

Introduction to Risk and Return

► **We have managed** to go through six chapters without directly addressing the problem of risk, but now the jig is up. We can no longer be satisfied with vague statements like “The opportunity cost of capital depends on the risk of the project.” We need to know how risk is defined, what the links are between risk and the opportunity cost of capital, and how the financial manager can cope with risk in practical situations.

In this chapter we concentrate on the first of these issues and leave the other two to Chapters 8 and 9. We start by summarizing more than 100 years of evidence

on rates of return in capital markets. Then we take a first look at investment risks and show how they can be reduced by portfolio diversification. We introduce you to beta, the standard risk measure for individual securities.

The themes of this chapter, then, are portfolio risk, security risk, and diversification. For the most part, we take the view of the individual investor. But at the end of the chapter we turn the problem around and ask whether diversification makes sense as a corporate objective.



7-1 Over a Century of Capital Market History in One Easy Lesson

Financial analysts are blessed with an enormous quantity of data. There are comprehensive databases of the prices of U.S. stocks, bonds, options, and commodities, as well as huge amounts of data for securities in other countries. We focus on a study by Dimson, Marsh, and Staunton that measures the historical performance of three portfolios of U.S. securities:¹

1. A portfolio of Treasury bills, that is, U.S. government debt securities maturing in less than one year.²
2. A portfolio of U.S. government bonds.
3. A portfolio of U.S. common stocks.

These investments offer different degrees of risk. Treasury bills are about as safe an investment as you can make. There is no risk of default, and their short maturity means that the prices of Treasury bills are relatively stable. In fact, an investor who wishes to lend money for, say, three months can achieve a perfectly certain payoff by purchasing a Treasury bill maturing in three months. However, the investor cannot lock in a *real* rate of return: There is still some uncertainty about inflation.

¹ See E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002).

² Treasury bills were not issued before 1919. Before that date the interest rate used is the commercial paper rate.

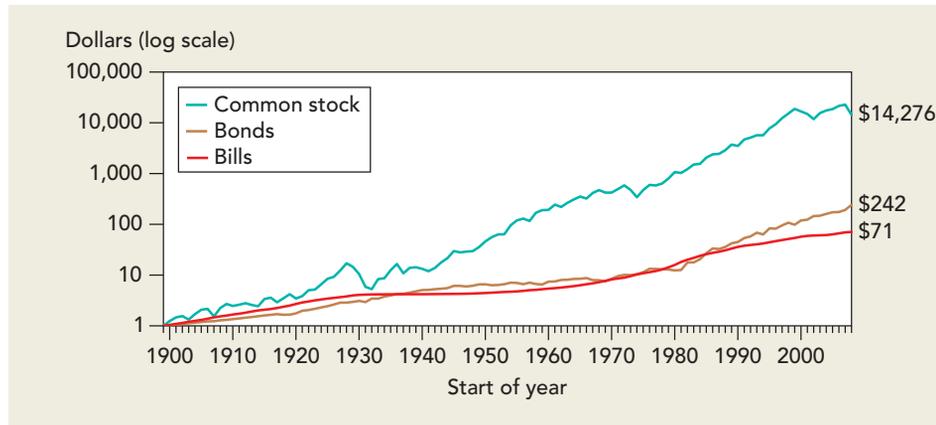


FIGURE 7.1

How an investment of \$1 at the start of 1900 would have grown by the end of 2008, assuming reinvestment of all dividend and interest payments.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

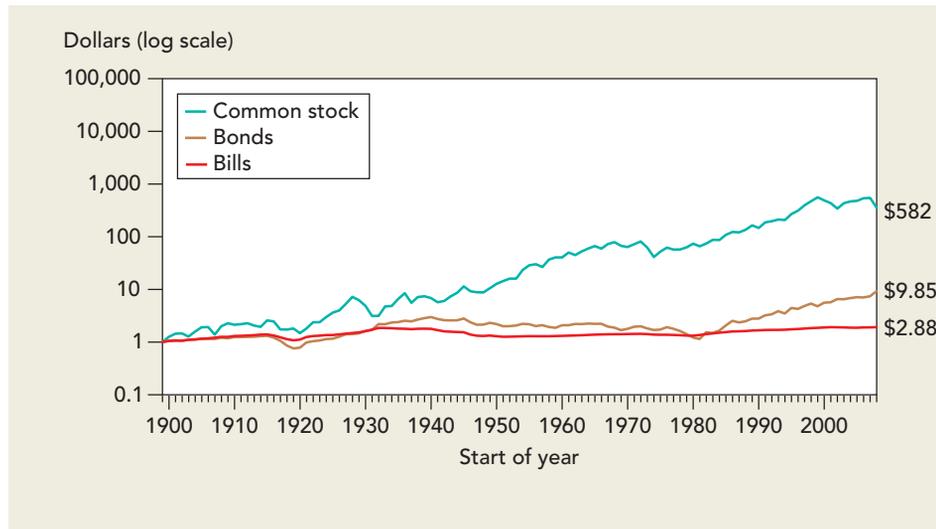


FIGURE 7.2

How an investment of \$1 at the start of 1900 would have grown in real terms by the end of 2008, assuming reinvestment of all dividend and interest payments. Compare this plot with Figure 7.1, and note how inflation has eroded the purchasing power of returns to investors.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

By switching to long-term government bonds, the investor acquires an asset whose price fluctuates as interest rates vary. (Bond prices fall when interest rates rise and rise when interest rates fall.) An investor who shifts from bonds to common stocks shares in all the ups and downs of the issuing companies.

Figure 7.1 shows how your money would have grown if you had invested \$1 at the start of 1900 and reinvested all dividend or interest income in each of the three portfolios.³ Figure 7.2 is identical except that it depicts the growth in the *real* value of the portfolio. We focus here on nominal values.

Investment performance coincides with our intuitive risk ranking. A dollar invested in the safest investment, Treasury bills, would have grown to \$71 by the end of 2008, barely enough to keep up with inflation. An investment in long-term Treasury bonds would have

³ Portfolio values are plotted on a log scale. If they were not, the ending values for the common stock portfolio would run off the top of the page.

TABLE 7.1 Average rates of return on U.S. Treasury bills, government bonds, and common stocks, 1900–2008 (figures in % per year).

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns*, (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

| | Average Annual Rate of Return | | Average Risk Premium (Extra Return versus Treasury Bills) |
|------------------|-------------------------------|------|---|
| | Nominal | Real | |
| Treasury bills | 4.0 | 1.1 | 0 |
| Government bonds | 5.5 | 2.6 | 1.5 |
| Common stocks | 11.1 | 8.0 | 7.1 |

produced \$242. Common stocks were in a class by themselves. An investor who placed a dollar in the stocks of large U.S. firms would have received \$14,276.

We can also calculate the rate of return from these portfolios for each year from 1900 to 2008. This rate of return reflects both cash receipts—dividends or interest—and the capital gains or losses realized during the year. Averages of the 109 annual rates of return for each portfolio are shown in Table 7.1.

Since 1900 Treasury bills have provided the lowest average return—4.0% per year in *nominal* terms and 1.1% in *real* terms. In other words, the average rate of inflation over this period was about 3% per year. Common stocks were again the winners. Stocks of major corporations provided an average nominal return of 11.1%. By taking on the risk of common stocks, investors earned a *risk premium* of $11.1 - 4.0 = 7.1\%$ over the return on Treasury bills.

You may ask why we look back over such a long period to measure average rates of return. The reason is that annual rates of return for common stocks fluctuate so much that averages taken over short periods are meaningless. Our only hope of gaining insights from historical rates of return is to look at a very long period.⁴

Arithmetic Averages and Compound Annual Returns

Notice that the average returns shown in Table 7.1 are arithmetic averages. In other words, we simply added the 109 annual returns and divided by 109. The arithmetic average is higher than the compound annual return over the period. The 109-year compound annual return for common stocks was 9.2%.⁵

The proper uses of arithmetic and compound rates of return from past investments are often misunderstood. Therefore, we call a brief time-out for a clarifying example.

Suppose that the price of Big Oil's common stock is \$100. There is an equal chance that at the end of the year the stock will be worth \$90, \$110, or \$130. Therefore, the return could be -10% , $+10\%$, or $+30\%$ (we assume that Big Oil does not pay a dividend). The *expected* return is $\frac{1}{3}(-10 + 10 + 30) = +10\%$.

⁴ We cannot be sure that this period is truly representative and that the average is not distorted by a few unusually high or low returns. The reliability of an estimate of the average is usually measured by its *standard error*. For example, the standard error of our estimate of the average risk premium on common stocks is 1.9%. There is a 95% chance that the *true* average is within plus or minus 2 standard errors of the 7.1% estimate. In other words, if you said that the true average was between 3.3 and 10.9%, you would have a 95% chance of being right. *Technical note:* The standard error of the average is equal to the standard deviation divided by the square root of the number of observations. In our case the standard deviation is 20.2%, and therefore the standard error is $20.2/\sqrt{109} = 1.9\%$.

⁵ This was calculated from $(1 + r)^{109} = 14,276$, which implies $r = .092$. *Technical note:* For lognormally distributed returns the annual compound return is equal to the arithmetic average return minus half the variance. For example, the annual standard deviation of returns on the U.S. market was about .20, or 20%. Variance was therefore $.20^2$, or .04. The compound annual return is about $.04/2 = .02$, or 2 percentage points less than the arithmetic average.

If we run the process in reverse and discount the expected cash flow by the expected rate of return, we obtain the value of Big Oil's stock:

$$PV = \frac{110}{1.10} = \$100$$

The expected return of 10% is therefore the correct rate at which to discount the expected cash flow from Big Oil's stock. It is also the opportunity cost of capital for investments that have the same degree of risk as Big Oil.

Now suppose that we observe the returns on Big Oil stock over a large number of years. If the odds are unchanged, the return will be -10% in a third of the years, $+10\%$ in a further third, and $+30\%$ in the remaining years. The arithmetic average of these yearly returns is

$$\frac{-10 + 10 + 30}{3} = +10\%$$

Thus the arithmetic average of the returns correctly measures the opportunity cost of capital for investments of similar risk to Big Oil stock.⁶

The average compound annual return⁷ on Big Oil stock would be

$$(.9 \times 1.1 \times 1.3)^{1/3} - 1 = .088, \text{ or } 8.8\%.$$

which is *less* than the opportunity cost of capital. Investors would not be willing to invest in a project that offered an 8.8% expected return if they could get an expected return of 10% in the capital markets. The net present value of such a project would be

$$NPV = -100 + \frac{108.8}{1.1} = -1.1$$

Moral: If the cost of capital is estimated from historical returns or risk premiums, use arithmetic averages, not compound annual rates of return.⁸

Using Historical Evidence to Evaluate Today's Cost of Capital

Suppose there is an investment project that you *know*—don't ask how—has the same risk as Standard and Poor's Composite Index. We will say that it has the same degree of risk as the *market portfolio*, although this is speaking somewhat loosely, because the index does not include all risky securities. What rate should you use to discount this project's forecasted cash flows?

⁶ You sometimes hear that the arithmetic average correctly measures the opportunity cost of capital for one-year cash flows, but not for more distant ones. Let us check. Suppose that you expect to receive a cash flow of \$121 in year 2. We know that one year hence investors will value that cash flow by discounting at 10% (the arithmetic average of possible returns). In other words, at the end of the year they will be willing to pay $PV_1 = 121/1.10 = \$110$ for the expected cash flow. But we already know how to value an asset that pays off \$110 in year 1—just discount at the 10% opportunity cost of capital. Thus $PV_0 = PV_1/1.10 = 110/1.1 = \100 . Our example demonstrates that the arithmetic average (10% in our example) provides a correct measure of the opportunity cost of capital regardless of the timing of the cash flow.

⁷ The compound annual return is often referred to as the *geometric average* return.

