1 What are interest rates?

1.1 Learning outcomes

After studying this text the learner should be able to:

1. Describe the context of interest rates.
2. Elucidate the bank margin and its role.
3. Describe what a rate of interest is and related concepts such as per cent, basis points and percentage points.
4. Describe the concept "time value of money".
5. Explain compound interest.
6. Define coupon rate and the price of a security.
7. Differentiate floating rate security and fixed rate security.
8. Differentiate primary and secondary market rates.
9. Differentiate nominal value and maturity value.
10. Differentiate yield rate and discount rate.
11. Describe the risk-free rate.
12. Elucidate bid and offer rates / prices and spread.
13. Distinguish between nominal and real interest rates.

1.2 Introduction

Before a study of this material is undertaken, it is important to have an understanding of the financial system – which provides the context of interest rates. We suggest "Financial System: An Introduction" which is available free at http://bookboon.com/en/financial-system-an-introduction-ebook. For those who are familiar with the system, we offer a brief reminder below.

Interest rates are the reward paid by a borrower (debtor) to a lender (creditor) for the use of money for a period, and they are expressed in percentage terms per annum (pa), for example, 6.525% pa, in order to make them comparable. Interest rates are also quite often referred to as the *price of money*. This is not helpful. One should rather refer to interest rates as being the rates (there are various) payable on debt and deposit obligations (a.k.a. instruments and securities) by the borrowers to the lenders, and that the prices of the debt and deposit obligations are derived from the cash flows payable on the obligations in the future – by discounting the cash flows by the rates payable.

Upfront we offer a significant statement: short-term interest rates are not determined by supply and demand; they are controlled by the central bank (and there is an especially good reason for this), and all other interest rates are a function of current short-term rates and expectations as to where they will be in the future. Supply and demand forces do enter the equation – to the extent that the central back reacts to these forces with its administratively-determined interest rate, the policy interest rate (PIR). They do play a role in the rate determination on longer term obligations, but the PIR remains the anchor. We will return to these issues many times.

The term *interest rate/s* can be quite confusing to those unfamiliar with the financial markets. There are many different interest rates; a few examples: call deposit rates, term deposit rates, repurchase agreement (repo) rates, base rates, policy rates, bank rates, government bond rates, corporate bond rates, negotiable certificates of deposit (NCD) rates, Treasury bill (TB) rates, corporate / commercial paper (CP) rates, fixed interest rates, floating interest rates, discount rates, coupon rates, real rates, nominal rates, effective rates, risk-free rates, and so on.

Confusing? Yes, but they are all related and there is a way demystify the terminology. This is the aim of this text. It also elucidates the significant role of interest rates in the economy. We begin with: interest rates apply only to debt and deposit instruments (there are a few exceptions, such as preference shares). To comprehend this, we need to provide a synopsis of the financial system. This is provided next. The organisation of this text is as follows:

- Financial system: a synopsis.
- Debt and deposits.
- The bank margin.
- Rate of interest.
- Time value of money.
- TVM and compound interest.
- Effective rate.
- Coupon rate.
- Price of a security: the principle.
- Price of a security: multiple future cash flows and yield to maturity.
- Other issues and terminology related to interest rates.

1.3 Financial system: a synopsis

We present Figure 1 as the backdrop to this brief discussion. Perusal of the figure will reveal:

First: Ultimate borrowers issue financial securities, meaning that they borrow funds and issue evidences thereof (a.k.a. securities, IOUs, instruments, obligations, etc.). There are only two: debt and shares / equities. The ultimate lenders lend their excess funds, meaning that they purchase securities (evidences of debt and shares). The ultimate lenders and borrowers are comprised of the same four sectors of the economy, as indicated. Some of them are lenders and borrowers at the same time (for example, government), but generally they are one or the other.

Second: Financial intermediaries interpose themselves between the ultimate lenders and borrowers by offering useful financial services. They have assets (buy securities) and liabilities (issue their own securities to fund their assets). The main financial intermediaries are:

- Banks (central bank and private sector banks): They buy debt securities and issue securities known as certificates of deposit (CDs) which are negotiable (i.e. marketable, called NCDs) or non-negotiable (NNCDs). They are overwhelmingly of a short-term nature. Note: The central bank's liabilities are not termed as such; we call them CDs for the sake of simplicity.
- Investment vehicles: They buy debt and shares and issue what may be called "participation interests" (PIs). Other names are *membership interests* and *units*.

Third: Debt securities are divided into long-term (LT) securities and short-term (ST) securities, and they are either marketable debt (MD) or non-marketable debt (NMD), i.e. the financial system has LT-MD, LT-NMD, ST-MD and ST-NMD. Marketable debt is marketable because secondary markets exist for them.

Fourth: Shares are issued by companies and are marketable (MS) or non-marketable (NMS). Debt and shares are issued in primary markets and traded in secondary markets, such as a stock exchange, making them marketable.

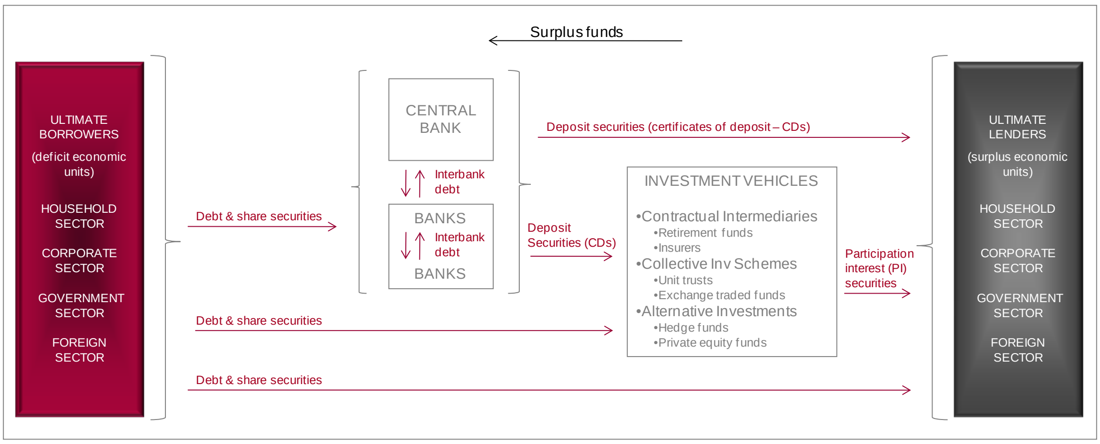

Figure 1.1: Financial system

An example will render the above comprehensible: A bank makes a mortgage loan to you to buy a house, and funds it by issuing CDs to a company:

- You are an ultimate borrower (a member of the household sector) and you issue an LT-NMD (an IOU), meaning you owe the bank.
- The bank buys your LT-NMD and issues CDs.
- The company (ultimate lender, a member of the corporate sector) buys the CDs.

