Time Value of Money Module

OBJECTIVES

After reading this Module, you will be able to:

1. Understand simple interest and compound interest.
2. Compute and use the future value of a single sum.
3. Compute and use the present value of a single sum.
4. Compute and use the future value of an ordinary annuity.
5. Compute and use the future value of an annuity due.
6. Compute and use the present value of an ordinary annuity.
7. Compute and use the present value of an annuity due.
8. Compute and use the present value of a deferred ordinary annuity.
9. Explain the conceptual issues regarding the use of present value in financial reporting.

Suppose someone asked you, "Would you rather have $100 today or $100 next year?" Your answer should be, "I'd rather have $100 today." This reply involves considering the **time value of money**. The difference in worth between the two amounts, the time value of money, is interest. **Interest is the cost of the use of money over time.** It is an expense to the borrower and revenue to the lender. Therefore, it is a very important element in the decision making related to the acquisition and disposal of many of the resources of a company.

Interest concepts are involved in the development of many values that a company reports on its financial statements. Also, managers need to understand the concept of interest when making decisions where cash paid or received *now* must be compared with amounts that will be received or paid in the *future*.

The cash flows at various dates, say some at three years from now, some at two years from now, and some at one year from now, cannot be added together to produce a relevant value. Future cash flows, before they can be added, must be converted to a common denominator by being restated to their present values as of a specific moment in time (often referred to as *time period zero*). The dollars to be received or paid three years from now have a *smaller* present value than those to be received or paid two years or one year from now. **The conversion of these future value amounts to the present value common denominator is known as discounting** and involves the removal of the interest or discount—the time value of money—from those dollars that would be received or paid three years, two years, or one year from now.

Instead of restating some of the cash inflows and outflows to their present values at time period zero, a common denominator is also achieved by stating them at a future value by adding the time value of money (interest) to these inflows and outflows. The future value of any series of inflows or outflows is the sum of these periodic amounts plus the compound interest calculated on the amounts.

A company uses the present value or the future value in many situations, such as (1) for measurement and reporting of some of its assets and liabilities, since many accounting pronouncements require the use of present value concepts in a number of measurement and reporting issues; and (2) when it accumulates information for decision making involving, for example, property, plant, and equipment acquisitions. We discuss these concepts in this Module and we apply them in various chapters when we discuss how a company records and reports (1) long-term notes payable and notes receivable when the interest rate is not specified or differs from the market rate at the time of the transaction, (2) assets acquired by the issuance of long-term debt securities that carry either no stated rate of interest, or a rate of interest that is different from the market rate at the time of the transaction, (3) bonds payable and investments in bonds and the amortization of bond premiums and discounts in each case, (4) long-term leases, (5) various aspects of employees' post-employment benefits, and (6) impairment of noncurrent assets.

Various compound interest techniques are used in the measurement of the values (costs) of these and other types of transactions. Most compound interest applications can be calculated by a longhand arithmetic process. However, quicker approaches and shortcuts to the solutions of the problems are available. In this Module we illustrate the basic principles of compound interest in a way that leads to the development of tables used to resolve issues introduced throughout this book. Note that many of the calculations are rounded.

Simple Interest and Compound Interest

Simple interest is interest on the original principal (amount originally received or paid) regardless of the number of time periods that have passed or the amount of interest that has been paid or accrued in the past. Interest rates are usually stated as an *annual* rate, which is adjusted for any other time period. Thus simple interest is calculated by the following equation:

Interest = Principal × Rate × Time

1 Understand simple interest and compound interest.

where time is either a fraction of a year or a multiple of years. If the term of a note is stated in days, say 90 days, the denominator of the time fraction in the preceding equation is usually stated in terms of a commercial year of 360 days rather than a full year of 365 days. In this practice the year is assumed to be a period of 12 months of 30 days each. For example, the simple interest on a $10,000, 90-day, 12% note given to a company by Allen Sanders is $300 ($10,000 × 0.12 × 90/360). However, if the term of this note is 15 months, the simple interest is $1,500 ($10,000 × 0.12 × 15/12). Observe that simple interest for more than one year is still calculated on only the principal amount (in this case $10,000).

Compound interest is the interest that accrues on both the principal and the past unpaid accrued interest. Simple interest of 12% for 15 months on the Allen Sanders note is $1,500. If, on the other hand, the 12% interest is *compounded quarterly* for 15 months (5 quarters), the total compound interest is $1,592.74, as we show in Example M-1. Note that in the compound interest computation, the future accumulated amount (value) at the end of each quarter becomes the principal sum used to compute the interest for the following period.

EXAMPLE M-1 Computation of Quarterly Compounded Interest

Period	Value at Beginning of Quarter*	×	Rate	×	Time	=	Compound Interest	Value at End of Quarter
1st quarter	$10,000.00	×	0.12	×	1/4		$ 300.00	$10,300.00
2nd quarter	10,300.00	×	0.12	×	1/4		309.00	10,609.00
3rd quarter	10,609.00	×	0.12	×	1/4		318.27	10,927.27
4th quarter	10,927.27	×	0.12	×	1/4		327.82	11,255.09
5th quarter	11,255.09	×	0.12	×	1/4		337.65	11,592.74
Compound interest on $10,000 at 12% compounded quarterly for 5 quarters							$1,592.74	

* This value is the amount on which interest is calculated.

To help solve the many business issues stated in the introductory section of this Module, accountants need to know the various types of compound interest computations. Although there are many variations, there are only four basic types:

1. **Future value (amount) of a single sum** at compound interest
2. **Present value of a single sum** due in the future
3. **Future value (amount) of an annuity,** a series of receipts or payments
4. **Present value of an annuity,** a series of receipts or payments

FUTURE VALUE OF A SINGLE SUM AT COMPOUND INTEREST

As we stated previously, the main objective of this Module is to explain shortcut methods to determine and apply the compound interest techniques. We will use the following step-by-step procedure, introducing the entire topic *only* with the future value of a single sum at compound interest:

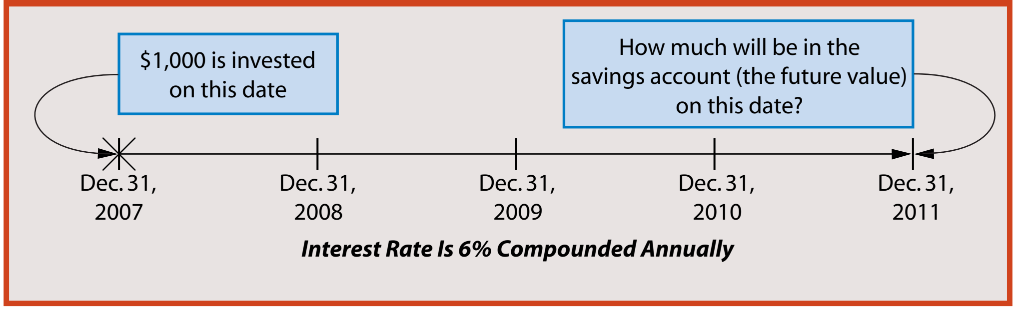
1. We diagram the idea or concept.
2. We make the computation using a longhand calculation.
3. We make the computation using formulas.
4. We discuss the method of constructing and using tables.
5. We illustrate the use of the tables to solve a compound interest problem.

2 Compute and use the future value of a single sum.

The Idea

The future value of a single sum at compound interest is the original sum plus the compound interest, stated as of a specific future date. It is also often referred to as the **future amount** of a single sum. For example, suppose you invest a single amount of $1,000 in a savings account on December 31, 2007. What will be the amount in the savings account on December 31, 2011 if interest at 6% is compounded annually each year? We show the issue graphically in Example M-2. Most compound interest calculations can be made by applying longhand arithmetic. We follow this procedure here only to clarify the various shortcut devices used.

EXAMPLE M-2 **Diagram of Future Value of a Single Sum**



The future value of $1,000 for four years at 6% a year can be calculated as we show in Example M-3. The single sum of $1,000 invested on December 31, 2007 has grown to $1,262.48 by December 31, 2011. This is the **future value.** The total interest of $262.48 for the four years is referred to as **compound interest.**

EXAMPLE M-3 **Calculation of Future Value of Single Sum at Compound Interest**

(1) Year	(2) Value at Beginning of Year	(3) Annual Compound Interest (Col. 2 × 0.06)	(4) Future Value at End of Year (Col. 2 + Col. 3)
2008	$1,000.00	$60.00	$1,060.00
2009	1,060.00	63.60	1,123.60
2010	1,123.60	67.42	1,191.02
2011	1,191.02	71.46	1,262.48

A slight variation of the longhand arithmetic approach is to determine what $1 invested on December 31, 2007 will amount to by December 31, 2011 if interest at 6% is compounded annually. Then this amount is multiplied by the principal sum to find the future value. In this case, $1 amounts to $1.26248 in four years. Knowing this fact, the value of 1,000 different $1 investments (or $1,000) at the end of four years can be calculated by multiplying the $1,000 by 1.26248 as follows: $1,000 × 1.26248 = $1,262.48. To avoid a significant rounding error in the final results, when solving this problem, the intermediate figures should *not* be rounded to the nearest cent.

Formula Approach

Each amount in column 4 of Example M-3 is 1.06 times the corresponding amount in column 2. The final future value is therefore $1,000 × 1.06 × 1.06 × 1.06 × 1.06 = $1,262.48. This means that 1.06 has been used as a multiplier four times; that is, 1.06 has been raised to the fourth power. The future value is therefore $1,000 multiplied by 1.06 to the fourth power:

Future Value = $\$1{,}000(1.06)^4 = \$1{,}262.48$

Thus the formula to compute the future value of a single sum at compound interest is:

$$f = p(1 + i)^n$$

where f = future value of a single sum at compound interest i for n periods
p = principal sum (present value)
i = interest rate for each of the stated time periods
n = number of time periods

It is important to understand that the interest rate i is the rate of interest applicable for the particular time period for which interest is compounded. For example, a stated annual rate of interest of 12% is

- 12% per year if interest is compounded annually
- 6% per one-half year if interest is compounded semiannually
- 3% per quarter if interest is compounded quarterly
- 1% per month if interest is compounded monthly

In general, **an interest rate per period (i) is the annual stated rate (sometimes called the nominal rate) divided by the number of compounding time periods in the year, and n is the number of time periods in the year multiplied by the number of years.**

The formula for the future value of 1 is:

$$f_{n,i} = (1 + i)^n$$

where $f_{n,i}$ is the future compound value of 1 ($1 or 1 of any other monetary unit) at interest rate i for n periods.

Using the preceding formula for the future value of 1, a short formula for the future compound value of any single amount at compound interest is:

$$f = p(f_{n,i})$$

The example of the future value of $1,000 invested at 6% with interest compounded annually can now be calculated in two steps:

Step 1 $f_{n = 4,\, i = 6\%} = (1.06)^4 = 1.2624796$

Step 2 $f = \$1{,}000(1.2624796) = \$1{,}262.48$

Recall that this is exactly the same as the *second* approach, which we used in the previous arithmetic method.

Table Approach

To develop additional shortcuts to the solution of the compound interest issue, tables for the future value of 1 have been constructed. These tables simply include calculations of the future values of 1 at different interest rates and for different time periods. They can be constructed by using the preceding formula with the desired interest rates and time periods. For example, suppose that you need tables of the future value of 1 at 2% and 14% for time periods 1 through 4 and for 40 years. The information for these can be calculated as follows:

$$f_{n=1,\, i=2\%} = (1.02)^1 = 1.020000 \qquad f_{n=1,\, i=14\%} = (1.14)^1 = 1.140000$$
$$f_{n=2,\, i=2\%} = (1.02)^2 = 1.040400 \qquad f_{n=2,\, i=14\%} = (1.14)^2 = 1.299600$$
$$f_{n=3,\, i=2\%} = (1.02)^3 = 1.061208 \qquad f_{n=3,\, i=14\%} = (1.14)^3 = 1.481544$$
$$f_{n=4,\, i=2\%} = (1.02)^4 = 1.082432 \qquad f_{n=4,\, i=14\%} = (1.14)^4 = 1.688960$$
$$f_{n=40,\, i=2\%} = (1.02)^{40} = 2.208040 \qquad f_{n=40,\, i=14\%} = (1.14)^{40} = 188.883514$$

This information can then be accumulated in a partial table as we show in Example M-4. In this kind of table the factors are shown without the use of the dollar sign. Each factor is an amount for a certain time period and rate. We provide more complete tables at the end of this Module.

EXAMPLE M-4 Future Value of 1 Table $(1 + i)^n$

Periods	2%	14%
1	1.020000	1.140000
2	1.040400	1.299600
3	1.061208	1.481544
4	1.082432	1.688960
.	.	.
.	.	.
.	.	.
40	2.208040	188.883514

Since the factors in Example M-4 and in Table 1 at the end of this Module are based on the formula $(1 + i)^n$, the table approach can be expressed as:

$$f = p(\text{Factor for } f_{n,i})$$

To calculate the future value that $1,000 will accumulate to in four years at 6% compounded annually, it is necessary to look up the table factor for $f_{n=4,\, i=6\%}$, namely, 1.262477; then, to arrive at the answer of $1,262.48, the calculation is: $f = \$1{,}000(1.262477) = \$1{,}262.48$.

Summary and Illustration

In addition to the straightforward situation of calculating the future value of a single sum at compound interest, you can solve other kinds of problems with the *future value of 1* table.

Example: Finding an Unstated Interest Rate

If $1,000 is invested on December 31, 2007 to earn compound interest and if the future value on December 31, 2014 is $2,998.70, what is the quarterly interest rate on the investment?

We show the facts in Example M-5. Using the table approach

$$f = p(\text{Factor for } f_{n,i})$$

and substituting in the formula the amounts shown in Example M-5, the factor is determined as follows:

$$\$2{,}998.70 = \$1{,}000(\text{Factor for } f_{n=28,\, i=?})$$

$$\text{Factor for } f_{n=28,\, i=?} = \frac{\$2{,}998.70}{\$1{,}000.00} = 2.99870$$

EXAMPLE M-5 **Diagram of Future Value of a Single Sum—Interest Rate to Be Determined**

The factor of 2.99870 is the future value of 1 for 28 time periods at an unknown interest rate. Using the future value of 1 table (Table 1) at the end of this Module, you look down the periods (n) column until you get to 28. Then you move horizontally on the $n = 28$ line to the column factor closest to 2.99870. If the value appears in the table, you can determine the interest rate (shown at the top of the column) that produces this value. In this case, 2.99870 is equal to 2.998703 (rounded) located in the 4% column; thus the quarterly interest rate is 4%. This is often referred to as being a stated annual rate of 16%; you should understand, however, that a quarterly rate of 4% compounded four times yields an effective rate of more than 16%. If the factor of 2.99870 does not appear in the table, an interpolation procedure is required to approximate the quarterly interest rate.[1] Calculators and computer software that compute the interest rate are widely available. ♦

You can solve other problems by using the future amount of 1 tables. Keep in mind, however, that most tables are incomplete. At times it will be necessary to construct tables for odd interest rates and time periods, or to use a calculator or computer software.