⁸ Our discussion above assumed that we *know* that the returns of -10 , $+10$, and $+30\%$ were equally likely. For an analysis of the effect of uncertainty about the expected return see I. A. Cooper, "Arithmetic Versus Geometric Mean Estimators: Setting Discount Rates for Capital Budgeting," *European Financial Management* 2 (July 1996), pp. 157–167; and E. Jaquier, A. Kane, and A. J. Marcus, "Optimal Estimation of the Risk Premium for the Long Run and Asset Allocation: A Case of Compounded Estimation Risk," *Journal of Financial Econometrics* 3 (2005), pp. 37–55. When future returns are forecasted to distant horizons, the historical arithmetic means are upward-biased. This bias would be small in most corporate-finance applications, however.

Clearly you should use the currently expected rate of return on the market portfolio; that is the return investors would forgo by investing in the proposed project. Let us call this market return r_m . One way to estimate r_m is to assume that the future will be like the past and that today's investors expect to receive the same "normal" rates of return revealed by the averages shown in Table 7.1. In this case, you would set r_m at 11.1%, the average of past market returns.

Unfortunately, this is *not* the way to do it; r_m is not likely to be stable over time. Remember that it is the sum of the risk-free interest rate r_f and a premium for risk. We know that r_f varies. For example, in 1981 the interest rate on Treasury bills was about 15%. It is difficult to believe that investors in that year were content to hold common stocks offering an expected return of only 11.1%.

If you need to estimate the return that investors expect to receive, a more sensible procedure is to take the interest rate on Treasury bills and add 7.1%, the average *risk premium* shown in Table 7.1. For example, in early 2009 the interest rate on Treasury bills was unusually low at .2%. Adding on the average risk premium, therefore, gives

$$\begin{aligned} r_m(2009) &= r_f(2009) + \text{normal risk premium} \\ &= .002 + .071 = .073, \text{ or } 7.3\% \end{aligned}$$

The crucial assumption here is that there is a normal, stable risk premium on the market portfolio, so that the expected *future* risk premium can be measured by the average past risk premium.

Even with over 100 years of data, we can't estimate the market risk premium exactly; nor can we be sure that investors today are demanding the same reward for risk that they were 50 or 100 years ago. All this leaves plenty of room for argument about what the risk premium *really* is.⁹

Many financial managers and economists believe that long-run historical returns are the best measure available. Others have a gut instinct that investors don't need such a large risk premium to persuade them to hold common stocks.¹⁰ For example, surveys of chief financial officers commonly suggest that they expect a market risk premium that is several percentage points below the historical average.¹¹

If you believe that the expected market risk premium is less than the historical average, you probably also believe that history has been unexpectedly kind to investors in the United States and that their good luck is unlikely to be repeated. Here are two reasons that history *may* overstate the risk premium that investors demand today.

Reason 1 Since 1900 the United States has been among the world's most prosperous countries. Other economies have languished or been wracked by war or civil unrest. By focusing on equity returns in the United States, we may obtain a biased view of what

⁹ Some of the disagreements simply reflect the fact that the risk premium is sometimes defined in different ways. Some measure the average difference between stock returns and the returns (or yields) on long-term bonds. Others measure the difference between the compound rate of growth on stocks and the interest rate. As we explained above, this is not an appropriate measure of the cost of capital.

¹⁰ There is some theory behind this instinct. The high risk premium earned in the market seems to imply that investors are extremely risk-averse. If that is true, investors ought to cut back their consumption when stock prices fall and wealth decreases. But the evidence suggests that when stock prices fall, investors spend at nearly the same rate. This is difficult to reconcile with high risk aversion and a high market risk premium. There is an active research literature on this "equity premium puzzle." See R. Mehra, "The Equity Premium Puzzle: A Review," *Foundations and Trends in Finance* 2 (2006), pp. 11–81, and R. Mehra, ed., *Handbook of the Equity Risk Premium* (Amsterdam: Elsevier Handbooks in Finance Series, 2008).

¹¹ It is difficult to interpret the responses to such surveys precisely. The best known is conducted every quarter by Duke University and *CFO* magazine and reported on at www.cfosurvey.org. On average since inception CFOs have predicted a 10-year return on U.S. equities of 3.7% in excess of the return on 10-year Treasury bonds. However, respondents appear to have interpreted the question as asking for their forecast of the *compound* annual return. In this case the comparable *expected* (arithmetic average) premium over *bills* is probably 2 or 3 percentage points higher at about 6%. For a description of the survey data, see J. R. Graham and C. Harvey, "The Long-Run Equity Risk Premium," *Finance Research Letters* 2 (2005), pp. 185–194.

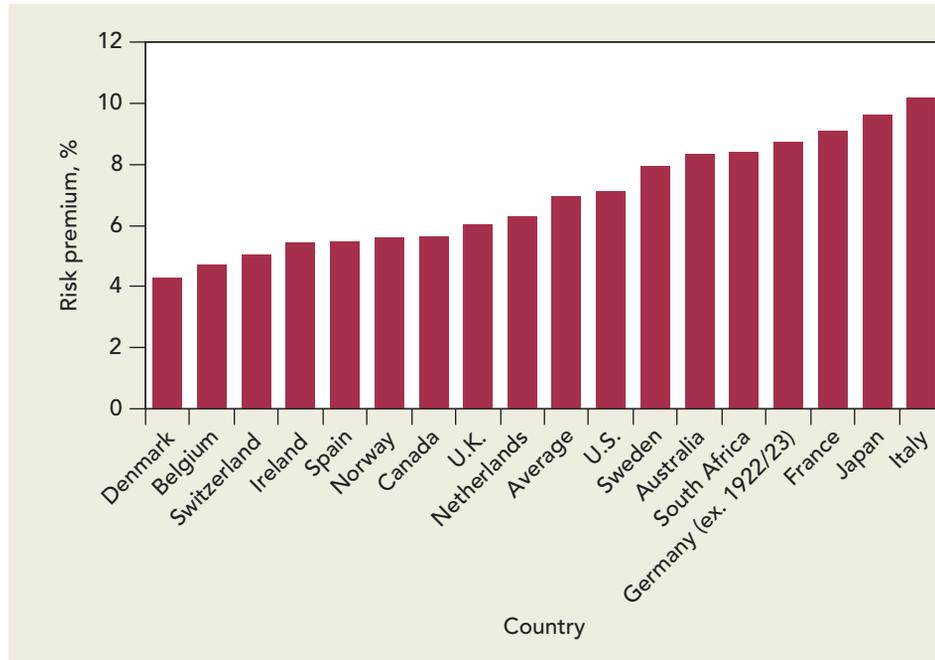


FIGURE 7.3

Average market risk premiums (nominal return on stocks minus nominal return on bills), 1900–2008.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

investors expected. Perhaps the historical averages miss the possibility that the United States could have turned out to be one of these less-fortunate countries.¹²

Figure 7.3 sheds some light on this issue. It is taken from a comprehensive study by Dimson, Marsh, and Staunton of market returns in 17 countries and shows the average risk premium in each country between 1900 and 2008. There is no evidence here that U.S. investors have been particularly fortunate; the U.S. was just about average in terms of returns.

In Figure 7.3 Danish stocks come bottom of the league; the average risk premium in Denmark was only 4.3%. The clear winner was Italy with a premium of 10.2%. Some of these differences between countries may reflect differences in risk. For example, Italian stocks have been particularly variable and investors may have required a higher return to compensate. But remember how difficult it is to make precise estimates of what investors expected. You probably would not be too far out if you concluded that the *expected* risk premium was the same in each country.¹³

Reason 2 Stock prices in the United States have for some years outpaced the growth in company dividends or earnings. For example, between 1950 and 2000 dividend yields in the United States fell from 7.2% to 1.1%. It seems unlikely that investors *expected* such a sharp decline in yields, in which case some part of the actual return during this period was *unexpected*.

Some believe that the low dividend yields at the turn of the century reflected optimism that the new economy would lead to a golden age of prosperity and surging profits, but others attribute the low yields to a reduction in the market risk premium. Perhaps the growth in mutual funds has made it easier for individuals to diversify away part of their risk, or perhaps pension funds and other financial institutions have found that they also could reduce

¹² This possibility was suggested in P. Jorion and W. N. Goetzmann, “Global Stock Markets in the Twentieth Century,” *Journal of Finance* 54 (June 1999), pp. 953–980.

¹³ We are concerned here with the difference between the nominal market return and the nominal interest rate. Sometimes you will see *real* risk premiums quoted—that is, the difference between the *real* market return and the *real* interest rate. If the inflation rate is i , then the real risk premium is $(r_m - r_f)/(1 + i)$. For countries such as Italy that have experienced a high degree of inflation, this real risk premium may be significantly lower than the nominal premium.

their risk by investing part of their funds overseas. If these investors can eliminate more of their risk than in the past, they may be content with a lower return.

To see how a rise in stock prices can stem from a fall in the risk premium, suppose that a stock is expected to pay a dividend next year of \$12 ($DIV_1 = 12$). The stock yields 3% and the dividend is expected to grow indefinitely by 7% a year ($g = .07$). Therefore the total return that investors expect is $r = 3 + 7 = 10\%$. We can find the stock's value by plugging these numbers into the constant-growth formula that we used in Chapter 4 to value stocks:

$$PV = DIV_1 / (r - g) = 12 / (.10 - .07) = \$400$$

Imagine that investors now revise downward their required return to $r = 9\%$. The dividend yield falls to 2% and the value of the stock rises to

$$PV = DIV_1 / (r - g) = 12 / (.09 - .07) = \$600$$

Thus a fall from 10% to 9% in the required return leads to a 50% rise in the stock price. If we include this price rise in our measures of past returns, we will be doubly wrong in our estimate of the risk premium. First, we will overestimate the return that investors required in the past. Second, we will fail to recognize that the return investors require in the future is lower than they needed in the past.

Dividend Yields and the Risk Premium

If there has been a downward shift in the return that investors have required, then past returns will provide an overestimate of the risk premium. We can't wholly get around this difficulty, but we can get another clue to the risk premium by going back to the constant-growth model that we discussed in Chapter 2. If stock prices are expected to keep pace with the growth in dividends, then the expected market return is equal to the dividend yield plus the expected dividend growth—that is, $r = DIV_1/P_0 + g$. Dividend yields in the United States have averaged 4.3% since 1900, and the annual growth in dividends has averaged 5.3%. If this dividend growth is representative of what investors *expected*, then the expected market return over this period was $DIV_1/P_0 + g = 4.3 + 5.3 = 9.6\%$, or 5.6% above the risk-free interest rate. This figure is 1.5% lower than the *realized* risk premium reported in Table 7.1.¹⁴

Dividend yields have averaged 4.3% since 1900, but, as you can see from Figure 7.4, they have fluctuated quite sharply. At the end of 1917, stocks were offering a yield of 9.0%; by 2000 the yield had plunged to just 1.1%. You sometimes hear financial managers suggest that in years such as 2000, when dividend yields were low, capital was relatively cheap. Is there any truth to this? Should companies be adjusting their cost of capital to reflect these fluctuations in yield?

Notice that there are only two possible reasons for the yield changes in Figure 7.4. One is that in some years investors were unusually optimistic or pessimistic about g , the future growth in dividends. The other is that r , the required return, was unusually high or low. Economists who have studied the behavior of dividend yields have concluded that very little of the variation is related to the subsequent rate of dividend growth. If they are right, the level of yields ought to be telling us something about the return that investors require.

This in fact appears to be the case. A reduction in the dividend yield seems to herald a reduction in the risk premium that investors can expect over the following few years. So, when yields are relatively low, companies may be justified in shaving their estimate

¹⁴ See E. F. Fama and K. R. French, "The Equity Premium," *Journal of Finance* 57 (April 2002), pp. 637–659. Fama and French quote even lower estimates of the risk premium, particularly for the second half of the period. The difference partly reflects the fact that they define the risk premium as the difference between market returns and the commercial paper rate. Except for the years 1900–1918, the interest rates used in Table 7.1 are the rates on U.S. Treasury bills.