Another way of seeing the financial system: There are six elements:

First: *Ultimate lenders* (= surplus economic units) and *ultimate borrowers* (= deficit economic units), i.e. the non-financial economic units that undertake the lending and borrowing process. The ultimate lenders lend to borrowers either directly or indirectly via financial intermediaries, by buying the securities they issue.

Second: *Financial intermediaries* which intermediate the lending and borrowing process. They interpose themselves between the lenders and borrowers, and earn a margin for the benefits of intermediation (including lower risk for the lenders). They buy the securities of the borrowers and issue their own to fund these (and thereby become intermediaries).

Third: *Financial instruments* (or securities, obligations, assets), which are created / issued by the ultimate borrowers and financial intermediaries to satisfy the financial requirements of the various participants. These instruments may be marketable (e.g. Treasury bills) or non-marketable (e.g. retirement annuities). There are two categories and two subcategories:

- Ultimate financial securities (issued by ultimate borrowers):
    - Debt securities.
    - Share (a.k.a. stock / equity) securities.

- Indirect financial securities (issued by financial intermediaries):
    - Deposit securities, a.k.a. certificates of deposit (CDs) (issued by banks).
    - Participation interests (PIs) (issued by investment vehicles).

Fourth: *Creation of money* (= bank deposits; bank notes are also deposits) by banks when they satisfy the demand for new bank credit. This is a unique feature of banks. Central banks have the tools to control money growth, which they do primarily to tame inflation.

Fifth: *Financial markets*, i.e. the institutional arrangements and conventions that exist for the issue and trading (dealing) of the financial instruments. The financial markets are:

- Money market (all ST-MD, ST-NMD and CDs), in other words the entire short-term debt and deposit market, marketable and non-marketable.
- Bond market (all LT-MD), in other words the marketable part of the long-term debt market.
- Share / stock / equity market (all MS).
- Foreign exchange market (the market for the exchange of currencies).
- Participation interests markets (there are a number, e.g. units of unit trusts, membership interest in a retirement fund).
- Derivatives markets (forwards, futures, swaps, options, etc.).

Sixth: *Interest rate / price discovery*, i.e. the establishment in the financial markets of the *rates of interest* on debt and deposit instruments, and the *prices* of share instruments.

As our interest in this text is interest rates and their discovery, we can ignore shares and PIs, which do not carry interest (there are exceptions, such as preferences shares, but we will ignore them in the interests of pedagogy). Thus, we are left with debt and deposits, and their markets.

1.4 Debt and deposits

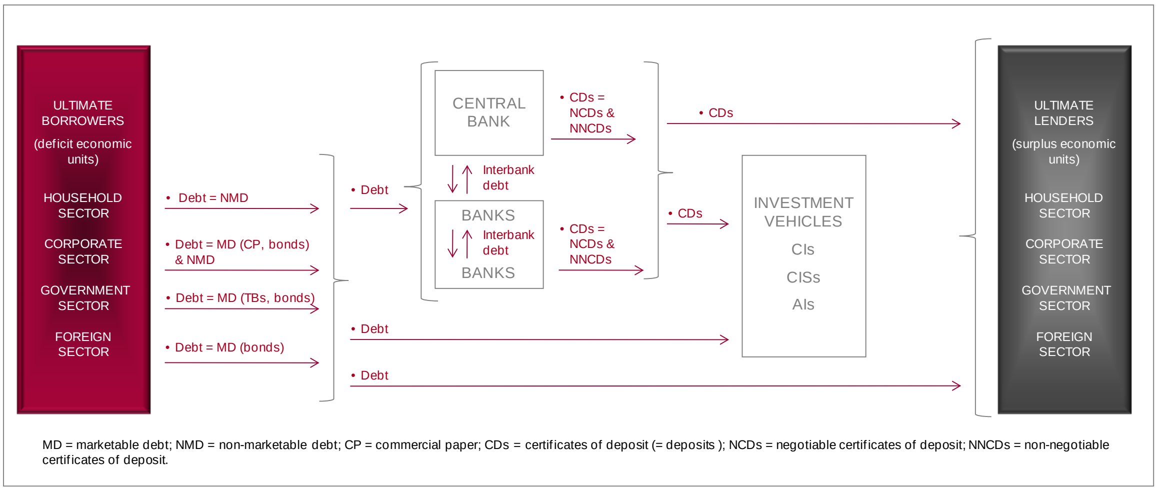

Figure 1.2: Debt and deposit securities

Debt securities are evidences of debt issued by the borrower to the lender. The lender may be a financial intermediary (bank, central bank or investment vehicle) or an ultimate lender (one of the four sectors). As may be seen in Figure 2, the following debt securities exist:

- Household sector:
    - MD (none, because they are not able to issue MD).
    - NMD (examples: leases, mortgage loans, overdraft facilities utilised).

- Corporate sector:
    - MD [commercial paper (CP), bankers' acceptances (BAs), promissory notes (PNs), corporate bonds].
    - NMD (examples: leases, mortgage loans, overdraft facilities utilised).

- Government sector:
    - MD [central government: Treasury bills (TBs), bonds].
    - NMD (issued by the lower levels of government, for example, local government bonds).

- Foreign sector:
    - MD [foreign commercial paper (CP), corporate bonds].
    - NMD (none as only the large foreign corporate entities are able to issue, and they issue MD).

Figure 2 shows: Deposit securities (CDs) are issued by banks to lenders, and the funds are used to purchase debt securities, in the form of MD and NMD. The vast majority are NMD, and specifically mortgage loans and overdraft facilities utilised. As we have seen, there are two categories of CDs: NCDs and NNCDs. The vast majority are NNCDs.

It is necessary at this time to make reference to the middle part of Figures 1 and 2, and enhanced in Figure 3: The interbank debt market (IBM). There are three parts to the IBM:

- Bank-to-bank interbank market (b2b IBM). This is where interbank claims and loans are settled, and this is effected over the accounts that banks are required to have with the central bank.
- Bank-to-central bank interbank market (b2cb IBM). This represents the reserve requirement amount, i.e. the requirement that banks are to hold a certain proportion of their deposits with the central bank (in most countries).
- Central bank-to-bank interbank market (cb2b IBM). This represents the (usually overnight) loans made by the central bank to the banks for monetary policy purposes.