Present Value of a Single Sum

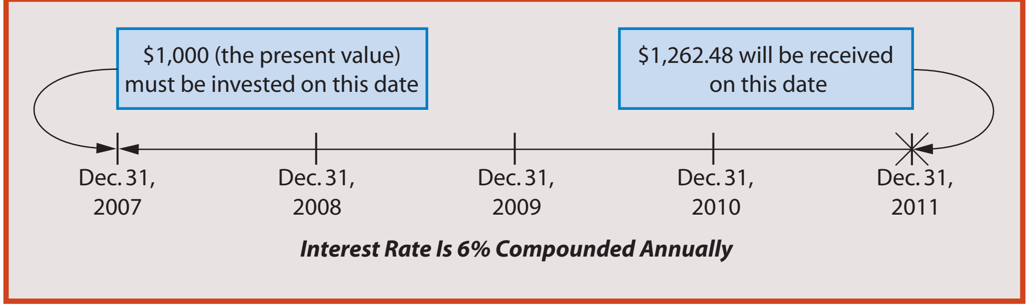
For the remaining compound interest techniques, we focus on the shortcut approach. After we discuss the idea, we state the formula and use factors derived from the formula.

The Idea

The present value is the principal that must be invested at time period zero to produce the known future value. Also, **discounting is the process of converting the future value to the present value.** For example, if $1,000 is worth $1,262.48 when it earns 6% compound interest per year for four years, then it follows that $1,262.48 to be received four years from now is worth $1,000 now at time period zero; that is, $1,000 is the present value of $1,262.48 discounted at 6% for four years. Example M-6 presents this information graphically.

1. You can use the following six steps to determine an interest rate by linear interpolation: (1) Calculate the compound interest factor as shown in the preceding example. (2) Look up in compound interest tables the two interest rates that yield the next largest and the next smallest factors from the calculated factor determined in step 1. (3) Determine (a) the difference between the two factors in step 2, and (b) the difference between the calculated factor from step 1 and the factor of the smaller interest rate from step 2. (4) Find the difference between the two interest rates found in step 2. (5) Apportion the difference in the interest rates in step 4 by multiplying it by a fraction: The numerator is the difference determined in step 3b and the denominator is the difference determined in step 3a. (6) The interest rate is then the lower rate found in step 2 *plus* the apportioned difference from step 5.

EXAMPLE M-6 Diagram of Present Value of a Single Sum



Shortcut Approaches

3 Compute and use the present value of a single sum.

While it is possible to calculate the present value of $1,262.48 to be received at the end of four years discounted at 6% by a longhand approach by reversing the process described in the calculation of the future value, we do not show this approach here. Instead we focus on the development of shortcut approaches to find the present value of a single sum. First we present the formula, then we explain how to create and use factors.

Formula Approach

Since the present value of a single future amount is the reciprocal value of the future value of a single sum, the formula for this calculation is:

$$p = f\,\frac{1}{(1+i)^n}$$

where p = present value of any given future value due in the future
f = future value
i = interest rate for each of the stated time periods
n = number of time periods

In this example the present value of $1,262.48 received at the end of 4 years discounted at 6% is $1,000, calculated as follows:

$$p = \$1{,}262.48\,\frac{1}{(1.06)^4} = \$1{,}000$$

The formula for the present value of 1 is:

$$p_{n,i} = \frac{1}{(1+i)^n}$$

where $p_{n,i}$ is the present value of 1 ($1 or 1 of any monetary unit) at interest rate i for n periods. It is now possible to express the formula for the present value of any given future amount as:

$$p = f(p_{n,i})$$

The example of the present value of $1,262.48 to be received four years from now with interest of 6% compounded annually can be calculated in two steps:

Step 1 $p_{n=4,\,i=6\%} = \dfrac{1}{(1.06)^4} = 0.792094$

Step 2 $p = \$1{,}262.48(0.792094) = \$1{,}000$

Table Approach

Using the formula for $p_{n,i}$, tables have been constructed for any interest rate and for any number of periods by simply substituting in the formula the selected various interest rates for the various time periods desired. Table 3 at the end of this Module shows the factors for the present value of 1 ($p_{n,i}$).

Since the factors in Table 3 are based on the formula $p_{n,i} = 1/(1 + i)^n$, the generalized table approach can be stated as:

$$p = f(\text{Factor for } p_{n,i})$$

To calculate the present value of \$1,262.48 to be received at the end of four years, discounted at 6%, look up the factor for $p_{n = 4,\ i = 6\%}$ in Table 3; it is 0.792094. Then the future value of \$1,262.48 is multiplied by this present value of 1 factor to obtain the present value amount of \$1,000, as follows: $p = \$1{,}262.48(0.792094) = \$1{,}000$.

Summary and Illustration

In addition to calculating the present value of a single sum using compound interest, you can solve other kinds of problems with the present value of 1 table.

Example: Finding an Unstated Interest Rate

Assuming that the present value of \$10,000 to be paid at the end of 10 years is \$3,855.43, what interest rate compounded annually is used in the calculation of the present value?

Example M-7 shows the known facts. Since both the present value and the future amount are known, this problem can be solved in two different ways: (1) by using the method we described in the future value section, or (2) by using the present value approach we describe here. Since we discussed the future value approach earlier in this Module, we use only the present value approach here to solve the problem. Using the table approach

$$p = f(\text{Factor for } p_{n,i})$$

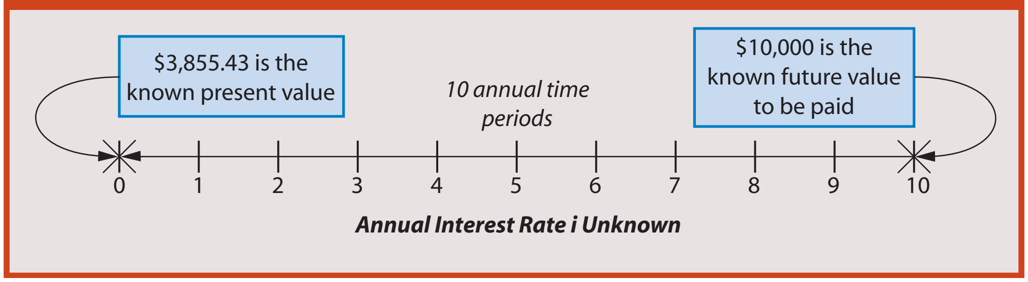
and substituting in the formula the known amounts shown in Example M-7, the factor is determined as follows:

$$\$3{,}855.43 = \$10{,}000.00\ (\text{Factor for } p_{n = 10,\ i = ?})$$

$$\text{Factor for } p_{n = 10,\ i = ?} = \frac{\$3{,}855.43}{\$10{,}000.00} = 0.385543$$

The factor of 0.385543 is the present value of 1 for 10 periods at an unknown interest rate. Using the present value of 1 table (Table 3), you look down the periods (n) column until you get to 10. Then you move horizontally on the $n = 10$ line to the column factor closest to 0.385543. If the amount appears in the table, you can determine the interest rate (shown at the top of the column) that produces this amount. In this case, 0.385543 is in the 10% column. Thus the annual rate is 10%. If the factor of 0.385543 does not appear in the table, an interpolation procedure is required to approximate the annual interest rate (see footnote 1). ♦

EXAMPLE M-7 Diagram of the Present Value of a Single Sum—Interest Rate to Be Determined



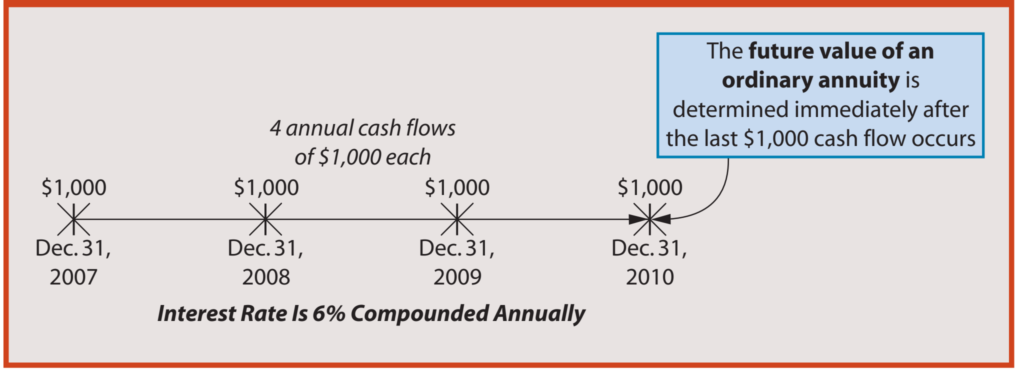
Measurements Involving an Annuity

An annuity is a series of equal cash flows (deposits, receipts, payments, or withdrawals), sometimes referred to as *rents*, made at regular intervals with interest compounded at a certain rate. The regular intervals between the cash flows may be any time period—for example, one year, a six-month period, one month, or even one day. In solving measurement problems involving the use of annuities, these four conditions must exist: (1) the periodic cash flows are equal in amount, (2) the time periods between the cash flows are the same length, (3) the interest rate is constant for each time period, and (4) the interest is compounded at the end of each time period.

Future Value of an Ordinary Annuity

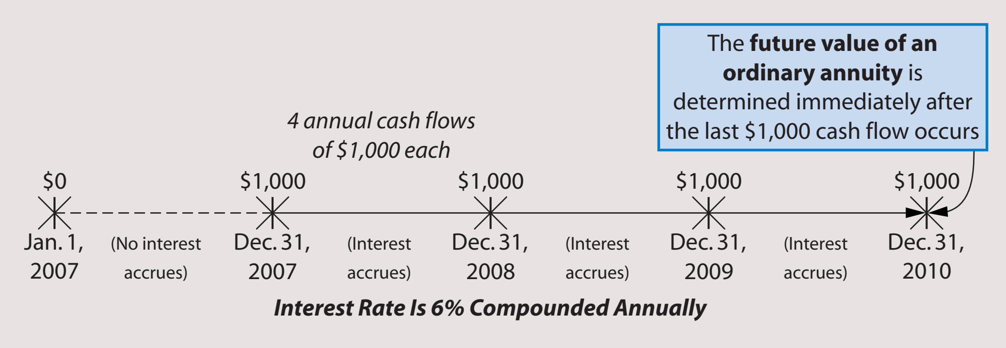
The future value of an ordinary annuity is determined *immediately* after the last cash flow in the series is made. For the first example, assume that Debbi Whitten wants to calculate the future value of four cash flows of $1,000, each with interest compounded annually at 6%, where the first $1,000 cash flow occurs on December 31, 2007 and the last $1,000 occurs on December 31, 2010. Example M-8 presents this information graphically.

EXAMPLE M-8 Diagram of Future Value of Ordinary Annuity



In drawing a **time line** such as that in Example M-8, some accountants prefer to add a beginning time segment to the left of the time when the first cash flow occurs. For example, they would draw the time line for the future amount of an ordinary annuity, as we show in Example M-9. This approach is acceptable if it is understood that the time from January 1, 2007 to December 31, 2007 (which is the period of time immediately *before* the first cash flow occurs) is not used to compute the future value of the ordinary annuity. It is similar to stating a decimal as .4 or 0.4. The zero in front of the decimal may help someone to understand the issue better, but does not change it. In the case of the future value of an ordinary annuity, however, placing the broken line segment to the left of the first cash flow may lead someone to think that the cash flows in an ordinary annuity *must occur* at the end of a given year. That statement is *not* true; the cash flows can occur, for example, on March 15 of each year, or November 5 of each year. For the calculation to be the future value of an *ordinary* annuity, the *future value* is determined *immediately after* the last cash flow in the series occurs. Because of the potential misinterpretation of the information, we prefer not to use the broken line segment to the left of the first cash flow in the time lines describing the future value of an ordinary annuity.

EXAMPLE M-9 Alternative Diagram of Future Value of an Ordinary Annuity



Shortcut Approaches

Formula Approach

4 Compute and use the future value of an ordinary annuity.

The formula for the future value of an ordinary annuity of any amount is:

$$F_0 = C\left[\frac{(1+i)^n - 1}{i}\right]$$

where F_0 = future value of an ordinary annuity of a series of cash flows of any amount
C = amount of each cash flow
n = number of cash flows (not the number of time periods)
i = interest rate for each of the stated time periods

In the example, the future value of an ordinary annuity of four cash flows of $1,000 each at 14% compounded annually is as follows:

$$F_0 = \$1{,}000\left[\frac{(1.06)^4 - 1}{0.06}\right] = \$4{,}374.62$$

The formula for the future value of an ordinary annuity with cash flows of 1 each is as follows:

$$F_{0_{n,i}} = \left[\frac{(1+i)^n - 1}{i}\right]$$

where $F_{0_{n,i}}$ is the future value of an *ordinary* annuity of n cash flows of 1 each at interest rate i.

With the preceding formula for $F_{0_{n,i}}$ it is possible to express another formula for the future value of an ordinary annuity of cash flows of any size in this manner:

$$F_0 = C(F_{0_{n,i}})$$

In a two-step approach, the future value of an ordinary annuity of four cash flows of $1,000 each at 14% compounded annually is calculated as follows:

Step 1 $F_{0_{n=4,\, i=6\%}} = \frac{(1.06)^4 - 1}{0.06} = 4.37462$

Step 2 $F_0 = \$1{,}000(4.37462) = \$4{,}374.62$

This two-step approach is used to solve the problem when factors are not available.

Table Approach

The formula for $F_{0_{n,i}}$ can be used to construct a table of the future value of any series of cash flows of 1 each for any interest rate. Here the number of cash flows of 1 and the interest rates are substituted into the formula

$$\frac{(1 + i)^n - 1}{i}$$

Table 2 at the end of this Module shows the factors for $F_{0_{n,i}}$. Turning to Table 2, observe the following:

1. The numbers in the first column (n) represent the number of cash flows.
2. The future values are always equal to or larger than the number of cash flows of 1. For example, the future value of four cash flows of 1 each at 6% is 4.374616. This figure comprises two elements: (a) the number of cash flows of 1 each *without* any interest, and (b) the compound interest on the cash flows, with the exception of the compound interest on the last cash flow in the series, which in the case of an ordinary annuity *does not* earn any interest.

Since Table 2 shows the calculation of $F_{0_{n,i}}$ or

$$\frac{(1 + i)^n - 1}{i}$$

values, the generalized table approach is as follows:

$$F_0 = C(\text{Factor for } F_{0_{n,i}})$$

To calculate the future value of an ordinary annuity of 4 cash flows of $1,000 each at 6%, you must look up the $F_{0_{n=4,\ i=6\%}}$ factor in the future value of an ordinary annuity of 1 table (Table 2); it is 4.374616. Then the amount of each cash flow, here $1,000, is multiplied by the Table 2 factor to obtain the future value of $4,374.62:

$$F_0 = \$1{,}000(4.374616) = \$4{,}374.62$$

Summary and Illustration

You can solve several kinds of problems using a future value of an ordinary annuity of 1 table, such as (1) calculating the future value when the cash flows and interest rate are known (the preceding problem); (2) calculating the value of each cash flow where the number of cash flows, interest rate, and future value are known; (3) calculating the number of cash flows when the amount of each cash flow, the interest rate, and the future value are known; and (4) calculating an unknown interest rate when the cash flows and the future value are known. To demonstrate the analysis used in the solution of all these problems, we show item (2) as follows.

Example: Determining the Amount of Each Cash Flow Needed to Accumulate a Fund to Retire Debt

At the beginning of 2007 the Rexson Company issued 10-year bonds with a face value of $1,000,000 due on December 31, 2016. The company will accumulate a fund to retire these bonds at maturity. It will make annual deposits to the fund beginning on December 31, 2007. How much must the company deposit each year, assuming that the fund will earn 12% interest compounded annually?