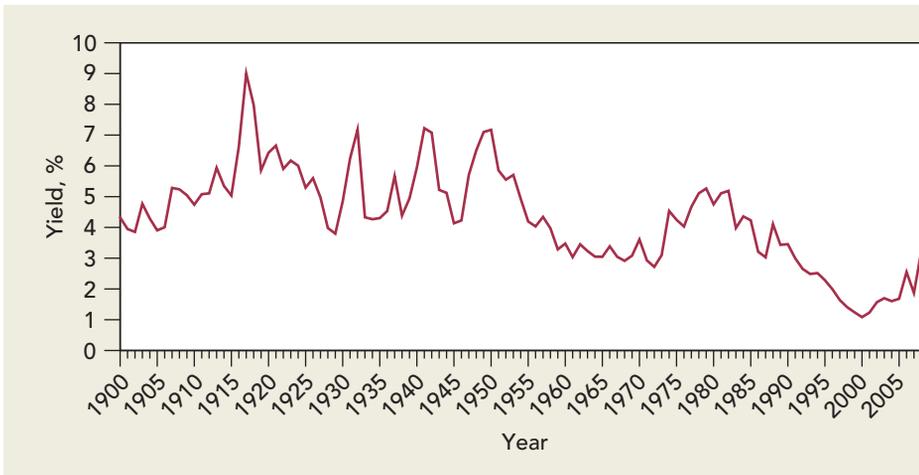


FIGURE 7.4

Dividend yields in the U.S.A. 1900–2008.

Source: R.J. Shiller, “Long Term Stock, Bond, Interest Rate and Consumption Data since 1871,” www.econ.yale.edu/~shiller/data.htm. Used with permission.

of required returns over the next year or so. However, changes in the dividend yield tell companies next to nothing about the expected risk premium over the next 10 or 20 years. It seems that, when estimating the discount rate for longer term investments, a firm can safely ignore year-to-year fluctuations in the dividend yield.

Out of this debate only one firm conclusion emerges: do not trust anyone who claims to *know* what returns investors expect. History contains some clues, but ultimately we have to judge whether investors on average have received what they expected. Many financial economists rely on the evidence of history and therefore work with a risk premium of about 7.1%. The remainder generally use a somewhat lower figure. Brealey, Myers, and Allen have no official position on the issue, but we believe that a range of 5% to 8% is reasonable for the risk premium in the United States.

7-2 Measuring Portfolio Risk

You now have a couple of benchmarks. You know the discount rate for safe projects, and you have an estimate of the rate for average-risk projects. But you *don't* know yet how to estimate discount rates for assets that do not fit these simple cases. To do that, you have to learn (1) how to measure risk and (2) the relationship between risks borne and risk premiums demanded.

Figure 7.5 shows the 109 annual rates of return for U.S. common stocks. The fluctuations in year-to-year returns are remarkably wide. The highest annual return was 57.6% in 1933—a partial rebound from the stock market crash of 1929–1932. However, there were losses exceeding 25% in five years, the worst being the –43.9% return in 1931.

Another way of presenting these data is by a histogram or frequency distribution. This is done in Figure 7.6, where the variability of year-to-year returns shows up in the wide “spread” of outcomes.

Variance and Standard Deviation

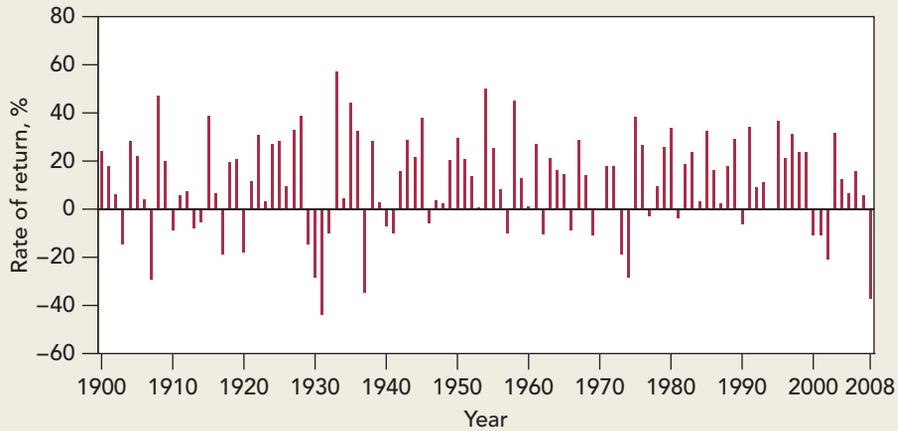
The standard statistical measures of spread are **variance** and **standard deviation**. The variance of the market return is the expected squared deviation from the expected return. In other words,

$$\text{Variance } (\tilde{r}_m) = \text{the expected value of } (\tilde{r}_m - r_m)^2$$

FIGURE 7.5

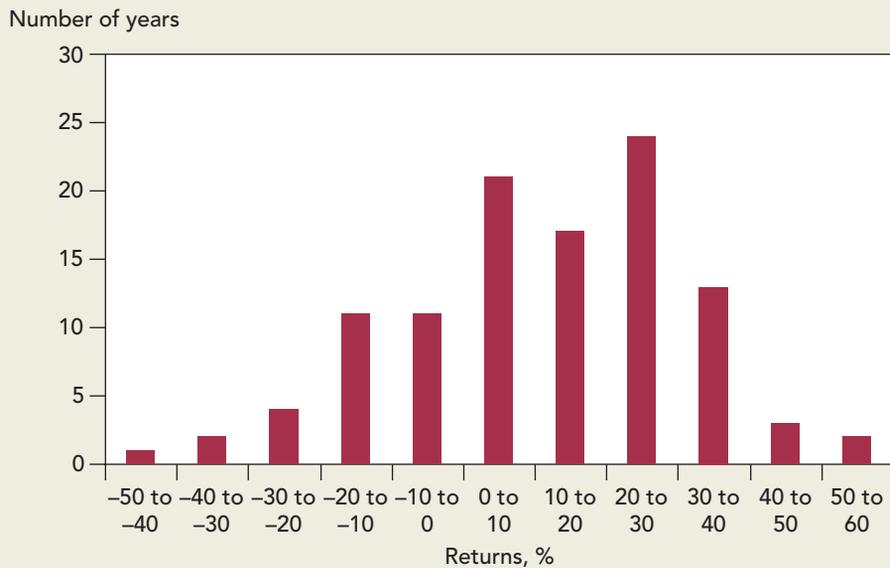
The stock market has been a profitable but extremely variable investment.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

**FIGURE 7.6**

Histogram of the annual rates of return from the stock market in the United States, 1900–2008, showing the wide spread of returns from investment in common stocks.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.



where \tilde{r}_m is the actual return and r_m is the expected return.¹⁵ The standard deviation is simply the square root of the variance:

$$\text{Standard deviation of } \tilde{r}_m = \sqrt{\text{variance}(\tilde{r}_m)}$$

Standard deviation is often denoted by σ and variance by σ^2 .

Here is a very simple example showing how variance and standard deviation are calculated. Suppose that you are offered the chance to play the following game. You start by

¹⁵ One more technical point. When variance is estimated from a sample of *observed* returns, we add the squared deviations and divide by $N - 1$, where N is the number of observations. We divide by $N - 1$ rather than N to correct for what is called *the loss of a degree of freedom*. The formula is

$$\text{Variance } (\tilde{r}_m) = \frac{1}{N - 1} \sum_{t=1}^N (\tilde{r}_{mt} - \bar{r}_m)^2$$

where \tilde{r}_{mt} is the market return in period t and \bar{r}_m is the mean of the values of \tilde{r}_{mt} .

| (1) Percent Rate of Return (\tilde{r}) | (2) Deviation from Expected Return ($\tilde{r} - r$) | (3) Squared Deviation ($\tilde{r} - r$) ² | (4) Probability | (5) Probability \times Squared Deviation |
|---|---|---|--------------------|---|
| +40 | +30 | 900 | .25 | 225 |
| +10 | 0 | 0 | .5 | 0 |
| -20 | -30 | 900 | .25 | 225 |
| Variance = expected value of $(\tilde{r} - r)^2 = 450$ | | | | |
| Standard deviation = $\sqrt{\text{variance}} = \sqrt{450} = 21$ | | | | |

TABLE 7.2

The coin-tossing game: Calculating variance and standard deviation.

investing \$100. Then two coins are flipped. For each head that comes up you get back your starting balance *plus* 20%, and for each tail that comes up you get back your starting balance *less* 10%. Clearly there are four equally likely outcomes:

- Head + head: You gain 40%.
- Head + tail: You gain 10%.
- Tail + head: You gain 10%.
- Tail + tail: You lose 20%.

There is a chance of 1 in 4, or .25, that you will make 40%; a chance of 2 in 4, or .5, that you will make 10%; and a chance of 1 in 4, or .25, that you will lose 20%. The game's expected return is, therefore, a weighted average of the possible outcomes:

$$\text{Expected return} = (.25 \times 40) + (.5 \times 10) + (.25 \times -20) = +10\%$$

Table 7.2 shows that the variance of the percentage returns is 450. Standard deviation is the square root of 450, or 21. This figure is in the same units as the rate of return, so we can say that the game's variability is 21%.

One way of defining uncertainty is to say that more things can happen than will happen. The risk of an asset can be completely expressed, as we did for the coin-tossing game, by writing all possible outcomes and the probability of each. In practice this is cumbersome and often impossible. Therefore we use variance or standard deviation to summarize the spread of possible outcomes.¹⁶

These measures are natural indexes of risk.¹⁷ If the outcome of the coin-tossing game had been certain, the standard deviation would have been zero. The actual standard deviation is positive because we *don't* know what will happen.

Or think of a second game, the same as the first except that each head means a 35% gain and each tail means a 25% loss. Again, there are four equally likely outcomes:

- Head + head: You gain 70%.
- Head + tail: You gain 10%.
- Tail + head: You gain 10%.
- Tail + tail: You lose 50%.

¹⁶ Which of the two we use is solely a matter of convenience. Since standard deviation is in the same units as the rate of return, it is generally more convenient to use standard deviation. However, when we are talking about the *proportion* of risk that is due to some factor, it is less confusing to work in terms of the variance.

¹⁷ As we explain in Chapter 8, standard deviation and variance are the correct measures of risk if the returns are normally distributed.

For this game the expected return is 10%, the same as that of the first game. But its standard deviation is double that of the first game, 42 versus 21%. By this measure the second game is twice as risky as the first.

Measuring Variability

In principle, you could estimate the variability of any portfolio of stocks or bonds by the procedure just described. You would identify the possible outcomes, assign a probability to each outcome, and grind through the calculations. But where do the probabilities come from? You can't look them up in the newspaper; newspapers seem to go out of their way to avoid definite statements about prospects for securities. We once saw an article headlined "Bond Prices Possibly Set to Move Sharply Either Way." Stockbrokers are much the same. Yours may respond to your query about possible market outcomes with a statement like this:

The market currently appears to be undergoing a period of consolidation. For the intermediate term, we would take a constructive view, provided economic recovery continues. The market could be up 20% a year from now, perhaps more if inflation continues low. On the other hand, . . .

The Delphic oracle gave advice, but no probabilities.

Most financial analysts start by observing past variability. Of course, there is no risk in hindsight, but it is reasonable to assume that portfolios with histories of high variability also have the least predictable future performance.

The annual standard deviations and variances observed for our three portfolios over the period 1900–2008 were:¹⁸

| Portfolio | Standard Deviation (σ) | Variance (σ^2) |
|------------------|---------------------------------|-------------------------|
| Treasury bills | 2.8 | 7.7 |
| Government bonds | 8.3 | 69.3 |
| Common stocks | 20.2 | 406.4 |

As expected, Treasury bills were the least variable security, and common stocks were the most variable. Government bonds hold the middle ground.

You may find it interesting to compare the coin-tossing game and the stock market as alternative investments. The stock market generated an average annual return of 11.1% with a standard deviation of 20.2%. The game offers 10% and 21%, respectively—slightly lower return and about the same variability. Your gambling friends may have come up with a crude representation of the stock market.

Figure 7.7 compares the standard deviation of stock market returns in 17 countries over the same 109-year period. Canada occupies low field with a standard deviation of 17.0%, but most of the other countries cluster together with percentage standard deviations in the low 20s.