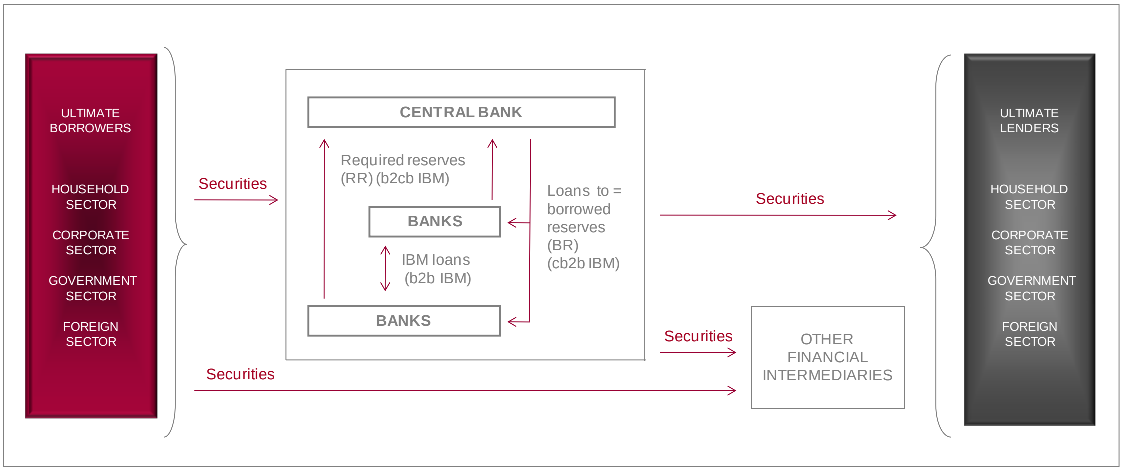

Figure 1.3: Interbank market

This significant market and its role in *interest rate determination* will be elucidated in detail later.

It will be evident that each ultimate borrower security [i.e. the different types of NMD and MD (CP, BAs, PNs, TBs, corporate and government bonds)] carries a different interest rate. Similarly, each NNCD and NCD carries a different rate of interest. This also applies to the IBM. However, the story is different in the IBM in that the genesis of interest rates is found here, and it is determined administratively to a significant degree, as we will show later.

Ignoring the influence of the central bank on interest rates (and therefore on inflation), each interest rate is dependent upon / influenced by:

- Term to maturity. The rate increases as the term to maturity increases.
- Risk profile of the issuer (borrower: ultimate borrower or bank). The rate rises as the risk rises.
- Marketability (all issues of securities take place in their *primary markets*, but only certain securities have *secondary markets* – where they can be sold or bought). The rate decreases as marketability increases.

One final point, which we have not indicated in Figures 1 and 2 (but rectify in Figure 4), is the existence of direct financing. Not all lending and borrowing takes place via the financial intermediaries. It can also occur directly (an example is a company or wealthy individual buying bonds with excess funds). However, the vast majority is undertaken via the financial intermediaries.

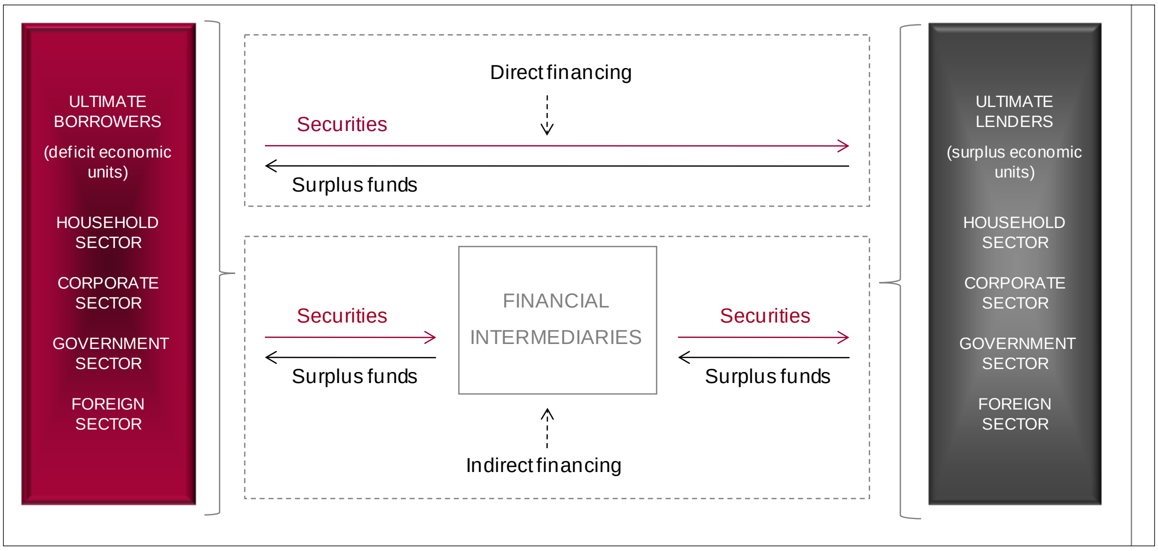

Figure 1.4: Direct and indirect financing

1.5 The bank margin

It is important at this stage to introduce the bank margin. In simple terms (see Figure 5; we will elucidate later) banks intermediate the lending and borrowing process, in the process transmuting NMD into NNCDs and NCDs. In essence, they are creating liquidity and reducing risk for the lenders (buyers of CDs = depositors) by taking on the information costs and providing diversification of assets.

For this service they charge a "fee" in the form of a lower rate of interest earned by the lender (the buyer of CDs) than they earn on the MD and NMD securities purchased (i.e. their loans / credit). This difference is the *bank margin* (I.e. – IP), and it is "sticky" in that it is jealously guarded by the banks as it represents a major part of their profits. It is kept at a reasonable number by competition in the banking sector.