Example M-10 shows the facts of the problem. The future value and the compound interest rate are known. The amount of each of the 10 deposits (cash flows) is the unknown factor. Starting with the formula

$$F_0 = C(\text{Factor for } F_0)$$

and then shifting the elements and substituting the known amount and applicable factor (from Table 2), the amount of each annual deposit is \$56,984.16, calculated as follows:

$$C = \frac{F_0}{\text{Factor for } F_{0_{n,i}}}$$

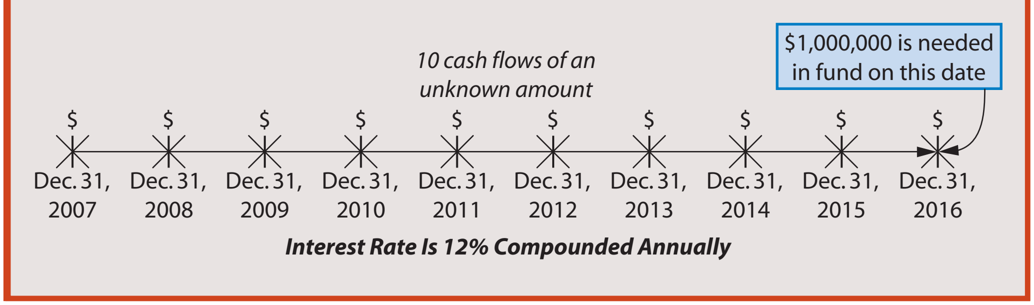
$$= \frac{F_0}{\text{Factor for } F_{0_{n=10,\ i=12\%}}}$$

$$= \frac{\$1,000,000}{17.548735}$$

$$= \$56,984.16$$

The 10 annual deposits of \$56,984.16, plus the compound interest, will accumulate to \$1,000,000 by December 31, 2016. ♦

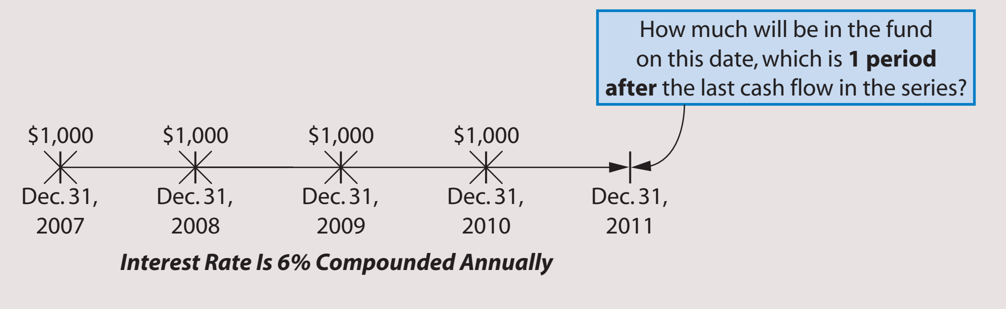
EXAMPLE M-10 **Future Value of an Ordinary Annuity—Amount of Cash Flows to Be Determined**



FUTURE VALUE OF AN ANNUITY DUE

The future value of an annuity due (F_d) is determined 1 period after the last cash flow in the series. For example, assume that Ronald Jacobson deposits in a fund four payments of \$1,000 each beginning December 31, 2007, with the last deposit being made on December 31, 2010. How much will be in the fund on December 31, 2010, 1 year after the final payment, if the fund earns interest at 6% compounded annually? Example M-11 shows the facts of this problem.

EXAMPLE M-11 **Diagram of Future Value of Annuity Due**



Solution Approach

5 Compute and use the future value of an annuity due.

By observing the information contained in Examples M-11 and M-8, you can determine a quick way to compute the future value of an annuity due.[2] When only the future value of an *ordinary* annuity table is available, you can use the factors by completing the following steps:

Step 1	In the *ordinary* annuity table (Table 2), look up the value of $n + 1$ cash flows at 6% or the value of 5 cash flows at 6%.	5.637093
Step 2	Subtract 1 without interest from the value obtained in step 1.	(1.000000)
	This is the converted future value factor for $F_{d_{n=4,\ i=6\%}}$	4.637093
Step 3	Multiply the amount of each cash flow, here \$1,000, by the converted factor for $F_{d_{n=4,\ i=6\%}}$ determined in step 2: F_d = \$1,000(4.637093) = \$4,637.09	

Tables of the future value of an annuity due of cash flows of 1 each are available in some finance books, but not in this book. Therefore, these values must be calculated using the tables for the future value of an *ordinary* annuity. As we showed previously, **the general rule is to use the future value of an ordinary annuity factor for $n + 1$ cash flows and subtract 1 from the factor.** (Note that we do include in this Module a present value of an annuity due table, as we discuss later.)

Present Value of an Annuity

The present value of an annuity is the present value of a series of equal cash flows that occur in the future. In other words, it is the amount that must be invested now and, if left to earn compound interest, will provide for a receipt or payment of a series of equal cash flows at regular intervals. Over time, the present value balance is *increased* periodically for interest and is *decreased* periodically for each receipt or payment. Thus, the last cash flow in the series exhausts the balance on deposit.

A company frequently uses the present value of an annuity concept to report many items in its financial statements, as we stated in the introduction to this Module. Because of the importance of the present value of an annuity, we will discuss the (1) present value of an ordinary annuity, (2) present value of an annuity due, and (3) present value of a deferred annuity.

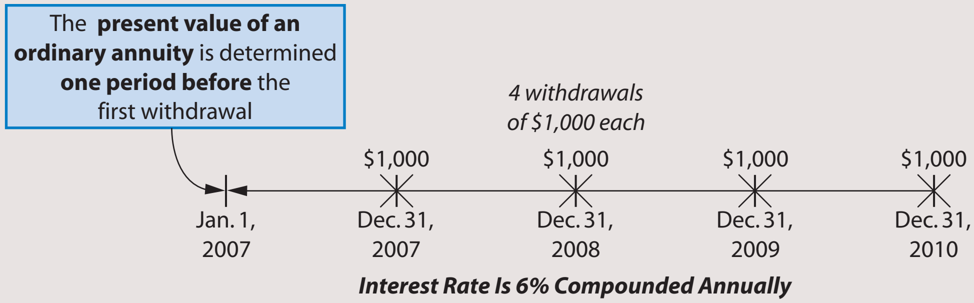
Present Value of an Ordinary Annuity

6 Compute and use the present value of an ordinary annuity.

The present value of an ordinary annuity is determined 1 period before the first cash flow in the series is made. For example, assume that Kyle Vasby wants to calculate the present value on January 1, 2007 of four future withdrawals (cash flows) of \$1,000, with the first withdrawal being made on December 31, 2007, 1 year after the determination of the present value. The applicable interest rate is 6% compounded annually. Example M-12 shows this information graphically.

2. An alternative approach is to multiply the future value of an ordinary annuity factor by 1 plus the interest rate. Thus, the future value in this example would be computed as \$1,000 × (4.374616 × 1.06) = \$4,637.09.

EXAMPLE M-12 **Diagram of Present Value of an Ordinary Annuity**



Solving by Determining the Present Value of a Series of Single Sums

The solution to this problem can be determined by using the present value of a single sum. For instance, the answer can be calculated in the following two steps: (1) determine the present value of four individual cash flows of 1 each for one, two, three, and four years, as we show in Example M-13; and (2) multiply the final results of the summation by $1,000.

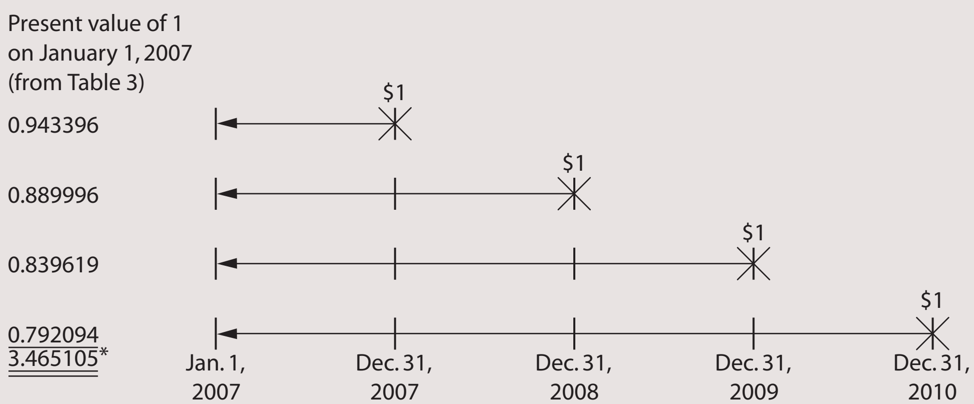
Step 1 The present value of four cash flows of 1 for one, two, three, and four years discounted at 6% is determined in Example M-13.

Step 2 Now it is possible to determine the present value of the four cash flows of $1,000 each by multiplying the $1,000 by 3.465105:

$\$1{,}000 \times 3.465105 = \$3{,}465.11$

The present value on January 1, 2007 is $3,465.11; or we can say that $3,465.11 must be invested on January 1, 2007 to provide for four withdrawals of $1,000 each starting on December 31, 2007, given an interest rate of 6%.

EXAMPLE M-13 **Present Value of Four Cash Flows of 1 for One, Two, Three, and Four Years at 6%**



*The value of 3.465105 is slightly smaller than the factor for $P_{0_{n=4,\,i=6\%}}$ of 3.465106 in Table 4 discussed later in this section; this is the result of rounding each of the four factors for $P_{n,\,i}$.

Shortcut Approaches

Formula Approach

Even though the preceding approach can be used, it is time-consuming for calculations involving a large number of cash flows. The formula for the present value of an ordinary annuity of any amount is:

$$P_0 = C\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$$

where P_0 = present value of an ordinary annuity of a series of cash flows of any amount
C = amount of each cash flow
n = number of cash flows (not the number of time periods)
i = interest rate for each of the stated time periods

In the example, the present value of an ordinary annuity of four cash flows of $1,000 each at 6% compounded annually can be calculated as follows:

$$P_0 = \$1{,}000\left[\frac{1-\frac{1}{(1.06)^4}}{0.06}\right] = \$3{,}465.11$$

Based on these calculations and formula observe that:

1. The results are the same as those produced in the first approach, $3,465.11.
2. The formula is developed from the formulas for both the future value of 1(f) and the present value of 1(p):

   $$(1 + i)^n = f$$

   $$\frac{1}{(1 + i)^n} = p$$

3. Thus the formula can be restated as follows:

   $$P_0 = C\left(\frac{1-p}{i}\right)$$

The formula for the present value of an ordinary annuity can be converted to that for a series of cash flows of 1 each as follows:

$$P_{0_{n,i}} = \left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$$

where $P_{0_{n,i}}$ is the present value of an ordinary annuity of n cash flows of 1 each at interest rate i. This formula can be expressed for the present value of an ordinary annuity of cash flows of *any size* as:

$$P_0 = C(P_{0_{n,i}})$$

In a two-step approach the present value of four future withdrawals (cash flows) of $1,000 each discounted at 6% is recalculated as follows:

Step 1 $$P_{0_{n=4,i=6\%}} = \left[\frac{1-\frac{1}{(1.06)^4}}{0.06}\right] = 3.46511$$

Step 2 $P_0 = \$1{,}000(3.46511) = \$3{,}465.11$

This calculation is exactly the same as that of the first formula, except that the process is divided into two steps. The two-step approach is the one used when tables of the present value of an ordinary annuity of 1 are available.

Table Approach

The formula for $P_{0_{n,i}}$ can be used to construct a table of the present value of any series of cash flows of 1 each for any interest rate. All that is necessary is to substitute in the formula the desired number of cash flows for the various required interest rates. Table 4 at the end of the Module shows the factors for $P_{0_{n,i}}$. Turning to Table 4, observe the following:

1. The numbers in the first column (n) represent the number of cash flows of 1 each. In this calculation the number of cash flows and time periods are equal.
2. The present value amounts are always smaller than the number of cash flows of 1. For example, the present value of three cash flows of 1 at 2% is 2.883883.

Since Table 4 shows the precalculation of $P_{0_{n,i}}$ or

$$\frac{1-\frac{1}{(1+i)^n}}{i}$$

the generalized table approach is as follows:

$$P_0 = C(\text{Factor for } P_{0_{n,i}})$$

Thus, to calculate the present value on January 1, 2007 of four future withdrawals (cash flows) of $1,000 discounted at 6%, with the first cash flow being withdrawn on December 31, 2007, it is necessary to look up the $P_{0_{n=4,\ i=6\%}}$ value in the present value of an ordinary annuity of 1 table (Table 4); it is 3.465106. This factor is then multiplied by $1,000 to determine the present value figure of $3,465.11:

$$P_0 = \$1{,}000(3.465106) = \$3{,}465.11$$

Over the 4 periods, the annuity yields interest each period as follows:

Period	Beginning Balance	Interest	Cash Flow	Ending Balance
1	$3,465.11	$207.91	$(1,000)	$2,673.02
2	2,673.02	160.38	(1,000)	1,833.40
3	1,833.40	110.00	(1,000)	943.40
4	943.40	56.60	(1,000)	0

Summary and Illustration

You can solve several kinds of problems by using the present value of an ordinary annuity of 1 table. We present one additional example: a problem involving the calculation of the periodic cash flows when the present value and interest rate are known.

Example: Determining the Value of Periodic Cash Flows When the Present Value Is Known

Suppose that on January 1, 2007 Rex Company borrows $100,000 to finance a plant expansion project. It plans to pay this amount back with interest at 12% in equal annual payments over a 10-year period, with the first payment due on December 31, 2007. What is the amount of each payment?