Of course, there is no reason to suppose that the market's variability should stay the same over more than a century. For example, Germany, Italy, and Japan now have much more stable economies and markets than they did in the years leading up to and including the Second World War.

¹⁸ In discussing the riskiness of *bonds*, be careful to specify the time period and whether you are speaking in real or nominal terms. The *nominal* return on a long-term government bond is absolutely certain to an investor who holds on until maturity; in other words, it is risk-free if you forget about inflation. After all, the government can always print money to pay off its debts. However, the real return on Treasury securities is uncertain because no one knows how much each future dollar will buy.

The bond returns were measured annually. The returns reflect year-to-year changes in bond prices as well as interest received. The *one-year* returns on long-term bonds are risky in *both* real and nominal terms.

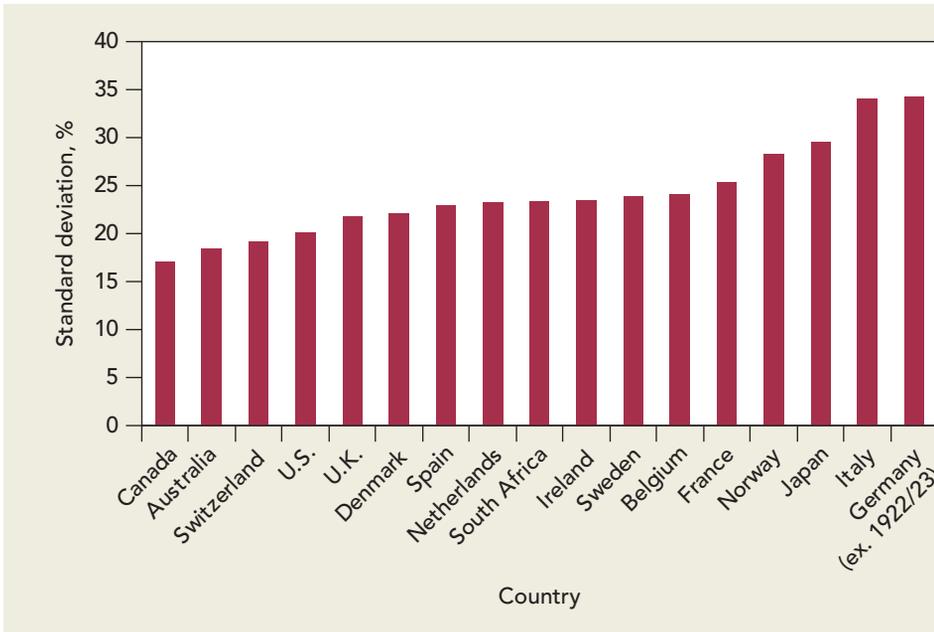


FIGURE 7.7

The risk (standard deviation of annual returns) of markets around the world, 1900–2008.

Source: E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

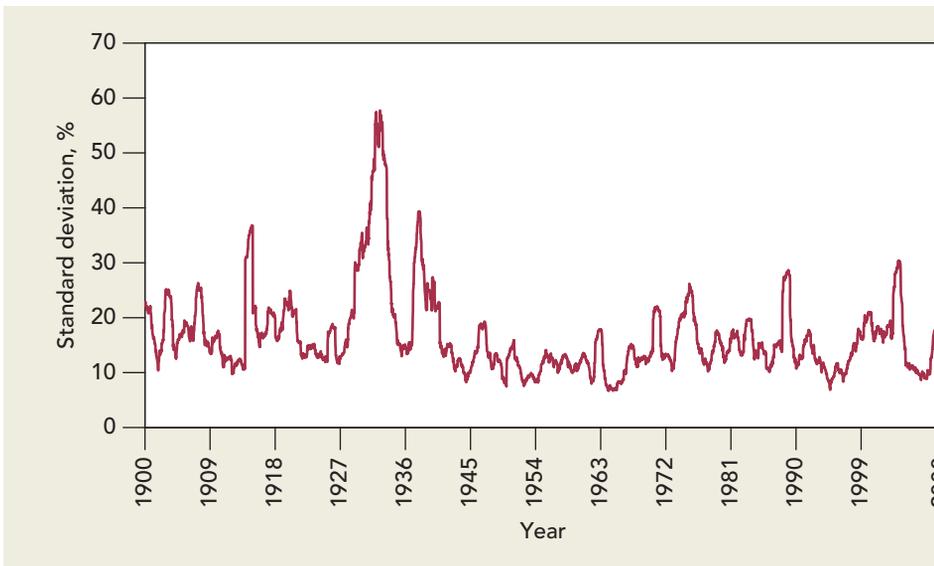


FIGURE 7.8

Annualized standard deviation of the preceding 52 weekly changes in the Dow Jones Industrial Average, 1900–2008.

Figure 7.8 does not suggest a long-term upward or downward trend in the volatility of the U.S. stock market.¹⁹ Instead there have been periods of both calm and turbulence. In 2005, an unusually tranquil year, the standard deviation of returns was only 9%, less than half the long-term average. The standard deviation in 2008 was about four times higher at 34%.

Market turbulence over shorter daily, weekly, or monthly periods can be amazingly high. On Black Monday, October 19, 1987, the U.S. market fell by 23% *on a single day*. The market

¹⁹ These estimates are derived from *weekly* rates of return. The weekly variance is converted to an annual variance by multiplying by 52. That is, the variance of the weekly return is one-fifty-second of the annual variance. The longer you hold a security or portfolio, the more risk you have to bear.

This conversion assumes that successive weekly returns are statistically independent. This is, in fact, a good assumption, as we will show in Chapter 13.

Because variance is approximately proportional to the length of time interval over which a security or portfolio return is measured, standard deviation is proportional to the square root of the interval.

standard deviation for the week surrounding Black Monday was equivalent to 89% per year. Fortunately volatility dropped back to normal levels within a few weeks after the crash.

At the height of the financial crisis in October and November 2008, the U.S. market standard deviation was running at a rate of about 70% per year. As we write this in August 2009, the standard deviation has fallen back to 25%.²⁰

Earlier we quoted 5% to 8% as a reasonable, normal range for the U.S. risk premium. The risk premium has probably increased as a result of the financial crisis. We hope that economic recovery and lower market volatility will allow the risk premium to fall back to normalcy.

How Diversification Reduces Risk

We can calculate our measures of variability equally well for individual securities and portfolios of securities. Of course, the level of variability over 100 years is less interesting for specific companies than for the market portfolio—it is a rare company that faces the same business risks today as it did a century ago.

Table 7.3 presents estimated standard deviations for 10 well-known common stocks for a recent five-year period.²¹ Do these standard deviations look high to you? They should. The market portfolio's standard deviation was about 13% during this period. Each of our individual stocks had higher volatility. Amazon was over four times more variable than the market portfolio.

Take a look also at Table 7.4, which shows the standard deviations of some well-known stocks from different countries and of the markets in which they trade. Some of these stocks are more variable than others, but you can see that once again the individual stocks are for the most part more variable than the market indexes.

This raises an important question: The market portfolio is made up of individual stocks, so why doesn't its variability reflect the average variability of its components? The answer is that *diversification reduces variability*.

TABLE 7.3

Standard deviations for selected U.S. common stocks, January 2004–December 2008 (figures in percent per year).

| Stock | Standard Deviation (σ) | Stock | Standard Deviation (σ) |
|-----------|---------------------------------|-------------------|---------------------------------|
| Amazon | 50.9 | Boeing | 23.7 |
| Ford | 47.2 | Disney | 19.6 |
| Newmont | 36.1 | Exxon Mobil | 19.1 |
| Dell | 30.9 | Campbell Soup | 15.8 |
| Starbucks | 30.3 | Johnson & Johnson | 12.5 |

TABLE 7.4

Standard deviations for selected foreign stocks and market indexes, January 2004–December 2008 (figures in percent per year).

| Standard Deviation (σ) | | | Standard Deviation (σ) | | |
|---------------------------------|-------|--------|---------------------------------|-------|--------|
| | Stock | Market | | Stock | Market |
| BP | 20.7 | 16.0 | LVMH | 20.6 | 18.3 |
| Deutsche Bank | 28.9 | 20.6 | Nestlé | 14.6 | 13.7 |
| Fiat | 35.7 | 18.9 | Nokia | 31.6 | 25.8 |
| Heineken | 21.0 | 20.8 | Sony | 33.9 | 16.6 |
| Iberia | 35.4 | 20.4 | Telefonica de Argentina | 58.6 | 40.0 |

²⁰ The standard deviations for 2008 and 2009 are the VIX index of market volatility, published by the Chicago Board Options Exchange (CBOE). We explain the VIX index in Chapter 21. In the meantime, you may wish to check the current level of the VIX on finance.yahoo or at the CBOE Web site.

²¹ These standard deviations are also calculated from monthly data.

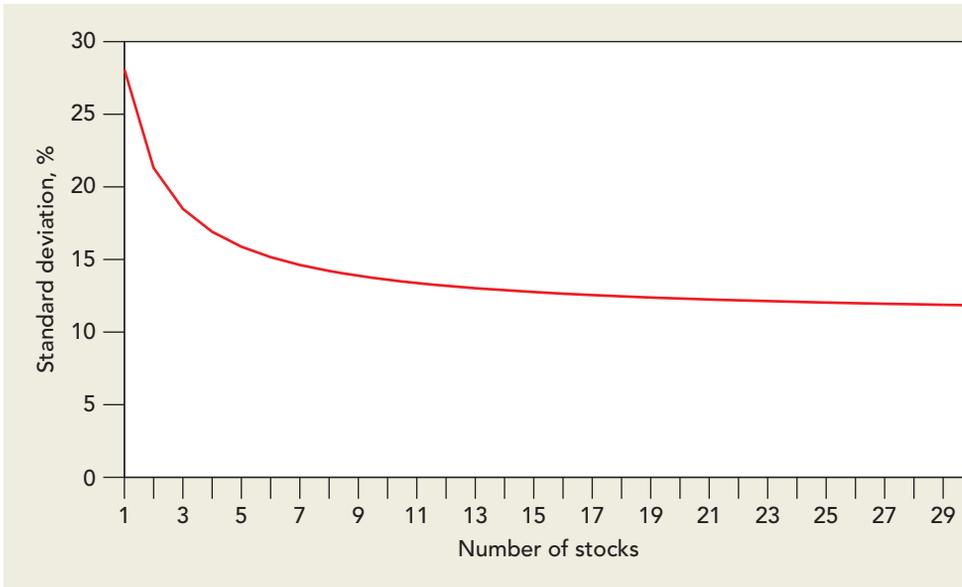


FIGURE 7.9

Average risk (standard deviation) of portfolios containing different numbers of stocks. The stocks were selected randomly from stocks traded on the New York Exchange from 2002 through 2007. Notice that diversification reduces risk rapidly at first, then more slowly.

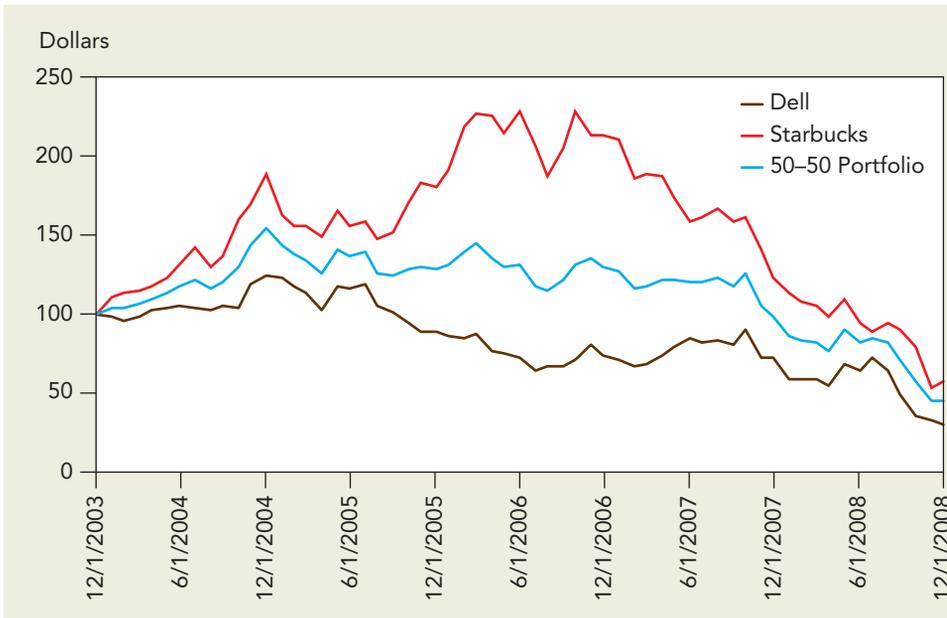


FIGURE 7.10

The value of a portfolio evenly divided between Dell and Starbucks was less volatile than either stock on its own. The assumed initial investment is \$100.