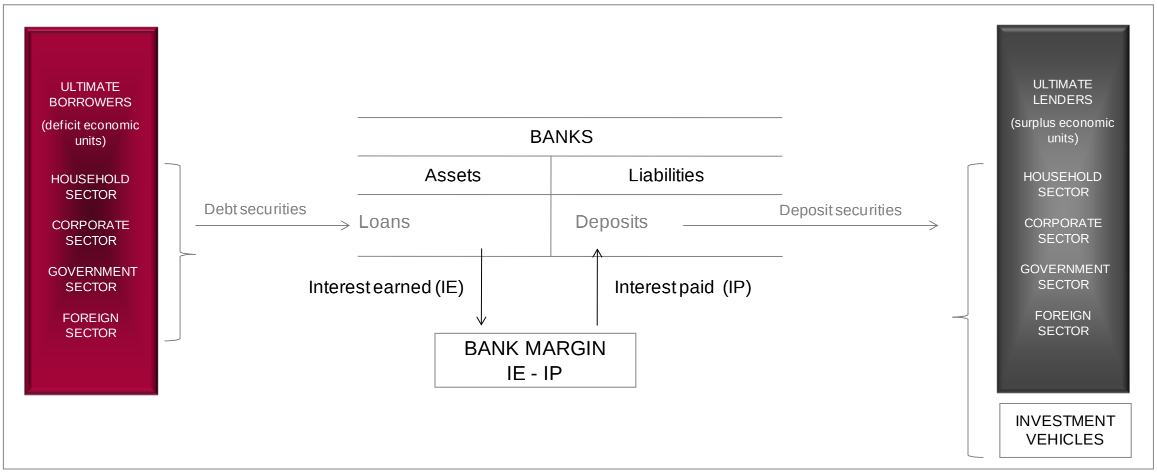

Figure 1.5: Bank margin

The bank margin is significant for another important reason: The central bank, by influencing the cost of the banks' liabilities ("at the margin"), via the policy interest rate (PIR), are able to directly influence the banks' lending rates. The benchmark bank lending rate is called *prime rate* (PR). PR is the high profile rate one sees in press announcements, and all bank lending rates are related to it. For example, a mortgage loan may be granted to a small borrower at PR+1%, and to a large borrower at PR-2%. Its prominence in monetary policy will become clear later.

1.6 Rate of interest

In order to concretise the understanding of interest, we need to go back to basics. The rate of interest is the price or fee paid by a borrower of money to the lender for the use of the money for a period, divided by the amount borrowed. The borrower is thus advancing consumption and paying for this privilege. From the perspective of the lender, the price or fee charged is his / her compensation for delaying consumption for the period of the loan.

Thus, seen simply, there are two elements to the rate of interest, the price or fee paid and the amount loaned / borrowed. An example:

Fee / price paid = LCC[1] 100
Amount loaned / borrowed = LCC 1 000.

The interest rate (ir) is as follows:

ir = fee paid / loan amount
= 100 / 1 000
= 0.10 (or 0.10 LCC per one unit of LCC loaned)
= 10%.

It should be evident that the rate of interest is a ratio, i.e. the ratio of (in this example) 100 / 1 000. Also, this is the rate of interest for the relevant period. As said above, in practice interest rates are expressed in pa terms, in order to make them comparable.

The term of the loan, the rate and the pa convention are important. For example, if the loan is for 91 days (t), and the 10% rate is for this period, the amount payable is LCC 100, but the rate is not 10% pa; it is (assuming the day-count convention is 365 days):

$$
\begin{aligned}
\text{Effective rate pa} &= \text{ir} \times (365 / \text{t}) \\
&= 0.1 \times (365 / 91) \\
&= 0.40110 \\
&= 40.11\%\ \text{pa}^{2}.
\end{aligned}
$$

If the 10% rate is a pa rate, then the amount payable on a LCC 1 000 loan after 91 days is:

$$
\begin{aligned}
\text{Interest payable (IP)} &= \text{loan amount} \times (\text{ir} \times \text{t} / 365) \\
&= \text{LCC } 1\,000 \times (0.1 \times 0.24932) \\
&= \text{LCC } 24.93.
\end{aligned}
$$

These numbers can be used to derive the ir pa:

ir pa = (IP / loan) × (365 / t)
= [(LCC 24.93 / LCC 1 000) × (365 / 91)]
= 0.1
= 10% pa.

Compounding and multiple regular future interest payments will be introduced later. The above discussion hints at the fact that money has a value over time. We cover this next.

1.7 Time value of money

The *time value of money* (TVM) concept, a significant concept in economics and finance, means that *money has a value over time*. It is founded on the notion that money represents a command over goods and services (i.e. consumption), and that if you delay consumption by lending part of your money supply to someone, you will expect compensation, otherwise you would not lend the money. What's the point? Even if you were inclined to lend the money to a friend compensation-free, this is a foolish idea, and there is a sound reason for this: the future is uncertain. There are two factors to consider in relation to the future: you cannot be certain that you will receive the money loaned and / or the compensation amount when they are due (= credit risk), and inflation may erode the value of the money lent (= inflation risk). As we know, the compensation amount is called interest.

Another way of looking[3] at this concept is that LCC 1 received today is worth more than LCC 1 received in the future. This of course is because the LCC can be invested and its value enhanced by the rate of return, the interest amount.

This is the basic tenet of the TVM concept, i.e. money has a *future value* (FV) and a *present value* (PV):

- FV is PV plus interest.
- PV is FV discounted at the relevant interest rate.

Another basic principle of the concept is that interest is compounded, i.e. interest that is earned is reinvested, and an essential assumption here is that interest earned is reinvested at the rate earned on the principal amount. The PV-FV concept is the foundation of all financial market mathematics.

From the previous section, we know that the principal amount (amount invested) is the PV and the FV is the sum of the PV and the interest amount (IA) earned, as follows.

FV = PV + IA.

This may be expressed as:

$$\begin{aligned} FV &= PV + [PV \times (ir \times t / 365)] \\ &= PV \times [1 + (ir \times t / 365)]. \end{aligned}$$

From this we are able to derive the PV formula:

$$PV = FV / [1 + (ir \times t / 365)].$$

Example: PV to FV:

PV = LCC 1 000 000
ir = 14% pa
t = 90 days

$$\begin{aligned} FV &= PV \times [1 + (ir \times t / 365)] \\ &= \text{LCC } 1\,000\,000 \times [1 + (0.14 \times 90 / 365)] \\ &= \text{LCC } 1\,000\,000 \times 1.03452055 \\ &= \text{LCC } 1\,034\,520.55. \end{aligned}$$

Example: FV to PV:

FV = LCC 1 350 000
ir = 12% pa
t = 120 days

$$\begin{aligned} PV &= FV / [1 + (ir \times t / 365)] \\ &= \text{LCC } 1\,350\,000 / [1 + (0.12 \times 120 / 365)] \\ &= \text{LCC } 1\,350\,000 / (1.0394521) \\ &= \text{LCC } 1\,298\,761.20. \end{aligned}$$

1.8 TMV and compound interest

Compound interest takes into account interest earned on interest and on the principal amount (i.e. the original amount of the investment / borrowing). It assumes always that the interest earned is reinvested at the original rate of interest from as soon as it is paid.

A simple example may be useful: a LCC 1 million investment for 2 years at 13% pa payable in arrears (see Figure 6).