Example M-14 shows the facts of the problem. The present value and the compound interest rate are known. The amount of each of the 10 cash flows is the unknown item and is $17,698.42, calculated as follows:

$$C = \frac{P_0}{\text{Factor for } P_{0_{n,i}}}$$

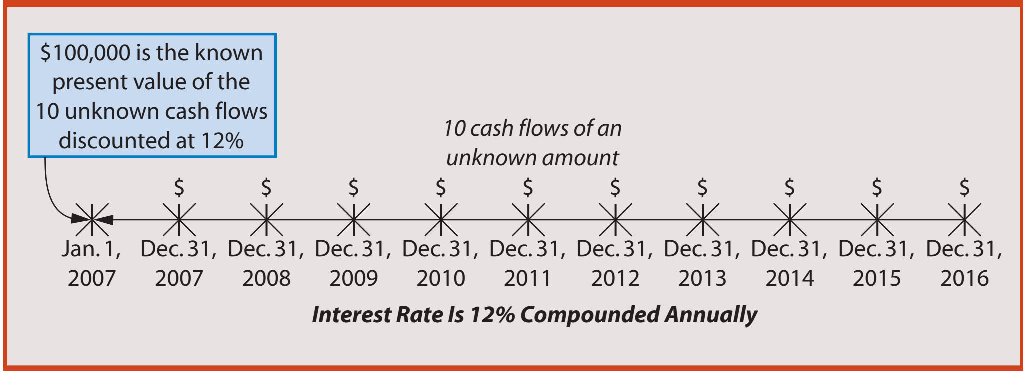
$$= \frac{P_0}{\text{Factor for } P_{0_{n=10,\ i=12\%}}}$$

$$= \frac{\$100{,}000}{5.605223}$$

$$= \$17{,}698.42$$

Remember that each of these payments of $17,698.42 includes (1) a payment of annual interest, and (2) a retirement of debt principal. For example, the interest for 2007 is $12,000 (12% × $100,000). Thus the amount of the payment on principal is $5,698.42 ($17,698.42 − $12,000). For the year 2008 the interest is $11,316.19 [12% × ($100,000 − $5,698.42)], and the retirement of principal is $6,382.23 ($17,698.42 − $11,316.19). The *last* payment of $17,698.42 on December 31, 2016, will be sufficient to retire the remaining principal and to pay the interest for the tenth year. ♦

EXAMPLE M-14 **Diagram of the Present Value of an Ordinary Annuity—Amount of Each Cash Flow to Be Determined**

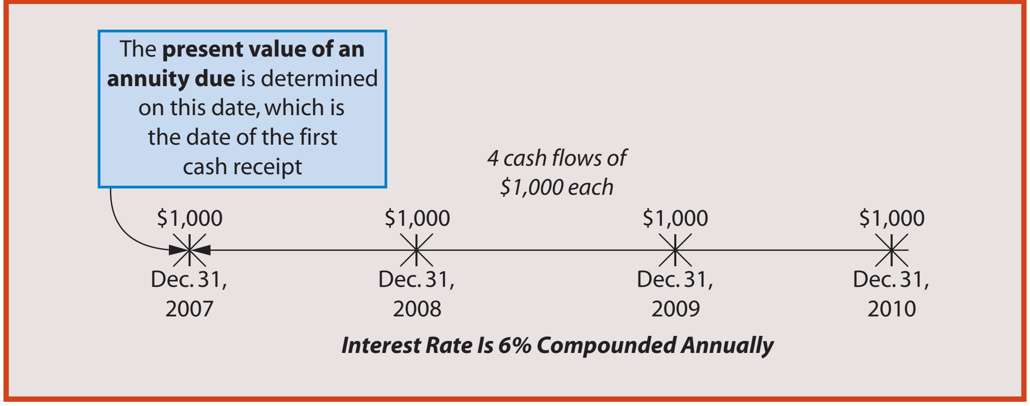


PRESENT VALUE OF AN ANNUITY DUE

7 Compute and use the present value of an annuity due.

The present value of an annuity due (P_d) is determined on the date of the first cash flow in the series. For example, assume that Barbara Livingston wants to calculate the present value of an annuity on December 31, 2007, which will permit four annual future receipts of $1,000 each, the first to be received on December 31, 2007. The interest rate is 6% compounded annually. Example M-15 shows the facts of this problem.

EXAMPLE M-15 Diagram of the Present Value of an Annuity Due



Shortcut Approaches

Formula Approach

The formula for the present value of an annuity due of any amount is:

$$P_d = C\left[\frac{1-\dfrac{1}{(1+i)^{n-1}}}{i}+1\right]$$

where P_d = present value of an ordinary annuity of a series of cash flows of any amount
C = amount of each cash flow
n = number of cash flows (not the number of time periods)
i = interest rate for each of the stated time periods

In the example, the present value of an annuity due of four cash flows of $1,000 each at 6% compounded annually is calculated as follows:

$$P_d = \$1{,}000\left[\frac{1-\dfrac{1}{1.06^3}}{0.06}+1\right] = \$3{,}673.01$$

The formula for the present value of an annuity due with cash flows of 1 each is:

$$P_{d_{n,i}} = C\left[\frac{1-\dfrac{1}{(1+i)^{n-1}}}{i}+1\right]$$

where $P_{d_{n,i}}$ is the present value of an annuity *due* of n cash flows of 1 each at interest rate i.

With the preceding formula for $P_{d_{n,i}}$ it is possible to express another formula for the future value of an ordinary annuity of cash flows of any size as:

$$P_d = C(P_{d_{n,i}})$$

In a two-step approach the present value of an annuity due of four cash flows of $1,000 each at 6% compounded annually is calculated as follows:

Step 1 $$P_{d_{n=4,i=6\%}} = \left[\frac{1-\frac{1}{1.06^3}}{0.06}+1\right] = 3.673012$$

Step 2 $P_d = \$1{,}000(3.673012) = \$3{,}673.01$

This two-step approach is used to solve the problem when factors are not available.

Table Approach

The formula for $P_{d_{n,i}}$ can be used to construct a table of the future value of any series of cash flows of 1 each for any interest rate. Table 5 at the end of this Module shows the factors for $P_{d_{n,i}}$. Since the factors in Table 5 are based on the formula for $P_{d_{n,i}}$ or

$$\frac{1-\frac{1}{(1+i)^{n-1}}}{i}+1$$

values, the generalized table approach is as follows:

$$P_d = C(\text{Factor for } P_{d_{n,i}})$$

To calculate the present value of an annuity due of four cash flows of $1,000 each at 6%, the $P_{d_{n=4,\,i=6\%}}$ factor is found in the present value of an annuity due table (Table 5); it is 3.673012. Then the amount of each cash flow, here $1,000, is multiplied by the Table 5 factor to obtain the present value of $3,673.01:

$$P_d = \$1{,}000(3.673012) = \$3{,}673.01$$

Alternative Table Approach

By observing the information contained in Examples M-15 and M-12, you can determine another way to compute the present value of an annuity due.[3] When only the present value of an *ordinary* annuity table is available, you can use the factors to determine the present value of an annuity due by completing the following steps:

Step 1	In the ordinary annuity table (Table 4), look up the present value of $n - 1$ cash flows at 6%, or the value of three cash flows at 6%.	2.673012
Step 2	Add 1 without interest to the value obtained in step 1.	1.000000
	This is the converted present value factor for $P_{d_{n=4,\,i=6\%}}$.	3.673012
Step 3	Multiply the amount of each cash flow, here $1,000, by the converted factor for $P_{d_{n=4,\,i=6\%}}$ determined in step 2:	
	$P_d = \$1{,}000(3.673012) = \$3{,}673.01$	

3. An alternative approach is to multiply the present value of an ordinary annuity factor by 1 plus the interest rate, which is consistent with the formula:

$$\frac{1-\frac{1}{(1+i)^n}}{i}\times(1+i)$$

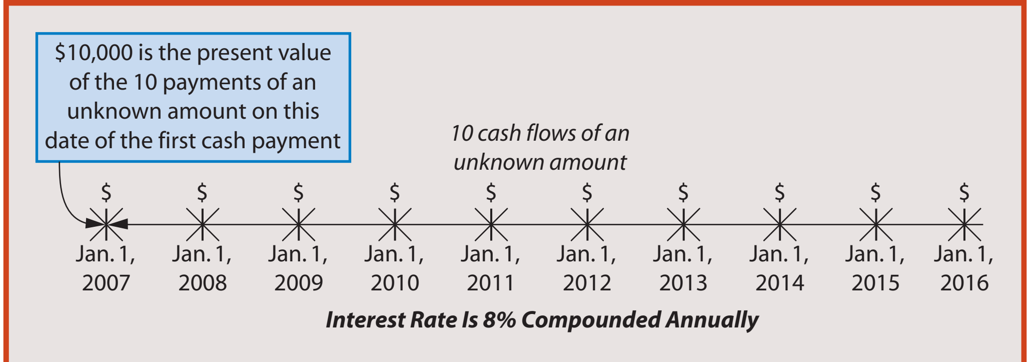
Thus, the present value in this example would be computed as $1,000 × (3.465106 × 1.06) = $3,673.01.

Thus, if the present value of an annuity due is calculated using tables for the present value of an *ordinary* annuity, **the general rule is to use present value of an ordinary annuity factor for $n - 1$ cash flows and add 1 to the factor.**

Another Application

Besides determining the present value of an annuity due where the amount of each cash flow is known, you can solve other types of problems by using the preceding approaches. Suppose, for example, that Katherine Spruill purchases on January 1, 2007 an item that costs $10,000. She agrees to pay for this item in 10 equal annual installments, with the first installment on January 1, 2007 as a down payment. The equal installments include interest at 8% on the unpaid balance at the beginning of each year. After the interest is deducted, the balance of each payment reduces the principal of the debt. This problem involves the present value of an annuity due. It requires the determination of the amount of each of 10 cash flows that have a present value of $10,000 when discounted at an annual rate of 8%. Example M-16 shows these facts graphically.

EXAMPLE M-16 **Diagram of the Present Value of an Annuity Due—Amount of Each Cash Flow to Be Determined**



The solution to this problem requires the rearrangement of the present value of an annuity due formula:

$$C = \frac{P_0}{\text{Factor for } P_{d_{n,i}}}$$

$$= \frac{\$10{,}000}{7.246888} = \$1{,}379.90$$

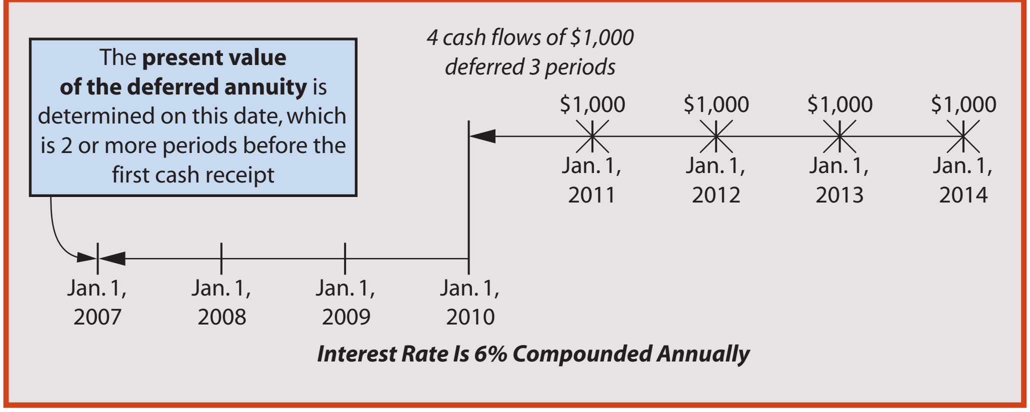
The down payment of $1,379.90 plus nine more payments of this same amount will retire the principal in nine years, plus pay interest at 8% on the balance of the principal outstanding at the beginning of each year.

Present Value of a Deferred Ordinary Annuity

8 Compute and use the present value of a deferred ordinary annuity.

The present value of a deferred ordinary annuity ($P_{deferred}$) is determined on a date two or more periods before the first cash flow in the series. Suppose, for example, that Helen Swain buys an annuity on January 1, 2007 that yields her four annual receipts of $1,000 each, with the first receipt on January 1, 2011. The interest rate is 6% compounded annually. What is the cost of the annuity—that is, what is the present value on January 1, 2007 of the four cash flows of $1,000 each to be received on January 1, 2011, 2012, 2013, and 2014—discounted at 6%? Example M-17 shows the facts of this problem diagrammatically.

EXAMPLE M-17 Diagram of the Present Value of a Deferred Ordinary Annuity



There are two ways to compute the present value of a deferred annuity. The first method involves a combination of the present value of an ordinary annuity (P_0) and the present value of a single sum due in the future (p). For the stated problem it is necessary to determine first the present value of an *ordinary* annuity of four cash flows of $1,000 each to find a single present value figure discounted to January 1, 2010. Note that because the present value of an ordinary annuity table is used, the present value of the four cash flows is computed on January 1, 2010, *not* January 1, 2011. That single sum is discounted for three more periods at 6% to arrive at the present value on January 1, 2007. Using the factors of $1 each, the present value is stated as follows:

$$P_{deferred} = C[(P_{0_{n,i}})(p_{k,i})]$$

where $P_{0_{n,i}}$ = present value of the ordinary annuity of the n cash flows of 1 at the given interest rate i

$p_{k,i}$ = present value of the single sum of 1 for k periods of deferment

Substituting appropriate factors from Tables 4 and 3, respectively, in this formula, the following solution is obtained:

$$\begin{aligned} P_{deferred} &= C[(P_{0_{n=4,\ i=6\%}})(p_{k=3,\ i=6\%})] \\ &= \$1{,}000[(3.465106)(0.839619)] \\ &= \$2{,}909.37 \end{aligned}$$

An alternative approach involves a combination of two ordinary annuities. For example, it is possible to calculate the present value of an ordinary annuity of $n + k$ cash flows of 1. From this amount is subtracted the present value of the k (the period of deferment, which is 3 in this example) cash flows of 1. This procedure removes the cash flows that were not available to be received; yet the discount factor for the three periods of deferments on the four cash flows that are to be received remains in the calculated factor. This difference is multiplied by the value of each cash flow to determine the final present value of the deferred annuity. Example M-18 illustrates this approach.

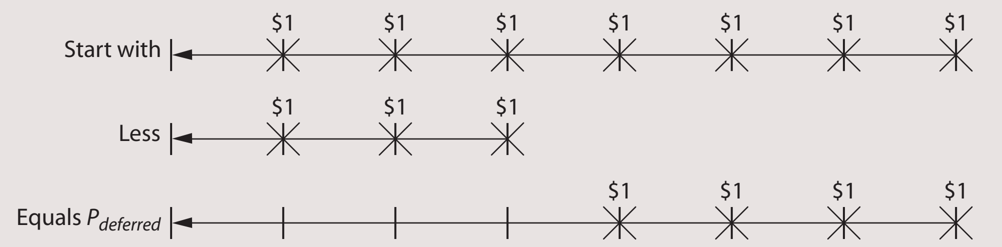
In effect, the present value of an ordinary annuity of $n + k$ cash flows, minus the present value of an ordinary annuity of the k cash flows, becomes a converted factor for the present value of a deferred annuity, as follows:

$$P_{deferred} = C(\text{Converted Factor for Present Value of Deferred Annuity of 1})$$

Using the factors from Table 4, the converted factor for the deferred ordinary annuity stated in the preceding problem is determined as follows:

$$P_{0_{n+k=7,\ i=6\%}}(5.582381) - P_{0_{k=3,\ i=6\%}}(2.673012) = 2.909369$$

EXAMPLE M-18 **Diagram of Converted Table Factor of Present Value of a Deferred Ordinary Annuity**



The present value of the four cash flows of $1,000 each, deferred three periods, is $2,909.37, calculated as follows:

$$P_{deferred} = \$1{,}000(2.909369) = \$2{,}909.37$$

Note that the two methods produce the same present value figure. Also, note that the period of deferment is *only* three periods and *not* four because the present value of an ordinary annuity table is used (see Example M-18 in the second approach). This assumption *is* required if the problem is to be solved by the use of *ordinary* annuity factors rather than annuity due factors.

Another Application

Besides determining the present value of a deferred annuity, other types of problems can be solved by using the previous approaches. For example, suppose that David Jones wants to invest $50,000 on January 1, 2007 so that he may withdraw 10 annual cash flows of equal amounts beginning January 1, 2013. If the fund earns 12% annual interest over its life, what will be the amount of each of the 10 withdrawals?