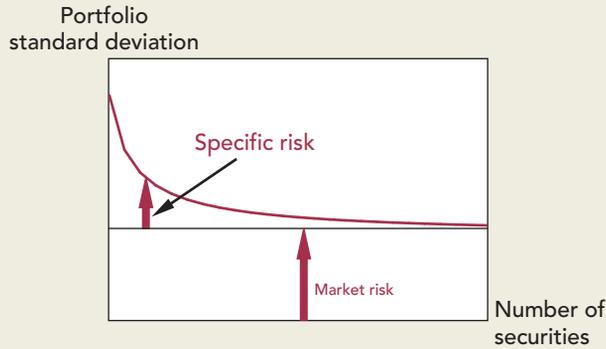
Even a little diversification can provide a substantial reduction in variability. Suppose you calculate and compare the standard deviations between 2002 and 2007 of one-stock portfolios, two-stock portfolios, five-stock portfolios, etc. You can see from Figure 7.9 that diversification can cut the variability of returns about in half. Notice also that you can get most of this benefit with relatively few stocks: The improvement is much smaller when the number of securities is increased beyond, say, 20 or 30.²²

Diversification works because prices of different stocks do not move exactly together. Statisticians make the same point when they say that stock price changes are less than perfectly correlated. Look, for example, at Figure 7.10, which plots the prices of Starbucks

²² There is some evidence that in recent years stocks have become individually more risky but have moved less closely together. Consequently, the benefits of diversification have increased. See J. Y. Campbell, M. Lettau, B. C. Malkiel, and Y. Xu, "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *Journal of Finance* 56 (February 2001), pp. 1–43.

FIGURE 7.11

Diversification eliminates specific risk. But there is some risk that diversification *cannot* eliminate. This is called *market risk*.



(top line) and Dell (bottom line) for the 60-month period ending December 2008. As we showed in Table 7.3, during this period the standard deviation of the monthly returns of both stocks was about 30%. Although the two stocks enjoyed a fairly bumpy ride, they did not move in exact lockstep. Often a decline in the value of Dell was offset by a rise in the price of Starbucks.²³ So, if you had split

your portfolio between the two stocks, you could have reduced the monthly fluctuations in the value of your investment. You can see from the blue line in Figure 7.10 that if your portfolio had been evenly divided between Dell and Starbucks, there would have been many more months when the return was just middling and far fewer cases of extreme returns. By diversifying between the two stocks, you would have reduced the standard deviation of the returns to about 20% a year.

The risk that potentially can be eliminated by diversification is called **specific risk**.²⁴ Specific risk stems from the fact that many of the perils that surround an individual company are peculiar to that company and perhaps its immediate competitors. But there is also some risk that you can't avoid, regardless of how much you diversify. This risk is generally known as **market risk**.²⁵ Market risk stems from the fact that there are other economywide perils that threaten all businesses. That is why stocks have a tendency to move together. And that is why investors are exposed to market uncertainties, no matter how many stocks they hold.

In Figure 7.11 we have divided risk into its two parts—specific risk and market risk. If you have only a single stock, specific risk is very important; but once you have a portfolio of 20 or more stocks, diversification has done the bulk of its work. For a reasonably well-diversified portfolio, only market risk matters. Therefore, the predominant source of uncertainty for a diversified investor is that the market will rise or plummet, carrying the investor's portfolio with it.

7-3 Calculating Portfolio Risk

We have given you an intuitive idea of how diversification reduces risk, but to understand fully the effect of diversification, you need to know how the risk of a portfolio depends on the risk of the individual shares.

Suppose that 60% of your portfolio is invested in Campbell Soup and the remainder is invested in Boeing. You expect that over the coming year Campbell Soup will give a return of 3.1% and Boeing, 9.5%. The expected return on your portfolio is simply a weighted average of the expected returns on the individual stocks.²⁶

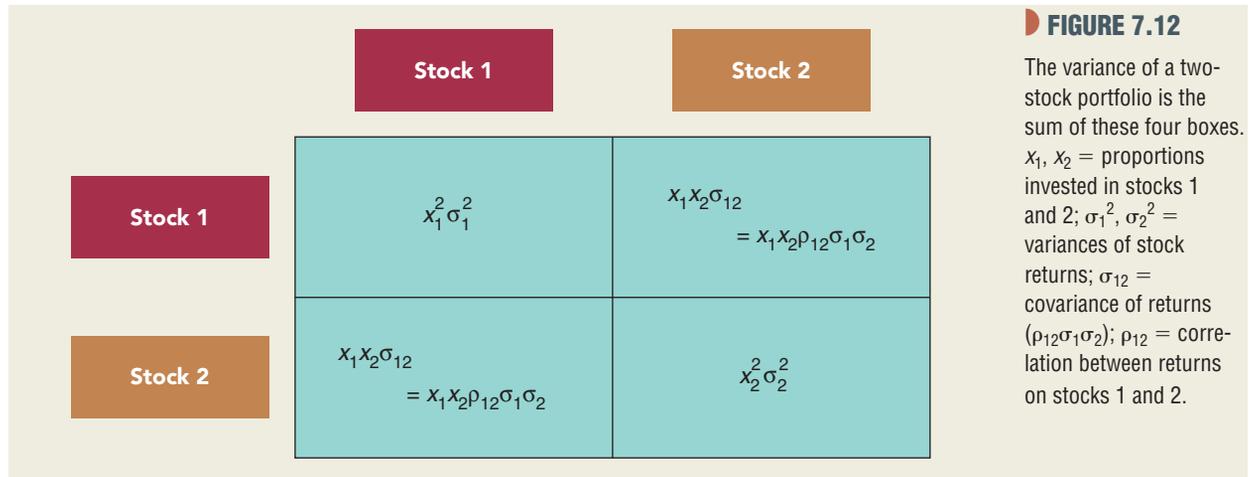
$$\text{Expected portfolio return} = (.60 \times 3.1) + (.40 \times 9.5) = 5.7\%$$

²³ Over this period the correlation between the returns on the two stocks was .29.

²⁴ Specific risk may be called *unsystematic risk*, *residual risk*, *unique risk*, or *diversifiable risk*.

²⁵ Market risk may be called *systematic risk* or *undiversifiable risk*.

²⁶ Let's check this. Suppose you invest \$60 in Campbell Soup and \$40 in Boeing. The expected dollar return on your Campbell holding is $.031 \times 60 = \$1.86$, and on Boeing it is $.095 \times 40 = \$3.80$. The expected dollar return on your portfolio is $1.86 + 3.80 = \$5.66$. The portfolio *rate* of return is $5.66/100 = 0.057$, or 5.7%.



Calculating the expected portfolio return is easy. The hard part is to work out the risk of your portfolio. In the past the standard deviation of returns was 15.8% for Campbell and 23.7% for Boeing. You believe that these figures are a good representation of the spread of possible *future* outcomes. At first you may be inclined to assume that the standard deviation of the portfolio is a weighted average of the standard deviations of the two stocks, that is, $(.60 \times 15.8) + (.40 \times 23.7) = 19.0\%$. That would be correct *only* if the prices of the two stocks moved in perfect lockstep. In any other case, diversification reduces the risk below this figure.

The exact procedure for calculating the risk of a two-stock portfolio is given in Figure 7.12. You need to fill in four boxes. To complete the top-left box, you weight the variance of the returns on stock 1 (σ_1^2) by the *square* of the proportion invested in it (x_1^2). Similarly, to complete the bottom-right box, you weight the variance of the returns on stock 2 (σ_2^2) by the *square* of the proportion invested in stock 2 (x_2^2).

The entries in these diagonal boxes depend on the variances of stocks 1 and 2; the entries in the other two boxes depend on their **covariance**. As you might guess, the covariance is a measure of the degree to which the two stocks “covary.” The covariance can be expressed as the product of the correlation coefficient ρ_{12} and the two standard deviations:²⁷

$$\text{Covariance between stocks 1 and 2} = \sigma_{12} = \rho_{12} \sigma_1 \sigma_2$$

For the most part stocks tend to move together. In this case the correlation coefficient ρ_{12} is positive, and therefore the covariance σ_{12} is also positive. If the prospects of the stocks were wholly unrelated, both the correlation coefficient and the covariance would be zero; and if the stocks tended to move in opposite directions, the correlation coefficient and the covariance would be negative. Just as you weighted the variances by the square of the proportion invested, so you must weight the covariance by the *product* of the two proportionate holdings x_1 and x_2 .

²⁷ Another way to define the covariance is as follows:

$$\text{Covariance between stocks 1 and 2} = \sigma_{12} = \text{expected value of } (\bar{r}_1 - r_1) \times (\bar{r}_2 - r_2)$$

Note that any security's covariance with itself is just its variance:

$$\begin{aligned} \sigma_{11} &= \text{expected value of } (\bar{r}_1 - r_1) \times (\bar{r}_1 - r_1) \\ &= \text{expected value of } (\bar{r}_1 - r_1)^2 = \text{variance of stock 1} = \sigma_1^2 \end{aligned}$$

Once you have completed these four boxes, you simply add the entries to obtain the portfolio variance:

$$\text{Portfolio variance} = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1x_2\rho_{12}\sigma_1\sigma_2)$$

The portfolio standard deviation is, of course, the square root of the variance.

Now you can try putting in some figures for Campbell Soup and Boeing. We said earlier that if the two stocks were perfectly correlated, the standard deviation of the portfolio would lie 40% of the way between the standard deviations of the two stocks. Let us check this out by filling in the boxes with $\rho_{12} = +1$.

| | Campbell Soup | Boeing |
|---------------|--|--|
| Campbell Soup | $x_1^2\sigma_1^2 = (.6)^2 \times (15.8)^2$ | $x_1x_2\rho_{12}\sigma_1\sigma_2$ $= (.6) \times (.4) \times 1 \times (15.8) \times (23.7)$ |
| Boeing | $x_1x_2\rho_{12}\sigma_1\sigma_2$ $= (.6) \times (.4) \times 1 \times (15.8) \times (23.7)$ | $x_2^2\sigma_2^2 = (.4)^2 \times (23.7)^2$ |

The variance of your portfolio is the sum of these entries:

$$\begin{aligned} \text{Portfolio variance} &= [(.6)^2 \times (15.8)^2] + [(.4)^2 \times (23.7)^2] + 2(.6 \times .4 \times 1 \times 15.8 \times 23.7) \\ &= 359.5 \end{aligned}$$

The standard deviation is $\sqrt{359.5} = 19\%$, or 40% of the way between 15.8 and 23.7.

Campbell Soup and Boeing do not move in perfect lockstep. If past experience is any guide, the correlation between the two stocks is about .18. If we go through the same exercise again with $\rho_{12} = .18$, we find

$$\begin{aligned} \text{Portfolio variance} &= [(.6)^2 \times (15.8)^2] + [(.4)^2 \times (23.7)^2] \\ &\quad + 2(.6 \times .4 \times .18 \times 15.8 \times 23.7) = 212.1 \end{aligned}$$

The standard deviation is $\sqrt{212.1} = 14.6\%$. The risk is now less than 40% of the way between 15.8 and 23.7. In fact, it is less than the risk of investing in Campbell Soup alone.