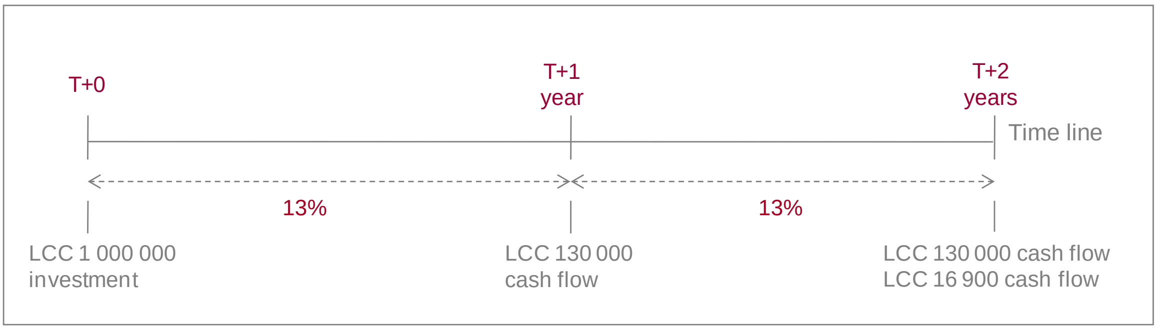

Figure 1.6: Compound interest (cash flows)

The LCC 1 million investment earns interest at 13% pa twice (LCC 130 000), i.e. at the end of each year. The first interest payment is also invested at 13% (the assumption as explained) for the last year [yielding LCC 16 900 (130 000 × 0.13)]. Thus the value of the investment at the end of the period of 2 years (FV) is:

$$\text{LCC } 1\,000\,000 + (2 \times \text{LCC } 130\,000) + \text{LCC } 16\,900 = \text{LCC } 1\,276\,900.$$

The compound interest formula is:

$$FV \quad = PV \times (1 + ir \,/\, not)^{y.not}$$

where

ir = interest rate pa
y = number of years
not = number of times interest is paid per annum.

Example:

PV = LCC 1 000 000
ir = 13% pa
y = 2
not = 1

$$
\begin{aligned}
FV &= \text{LCC } 1\,000\,000 \times (1 + 0.13 / 1)^{2\times1} \\
&= \text{LCC } 1\,000\,000 \times (1.13)^{2} \\
&= \text{LCC } 1\,000\,000 \times (1.2769) \\
&= \text{LCC } 1\,276\,900.00.
\end{aligned}
$$

Another example:

PV = LCC 1 000 000
ir = 15% pa
y = 1
not = 12 (i.e. monthly)

$$
\begin{aligned}
FV &= \text{LCC } 1\,000\,000 \times (1 + 0.15 / 12)^{1\times12} \\
&= \text{LCC } 1\,000\,000 \times (1.0125)^{12} \\
&= \text{LCC } 1\,000\,000 \times (1.16075452) \\
&= \text{LCC } 1\,160\,754.52.
\end{aligned}
$$

Yet another example:

PV = LCC 1 000 000
ir = 15% pa
y = 3
not = 2 (i.e. six-monthly)

$$
\begin{aligned}
FV &= \text{LCC } 1\,000\,000 \times (1 + 0.15 / 2)^{3\times2} \\
&= \text{LCC } 1\,000\,000 \times (1.075)^{6} \\
&= \text{LCC } 1\,000\,000 \times (1.54330153) \\
&= \text{LCC } 1\,543\,301.53.
\end{aligned}
$$

The PV of an investment may be derived from the FV:

$$PV = FV / (1 + ir / not)^{y.not}.$$

An example: What amount must be invested now (PV) at 12% pa compounded semi-annually to end up at LCC 1 million in 3 years' time? The answer is:

$$\begin{aligned} PV &= LCC\ 1\ 000\ 000 / (1 + 0.12 / 2)^{3.2} \\ &= LCC\ 1\ 000\ 000 / (1.06)^{6} \\ &= LCC\ 1\ 000\ 000 / 1.41851911 \\ &= LCC\ 704\ 960.54. \end{aligned}$$

This is a significant formula in economics and finance. Borrowings and investments have future cash flows (FVs). This formula enables one to calculate the price (= PV) of an investment with future cash flows. Note that the interest rate is part of the denominator, which means that when a rate rises, the price (PV) of the investment falls. The converse obviously holds.

Note also that this formula is applied differently in the case of financial instruments with multiple regular cash flows (FVs). As we will show later, each cash flow is discounted to PV and then added.

1.9 Effective rate

Rates of interest pa in the financial markets are quoted with the interest frequency stated. These rates are referred to as the *nominal* rates. For example, a rate may be quoted as 13.5% pa with interest payable *monthly*, or a rate may be quoted as 12% pa with interest payable *quarterly*.

The terminology used in the market for these two rates are 13.5% *nacm* (nominal annual compounded monthly) and 12% *nacq* (nominal annual compounded quarterly). In the case where interest is payable six-monthly and at the end of a year, the terminology would be *nacs* (nominal annual compounded semi-annually) and *naca* (nominal annual compounded annually).

In order to compare these rates, the term *effective rate* is applied. *Nominal rates* are converted to *effective rates* with the use of the following formula:

$$ir_e = [(1 + ir_n / t)^t - 1]$$

where

ir_e = effective rate

ir_n = nominal rate

t = number of interest periods per annum.

An example: A 12% *nacm rate* converts to an *effective rate* as follows:

$$\begin{aligned} ir_e &= (1 + ir_n / t)^t - 1 \\ &= (1 + 0.12 / 12)^{12} - 1 \\ &= (1 + 0.01)^{12} - 1 \\ &= 1.12683 - 1 \\ &= 0.12683 \\ &= 12.68\%. \end{aligned}$$

Another example: A 12% *nacq rate* converts to an *effective rate* as follows:

$$\begin{aligned} ir_e &= (1 + ir_n / t)^t - 1 \\ &= (1 + 0.12 / 4)^{4} - 1 \\ &= (1 + 0.03)^{4} - 1 \\ &= 1.12550 - 1 \\ &= 0.12550 \\ &= 12.55\%. \end{aligned}$$

It will be evident that a 12% *naca* rate will be equal to an *effective rate* of 12%. Thus, the more interest periods involved, the higher the effective rate will be.

1.10 Coupon rate

Ninety-nine per cent of bonds and long-term NCDs have a *coupon rate* printed on the face of the certificate (or on the computer generated letter / printout in the age of dematerialisation). This is the fixed rate of interest payable to the registered holders of the bonds on the specified interest payment dates. The payment dates may be monthly, quarterly, semi-annually or annually. Semi-annually is the most common.