Example M-19 shows the facts of this problem. A simpler method that can be used to solve this problem is a variation of the second suggested solution. Here, the value of **C** can be determined from the following expression of the present value of a deferred annuity formula:

$$C = \frac{P_{deferred}}{\text{Converted Factor for Present Value of Deferred Annuity of 1}}$$

Using Table 4, the converted factor for 10 cash flows of 1 each, deferred 5 periods at 12%, is as follows:

$$\begin{aligned}\text{Converted Factor} &= P_{0_{n+k=15,\ i=12\%}}(6.810864) - P_{0_{k=5,\ i=12\%}}(3.604776)\\ &= 3.206088\end{aligned}$$

Then the amount of each cash flow is

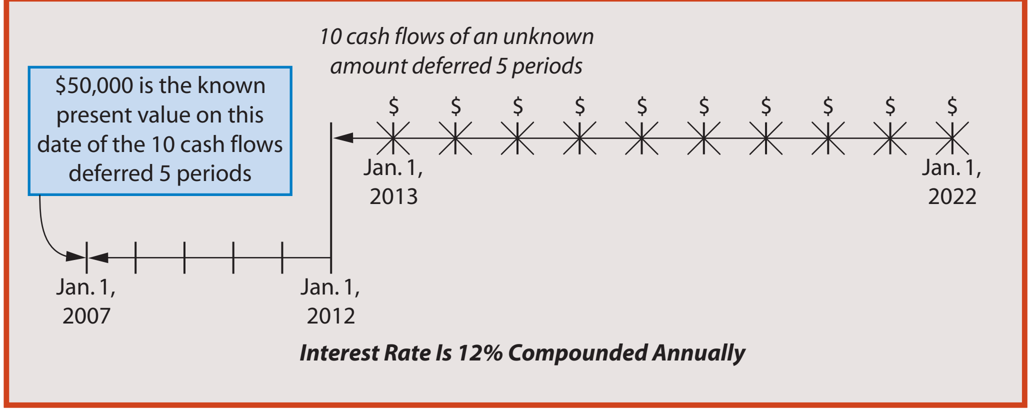
$$C = \frac{\$50{,}000}{3.206088} = \$15{,}595.33$$

The accuracy of the answer produced by the second approach can be tested using the amount of each cash flow and the solution from the first approach. The present value of 10 cash flows of $15,595.33 deferred 5 periods and discounted at 12% must be $50,000 if the first solution is correct. The proof can be calculated as follows:

$$\begin{aligned}P_{deferred} &= \$15{,}595.33[(5.650223)(0.567427)]\\ &= \$50{,}000\end{aligned}$$

A slight rounding-error difference may occur with this method because the solution requires the multiplication of two factors, $P_{0_{n,i}}$ and $p_{k,i}$, which are rounded.

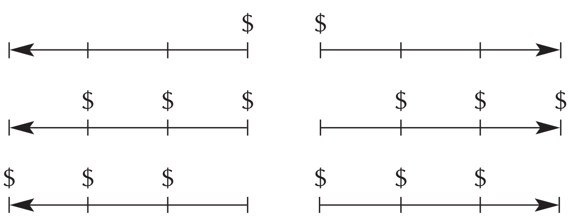
EXAMPLE M-19 **Diagram of the Present Value of a Deferred Annuity—Amount of Each Cash Flow to Be Determined**



Summary of Present and Future Value Calculations

The present and future value calculations discussed in this Module may be summarized by the following diagrams:

	Present Value	Future Value
of a single sum for 3 periods		
of a 3-payment ordinary annuity		
of a 3-payment annuity due		



Conceptual Evaluation of Present Value Techniques in Financial Reporting

9 Explain the conceptual issues regarding the use of present value in financial reporting.

Accounting principles have evolved without a unifying objective or rationale for determining when present value techniques should and should not be used. Among the issues are the use of present value for the initial valuation of assets and liabilities, the amortization of those assets and liabilities, and any subsequent revaluation when interest rates change.

Present values are used in generally accepted accounting principles for certain monetary items. A monetary item is money or a claim to money that is not affected by changes in the prices of specific goods or services. For example, a note payable is a monetary item, whereas a warranty payable is a nonmonetary item. Monetary items for which present values are used in generally accepted accounting principles include bonds payable and bond investments, long-term notes payable and receivable, leases, and postretirement benefits (e.g., pensions). Present value is not used for items such as deferred income taxes. Some accountants argue that present value should be used for nonmonetary items such as property, plant, and equipment. However, accounting principles have not been extended to the use of present value for these nonmonetary items, except for the impairment of noncurrent assets. Therefore, present values are not used for warranties, unearned revenue, compensated absences, or for nonmonetary assets. We discuss each of these topics in this book.

Most accountants would argue that the use of present value creates a *relevant* accounting measurement. For example, in the situations we discussed earlier, present value amounts are more relevant than, say, the total of the undiscounted cash flows because they represent the equivalent current cash amount. However, the use of present value may create measurements that are less *reliable* (especially if used for nonmonetary items) because the computation requires:

1. The estimation of the future cash flows, including the timing, amount, and risk of those cash flows.
2. The estimation of the interest rate. Interest rates that could be used include the historical rate, the current rate, the average expected rate, the weighted average cost of capital, or the incremental borrowing rate.[4]
3. The degree to which the cash flows from the individual assets may be added (and the liabilities subtracted) to give a measure of the value of the company.

In 2000, the FASB issued **FASB Statement of Concepts No. 7,** "Using Cash Flow Information and Present Value in Accounting Measurements."[5] The *Statement* provides a framework for using future cash flows as the basis for an accounting measurement of both assets and liabilities. It provides general principles governing the use of present value, as well as the objectives of present value accounting measurements. The *Statement* does not address recognition issues, and therefore does not address when fair value should be based on present value, or when assets or liabilities should be remeasured using present value. It describes five elements that together may be used to determine the value of various assets and liabilities:

1. An estimate of the future cash flow(s) and the timing of those cash flows
2. Estimates about variations in the amount or timing of those cash flows
3. The risk-free interest rate
4. An increase in the interest for any expected risk
5. Other factors, including a lack of liquidity and market imperfections

The methodology introduced in the *Statement* permits development of a fair value using cash flow information even if uncertainties exist about the timing and/or amount of the cash flows. Present value calculations have typically been based on a single set of cash flows and a single discount rate. The *Statement* introduces the concept of "expected cash flows" when using present value techniques for accounting measurements.

Expected cash flows are a probability-weighted average of the range of possible estimated cash flow amounts and/or estimated timing of cash flows. For example, in regard to differing expected amounts, a company may estimate that there is a 20% probability that the cash flow in a given year will be $1,000, a 50% probability that it will be $1,200, and a 30% probability that it will be $1,400. The company would use the expected cash flow of $1,220 [($1,000 × 0.20) + ($1,200 × 0.50) + ($1,400 × 0.30)] in its present value calculations. Or in regard to the timing of its cash flows, a company might determine that it has a 30% probability of receiving $1,000 in one year but a 70% probability of receiving $1,000 in two years. The present value would be calculated as [(30% × the present value of $1,000 in one year) + (70% × the present value of $1,000 in two years)].

The *Statement* also discusses the use of present value to estimate the fair value for a transaction between willing parties or to develop entity-specific measurements. The entity-specific value (or value in use) is the value of an asset or liability to a particular entity. In other words, the measurement substitutes the entity's assumptions for those of the market place. The FASB concluded that an entity-specific measurement might be appropriate in some situations. As with all *FASB Statements of Concepts,* the conclusions do not create specific GAAP, but will provide guidance for the development of future *FASB Statements of Financial Accounting Standards.* The FASB has also issued an *Exposure Draft* on fair value measurements that would clarify the use of present value techniques to estimate fair value.[6]

4. R. Aggarwal and C. H. Gibson, *Discounting in Financial Accounting and Reporting* (Morristown, N.J.: Financial Executives Research Foundation, 1989), p. 45.
5. "Using Cash Flow Information and Present Value in Accounting Measurements," *FASB Statement of Financial Accounting Concepts No. 7* (Norwalk, Conn.: FASB, 2000).
6. "Fair Value Measurements," *FASB Proposed Statement of Financial Accounting Standards* (Norwalk, Conn.: FASB, 2004).

SUMMARY

At the beginning of the Module, we identified several objectives you would accomplish after reading the Module. The objectives are listed below, each followed by a brief summary of the key points in the Module discussion.

1. **Understand simple interest and compound interest.** Simple interest is interest on the original principal regardless of the number of time periods that have passed or the amount of interest that has been paid or accrued in the past. Compound interest is the interest that accrues on both the principal and the past unpaid accrued interest.
2. **Compute and use the future value of a single sum.** The future value of a single sum is the original sum plus the compound interest, stated as of a specific future date. The future value may be computed using a formula approach or a table approach (Table 1 at the end of this Module).
3. **Compute and use the present value of a single sum.** The present value is the principal that must be invested at time period zero to produce the known future value. Discounting is the process of converting the future value to the present value. The present value may be computed using a formula approach or a table approach (Table 3).
4. **Compute and use the future value of an ordinary annuity.** An annuity is a series of equal cash flows made at regular intervals with interest compounded at a certain rate. The future value of an ordinary annuity is determined immediately after the last cash flow in the series is made. The future value may be computed using a formula approach or a table approach (Table 2).
5. **Compute and use the future value of annuity due.** The future value of an annuity due is determined one period after the last cash flow in the series. The general rule for determining the future value of an annuity due factor is to take the future value of an ordinary annuity factor (Table 2) for $n + 1$ cash flows and subtract 1 from the factor.
6. **Compute and use the present value of an ordinary annuity.** The present value of an ordinary annuity is determined one period before the first cash flow in the series is made. The present value may be computed using a formula approach or a table approach (Table 4).
7. **Compute and use the present value of annuity due.** The present value of an annuity due is determined on the date of the first cash flow in the series. The present value may be computed using a formula approach or a table approach (Table 5). The general rule for determining the present value of an annuity due factor is to take the present value of an ordinary annuity factor (Table 4) for $n - 1$ cash flows and add 1 to the factor.
8. **Compute and use the present value of a deferred ordinary annuity.** The present value of a deferred ordinary annuity is determined on a date two or more periods before the first cash flow in the series. The general rule for determining the present value of a deferred ordinary annuity factor is to take the factor for the present value of an ordinary annuity of $n + k$ cash flows and subtract the factor for the present value of an ordinary annuity of the k cash flows.
9. **Explain the conceptual issues regarding the use of present value in financial statements.** The conceptual issues include the lack of a unifying objective or rationale for determining when present value techniques should and should not be used. These include the use of present value for the initial valuation of assets and liabilities, the amortization of those assets and liabilities, and any subsequent revaluation when interest rates change. Another issue is that present value may create measurements that are less reliable. In 2000, the FASB issued *FASB Statement of Concepts No. 7*, "Using Cash Flow Information and Present Value in Accounting Measurements." The *Statement* identifies five elements that together may be used to determine the value of various assets and liabilities: (1) an estimate of the future cash flow(s) and the timing of those cash flows, (2) estimates about variations in the amount or timing of those cash flows, (3) the risk-free interest rate, (4) an increase in the interest for any expected risk, and (5) other factors, including a lack of liquidity and market imperfections. The FASB has issued an *Exposure Draft* that would clarify the use of present value to estimate fair value.

QUESTIONS

QM-1 Define *interest*. Explain how the cost of interest is similar to the price of any merchandise item.

QM-2 Discuss the following concepts of interest: *simple interest, compound interest, time value of money, discount*.

QM-3 Distinguish between the *future value of 1* and the *future value of an ordinary annuity of 1*.

QM-4 What is the interest rate per period and the frequency of compounding per year in each of the following?
a. 18% compounded semiannually
b. 16% compounded quarterly
c. 15% compounded monthly

QM-5 Distinguish between the *future value of 1* and the *present value of 1* and between the *present value of 1* and the *present value of an ordinary annuity of 1*.

QM-6 Distinguish between the *future value of an ordinary annuity* and the *future value of an annuity due*. Draw a time line of each.

QM-7 Distinguish between the *present value of an annuity due* and the *present value of a deferred annuity*. Draw a time line of each.

QM-8 Explain how to solve each of the following without tables (in each case use the quickest approach possible):

a. The present value of $10,000 for four years at 10% compounded annually
b. The present value of $5,000 for five years at 10% [start with information developed in (a)]
c. The future value of five cash flows of an ordinary annuity of $3,000 each at 10% compound interest

QM-9 Potter wishes to deposit a sum that at 12% interest, compounded semiannually, will permit two withdrawals: $40,000 at the end of 4 years and $50,000 at the end of 10 years. Analyze the problem to determine the required deposit, stating the procedure to follow and the tables to use in developing the solution.

QM-10 The following factors are taken from the compound interest tables for the same number of time periods and/or cash flows for the same interest rate:

a. 8.137249
b. 50.980352
c. 6.265060
d. 7.142168
e. 0.122892

Identify each of the five compound interest table factors without reference to the tables. Discuss briefly.

QM-11 Explain how to determine the converted table factor for any deferred annuity by using the present value of an ordinary annuity table.

QM-12 Samuel Ames owes $20,000 to a friend. He wants to know how much he would have to pay if he paid the debt in three annual installments at the end of each year, which would include interest at 14%. Draw a time line for the problem. Indicate what table to use. Look up the table value and place in a brief formula, but do not solve.

QM-13 Starting with the given value for $(1.16)^{10} = 4.411435$, describe the fastest way to solve each of the following:

a. $p_{n=10,i=16\%}$
b. $f_{n=20,i=16\%}$
c. $F_{0_{n=10,i=16\%}}$
d. $P_{0_{n=10,i=16\%}}$
e. $F_{0_{n=20,i=16\%}}$

MULTIPLE CHOICE (AICPA Adapted)

Select the best answer for each of the following.

Items 1 through 4 require use of present value tables. The following are the present value factors of $1 discounted at 8% for one to five periods. Each item is based on 8% interest compounded annually from day of deposit to day of withdrawal.