The greatest payoff to diversification comes when the two stocks are negatively correlated. Unfortunately, this almost never occurs with real stocks, but just for illustration, let us assume it for Campbell Soup and Boeing. And as long as we are being unrealistic, we might as well go whole hog and assume perfect negative correlation ($\rho_{12} = -1$). In this case,

$$\begin{aligned} \text{Portfolio variance} &= [(.6)^2 \times (15.8)^2] + [(.4)^2 \times (23.7)^2] \\ &\quad + 2(.6 \times .4 \times (-1) \times 15.8 \times 23.7) = 0 \end{aligned}$$

When there is perfect negative correlation, there is always a portfolio strategy (represented by a particular set of portfolio weights) that will completely eliminate risk.²⁸ It's too bad perfect negative correlation doesn't really occur between common stocks.

General Formula for Computing Portfolio Risk

The method for calculating portfolio risk can easily be extended to portfolios of three or more securities. We just have to fill in a larger number of boxes. Each of those down the

²⁸ Since the standard deviation of Boeing is 1.5 times that of Campbell Soup, you need to invest 1.5 times more in Campbell Soup to eliminate risk in this two-stock portfolio.

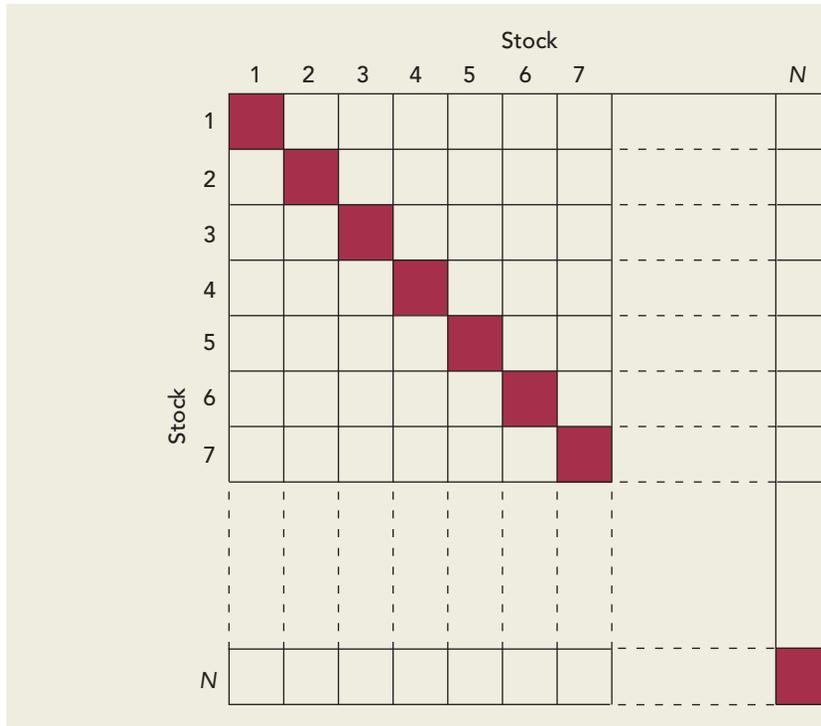


FIGURE 7.13
To find the variance of an N -stock portfolio, we must add the entries in a matrix like this. The diagonal cells contain variance terms ($x_i^2 \sigma_i^2$) and the off-diagonal cells contain covariance terms ($x_i x_j \sigma_{ij}$).

diagonal—the shaded boxes in Figure 7.13—contains the variance weighted by the square of the proportion invested. Each of the other boxes contains the covariance between that pair of securities, weighted by the product of the proportions invested.²⁹

Limits to Diversification

Did you notice in Figure 7.13 how much more important the covariances become as we add more securities to the portfolio? When there are just two securities, there are equal numbers of variance boxes and of covariance boxes. When there are many securities, the number of covariances is much larger than the number of variances. Thus the variability of a well-diversified portfolio reflects mainly the covariances.

Suppose we are dealing with portfolios in which equal investments are made in each of N stocks. The proportion invested in each stock is, therefore, $1/N$. So in each variance box we have $(1/N)^2$ times the variance, and in each covariance box we have $(1/N)^2$ times the covariance. There are N variance boxes and $N^2 - N$ covariance boxes. Therefore,

$$\begin{aligned} \text{Portfolio variance} &= N \left(\frac{1}{N} \right)^2 \times \text{average variance} \\ &\quad + (N^2 - N) \left(\frac{1}{N} \right)^2 \times \text{average covariance} \\ &= \frac{1}{N} \times \text{average variance} + \left(1 - \frac{1}{N} \right) \times \text{average covariance} \end{aligned}$$

²⁹ The formal equivalent to “add up all the boxes” is

$$\text{Portfolio variance} = \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij}$$

Notice that when $i = j$, σ_{ij} is just the variance of stock i .

Notice that as N increases, the portfolio variance steadily approaches the average covariance. If the average covariance were zero, it would be possible to eliminate *all* risk by holding a sufficient number of securities. Unfortunately common stocks move together, not independently. Thus most of the stocks that the investor can actually buy are tied together in a web of positive covariances that set the limit to the benefits of diversification. Now we can understand the precise meaning of the market risk portrayed in Figure 7.11. It is the average covariance that constitutes the bedrock of risk remaining after diversification has done its work.

7-4 How Individual Securities Affect Portfolio Risk

We presented earlier some data on the variability of 10 individual U.S. securities. Amazon had the highest standard deviation and Johnson & Johnson had the lowest. If you had held Amazon on its own, the spread of possible returns would have been more than four times greater than if you had held Johnson & Johnson on its own. But that is not a very interesting fact. Wise investors don't put all their eggs into just one basket: They reduce their risk by diversification. They are therefore interested in the effect that each stock will have on the risk of their portfolio.

This brings us to one of the principal themes of this chapter. *The risk of a well-diversified portfolio depends on the market risk of the securities included in the portfolio.* Tattoo that statement on your forehead if you can't remember it any other way. It is one of the most important ideas in this book.

Market Risk Is Measured by Beta

If you want to know the contribution of an individual security to the risk of a well-diversified portfolio, it is no good thinking about how risky that security is if held in isolation—you need to measure its *market risk*, and that boils down to measuring how sensitive it is to market movements. This sensitivity is called **beta** (β).

Stocks with betas greater than 1.0 tend to amplify the overall movements of the market. Stocks with betas between 0 and 1.0 tend to move in the same direction as the market, but not as far. Of course, the market is the portfolio of all stocks, so the “average” stock has a beta of 1.0. Table 7.5 reports betas for the 10 well-known common stocks we referred to earlier.

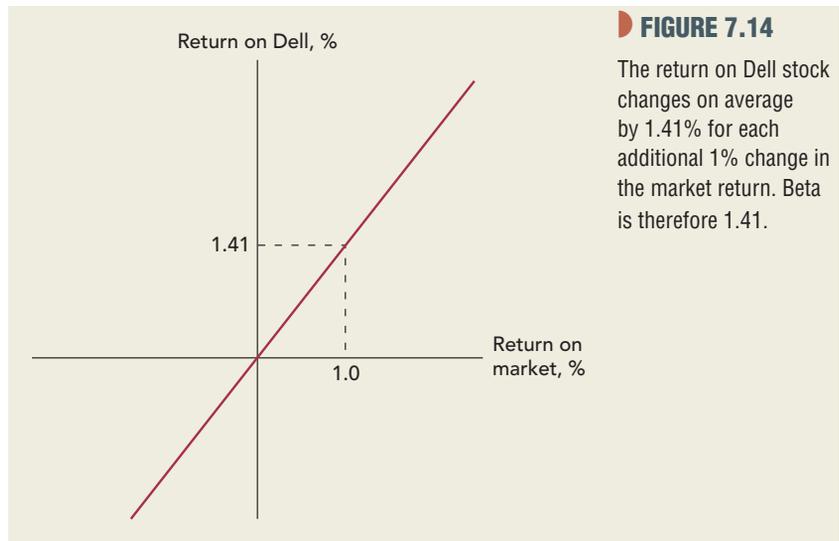
Over the five years from January 2004 to December 2008, Dell had a beta of 1.41. If the future resembles the past, this means that *on average* when the market rises an extra 1%, Dell's stock price will rise by an extra 1.41%. When the market falls an extra 2%, Dell's stock prices will fall an extra $2 \times 1.41 = 2.82\%$. Thus a line fitted to a plot of Dell's returns versus market returns has a slope of 1.41. See Figure 7.14.

Of course Dell's stock returns are not perfectly correlated with market returns. The company is also subject to specific risk, so the actual returns will be scattered about the line in Figure 7.14. Sometimes Dell will head south while the market goes north, and vice versa.

TABLE 7.5
Betas for selected U.S.
common stocks, January
2004–December 2008.

| Stock | Beta (β) | Stock | Beta (β) |
|-----------|------------------|-------------------|------------------|
| Amazon | 2.16 | Disney | .96 |
| Ford | 1.75 | Newmont | .63 |
| Dell | 1.41 | Exxon Mobil | .55 |
| Starbucks | 1.16 | Johnson & Johnson | .50 |
| Boeing | 1.14 | Campbell Soup | .30 |

Of the 10 stocks in Table 7.5 Dell has one of the highest betas. Campbell Soup is at the other extreme. A line fitted to a plot of Campbell Soup's returns versus market returns would be less steep: Its slope would be only .30. Notice that many of the stocks that have high standard deviations also have high betas. But that is not always so. For example, Newmont, which has a relatively high standard deviation, has joined the low-beta stocks in the right-hand column of Table 7.5. It seems that while Newmont is a risky investment if held on its own, it makes a relatively low contribution to the risk of a diversified portfolio.



Just as we can measure how the returns of a U.S. stock are affected by fluctuations in the U.S. market, so we can measure how stocks in other countries are affected by movements in *their* markets. Table 7.6 shows the betas for the sample of stocks from other countries.

Why Security Betas Determine Portfolio Risk

Let us review the two crucial points about security risk and portfolio risk:

- Market risk accounts for most of the risk of a well-diversified portfolio.
- The beta of an individual security measures its sensitivity to market movements.

It is easy to see where we are headed: In a portfolio context, a security's risk is measured by beta. Perhaps we could just jump to that conclusion, but we would rather explain it. Here is an intuitive explanation. We provide a more technical one in footnote 31.

Where's Bedrock? Look back to Figure 7.11, which shows how the standard deviation of portfolio return depends on the number of securities in the portfolio. With more securities, and therefore better diversification, portfolio risk declines until all specific risk is eliminated and only the bedrock of market risk remains.

Where's bedrock? It depends on the average beta of the securities selected.

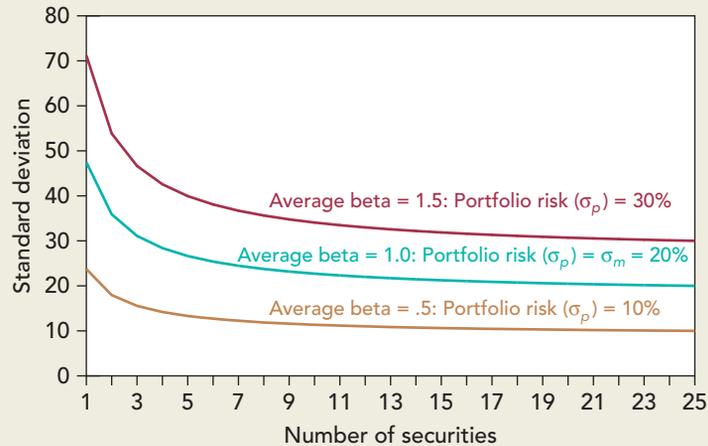
Suppose we constructed a portfolio containing a large number of stocks—500, say—drawn randomly from the whole market. What would we get? The market itself, or a portfolio *very* close to it. The portfolio beta would be 1.0, and the correlation with the market would be 1.0. If the standard deviation of the market were 20% (roughly its average for 1900–2008), then the portfolio standard deviation would also be 20%. This is shown by the green line in Figure 7.15.

| Stock | Beta (β) | Stock | Beta (β) |
|---------------|------------------|-------------------------|------------------|
| BP | .49 | LVMH | .86 |
| Deutsche Bank | 1.07 | Nestlé | .35 |
| Fiat | 1.11 | Nokia | 1.07 |
| Heineken | .53 | Sony | 1.32 |
| Iberia | .59 | Telefonica de Argentina | .42 |

TABLE 7.6
Betas for selected foreign stocks, January 2004–December 2008 (beta is measured relative to the stock's home market).