The origin of the word *coupon* is the bond certificates of decades ago which had coupons attached. These bonds were issued to bearer and they had a coupon for each interest payment. On interest dates the holder detached the relevant coupon and presented it to the issuer (mainly the government) for payment of the interest.

The modern equivalent of the physical coupon is the coupon rate (cr) printed on the face of the certificate. For example, a LCC 1 million bond may have a coupon of 12.0% pa and interest payment dates of 30 June and 30 December. On these interest dates an amount of LCC 60 000 would be paid to the registered holders:

$$\begin{aligned} \text{Interest payable} &= (\text{cr} / \text{not}) \times \text{LCC } 1\,000\,000 \\ &= 0.12 / 2 \times \text{LCC } 1\,000\,000 \\ &= \text{LCC } 60\,000.00. \end{aligned}$$

The bonds that do not have a coupon rate are:

- Variable rate bonds (such as the inflation-linked bonds).
- Zero coupon bonds.
- Islamic bonds.

Zero coupon bonds are issued for periods of longer than a year and only the nominal / face value (FV) is payable on the maturity date. This of course means that zero coupon bonds are issued at a *discount*, and that the interest earned = FV – PV.

It is to be noted that the coupon earned by the holder is not necessarily the *actual* rate that s/he is earning. The coupon rate is the rate earned only if the bond is issued or trades at a price of 1.0 or 100%. In most cases bonds are issued and trade at a price *premium* (for example 102.4%) or a price *discount* (for example 92.8%). We take this further in the next section.

1.11 Price of a security: the principle

The price of a fixed interest rate security is inversely related to the market interest rate for the security. The best example to demonstrate this is that of a bond that has a fixed rate payable but has no maturity date: the perpetual bond. The price of this bond is:

Price = cr / ir

where

cr = coupon payment (assumed to be annual)
ir = interest rate (at which the perpetual bond trades).

It should be clear that when cr = ir, the price is 1.0 or 100%. However, in the case of a perpetual bond that has an annual coupon of 10% pa, but is trading at 9% pa, the price is:

Price = 10% / 9%
= 1.1111111.

The principal at work here is that when the market rate for the perpetual bond falls from 10% pa to 9% pa, the buyers are prepared to earn 9% pa in perpetuity. This means that they are prepared to pay a *price* for the security that will yield them 9% pa. On a LCC 1 million nominal / face value perpetual bond the annual income is LCC 100 000 (= cr = 10% pa). Thus, the buyers will be prepared to pay LCC 1 111 111.11 for the bond:

Consideration = (10% / 9%) × LCC 1 000 000
= 1.1111111 × LCC 1 000 000
= LCC 1 111 111.11.

Another example is called for:

Security	= government bond
Nominal value	= LCC 1 000 000
Coupon rate (cr, i.e. fixed rate for the period)	= 15% pa
Coupon payable	= in arrears, on maturity
Term to maturity	= 365 days
Market rate	= 15% pa.

The price of the bond on the issue date is 1.0 or 100%, i.e. the investor pays LCC 1 000 000 for the bond. If s/he holds the security for the period of 365 days, s/he will earn the coupon:

Coupon = LCC 1 000 000 × 15.0 / 100
= LCC 1 000 000 × 0.15
= LCC 150 000.

However, if the *interest rate* for the bond in the secondary market falls to 7.5% pa on the same day (the day of issue), the price of the bond will be 2.0 or 200%. This is because there are buyers that are willing to accept a fixed interest rate of 7.5% pa for the period. (Remember that the coupon rate of 15% pa does not change.) In terms of the formula shown above the price of the bond changes to:

Price = cr / ir
= 15.0% / 7.5%
= 2.0.

The consideration payable is:

Consideration = nominal value × price
= LCC 1 000 000 × 2.0
= LCC 2 000 000.00.

It will be evident that the buyer will earn:

Rate earned = coupon / LCC 2 000 000
= LCC 150 000 / LCC 2 000 000
= 0.075
= 7.5% pa.

This is the market rate at which s/he bought the bond. This demonstrates the principle: price and market rates are inversely related. We now turn to reality: most bonds worldwide have longer terms and have multiple regular interest payments.

1.12 Price of a security: multiple future cash flows and yield to maturity

As said, the vast majority of bonds have longer terms to maturity (up to 30 years) and have multiple and regular (usually twice pa) coupon interest payments. It is best to elucidate with an example However, before we do so we need to introduce the concept yield to maturity (ytm). Although in the bond markets of the world the broker-dealers refer to a "rate" on a long-term security, they are actually referring to its ytm.

Ytm is a measure of the *rate of return* on a bond that has a number of coupons paid over a number of years and a face value payable at maturity. It may also be described as the price that buyers are prepared to pay now (PV) for a stream of regular payments and a lump sum at the end of the period for which the bond is issued. It is an *average rate* earned per annum over the period.

Formally described, the ytm is the *discount rate that equates the future coupon payments and principal amount of a bond with the market price*. Another way of stating this is: the *price* is merely the *discounted value of the income stream (i.e. the coupon payments and redemption amount), discounted at the market yield (ytm).*

The following example will illuminate the PV-FV of a longer term bond or NCD. We choose a 3-year maturity and annual interest payments to elucidate:

Settlement date:	30 / 9 / 2014
Maturity date:	30 / 9 / 2017
Coupon rate (cr):	9% pa
Nominal / face value:	LCC 1 000 000
Interest date:	30 / 9
Ytm (i.e. market rate)	8% pa (payable annually in arrears).

The cash flows and their discounted values (the ytm is used) are as shown in Table 1.

Date	Coupon payment (C)	Nominal / face value	Compounding periods (cp)	Present value $C / (1 + ytm)^{cp}$
30/9/2015	LCC 90 000	-	1	LCC 83 333.33
30/9/2016	LCC 90 000	-	2	LCC 77 160.49
30/9/2017	LCC 90 000	-	3	LCC 71 444.90
30/9/2017	-	LCC 1 000 000	3	LCC 793 832.24
Total	LCC 270 000	LCC 1 000 000		LCC 1 025 770.96
C = coupon payment. cr = coupon rate. cp = compounding periods (years).				

Table 1.1: Cash flows and discounted values

The value now of the bond is LCC 1 025 770.96, and the price of the bond is 1.02577096. The price is calculates as follows:

$$\text{Price (PV)} = [cr / (1 + ytm)^1] + [cr / (1 + ytm)^2] + [cr / (1 + ytm)^3] + [1 / (1 + ytm)^3]$$

where:

cr = coupon rate pa (expressed as a fraction of 1)
ytm = yield to maturity (expressed as a fraction of 1).