Periods	Present Value of $1 Discounted at 8% per Period
1	0.926
2	0.857
3	0.794
4	0.735
5	0.681

MM-1 What amount should be deposited in a bank today to grow to $1,000 three years from today?

a. $\dfrac{\$1,000}{0.794}$
b. $1,000 × 0.926 × 3
c. ($1,000 × 0.926) + ($1,000 × 0.857) + ($1,000 × 0.794)
d. $1,000 × 0.794

MM-2 What amount should an individual have in his bank account today, before withdrawal, if he needs $2,000 each year for four years, with the first withdrawal to be made today and each subsequent withdrawal at one-year intervals? (He is to have exactly a zero balance in his bank account after the fourth withdrawal.)

a. $2,000 + ($2,000 × 0.926) + ($2,000 × 0.857) + ($2,000 × 0.794)
b. $\dfrac{\$2,000}{0.735} \times 4$
c. ($2,000 × 0.926) + ($2,000 × 0.857) + ($2,000 × 0.794) + ($2,000 × 0.735)
d. $\dfrac{\$2,000}{0.926} \times 4$

MM-3 If an individual put $3,000 in a savings account today, what amount of cash will be available two years from today?

a. $3,000 × 0.857
b. $3,000 × 0.857 × 2
c. $\dfrac{\$3,000}{0.857}$
d. $\dfrac{\$3,000}{0.926} \times 2$

MM-4 What is the present value today of $4,000 to be received six years from today?

a. $4,000 × 0.926 × 6
b. $4,000 × 0.794 × 2
c. $4,000 × 0.681 × 0.926
d. Cannot be determined from the information given

MM-5 On January 1, 2007 Kern Company sold a machine to Burns Company. Burns signed a non-interest-bearing note requiring payment of $30,000 annually for seven years. The first payment was made on January 1, 2007. The prevailing rate of interest for this type of note at the date of issuance was 10%. Information on present value factors is as follows:

Periods	Present Value of 1 at 10%	Present Value of Ordinary Annuity of 1 at 10%
6	0.56	4.36
7	0.51	4.87

Kern should record the sale in January 2007 at

a. $107,100
b. $130,800
c. $146,100
d. $160,800

MM-6 On May 1, 2007, a company purchased a new machine that it does not have to pay for until May 1, 2009. The total payment on May 1, 2009 will include both principal and interest. Assuming interest at a 10% rate, the cost of the machine would be the total payment multiplied by what time value of money concept?

a. Future value of annuity of 1
b. Future value of 1
c. Present value of annuity of 1
d. Present value of 1

MM-7 An office equipment representative has a machine for sale or lease. If you buy the machine, the cost is $7,596. If you lease the machine, you will have to sign a noncancellable lease and make five payments of $2,000 each. The first payment will be paid on the first day of the lease. At the time of the last payment you will receive title to the machine. The present value of an ordinary annuity of $1 is as follows:

Present Value Number of Periods	10%	12%	16%
1	0.909	0.893	0.862
2	1.736	1.690	1.605
3	2.487	2.402	2.246
4	3.170	3.037	2.798
5	3.791	3.605	3.274

The interest rate implicit in this lease is approximately

a. 10%
b. 12%
c. Between 10% and 12%
d. 16%

MM-8 An accountant wishes to find the present value of an annuity of $1 payable at the beginning of each period at 10% for eight periods. He has only one present value table, which shows the present value of an annuity of $1 payable at the end of each period. To compute the present value factor he needs, the accountant would use the present value factor in the 10% column for

a. Seven periods
b. Seven periods and add 1
c. Eight periods
d. Nine periods and subtract 1

MM-9 On July 1, 2007, James Rago signed an agreement to operate as a franchisee of Fast Foods, Inc., for an initial franchise fee of $60,000. Of this amount, $20,000 was paid when the agreement was signed and the balance is payable in four equal annual payments of $10,000 beginning July 1, 2008. The agreement provides that the down payment is not refundable and no future services are required of the franchisor. Rago's credit rating indicates that he can borrow money at 14% for a loan of this type. Information on present and future value factors is as follows:

Present value of $1 at 14% for four periods	0.59
Future value of $1 at 14% for four periods	1.69
Present value of an ordinary annuity of $1 at 14% for four periods	2.91

Rago should record the acquisition cost of the franchise on July 1, 2007 at

a. $43,600
b. $49,100
c. $60,000
d. $67,600

MM-10 For which of the following transactions would the use of the present value of an annuity due concept be appropriate in calculating the present value of the asset obtained or liability owed at the date of incurrence?

a. A capital lease is entered into with the initial lease payment due one month subsequent to the signing of the lease agreement.
b. A capital lease is entered into with the initial lease payment due upon the signing of the lease agreement.
c. A 10-year, 8% bond is issued on January 2, with interest payable semiannually on July 1 and January 1 yielding 7%.
d. A 10-year, 8% bond is issued on January 2, with interest payable semiannually on July 1 and January 1 yielding 9%.

EXERCISES

EM-1 ***Future Value of an Investment and Compound Interest*** Using the future value tables, solve the following:

Required

1. What is the value on January 1, 2014 of $40,000 deposited on January 1, 2007 which accumulates interest at 12% compounded annually?
2. What is the value on January 1, 2013 of $10,000 deposited on July 1, 2007 which accumulates interest at 16% compounded quarterly?
3. What is the compound interest on an investment of $6,000 left on deposit for five years at 10% compounded annually?

EM-2 ***Future Value of an Investment*** Hugh Colson deposited $20,000 in a special savings account that provides for interest at the annual rate of 12% compounded semiannually if the deposit is maintained for four years.

Required

Calculate the balance of the savings account at the end of the four-year period.

EM-3 ***Present Value of a Sum and Compound Discount*** Using the present value tables, solve the following problems:

Required

1. What is the present value on January 1, 2007 of $30,000 due on January 1, 2012 and discounted at 12% compounded annually?
2. What is the present value on July 1, 2007 of $8,000 due January 1, 2012 and discounted at 16% compounded quarterly?
3. What is the compound discount on $8,000 due at the end of five years at 10% compounded annually?

EM-4 ***Future Value of Annuity*** Using appropriate tables, solve the following future value of annuity problems:

Required

1. What is the future value on December 31, 2013 of seven cash flows of $10,000, with the first cash payment being made on December 31, 2007 and interest at 12% being compounded annually?
2. What is the future value on December 31, 2014 of seven cash flows of $10,000, with the first cash payment made on December 31, 2007 and interest at 12% being compounded annually?

EM-5 ***Present Value of an Annuity*** Samuel David wants to make five equal annual withdrawals of $8,000 from a fund that will earn interest at 10% compounded annually.

Required

How much would David have to invest on:

1. January 1, 2007 if the first withdrawal is made on January 1, 2008?
2. January 1, 2007 if the first withdrawal is made on January 1, 2007?

EM-6 ***Amount of Each Cash Flow*** Six equal annual contributions are made to a fund, with the first deposit on December 31, 2007.

Required

Using the future value tables, determine the equal contributions that, if invested at 10% compounded annually, will produce a fund of $30,000, assuming that this sum is desired on December 31, 2012.

EM-7 ***Amount of an Annuity*** Beginning December 31, 2011, five equal annual withdrawals are to be made.

Required

Using the appropriate tables, determine the equal annual withdrawals if $25,000 is invested at an interest of 12% compounded annually on

1. December 31, 2010
2. December 31, 2011
3. December 31, 2007

EM-8 ***Amount of Each Cash Flow*** R. Lee Rouse borrows $10,000 that is to be repaid in 24 equal monthly installments payable at the end of each subsequent month with interest at the rate of 1½% a month.

Required

Using the appropriate table, calculate the equal installments.

EM-9 ***Amount of Each Cash Flow*** On January 1, 2007 Charles Jamison borrows $40,000 from his father to open a business. The son is the beneficiary of a trust created by his favorite aunt from which he will receive $25,000 on January 1, 2017. He signs an agreement to make this amount payable to his father and, further, to pay his father equal annual amounts from January 1, 2008 to January 1, 2016, inclusive, in retirement of the debt. Interest is 12%.

Required

What are the annual payments?

EM-10 ***Amount of an Annuity*** Beginning with January 1, 2007, five equal deposits are to be made in a fund.

Required

Using the appropriate tables, determine the equal deposits if interest at 10% is compounded annually and if $200,000 must be in the fund on

1. January 1, 2012
2. January 1, 2013

EM-11 ***Series of Compound Interest Techniques*** The following are several situations involving compound interest.

Required

Using the appropriate table, solve each of the following:

1. Hope Dearborn invests $40,000 on January 1, 2007 in a savings account that earns interest of 8% compounded semiannually. What will be the amount in the fund on December 31, 2012?

2. Ben Johnson receives a bonus of $5,000 each year on December 31. He starts depositing his bonus on December 31, 2007 in a savings account that earns interest of 12% compounded annually. What will be the amount in the fund on December 31, 2011 after he deposits his bonus received on that date?
3. Ron Sewert owes $30,000 on a non-interest-bearing note due January 1, 2017. He offers to pay the amount on January 1, 2007 provided that it is discounted at 10% on a compound annual discount basis. What would he have to pay on January 1, 2007 under this assumption?
4. June Stickney purchased an annuity on January 1, 2007 which, at a 12% annual rate, would yield $6,000 each June 30 and December 31 for the next six years. What was the cost of the annuity to Stickney?
5. Five equal annual contributions are to be made to a fund, the first deposit on December 31, 2007. Determine the equal contributions that, if invested at 10% compounded annually, will produce a fund of $30,000 on December 31, 2012.
6. Beginning on December 31, 2008, six equal annual withdrawals are to be made. Determine the equal annual withdrawals if $11,000 is invested at 10% interest compounded annually on December 31, 2007.

EM-12 ***Amount of an Annuity*** John Goodheart wishes to provide for six annual withdrawals of $3,000 each beginning January 1, 2017. He wishes to make 10 annual deposits beginning January 1, 2007, with the last deposit to be made on January 1, 2016.

Required
If the fund earns interest compounded annually at 10%, how much is each of the 10 deposits?

EM-13 ***Present Value of Leased Asset*** On January 1, 2007 Ashly Farms leased a hay baler from Agrico Tractor Company. Ashly was having cash flow problems, so Agrico drew up the lease to allow Ashly to reestablish itself. The lease requires Ashly to make $3,000 payments on January 1 of each year for five years beginning in 2007. The interest rate is 12%.

Required
Calculate the present value of the cost of the lease payments to Ashly on January 1, 2007.

EM-14 ***Number of Cash Flows*** On July 1, 2007 Boston Company purchased a machine at a cost of $80,000. It paid $56,046.06 in cash and signed a 10% note for the difference. This note is to be paid off in annual installments of $5,000 each, payable each July 1, beginning immediately. The $5,000 includes a payment of interest on the balance of the principal at the beginning of each period and a payment on the principal.

Required
Calculate the number of annual payments to be made by Boston Company.

PROBLEMS

PM-1 ***Future Value of an Investment*** Using the future value tables, solve the following:

Required
1. What is the future value on December 31, 2011 of a deposit of $35,000 made on December 31, 2007 assuming interest of 10% compounded annually?
2. What is the future value on December 31, 2011 of a deposit of $10,000 made on December 31, 2007 assuming interest of 16% compounded quarterly?
3. What is the future value on December 31, 2011 of a deposit of $25,000 made on December 31, 2007 assuming interest of 12% compounded semiannually?

PM-2 ***Present Value Issues*** Using the present value tables, solve the following:

Required
1. What is the present value on January 1, 2007 of $30,000 due on January 1, 2011 and discounted at 10% compounded annually?
2. What is the present value on January 1, 2007 of $40,000 due on January 1, 2011 and discounted at 11% compounded semiannually?
3. What is the present value on January 1, 2007 of $50,000 due on January 1, 2011 and discounted at 16% compounded quarterly?

PM-3 ***Future Value Issues*** Using the future values tables, solve the following:

Required
1. What is the future value on December 31, 2016 of 10 cash flows of $20,000 with the first cash payment made on December 31, 2007 and interest at 10% being compounded annually?

2. What is the future value on June 30, 2017 of 20 cash flows of $15,000 with the first cash payment made on December 31, 2007 and the annual interest rate of 10% being compounded semiannually?
3. What is the future value on December 31, 2017 of 20 cash flows of $15,000 with the first cash payment made on December 31, 2007 and the annual interest rate of 10% being compounded semiannually?

PM-4 ***Amount of Each Cash Flow*** On December 31, 2014 Michael McDowell desires to have $60,000. He plans to make six deposits in a fund to provide this amount. Interest is compounded annually at 12%.

Required
Compute the equal annual amounts that McDowell must deposit assuming that he makes the first deposit on
1. December 31, 2009
2. December 31, 2008

PM-5 ***Value of an Annuity*** John Joshua wants to make five equal annual withdrawals of $20,000 from a fund that will earn interest at 12% compounded annually.

Required
How much would Joshua have to invest on January 1, 2007 if he makes the first withdrawal on
1. January 1, 2008?
2. January 1, 2007?
3. January 1, 2012?

PM-6 ***Value of an Annuity*** Ralph Benke wants to make eight equal semiannual withdrawals of $8,000 from a fund that will earn interest at 11% compounded semiannually.

Required
How much would Benke have to invest on:
1. January 1, 2007 if the first withdrawal is made on July 1, 2007?
2. July 1, 2007 if the first withdrawal is made on July 1, 2007?
3. January 1, 2007 if the first withdrawal is made on January 1, 2010?

PM-7 ***Various Compound Interest Issues*** You are given the following situations:

1. Thomas Petry owes a debt of $7,000 from the purchase of a boat. The debt bears interest of 12% payable annually. Petry will pay the debt and interest in five annual installments beginning in one year. Calculate the equal annual installments that will pay off the debt and interest at 12% on the unpaid balance.
2. On January 1, 2007 John Cothran offers to buy Ruth House's used tractor and equipment for $4,000 payable in 12 equal semiannual installments, which are to include payment of 10% interest on the unpaid balance and payment of a portion of the principal, with the first installment to be made on January 1, 2007. Calculate the amount of each of these installments.
3. Nadine Love invests in a $60,000 annuity at 12% compounded annually on March 1, 2007. The first of 15 receipts from the annuity is payable to Love on March 1, 2017, 10 years after the annuity is purchased and on the date Love expects to retire. Calculate the amount of each of the 15 equal annual receipts.

Required
Using the appropriate tables, solve each of the preceding situations.

PM-8 ***Value of an Annuity*** Using the appropriate tables, solve each of the following:

Required
1. Beginning December 31, 2008, five equal withdrawals are to be made. Determine the equal annual withdrawals if $30,000 is invested at 10% interest compounded annually on December 31, 2007.
2. Ten payments of $3,000 are due at annual intervals beginning June 30, 2008. What amount will be accepted in cancellation of this series of payments on June 30, 2007 assuming a discount rate of 14% compounded annually?
3. Ten payments of $2,000 are due at annual intervals beginning December 31, 2007. What amount will be accepted in cancellation of this series of payments on January 1, 2007 assuming a discount rate of 12% compounded annually?

PM-9 ***Amount of Each Cash Flow*** On January 1, 2007 Philip Holding invests $40,000 in an annuity to provide eight equal semiannual payments. Interest is 10%, compounded semiannually.

Required
Compute the equal semiannual amounts that Holding will receive, assuming that the first withdrawal is to be received on
1. July 1, 2007
2. January 1, 2007
3. July 1, 2010
4. January 1, 2012

PM-10 ***Number of Cash Flows*** The following are two independent situations.

1. Houser wishes to accumulate a fund of $40,000 for the purchase of a house and lot. He plans to deposit $4,000 semiannually at the end of each six months. Assuming interest at 14% a year compounded semiannually, how many deposits of $4,000 each will be required and what is the amount of the last deposit?
2. On January 1, 2007 Joan Campbell borrows $20,000 from Susan Rone and agrees to repay this amount in payments of $4,000 a year until the debt is paid in full. Payments are to be of an equal amount and are to include interest at 12% on the unpaid balance of principal at the beginning of each period. Assuming that the first payment is to be made on January 1, 2008, determine the number of payments of $4,000 each to be made and the amount of the final payment.

Required
Using the appropriate tables, solve each of the preceding situations.

PM-11 ***Serial Installments; Amounts Applicable to Interest and Principal*** Ronald McDuffie purchases a new car at a cost of $14,400. He pays $3,000 down and issues an installment note payable by which he promises to pay the balance in 18 equal monthly installments, which include interest at an annual rate of 18% on the remaining unpaid balance at the beginning of each month, starting with the first month after the purchase.

Required
1. Compute the equal installment payments.
2. Compute the interest that will be paid for each of the first two periods. Indicate the amount of each payment that will be a reduction of principal.

PM-12 ***Determining Loan Repayments*** Rockness needs $40,000 to pay off a loan due on December 31, 2016. His plans included the making of 10 annual deposits beginning on December 31, 2007 in accumulating a fund to pay off the loan. Without making a precise calculation, Rockness made three annual deposits of $4,000 each on December 31, 2007, 2008, and 2009, which have been earning interest at 10% compounded annually.