FIGURE 7.15

The green line shows that a well-diversified portfolio of randomly selected stocks ends up with $\beta = 1$ and a standard deviation equal to the market's—in this case 20%. The upper red line shows that a well-diversified portfolio with $\beta = 1.5$ has a standard deviation of about 30%—1.5 times that of the market. The lower brown line shows that a well-diversified portfolio with $\beta = .5$ has a standard deviation of about 10%—half that of the market.



But suppose we constructed the portfolio from a large group of stocks with an average beta of 1.5. Again we would end up with a 500-stock portfolio with virtually no specific risk—a portfolio that moves almost in lockstep with the market. However, *this* portfolio's standard deviation would be 30%, 1.5 times that of the market.³⁰ A well-diversified portfolio with a beta of 1.5 will amplify every market move by 50% and end up with 150% of the market's risk. The upper red line in Figure 7.15 shows this case.

Of course, we could repeat the same experiment with stocks with a beta of .5 and end up with a well-diversified portfolio half as risky as the market. You can see this also in Figure 7.15.

The general point is this: The risk of a well-diversified portfolio is proportional to the portfolio beta, which equals the average beta of the securities included in the portfolio. This shows you how portfolio risk is driven by security betas.

Calculating Beta A statistician would define the beta of stock i as

$$\beta_i = \sigma_{im} / \sigma_m^2$$

where σ_{im} is the *covariance* between the stock returns and the market returns and σ_m^2 is the variance of the returns on the market. It turns out that this ratio of covariance to variance measures a stock's contribution to portfolio risk.³¹

³⁰ A 500-stock portfolio with $\beta = 1.5$ would still have some specific risk because it would be unduly concentrated in high-beta industries. Its actual standard deviation would be a bit higher than 30%. If that worries you, relax; we will show you in Chapter 8 how you can construct a fully diversified portfolio with a beta of 1.5 by borrowing and investing in the market portfolio.

³¹ To understand why, skip back to Figure 7.13. Each row of boxes in Figure 7.13 represents the contribution of that particular security to the portfolio's risk. For example, the contribution of stock 1 is

$$x_1 x_1 \sigma_{11} + x_1 x_2 \sigma_{12} + \dots = x_1 (x_1 \sigma_{11} + x_2 \sigma_{12} + \dots)$$

where x_i is the proportion invested in stock i , and σ_{ij} is the covariance between stocks i and j (note: σ_{ii} is equal to the variance of stock i). In other words, the contribution of stock 1 to portfolio risk is equal to the relative size of the holding (x_1) times the average covariance between stock 1 and all the stocks in the portfolio. We can write this more concisely by saying that the contribution of stock 1 to portfolio risk is equal to the holding size (x_1) times the covariance between stock 1 and the entire portfolio (σ_{1p}).

To find stock 1's *relative* contribution to risk we simply divide by the portfolio variance to give $x_1 (\sigma_{1p} / \sigma_p^2)$. In other words, it is equal to the holding size (x_1) times the beta of stock 1 relative to the portfolio (σ_{1p} / σ_p^2).

We can calculate the beta of a stock relative to *any* portfolio by simply taking its covariance with the portfolio and dividing by the portfolio's variance. If we wish to find a stock's beta *relative to the market portfolio* we just calculate its covariance with the market portfolio and divide by the variance of the market:

$$\text{Beta relative to market portfolio} = \frac{\text{covariance with the market}}{\text{variance of market}} = \frac{\sigma_{im}}{\sigma_m^2}$$

(or, more simply, beta)

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---------|------------------|---------------------|---------------|--|--|--------------------------------------|---|
| | | | | Deviation from average | Deviation from average Anchovy Q return | Squared deviation from average | Product of deviations from average returns (cols 4 × 5) |
| Month | Market return | Anchovy Q return | market return | market return | return | market return | |
| 1 | -8% | -11% | -10 | -13 | 100 | 130 | |
| 2 | 4 | 8 | 2 | 6 | 4 | 12 | |
| 3 | 12 | 19 | 10 | 17 | 100 | 170 | |
| 4 | -6 | -13 | -8 | -15 | 64 | 120 | |
| 5 | 2 | 3 | 0 | 1 | 0 | 0 | |
| 6 | 8 | 6 | 6 | 4 | 36 | 24 | |
| Average | 2 | 2 | | Total | 304 | 456 | |
| | | | | Variance = $\sigma_m^2 = 304/6 = 50.67$ | | | |
| | | | | Covariance = $\sigma_{im} = 456/6 = 76$ | | | |
| | | | | Beta (β) = $\sigma_{im}/\sigma_m^2 = 76/50.67 = 1.5$ | | | |

TABLE 7.7 Calculating the variance of the market returns and the covariance between the returns on the market and those of Anchovy Queen. Beta is the ratio of the variance to the covariance (i.e., $\beta = \sigma_{im}/\sigma_m^2$).

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Here is a simple example of how to do the calculations. Columns 2 and 3 in Table 7.7 show the returns over a particular six-month period on the market and the stock of the Anchovy Queen restaurant chain. You can see that, although both investments provided an average return of 2%, Anchovy Queen's stock was particularly sensitive to market movements, rising more when the market rises and falling more when the market falls.

Columns 4 and 5 show the deviations of each month's return from the average. To calculate the market variance, we need to average the squared deviations of the market returns (column 6). And to calculate the covariance between the stock returns and the market, we need to average the product of the two deviations (column 7). Beta is the ratio of the covariance to the market variance, or $76/50.67 = 1.50$. A diversified portfolio of stocks with the same beta as Anchovy Queen would be one-and-a-half times as volatile as the market.

7-5 Diversification and Value Additivity

We have seen that diversification reduces risk and, therefore, makes sense for investors. But does it also make sense for the firm? Is a diversified firm more attractive to investors than an undiversified one? If it is, we have an *extremely* disturbing result. If diversification is an appropriate corporate objective, each project has to be analyzed as a potential addition to the firm's portfolio of assets. The value of the diversified package would be greater than the sum of the parts. So present values would no longer add.

Diversification is undoubtedly a good thing, but that does not mean that firms should practice it. If investors were *not* able to hold a large number of securities, then they

might want firms to diversify for them. But investors *can* diversify.³² In many ways they can do so more easily than firms. Individuals can invest in the steel industry this week and pull out next week. A firm cannot do that. To be sure, the individual would have to pay brokerage fees on the purchase and sale of steel company shares, but think of the time and expense for a firm to acquire a steel company or to start up a new steel-making operation.

You can probably see where we are heading. If investors can diversify on their own account, they will not pay any *extra* for firms that diversify. And if they have a sufficiently wide choice of securities, they will not pay any *less* because they are unable to invest separately in each factory. Therefore, in countries like the United States, which have large and competitive capital markets, diversification does not add to a firm's value or subtract from it. The total value is the sum of its parts.

This conclusion is important for corporate finance, because it justifies adding present values. The concept of *value additivity* is so important that we will give a formal definition of it. If the capital market establishes a value $PV(A)$ for asset A and $PV(B)$ for B, the market value of a firm that holds only these two assets is

$$PV(AB) = PV(A) + PV(B)$$

A three-asset firm combining assets A, B, and C would be worth $PV(ABC) = PV(A) + PV(B) + PV(C)$, and so on for any number of assets.

We have relied on intuitive arguments for value additivity. But the concept is a general one that can be proved formally by several different routes.³³ The concept seems to be widely accepted, for thousands of managers add thousands of present values daily, usually without thinking about it.

³² One of the simplest ways for an individual to diversify is to buy shares in a mutual fund that holds a diversified portfolio.

³³ You may wish to refer to the Appendix to Chapter 31, which discusses diversification and value additivity in the context of mergers.

SUMMARY

Our review of capital market history showed that the returns to investors have varied according to the risks they have borne. At one extreme, very safe securities like U.S. Treasury bills have provided an average return over 109 years of only 4.0% a year. The riskiest securities that we looked at were common stocks. The stock market provided an average return of 11.1%, a premium of 7.1% over the safe rate of interest.

This gives us two benchmarks for the opportunity cost of capital. If we are evaluating a safe project, we discount at the current risk-free rate of interest. If we are evaluating a project of average risk, we discount at the expected return on the average common stock. Historical evidence suggests that this return is 7.1% above the risk-free rate, but many financial managers and economists opt for a lower figure. That still leaves us with a lot of assets that don't fit these simple cases. Before we can deal with them, we need to learn how to measure risk.

Risk is best judged in a portfolio context. Most investors do not put all their eggs into one basket: They diversify. Thus the effective risk of any security cannot be judged by an examination of that security alone. Part of the uncertainty about the security's return is diversified away when the security is grouped with others in a portfolio.

Risk in investment means that future returns are unpredictable. This spread of possible outcomes is usually measured by standard deviation. The standard deviation of the *market portfolio*, generally represented by the Standard and Poor's Composite Index, is around 15% to 20% a year.

Most individual stocks have higher standard deviations than this, but much of their variability represents *specific* risk that can be eliminated by diversification. Diversification cannot eliminate *market* risk. Diversified portfolios are exposed to variation in the general level of the market.

A security's contribution to the risk of a well-diversified portfolio depends on how the security is liable to be affected by a general market decline. This sensitivity to market movements is known as *beta* (β). Beta measures the amount that investors expect the stock price to change for each additional 1% change in the market. The average beta of all stocks is 1.0. A stock with a beta greater than 1 is unusually sensitive to market movements; a stock with a beta below 1 is unusually insensitive to market movements. The standard deviation of a well-diversified portfolio is proportional to its beta. Thus a diversified portfolio invested in stocks with a beta of 2.0 will have twice the risk of a diversified portfolio with a beta of 1.0.

One theme of this chapter is that diversification is a good thing *for the investor*. This does not imply that *firms* should diversify. Corporate diversification is redundant if investors can diversify on their own account. Since diversification does not affect the value of the firm, present values add even when risk is explicitly considered. Thanks to *value additivity*, the net present value rule for capital budgeting works even under uncertainty.

In this chapter we have introduced you to a number of formulas. They are reproduced in the endpapers to the book. You should take a look and check that you understand them.

Near the end of Chapter 9 we list some Excel functions that are useful for measuring the risk of stocks and portfolios.

For international evidence on market returns since 1900, see:

E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002). More recent data is available in The Credit Suisse Global Investment Returns Yearbook at www.tinyurl.com/DMSyearbook.

The Ibbotson Yearbook is a valuable record of the performance of U.S. securities since 1926:

Ibbotson Stocks, Bonds, Bills, and Inflation 2009 Yearbook (Chicago, IL: Morningstar, Inc., 2009).

Useful books and reviews on the equity risk premium include:

B. Cornell, *The Equity Risk Premium: The Long-Run Future of the Stock Market* (New York: Wiley, 1999).

W. Goetzmann and R. Ibbotson, *The Equity Risk Premium: Essays and Explorations* (Oxford University Press, 2006).

R. Mehra (ed.), *Handbook of Investments: Equity Risk Premium 1* (Amsterdam, North-Holland, 2007).

R. Mehra and E. C. Prescott, "The Equity Risk Premium in Prospect," in *Handbook of the Economics of Finance*, eds. G. M. Constantinides, M. Harris, and R. M. Stulz (Amsterdam, North-Holland, 2003).



FURTHER READING



Select problems are available in McGraw-Hill Connect. Please see the preface for more information.

BASIC

1. A game of chance offers the following odds and payoffs. Each play of the game costs \$100, so the net profit per play is the payoff less \$100.