Using the same numbers as above (coupon rate = 9% pa, ytm = 8% pa):

Price (PV) = (0.09 / 1.08) + (0.09 / 1.166400) + (0.09 / 1.259712) + (1 / 1.259712)
= 0.08333333 + 0.07716049 + 0.0714449 + 0.79383224
= 1.02577096.

It will be apparent that the coupon rate (0.09) for the periods and the face value (1) that takes place at maturity (all FVs) are discounted at the ytm to PV. Because the coupon rate is higher than the ytm, the price is *higher than 1* (a premium to par). Where the coupon rate is equal to the ytm (assume 9% pa) the price is *equal to 1* (par):

Price (PV) = (0.09 / 1.09) + (0.09 / 1.1881) + (0.09 / 1.295029) + (1 / 1.295029)
= 0.082569 + 0.075751 + 0.069497 + 0.772183
= 1.000000.

As noted, when the coupon rate is lower than the ytm (assume coupon rate = 9% pa, ytm = 11% pa), the price is *lower than 1* (i.e. at a discount to par):

Price (PV) = (0.09 / 1.11) + (0.09 / 1.232100) + (0.09 / 1.367631) + (1 / 1.367631)
= 0.081081 + 0.073046 + 0.065807 + 0.731191
= 0.951125.

The *inverse relationship* between ytm and price is clear. This is because the ytm is the *denominator* in the formula. Thus, if the ytm falls, the price of the bond rises. It follows that if the ytm increases the price falls. Another way of seeing this phenomenon is the logic of: As the ytm rises the future cash flows are worth less when discounted to present value, pulling down the price.

In reality bonds are slightly more complicated but the principle remains the same. The majority (by far) of bonds issued in the bond market have coupons that are payable six-monthly in arrears, and they are issued and traded for periods that are broken, i.e. issues and secondary market settlement dates are between interest payment dates.

In the case where interest payments are made six-monthly in arrears (ignoring settlement between interest payment dates), the coupon rate is halved and the compounding periods are doubled (assume a three-year bond):

Price (PV) $= [(cr / 2) / (1 + ytm / 2)^1] + [(cr / 2) / (1 + ytm / 2)^2] +$
$[(cr / 2) / (1 + ytm / 2)^3] + [(cr / 2) / (1 + ytm / 2)^4] +$
$[(cr / 2) / (1 + ytm / 2)^5] + [(cr / 2) / (1 + ytm / 2)^6) +$
$[1 / (1 + ytm / 2)^6]$.

The bond formula is usually written as:

$$\text{Price} = \sum_{t=1}^{n} [cr / (1 + ytm)^{t}] + [1 / (1 + ytm)^{n}]$$

where

cr = coupon rate (cr / 2 if six-monthly)
ytm = yield to maturity (ytm / 2 if six-monthly)
n = number of periods (years × 2 if six-monthly).

1.13 Other issues and terminology related to interest rates

1.13.1 Basis points, percentage points

It has been uttered that "interest rates have increased by one per cent", or "the central bank cut rates by a half per cent". Both expressions are incorrect, because a *percentage change* implies the change in interest rates from *one level* to *another level*. For example, if a rate of interest changes from 10.5% pa to 11.5% pa, then the *percentage increase* is 9.52% [(11.5 / 10.5) – 1) × 100], not 1%.

The correct terminology is the interest rate increased by *100 basis points* or *1 percentage point*. The basis point concept was developed to explain small movements in interest rates. Thus, a basis point is equal to 1 / 100 of a percentage point.

1.13.2 Floating rate and fixed rate securities

We saw above that a fixed rate security (a.k.a. fixed interest security) is a security (evidence of debt), which carries a fixed rate of interest, i.e. the rate of interest payable by the issuer (borrower) remains unchanged throughout the life of the security, irrespective of the rate at which the security *trades* in the secondary market.

On the other hand, a floating rate security is a security on which the rate of interest changes daily or less frequently (depending on the deal). An example of a true floating rate security is a "call deposit", i.e. the interest rate on the deposit can change daily. Another example is a 3-year corporate bond issued at the 91-day TB rate + 100 basis points. It re-prices every 91 days with the issue of new 91-day TBs. In other words a floating rate debt security is a debt on which the rate is benchmarked on a well-publicised rate.

Apart from the TB rate, the most often used benchmark rates are:

- A well-publicised interbank rate. An example is the UK LIBOR rate (London interbank offer rate), which in theory can change daily.
- The prime lending rate (PR) of the banks (a.k.a. base rate, bank rate, etc.).
- The policy interest rate (PIR) of the central bank (a.k.a. discount rate, repo rate, bank rate, key interest rate, etc.).

1.13.3 Primary and secondary market rates

As said earlier the *primary market* is the market for the issue of new securities and the *secondary market* is the market for the trading of existing securities (i.e. securities that are already in issue). The rate in the primary market can be called an *issue rate* or a *primary issue rate,* but usually the former. The rate in the secondary market is called the *secondary market rate* or just the *market rate* (usually the latter). For example, when a new government bond is issued it is issued at an *issue rate*. When it trades in the secondary market it trades at the *market rate* (the ytm).

Primary market rates are unimportant (in terms of analysis), compared with secondary market rates, because they are issue rates that applied at a particular time, and they in any case are established with reference to the secondary market rates. Also, issues of particular securities are not made on all days. Secondary market rates are important and studied by analysts and academics because they are discovered / established in some cases every second of the day. Because of this they provide *time series*' of various rates.

However, this does not apply to certain high profile primary rates that originate in markets that do not have secondary markets: Examples are the call money rates paid by banks, the prime lending rate (PR) of banks, the mortgage rate of banks and the policy interest rate (PIR) of the central bank (which of course are all closely linked).

1.13.4 Nominal value and maturity value

We have covered *nominal value* and hinted at the concept *maturity value*. We need here to distinguish between money market (all short-term) and bond market (marketable long-term) securities. As we have seen, the plain vanilla bond is one that pays a *coupon* periodically for a number of years on an amount. This *amount* is the nominal value (a.k.a. face value) of the bond. An example will elucidate:

Security	= government bond
Nominal value	= LCC 1 000 000
Coupon rate (i.e. fixed rate for the period)	= 15% pa
Coupon payable	= annually in arrears
Term to maturity	= 5 years
Market rate	= 10% pa.