Required
What is the equal amount of each of the next seven deposits for the period December 31, 2010 to December 31, 2016 to reach the fund objective, assuming that the fund will continue to earn interest at 10% compounded annually?

PM-13 ***Purchase of Asset*** William Thomas intends to purchase a tractor on credit. Two local implement dealers have offered him the following payment plans for identical tractors:
1. Redd Truck & Tractor's plan calls for five annual payments of $10,350, with the first payment now and the remaining payments at the beginning of each of the next four years.
2. Greene Farm Implements requires semiannual payments of $5,750 at the end of each of the next 10 semiannual periods, with the first payment to be in six months.

Required
Determine which of the preceding plans offers Thomas the lower present value. The applicable annual interest rate is 10% for both alternatives.

PM-14 ***Fund to Retire Bonds*** At the beginning of 2007 Shanklin Company issued 10-year bonds with a face value of $1,000,000 due on December 31, 2016. The company wants to accumulate a fund to retire these bonds at maturity by making annual deposits beginning on December 31, 2007.

Required
How much must the company deposit each year, assuming that the fund will earn 12% interest a year compounded annually?

PM-15 ***Asset Purchase Price*** BWP, Inc., is considering the purchase of an asset. BWP's required rate of return on new assets is 12%. The expected net cash inflows generated by the new asset are as follows:

Years	Amount	Nature of the Cash Inflows
1–4	$3,000	Net operating revenues
5–9	2,500	Net operating revenues
10	2,000	Net operating revenues
10	1,000	Sale of asset

Required
Given that the net cash inflows can be realized, what is the maximum amount BWP should be willing to pay for the new asset? Assume that each cash inflow occurs at the end of the year. (*Contributed by Norma C. Powell*)

PM-16 ***Acquisition of Asset*** SuMar Company purchased a new piece of machinery by paying $2,000 down and agreeing to pay $1,000 at the end of each year for five years. The appropriate interest rate is 8%.

Required
1. What is the cost of the machinery?

2. Prepare the journal entry to record the purchase of the machinery.
3. Prepare a table that shows the interest and ending balance of the liability each year. (*Contributed by Norma C. Powell*)

PM-17 ***Present Value Issues*** Nello Construction Company has just purchased several major pieces of road-building equipment. Since the purchase price is so large, the equipment company is giving Nello an option of choosing one of four different payment plans:

1. $600,000 immediately in cash.
2. $200,000 down payment now; $65,000 per year for 12 years, beginning at the end of the current year.
3. $200,000 down payment now; $25,000 per year for 3 years beginning at the end of the current year; $75,000 per year for 11 years beginning at the end of the fourth year after the purchase.
4. $80,000 now and at the beginning of each of the next 13 years.

Required
You have been asked by the Nello Construction Company to decide which payment plan will provide the smallest present value. The expected effective interest rate during the future periods stated above is 12%.

PM-18 **AICPA Adapted** ***Comprehensive***

Part a. Reproduced in the following table are the first three lines from the 2% columns of each of several tables of mathematical values. For each of the following items, you are to select from among these fragmentary tables the one from which the amount required can be obtained *most directly* (assuming that the complete table was available in each instance):

Periods	Table A	Table B	Table C	Table D	Table E	Table F
0	1.0000		1.0000			
1	0.9804	1.0200	1.0200	1.0000	0.9804	1.0200
2	0.9612	2.0604	1.0404	0.4950	1.9416	0.5150
3		3.1216		0.3268	2.8839	0.3468

1. The amount to which a single sum would accumulate at compound interest by the end of a specified period (interest compounded annually).
2. The amount that must be appropriated at the end of each of a specific number of years to provide for the accumulation, at annually compounded interest, of a certain sum.
3. The amount that must be deposited in a fund that will earn interest at a specified rate, compounded annually, in order to make possible the withdrawal of certain equal sums annually over a specified period starting one year from date of deposit.
4. The amount of interest that will accumulate on a single deposit by the end of a specified period (interest compounded semiannually).
5. The amount, net of compound discount, that if paid now would settle a debt of larger amount due at a specified future date.

Part b. The following tables of values at 10% interest may be used as needed to answer the questions in this part of the problem.

Periods	Future Value of 1 at Compound Interest	Present Value of 1 at Compound Interest	Future Value of Annuity of 1 at End of Each Period	Present Value of Annuity of 1 at End of Each Period
1	1.100	0.9091	1.0000	0.9091
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
6	1.7716	0.5645	7.7156	4.3553
7	1.9487	0.5132	9.4872	4.8684
8	2.1436	0.4665	11.4359	5.3349
9	2.3579	0.4241	13.5795	5.7590
10	2.5937	0.3855	15.9374	6.1446
11	2.8531	0.3505	18.5312	6.4951
12	3.1384	0.3186	21.3843	6.8137
13	3.4523	0.2897	24.5227	7.1034
14	3.7975	0.2633	27.9750	7.3667
15	4.1772	0.2394	31.7725	7.6061
16	4.5950	0.2176	35.9497	7.8237

1. Your client has made annual payments of $2,500 into a fund at the close of each year for the past three years. The fund balance immediately after the third payment totaled $8,275. He has asked you how many more $2,500 annual payments

will be required to bring the fund to $22,500, assuming that the fund continues to earn interest at 10% compounded annually. Compute the number of full payments required and the amount of the final payment if it does not require the entire $2,500. Carefully label all computations supporting your answer.

2. Your client wishes to provide for the payment of an obligation of $200,000 due on July 1, 2014. He plans to deposit $20,000 in a special fund each July 1 for 7 years, starting July 1, 2008. He wishes to make an initial deposit on July 1, 2007 of an amount that, with its accumulated interest, will bring the fund up to $200,000 at the maturity of the obligation. He expects that the fund will earn interest at the rate of 10% compounded annually. Compute the amount to be deposited July 1, 2007. Carefully label all computations supporting your answer.

PM-19 ***Comprehensive*** The following are three independent situations:

1. M. Herman has decided to set up a scholarship fund for students. She is willing to deposit $5,000 in a trust fund at the end of each year for 10 years. She wants the trust fund to then pay annual scholarships at the end of each year for 30 years.
2. Charles Jordy is planning to save for his retirement. He has decided that he can save $3,000 at the end of each year for the next 10 years, $5,000 at the end of each year for years 11 through 20, and $10,000 at the end of each year for years 21 through 30.
3. Patricia Karpas has $200,000 in savings on the day she retires. She intends to spend $2,000 per month traveling around the world for the next two years, during which time her savings will earn 18%, compounded monthly. For the next five years, she intends to spend $6,000 every six months, during which time her savings will earn 12%, compounded semiannually. For the rest of her life expectancy of 15 years, she wants an annuity to cover her living costs. During this period her savings will earn 10% compounded annually. Assume that all payments occur at the end of each period.

Required

1. In Situation 1, how much will the annual scholarships be if the fund can earn 6%? 10%?
2. In Situation 2,
   (a) How much will Jordy have at the end of 30 years if his savings can earn 10%? 6%?
   (b) If Jordy expects to live for 20 years in retirement, how much can he spend each year if his savings earn 10%? 6%?
   (c) How much would Jordy need to invest today to have the same amount available at the time he retires as calculated in 2(a) at 10%? 6%?
3. In Situation 3, how much will Karpas's annuity be?

CASES

CM-1 Cost of Insurance Plans

The Johnson Company is considering three different time periods for an insurance policy on its main office building. The premiums on a fire insurance policy covering the building for the amount of $2,000,000 on a one-year, three-year, and five-year basis are as follows:

One year	$ 4,480
Three years	11,200
Five years	17,920

In each case the entire premium for the full term of the policy is payable at the beginning of the year in which the policy is purchased.

Required

Evaluate the annual cost of each insurance plan for the insured, assuming that money is worth 12% compounded annually. Which plan do you recommend? State the savings for the company.

CM-2 Acquisition of Equipment

The manager of the Taylor Company has consulted you, the controller, as to which of the following plans you would recommend in acquiring the use of a piece of heavy equipment:

1. Purchase the equipment and pay immediately a cash price of $36,800. The service life of the heavy equipment is estimated to be five years, with a resale value at the end of that time of $5,500.
2. Lease the equipment at the rate of $9,100 per year for five years, payable at the beginning of each year.

Required

Assuming that the time value of money is 12%, evaluate the two alternatives and indicate which plan you would recommend to the manager, stating the value of savings to the company.

CM-3 Effective Interest in Various Situations

On March 1, 2007 the White Company purchased $400,000 worth of inventory on credit with terms of 1/20, n/60. In the past, White has always followed the policy of making payment one month (30 days) after the goods are purchased.

A new member of White's staff has indicated that the company he previously worked for never passed up its cash discounts, and he wonders if this is not a sound policy. It was pointed out, however, that if White were to pay the bill on March 20 rather than on March 30, it would have to borrow the necessary funds for the 10 extra days. White's borrowing terms with a local bank were estimated to be at 14% (annual rate), with a 15% compensating balance (a requirement by the bank that White maintain an amount in its account equal to 15% of the loan) for the term of the loan. Most members of White's staff felt that it made little sense to take out a 14% loan with a compensating balance of 15% in order to save 1% on $400,000 by paying the account 10 days earlier than planned.

Required

1. In terms of simple effective annual interest cost, explain whether it would be to White's advantage to borrow the amount necessary to take the 1% discount by paying the bill 10 days early.
2. It has also been pointed out to White that if it does not take advantage of the cash discount, it should wait the entire 60-day period to pay the full bill rather than pay within 30 days. Explain how your answer to Requirement 1 would change if White undertook this policy.
3. Your answer to Requirement 2 indicates that, in relation to Requirement 1, it has become either more desirable or less desirable to borrow in order to take advantage of the 1% cash discount.
   a. If you said *more desirable*, explain why.
   b. If you said *less desirable*, make a similar explanation.

CM-4 Future Value of Single Investment and Annuity

Jane Dough was a teller in a large northeastern bank. She was single and approaching age 30, and she considered herself an honest and upright citizen. After considering what she might do to build a retirement plan for the future, she decided to embezzle $1,500,000. Subsequently she gave herself up to the authorities but did not return the $1,500,000. She was tried, convicted, and sentenced to 20 years in prison. After completing her 20-year term, she returned the $1,500,000 that she had stolen. She then decided to take a world cruise. On the ship someone asked her how she had accumulated enough money to afford the trip. She replied, "Do you know how much *interest* $1,500,000 will earn in 20 years if invested at an annual rate of 16% compounded quarterly?"

Required

1. Determine the answer to Jane Dough's question. The table factor for $f_{n=40,\ i=4\%}$ is 4.801021.
2. Evaluate Jane's retirement decision, assuming that she could have earned $21,000 each year for each of the 20 years she was in prison. Assume that $11,000 is required each year to cover living expenses and that she could have invested the remaining $10,000 at the end of each year to earn interest at 16% compounded annually.

CM-5 Value of a Note

You have just been promoted to manager at a national CPA firm. On your first job a new accountant approaches you with the following situation: He has discovered that the president of the client company has a brother who is both the major stockholder and the president of a local bank. Your client has a $300,000, five-year note payable to the bank at 4% interest compounded annually. Since the going interest rate is 16%, the accountant suggests that the note be recorded at its present value using this going rate. The president says that the effective liability is $300,000 and should be reported on the balance sheet at this figure. The note was issued on January 1, 2007 and is due on January 1, 2012.

Required

1. Explain who is correct.
2. At what amount should the company have valued the note on January 1, 2007, assuming that the accountant's assessment is correct?

CM-6 Future Value and Present Value Issues

Jean Perry has a $25,000 whole-life insurance policy that she began many years ago. She is presently 55 years old. One of the benefits of the policy is that Perry can borrow up to a given amount at 12% interest (2% below the current rate), with the principal due two years after the loan is made. The policy states that should Perry default on the principal payment, it will simply be deducted from the amount given her beneficiary when she dies. However, the interest will continue to accrue as long as the note is not paid. Perry has just borrowed $5,000 on this policy to take a vacation in Hawaii.

Required

Assuming that a woman of Perry's health is expected to live to be 72, explain whether it would be financially advantageous for Perry to repay the principal on the loan in two years. (Calculations are not required.)

COMPOUND INTEREST TABLES

Table 1: Future Value of 1: $f_{n,i} = (1 + i)^n$

Table 2: Future Value of an Ordinary Annuity of 1: $F_{0_{n,i}} = \dfrac{(1+i)^n - 1}{i}$

Table 3: Present Value of 1: $p_{n,i} = \dfrac{1}{(1 + i)^n}$

Table 4: Present Value of an Ordinary Annuity of 1: $P_{0_{n,i}} = \dfrac{1 - \dfrac{1}{(1+i)^n}}{i}$

Table 5: Present Value of Annuity Due: $P_{d_{n,i}} = \dfrac{1 - \dfrac{1}{(1+i)^{n-1}}}{i} + 1$

Table 1 FUTURE VALUE OF 1: $f_{n,i} = (1 + i)^n$

n	1.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%
1	1.015000	1.040000	1.045000	1.050000	1.055000	1.060000	1.070000
2	1.030225	1.081600	1.092025	1.102500	1.113025	1.123600	1.144900
3	1.045678	1.124864	1.141166	1.157625	1.174241	1.191016	1.225043
4	1.061364	1.169859	1.192519	1.215506	1.238825	1.262477	1.310796
5	1.077284	1.216653	1.246182	1.276282	1.306960	1.338226	1.402552
6	1.093443	1.265319	1.302260	1.340096	1.378843	1.418519	1.500730
7	1.109845	1.315932	1.360862	1.407100	1.454679	1.503630	1.605781
8	1.126493	1.368569	1.422101	1.477455	1.534687	1.593848	1.718186
9	1.143390	1.423312	1.486095	1.551328	1.619094	1.689479	1.838459
10	1.160541	1.480244	1.552969	1.628895	1.708144	1.790848	1.967151
11	1.177949	1.539454	1.622853	1.710339	1.802092	1.898299	2.104852
12	1.195618	1.601032	1.695881	1.795856	1.901207	2.012196	2.252192
13	1.213552	1.665074	1.772196	1.885649	2.005774	2.132928	2.409845
14	1.231756	1.731676	1.851945	1.979932	2.116091	2.260904	2.578534
15	1.250232	1.800944	1.935282	2.078928	2.232476	2.396558	2.759032
16	1.268986	1.872981	2.022370	2.182875	2.355263	2.540352	2.952164
17	1.288020	1.947900	2.113377	2.292018	2.484802	2.692773	3.158815
18	1.307341	2.025817	2.208479	2.406619	2.621466	2.854339	3.379932
19	1.326951	2.106849	2.307860	2.526950	2.765647	3.025600	3.616528
20	1.346855	2.191123	2.411714	2.653298	2.917757	3.207135	3.869684
21	1.367058	2.278768	2.520241	2.785963	3.078234	3.399564	4.140562
22	1.387564	2.369919	2.633652	2.925261	3.247537	3.603537	4.430402
23	1.408377	2.464716	2.752166	3.071524	3.426152	3.819750	4.740530
24	1.429503	2.563304	2.876014	3.225100	3.614590	4.048935	5.072367
25	1.450945	2.665836	3.005434	3.386355	3.813392	4.291871	5.427433
26	1.472710	2.772470	3.140679	3.555673	4.023129	4.549383	5.807353
27	1.494800	2.883369	3.282010	3.733456	4.244401	4.822346	6.213868
28	1.517222	2.998703	3.429700	3.920129	4.477843	5.111687	6.648838
29	1.539981	3.118651	3.584036	4.116136	4.724124	5.418388	7.114257
30	1.563080	3.243398	3.745318	4.321942	4.983951	5.743491	7.612255