PROBLEM SETS

| Probability | Payoff | Net Profit |
|-------------|--------|------------|
| .10 | \$500 | \$400 |
| .50 | 100 | 0 |
| .40 | 0 | -100 |

What are the expected cash payoff and expected rate of return? Calculate the variance and standard deviation of this rate of return.



- The following table shows the nominal returns on the U.S. stocks and the rate of inflation.
 - What was the standard deviation of the market returns?
 - Calculate the average real return.

| Year | Nominal Return (%) | Inflation (%) |
|------|--------------------|---------------|
| 2004 | +12.5 | +3.3 |
| 2005 | +6.4 | +3.4 |
| 2006 | +15.8 | +2.5 |
| 2007 | +5.6 | +4.1 |
| 2008 | -37.2 | +0.1 |



- During the boom years of 2003–2007, ace mutual fund manager Diana Sauros produced the following percentage rates of return. Rates of return on the market are given for comparison.

| | 2003 | 2004 | 2005 | 2006 | 2007 |
|------------|-------|-------|------|-------|------|
| Ms. Sauros | +39.1 | +11.0 | +2.6 | +18.0 | +2.3 |
| S&P 500 | +31.6 | +12.5 | +6.4 | +15.8 | +5.6 |

Calculate the average return and standard deviation of Ms. Sauros's mutual fund. Did she do better or worse than the market by these measures?

- True or false?
 - Investors prefer diversified companies because they are less risky.
 - If stocks were perfectly positively correlated, diversification would not reduce risk.
 - Diversification over a large number of assets completely eliminates risk.
 - Diversification works only when assets are uncorrelated.
 - A stock with a low standard deviation always contributes less to portfolio risk than a stock with a higher standard deviation.
 - The contribution of a stock to the risk of a well-diversified portfolio depends on its market risk.
 - A well-diversified portfolio with a beta of 2.0 is twice as risky as the market portfolio.
 - An undiversified portfolio with a beta of 2.0 is less than twice as risky as the market portfolio.
- In which of the following situations would you get the largest reduction in risk by spreading your investment across two stocks?
 - The two shares are perfectly correlated.
 - There is no correlation.
 - There is modest negative correlation.
 - There is perfect negative correlation.

6. To calculate the variance of a three-stock portfolio, you need to add nine boxes:

| | | |
|--|--|--|
| | | |
| | | |
| | | |

Use the same symbols that we used in this chapter; for example, x_1 = proportion invested in stock 1 and σ_{12} = covariance between stocks 1 and 2. Now complete the nine boxes.

7. Suppose the standard deviation of the market return is 20%.
- What is the standard deviation of returns on a well-diversified portfolio with a beta of 1.3?
 - What is the standard deviation of returns on a well-diversified portfolio with a beta of 0?
 - A well-diversified portfolio has a standard deviation of 15%. What is its beta?
 - A poorly diversified portfolio has a standard deviation of 20%. What can you say about its beta?
8. A portfolio contains equal investments in 10 stocks. Five have a beta of 1.2; the remainder have a beta of 1.4. What is the portfolio beta?
- 1.3.
 - Greater than 1.3 because the portfolio is not completely diversified.
 - Less than 1.3 because diversification reduces beta.
9. What is the beta of each of the stocks shown in Table 7.8?

| Stock Return if Market Return Is: | | |
|-----------------------------------|------|------|
| Stock | -10% | +10% |
| A | 0 | +20 |
| B | -20 | +20 |
| C | -30 | 0 |
| D | +15 | +15 |
| E | +10 | -10 |

TABLE 7.8
See Problem 9.

INTERMEDIATE

10. Here are inflation rates and U.S. stock market and Treasury bill returns between 1929 and 1933:

| Year | Inflation | Stock Market Return | T-Bill Return |
|------|-----------|---------------------|---------------|
| 1929 | -.2 | -14.5 | 4.8 |
| 1930 | -6.0 | -28.3 | 2.4 |
| 1931 | -9.5 | -43.9 | 1.1 |
| 1932 | -10.3 | -9.9 | 1.0 |
| 1933 | .5 | 57.3 | .3 |

- What was the real return on the stock market in each year?
- What was the average real return?
- What was the risk premium in each year?
- What was the average risk premium?
- What was the standard deviation of the risk premium?

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11. Each of the following statements is dangerous or misleading. Explain why.
 - a. A long-term United States government bond is always absolutely safe.
 - b. All investors should prefer stocks to bonds because stocks offer higher long-run rates of return.
 - c. The best practical forecast of future rates of return on the stock market is a 5- or 10-year average of historical returns.
12. Hippique s.a., which owns a stable of racehorses, has just invested in a mysterious black stallion with great form but disputed bloodlines. Some experts in horseflesh predict the horse will win the coveted Prix de Bidet; others argue that it should be put out to grass. Is this a risky investment for Hippique shareholders? Explain.
13. Lonesome Gulch Mines has a standard deviation of 42% per year and a beta of $+ .10$. Amalgamated Copper has a standard deviation of 31% a year and a beta of $+ .66$. Explain why Lonesome Gulch is the safer investment for a diversified investor.
14. Hyacinth Macaw invests 60% of her funds in stock I and the balance in stock J. The standard deviation of returns on I is 10%, and on J it is 20%. Calculate the variance of portfolio returns, assuming
 - a. The correlation between the returns is 1.0.
 - b. The correlation is $.5$.
 - c. The correlation is 0.
15.
 - a. How many variance terms and how many covariance terms do you need to calculate the risk of a 100-share portfolio?
 - b. Suppose all stocks had a standard deviation of 30% and a correlation with each other of $.4$. What is the standard deviation of the returns on a portfolio that has equal holdings in 50 stocks?
 - c. What is the standard deviation of a fully diversified portfolio of such stocks?
16. Suppose that the standard deviation of returns from a typical share is about $.40$ (or 40%) a year. The correlation between the returns of each pair of shares is about $.3$.
 - a. Calculate the variance and standard deviation of the returns on a portfolio that has equal investments in 2 shares, 3 shares, and so on, up to 10 shares.
 - b. Use your estimates to draw a graph like Figure 7.11. How large is the underlying market risk that cannot be diversified away?
 - c. Now repeat the problem, assuming that the correlation between each pair of stocks is zero.
17. Table 7.9 shows standard deviations and correlation coefficients for eight stocks from different countries. Calculate the variance of a portfolio with equal investments in each stock.
18. Your eccentric Aunt Claudia has left you \$50,000 in Canadian Pacific shares plus \$50,000 cash. Unfortunately her will requires that the Canadian Pacific stock not be sold for one year and the \$50,000 cash must be entirely invested in one of the stocks shown in Table 7.9. What is the safest attainable portfolio under these restrictions?
19. There are few, if any, real companies with negative betas. But suppose you found one with $\beta = -.25$.
 - a. How would you expect this stock's rate of return to change if the overall market rose by an extra 5%? What if the market fell by an extra 5%?
 - b. You have \$1 million invested in a well-diversified portfolio of stocks. Now you receive an additional \$20,000 bequest. Which of the following actions will yield the safest overall portfolio return?
 - i. Invest \$20,000 in Treasury bills (which have $\beta = 0$).
 - ii. Invest \$20,000 in stocks with $\beta = 1$.
 - iii. Invest \$20,000 in the stock with $\beta = -.25$.
 Explain your answer.

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| Correlation Coefficients | | | | | | | | | |
|--------------------------|----|------------------|---------------|------|----------|------|--------|-------------|--------------------|
| | BP | Canadian Pacific | Deutsche Bank | Fiat | Heineken | LVMH | Nestlé | Tata Motors | Standard Deviation |
| BP | 1 | 0.19 | 0.23 | 0.20 | 0.34 | 0.30 | 0.16 | 0.09 | 22.2% |
| Canadian Pacific | | 1 | 0.43 | 0.31 | 0.39 | 0.34 | 0.17 | 0.40 | 23.9 |
| Deutsche Bank | | | 1 | 0.74 | 0.73 | 0.73 | 0.49 | 0.68 | 29.2 |
| Fiat | | | | 1 | 0.66 | 0.64 | 0.47 | 0.53 | 35.7 |
| Heineken | | | | | 1 | 0.64 | 0.51 | 0.50 | 18.9 |
| LVMH | | | | | | 1 | 0.52 | 0.60 | 20.8 |
| Nestlé | | | | | | | 1 | 0.43 | 15.4 |
| Tata Motors | | | | | | | | 1 | 43.0 |

TABLE 7.9 Standard deviations of returns and correlation coefficients for a sample of eight stocks.

Note: Correlations and standard deviations are calculated using returns in each country's own currency; in other words, they assume that the investor is protected against exchange risk.

20. You can form a portfolio of two assets, A and B, whose returns have the following characteristics:

| Stock | Expected Return | Standard Deviation | Correlation |
|-------|-----------------|--------------------|-------------|
| A | 10% | 20% | .5 |
| B | 15 | 40 | |

If you demand an expected return of 12%, what are the portfolio weights? What is the portfolio's standard deviation?

CHALLENGE

21. Here are some historical data on the risk characteristics of Dell and McDonald's:

| | Dell | McDonald's |
|---|------|------------|
| β (beta) | 1.41 | .77 |
| Yearly standard deviation of return (%) | 30.9 | 17.2 |

Assume the standard deviation of the return on the market was 15%.

- The correlation coefficient of Dell's return versus McDonald's is .31. What is the standard deviation of a portfolio invested half in Dell and half in McDonald's?
 - What is the standard deviation of a portfolio invested one-third in Dell, one-third in McDonald's, and one-third in risk-free Treasury bills?
 - What is the standard deviation if the portfolio is split evenly between Dell and McDonald's and is financed at 50% margin, i.e., the investor puts up only 50% of the total amount and borrows the balance from the broker?
 - What is the *approximate* standard deviation of a portfolio composed of 100 stocks with betas of 1.41 like Dell? How about 100 stocks like McDonald's? (*Hint*: Part (d) should not require anything but the simplest arithmetic to answer.)
22. Suppose that Treasury bills offer a return of about 6% and the expected market risk premium is 8.5%. The standard deviation of Treasury-bill returns is zero and the standard

deviation of market returns is 20%. Use the formula for portfolio risk to calculate the standard deviation of portfolios with different proportions in Treasury bills and the market. (*Note:* The covariance of two rates of return must be zero when the standard deviation of one return is zero.) Graph the expected returns and standard deviations.

23. Calculate the beta of each of the stocks in Table 7.9 relative to a portfolio with equal investments in each stock.

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You can download data for the following questions from the Standard & Poor's Market Insight Web site (www.mhhe.com/edumarketinsight)—see the “Monthly Adjusted Prices” spreadsheet—or from finance.yahoo.com. Refer to the useful Spreadsheet Functions box near the end of Chapter 9 for information on Excel functions.

1. Download to a spreadsheet the last three years of monthly adjusted stock prices for Coca-Cola (KO), Citigroup (C), and Pfizer (PFE).
 - a. Calculate the monthly returns.
 - b. Calculate the monthly standard deviation of those returns (see Section 7-2). Use the Excel function STDEVP to check your answer. Find the annualized standard deviation by multiplying by the square root of 12.
 - c. Use the Excel function CORREL to calculate the correlation coefficient between the monthly returns for each pair of stocks. Which pair provides the greatest gain from diversification?
 - d. Calculate the standard deviation of returns for a portfolio with equal investments in the three stocks.
2. Download to a spreadsheet the last five years of monthly adjusted stock prices for each of the companies in Table 7.5 and for the Standard & Poor's Composite Index (S&P 500).
 - a. Calculate the monthly returns.
 - b. Calculate beta for each stock using the Excel function SLOPE, where the “y” range refers to the stock return (the dependent variable) and the “x” range is the market return (the independent variable).
 - c. How have the betas changed from those reported in Table 7.5?
3. A large mutual fund group such as Fidelity offers a variety of funds. They include *sector funds* that specialize in particular industries and *index funds* that simply invest in the market index. Log on to www.fidelity.com and find first the standard deviation of returns on the Fidelity Spartan 500 Index Fund, which replicates the S&P 500. Now find the standard deviations for different sector funds. Are they larger or smaller than the figure for the index fund? How do you interpret your findings?

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