When this bond has less than 12 months to maturity, on maturity it will pay LCC 1 000 000 + the coupon amount of LCC 150 000 = LCC 1 150 000 to the holder. This is the *maturity value* (MV). If someone buys this bond when it has 85 days to maturity at a rate (*rate*, no longer *ytm*, applies here) of 9.5% pa, she will pay:

Consideration (PV) = MV (= FV) discounted at 9.5% pa
= LCC 1 150 000 / [(1 + (0.095 × 85 / 365)].
= LCC 1 150 000 / 1.02212329
= LCC 1 125 108.89.

Because this bond has less than a year to maturity, it falls into the money market, and is also called an *interest add-on* security. In the money market here are two main types of fixed interest securities (they have one interest payment):

- Interest add-on securities.
- Discount securities (covered later).

An example of the former is the bond covered above. Another is the NCD. A buyer of a new NCD (i.e. a depositor) will deposit at the bank LCC 1 000 000 at a rate of 8.25% pa for 182 days. The maturity value (MV = FV) of the NCD is:

MV (FV) = PV × [1 + (0.0825 × 182 / 365)]
= LCC 1 000 000 × 1.04113699
= LCC 1 041 136.99.

If this NCD is sold in the secondary market after 10 days (i.e. has 172 days to maturity) at 7.5% pa, the calculation of the consideration (PV) is done according to the PV-FV formula presented above in the case of the short bond:

Consideration (PV) = MV (= FV) discounted at 7.5% pa
= LCC 1 041 136.99 / [(1 + (0.075 × 172 / 365)].
= LCC 1 041 136.99 / 1.03534247
= LCC 1 005 596.72.

1.13.5 Yield rate and discount rate

So far we have worked with *yields*, which are the *actual rates of return* on securities. A variation is *ytm* in the case of bonds, which is an average yield / return.

We said above that in the money market we find *discount* securities. An example is the TB. A TB with a nominal / face value of LCC 1 000 000 matures at LCC 1 000 000 (= FV), but it is issued and traded at a *discount rate*. This TB can for example trade at LCC 950 000 (depending on the discount rate), calculated according to (dr = discount rate; d = days to maturity):

PV = FV × [1 – (dr × d / 365)].

If a LCC 1 000 000 (= FV) TB has 91 days to run and trades at 11.0% pa, its consideration is:

Consideration (PV) = LCC 1 000 000 × [1 – (0.11 × 91 / 365)]
= LCC 1 000 000 × (1 – 0.02742466)
= LCC 1 000 000 × 0.97257534
= LCC 972 575.34.

From the above it will be apparent that there is a fundamental difference between a *discount* rate and a *yield* rate and therefore between a *discount* amount and a *yield* amount. A yield amount is based on the PV, and the FV is the sum of the two. The discount amount, on the other hand, is based on the FV, and the PV is the difference between the two. It follows that the yield rate of interest is always expressed as a percentage of the PV, while the discount rate is expressed as a percentage of the FV.

It will be evident that the FV of the LCC 1 000 000 TB is also a MV. Thus, we can arrive at the same consideration as above if we convert the discount rate to a yield rate using the conversion formula:

$$
\begin{aligned}
ir &= dr / [1 - (dr \times d / 365)] \\
&= 0.11 / [1 - (0.11 \times 91 / 365)] \\
&= 0.11 / (1 - 0.02742466) \\
&= 0.11 / 0.97257534 \\
&= 0.1131 \\
&= 11.31\%.
\end{aligned}
$$

The proof (refer to the last consideration calculation above):

$$
\begin{aligned}
\text{Consideration} &= MV / [1 + (0.1131 \times 91 / 365)] \\
&= \text{LCC } 1\,000\,000 / 1.02819753 \\
&= \text{LCC } 972\,575.34.
\end{aligned}
$$

[4]

In conclusion, the yield interest rate to discount rate conversion formula:

$$dr = ir / [1 + (ir \times d / 365)].$$

1.13.6 Risk-free rate

Some textbooks are confusing on the subject of the risk-free rate (*rfr*). Some define it as the rate on 3-month TBs, while others say it does not exist. Our view is that there are many rfrs and they can be found on the government security (bonds and TBs) yield curve. The government security yield curve is a snapshot of all rates on government securities in issue (ytms on bonds and yields on TBs) at a specific point in time. A yield curve, also called *the term structure of interest rates*, presents the relationship between term to maturity and rates at a point in time. It is discussed in detail later.

Why is the rfr important? It is a rate that is used in many financial market calculations, especially in the derivative markets. It also represents the basis of the rate that an investor should accept on risky assets (i.e. non-government securities, such as shares, corporate bonds, etc.). Thus, the rate on a risky security, a.k.a. required rate of return (rrr), is equal to the rfr plus a risk premium (rp):

$$rrr = rfr + rp$$

The investor has to decide what the rp should be. There are many studies on the size of the rp, such as the capital asset pricing model (CAPM).

What does risk-free mean? Government securities are considered risk-free because they have the ability to raise revenue (tax and issue securities), and thus always service debt and honour maturities. As is well known, some countries' government securities are not risk-free, but such countries are few.

1.13.7 Bid and offer rates / prices and spread

There are two debt / deposit market types:

- Order driven markets.
- Quote driven markets.

In order-driven financial markets, such as share exchanges, sellers or buyers place *orders* to sell or buy shares with their brokers. In the age of share exchanges' Automated Trading Systems (ATS), the ATS's central order book arranges the sell and buy prices according to best price followed by the inferior prices. Deals are struck by the ATS when the buy and sell prices coincide.

In the debt and deposit markets, the main market type is *quote-driven* (there is an element of order placing), in the sense that the market (certainly in most bond markets) is "made". This means that market makers (usually the banks) quote both buy and sell rates / prices simultaneously. They are the *bid* (the market maker's buying rate; the selling rate from the perspective of the client) and *offer* rates (the market maker's selling rate; the buying rate from the viewpoint of the client). In some countries these rates are known as *bid* and *ask* rates.

The bid price is always lower than the offer price and the difference is called the *spread*. The spread is the compensation for the market maker for the risk taken in quoting firm bid and offer prices simultaneously. *Firm* means that the market maker is prepared to deal at the prices quoted in a given volume (which disclosed by the client). We will return to this issue later, as it is an important part of price discovery of rates.

1.13.8 Nominal and real interest rates

Any interest rate published in the media is a *nominal* interest rate (nir). An interest rate adjusted for inflation (π) is a *real* interest rate (rir), and it reflects the true cost of borrowing / the true earning rate. The first person to "split" the rate is Prof Irving Fisher, and the equation of the real interest rate, named for him (*Fisher equation*), is:

$$\text{rir} = \text{nir} - \pi.$$

The inflation rate used can be the current rate if it is low and has been level for some time, or the expected rate if this is not the case. We will return to this issue later.

1.14 References

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