeds in specific stocks like Pioneer. But how do I decide whether Pioneer really improves the portfolio? Even if it does, how much should I buy?

Marsha: Just maximize the Sharpe ratio, dear.

John: I've got it! The answer is yes!

Marsha: What's the question?

John: You asked me to marry you. The answer is yes. Where should we go on our honeymoon?

Marsha: How about Australia? I'd love to visit the Sydney Futures Exchange.

QUESTIONS

1. Table 8.4 reproduces John's notes on Pioneer Gypsum and Global Mining. Calculate the expected return, risk premium, and standard deviation of a portfolio invested partly in the market and partly in Pioneer. (You can calculate the necessary inputs from the betas and standard deviations given in the table.) Does adding Pioneer to the market benchmark improve the Sharpe ratio? How much should John invest in Pioneer and how much in the market?
2. Repeat the analysis for Global Mining. What should John do in this case? Assume that Global accounts for .75% of the S&P index.

TABLE 8.4 John's notes on Pioneer Gypsum and Global Mining.

	Pioneer Gypsum	Global Mining
Expected return	11.0%	12.9%
Standard deviation	32%	20%
Beta	.65	1.22
Stock price	\$87.50	\$105.00