n	8.0%	9.0%	10.0%	12.0%	14.0%	16.0%	18.0%
1	1.080000	1.090000	1.100000	1.120000	1.140000	1.160000	1.180000
2	1.166400	1.188100	1.210000	1.254400	1.299600	1.345600	1.392400
3	1.259712	1.295029	1.331000	1.404928	1.481544	1.560896	1.643032
4	1.360489	1.411582	1.464100	1.573519	1.688960	1.810639	1.938778
5	1.469328	1.538624	1.610510	1.762342	1.925415	2.100342	2.287758
6	1.586874	1.677100	1.771561	1.973823	2.194973	2.436396	2.699554
7	1.713824	1.828039	1.948717	2.210681	2.502269	2.826220	3.185474
8	1.850930	1.992563	2.143589	2.475963	2.852586	3.278415	3.758859
9	1.999005	2.171893	2.357948	2.773079	3.251949	3.802961	4.435454
10	2.158925	2.367364	2.593742	3.105848	3.707221	4.411435	5.233836
11	2.331639	2.580426	2.853117	3.478550	4.226232	5.117265	6.175926
12	2.518170	2.812665	3.138428	3.895976	4.817905	5.936027	7.287593
13	2.719624	3.065805	3.452271	4.363493	5.492411	6.885791	8.599359
14	2.937194	3.341727	3.797498	4.887112	6.261349	7.987518	10.147244
15	3.172169	3.642482	4.177248	5.473566	7.137938	9.265521	11.973748
16	3.425943	3.970306	4.594973	6.130394	8.137249	10.748004	14.129023
17	3.700018	4.327633	5.054470	6.866041	9.276464	12.467685	16.672247
18	3.996019	4.717120	5.559917	7.689966	10.575169	14.462514	19.673251
19	4.315701	5.141661	6.115909	8.612762	12.055693	16.776517	23.214436
20	4.660957	5.604411	6.727500	9.646293	13.743490	19.460759	27.393035
21	5.033834	6.108808	7.400250	10.803848	15.667578	22.574481	32.323781
22	5.436540	6.658600	8.140275	12.100310	17.861039	26.186398	38.142061
23	5.871464	7.257874	8.954302	13.552347	20.361585	30.376222	45.007632
24	6.341181	7.911083	9.849733	15.178629	23.212207	35.236417	53.109006
25	6.848475	8.623081	10.834706	17.000064	26.461916	40.874244	62.668627
26	7.396353	9.399158	11.918177	19.040072	30.166584	47.414123	73.948980
27	7.988061	10.245082	13.109994	21.324881	34.389906	55.000382	87.259797
28	8.627106	11.167140	14.420994	23.883866	39.204493	63.800444	102.966560
29	9.317275	12.172182	15.863093	26.749930	44.693122	74.008515	121.500541
30	10.062657	13.267678	17.449402	29.959922	50.950159	85.849877	143.370638

Table 2 FUTURE VALUE OF AN ORDINARY ANNUITY OF 1: $F_{0_{n,i}} = \frac{(1+i)^n - 1}{i}$

n	1.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.015000	2.040000	2.045000	2.050000	2.055000	2.060000	2.070000
3	3.045225	3.121600	3.137025	3.152500	3.168025	3.183600	3.214900
4	4.090903	4.246464	4.278191	4.310125	4.342266	4.374616	4.439943
5	5.152267	5.416323	5.470710	5.525631	5.581091	5.637093	5.750739
6	6.229551	6.632975	6.716892	6.801913	6.888051	6.975319	7.153291
7	7.322994	7.898294	8.019152	8.142008	8.266894	8.393838	8.654021
8	8.432839	9.214226	9.380014	9.549109	9.721573	9.897468	10.259803
9	9.559332	10.582795	10.802114	11.026564	11.256260	11.491316	11.977989
10	10.702722	12.006107	12.288209	12.577893	12.875354	13.180795	13.816448
11	11.863262	13.486351	13.841179	14.206787	14.583498	14.971643	15.783599
12	13.041211	15.025805	15.464032	15.917127	16.385591	16.869941	17.888451
13	14.236830	16.626838	17.159913	17.712983	18.286798	18.882138	20.140643
14	15.450382	18.291911	18.932109	19.598632	20.292572	21.015066	22.550488
15	16.682138	20.023588	20.784054	21.578564	22.408663	23.275970	25.129022
16	17.932370	21.824531	22.719337	23.657492	24.641140	25.672528	27.888054
17	19.201355	23.697512	24.741707	25.840366	26.996403	28.212880	30.840217
18	20.489376	25.645413	26.855084	28.132385	29.481205	30.905653	33.999033
19	21.796716	27.671229	29.063562	30.539004	32.102671	33.759992	37.378965
20	23.123667	29.778079	31.371423	33.065954	34.868318	36.785591	40.995492
21	24.470522	31.969202	33.783137	35.719252	37.786076	39.992727	44.865177
22	25.837580	34.247970	36.303378	38.505214	40.864310	43.392290	49.005739
23	27.225144	36.617889	38.937030	41.430475	44.111847	46.995828	53.436141
24	28.633521	39.082604	41.689196	44.501999	47.537998	50.815577	58.176671
25	30.063024	41.645908	44.565210	47.727099	51.152588	54.864512	63.249038
26	31.513969	44.311745	47.570645	51.113454	54.965981	59.156383	68.676470
27	32.986678	47.084214	50.711324	54.669126	58.989109	63.705766	74.483823
28	34.481479	49.967583	53.993333	58.402583	63.233510	68.528112	80.697691
29	35.998701	52.966286	57.423033	62.322712	67.711354	73.639798	87.346529
30	37.538681	56.084938	61.007070	66.438848	72.435478	79.058186	94.460786

n	8.0%	9.0%	10.0%	12.0%	14.0%	16.0%	18.0%
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.080000	2.090000	2.100000	2.120000	2.140000	2.160000	2.180000
3	3.246400	3.278100	3.310000	3.374400	3.439600	3.505600	3.572400
4	4.506112	4.573129	4.641000	4.779328	4.921144	5.066496	5.215432
5	5.866601	5.984711	6.105100	6.352847	6.610104	6.877135	7.154210
6	7.335929	7.523335	7.715610	8.115189	8.535519	8.977477	9.441968
7	8.922803	9.200435	9.487171	10.089012	10.730491	11.413873	12.141522
8	10.636628	11.028474	11.435888	12.299693	13.232760	14.240093	15.326996
9	12.487558	13.021036	13.579477	14.775656	16.085347	17.518508	19.085855
10	14.486562	15.192930	15.937425	17.548735	19.337295	21.321469	23.521309
11	16.645487	17.560293	18.531167	20.654583	23.044516	25.732904	28.755144
12	18.977126	20.140720	21.384284	24.133133	27.270749	30.850169	34.931070
13	21.495297	22.953385	24.522712	28.029109	32.088654	36.786196	42.218663
14	24.214920	26.019189	27.974983	32.392602	37.581065	43.671987	50.818022
15	27.152114	29.360916	31.772482	37.279715	43.842414	51.659505	60.965266
16	30.324283	33.003399	35.949730	42.753280	50.980352	60.925026	72.939014
17	33.750226	36.973705	40.544703	48.883674	59.117601	71.673030	87.068036
18	37.450244	41.301338	45.599173	55.749715	68.394066	84.140715	103.740283
19	41.446263	46.018458	51.159090	63.439681	78.969235	98.603230	123.413534
20	45.761964	51.160120	57.274999	72.052442	91.024928	115.379747	146.627970
21	50.422921	56.764530	64.002499	81.698736	104.768418	134.840506	174.021005
22	55.456755	62.873338	71.402749	92.502584	120.435996	157.414987	206.344785
23	60.893296	69.531939	79.543024	104.602894	138.297035	183.601385	244.486847
24	66.764759	76.789813	88.497327	118.155241	158.658620	213.977607	289.494479
25	73.105940	84.700896	98.347059	133.333870	181.870827	249.214024	342.603486
26	79.954415	93.323977	109.181765	150.333934	208.332743	290.088267	405.272113
27	87.350768	102.723135	121.099942	169.374007	238.499327	337.502390	479.221093
28	95.338830	112.968217	134.209936	190.698887	272.889233	392.502773	566.480890
29	103.965936	124.135356	148.630930	214.582754	312.093725	456.303216	669.447450
30	113.283211	136.307539	164.494023	241.332684	356.786847	530.311731	790.947991

Table 3 PRESENT VALUE OF 1: $p_{n,i} = \dfrac{1}{(1+i)^n}$

n	1.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%
1	0.985222	0.961538	0.956938	0.952381	0.947867	0.943396	0.934579
2	0.970662	0.924556	0.915730	0.907029	0.898452	0.889996	0.873439
3	0.956317	0.888996	0.876297	0.863838	0.851614	0.839619	0.816298
4	0.942184	0.854804	0.838561	0.822702	0.807217	0.792094	0.762895
5	0.928260	0.821927	0.802451	0.783526	0.765134	0.747258	0.712986
6	0.914542	0.790315	0.767896	0.746215	0.725246	0.704961	0.666342
7	0.901027	0.759918	0.734828	0.710681	0.687437	0.665057	0.622750
8	0.887711	0.730690	0.703185	0.676839	0.651599	0.627412	0.582009
9	0.874592	0.702587	0.672904	0.644609	0.617629	0.591898	0.543934
10	0.861667	0.675564	0.643928	0.613913	0.585431	0.558395	0.508349
11	0.848933	0.649581	0.616199	0.584679	0.554911	0.526788	0.475093
12	0.836387	0.624597	0.589664	0.556837	0.525982	0.496969	0.444012
13	0.824027	0.600574	0.564272	0.530321	0.498561	0.468839	0.414964
14	0.811849	0.577475	0.539973	0.505068	0.472569	0.442301	0.387817
15	0.799852	0.555265	0.516720	0.481017	0.447933	0.417265	0.362446
16	0.788031	0.533908	0.494469	0.458112	0.424581	0.393646	0.338735
17	0.776385	0.513373	0.473176	0.436297	0.402447	0.371364	0.316574
18	0.764912	0.493628	0.452800	0.415521	0.381466	0.350344	0.295864
19	0.753607	0.474642	0.433302	0.395734	0.361579	0.330513	0.276508
20	0.742470	0.456387	0.414643	0.376889	0.342729	0.311805	0.258419
21	0.731498	0.438834	0.396787	0.358942	0.324862	0.294155	0.241513
22	0.720688	0.421955	0.379701	0.341850	0.307926	0.277505	0.225713
23	0.710037	0.405726	0.363350	0.325571	0.291873	0.261797	0.210947
24	0.699544	0.390121	0.347703	0.310068	0.276657	0.246979	0.197147
25	0.689206	0.375117	0.332731	0.295303	0.262234	0.232999	0.184249
26	0.679021	0.360689	0.318402	0.281241	0.248563	0.219810	0.172195
27	0.668986	0.346817	0.304691	0.267848	0.235605	0.207368	0.160930
28	0.659099	0.333477	0.291571	0.255094	0.223322	0.195630	0.150402
29	0.649359	0.320651	0.279015	0.242946	0.211679	0.184557	0.140563
30	0.639762	0.308319	0.267000	0.231377	0.200644	0.174110	0.131367

n	8.0%	9.0%	10.0%	12.0%	14.0%	16.0%	18.0%
1	0.925926	0.917431	0.909091	0.892857	0.877193	0.862069	0.847458
2	0.857339	0.841680	0.826446	0.797194	0.769468	0.743163	0.718184
3	0.793832	0.772183	0.751315	0.711780	0.674972	0.640658	0.608631
4	0.735030	0.708425	0.683013	0.635518	0.592080	0.552291	0.515789
5	0.680583	0.649931	0.620921	0.567427	0.519369	0.476113	0.437109
6	0.630170	0.596267	0.564474	0.506631	0.455587	0.410442	0.370432
7	0.583490	0.547034	0.513158	0.452349	0.399637	0.353830	0.313925
8	0.540269	0.501866	0.466507	0.403883	0.350559	0.305025	0.266038
9	0.500249	0.460428	0.424098	0.360610	0.307508	0.262953	0.225456
10	0.463193	0.422411	0.385543	0.321973	0.269744	0.226684	0.191064
11	0.428883	0.387533	0.350494	0.287476	0.236617	0.195417	0.161919
12	0.397114	0.355535	0.318631	0.256675	0.207559	0.168463	0.137220
13	0.367698	0.326179	0.289664	0.229174	0.182069	0.145227	0.116288
14	0.340461	0.299246	0.263331	0.204620	0.159710	0.125195	0.098549
15	0.315242	0.274538	0.239392	0.182696	0.140096	0.107927	0.083516
16	0.291890	0.251870	0.217629	0.163122	0.122892	0.093041	0.070776
17	0.270269	0.231073	0.197845	0.145644	0.107800	0.080207	0.059980
18	0.250249	0.211994	0.179859	0.130040	0.094561	0.069144	0.050830
19	0.231712	0.194490	0.163508	0.116107	0.082948	0.059607	0.043077
20	0.214548	0.178431	0.148644	0.103667	0.072762	0.051385	0.036506
21	0.198656	0.163698	0.135131	0.092560	0.063826	0.044298	0.030937
22	0.183941	0.150182	0.122846	0.082643	0.055988	0.038188	0.026218
23	0.170315	0.137781	0.111678	0.073788	0.049112	0.032920	0.022218
24	0.157699	0.126405	0.101526	0.065882	0.043081	0.028380	0.018829
25	0.146018	0.115968	0.092296	0.058823	0.037790	0.024465	0.015957
26	0.135202	0.106393	0.083905	0.052521	0.033149	0.021091	0.013523
27	0.125187	0.097608	0.076278	0.046894	0.029078	0.018182	0.011460
28	0.115914	0.089548	0.069343	0.041869	0.025507	0.015674	0.009712
29	0.107328	0.082155	0.063039	0.037383	0.022375	0.013512	0.008230
30	0.099377	0.075371	0.057309	0.033378	0.019627	0.011648	0.006975

Table 4 PRESENT VALUE OF AN ORDINARY ANNUITY OF 1: $P_{0_{n,i}} = \frac{1 - \frac{1}{(1+i)^n}}{i}$

n	1.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%
1	0.985222	0.961538	0.956938	0.952381	0.947867	0.943396	0.934579
2	1.955883	1.886095	1.872668	1.859410	1.846320	1.833393	1.808018
3	2.912200	2.775091	2.748964	2.723248	2.697933	2.673012	2.624316
4	3.854385	3.629895	3.587526	3.545951	3.505150	3.465106	3.387211
5	4.782645	4.451822	4.389977	4.329477	4.270284	4.212364	4.100197
6	5.697187	5.242137	5.157872	5.075692	4.995530	4.917324	4.766540
7	6.598214	6.002055	5.892701	5.786373	5.682967	5.582381	5.389289
8	7.485925	6.732745	6.595886	6.463213	6.334566	6.209794	5.971299
9	8.360517	7.435332	7.268790	7.107822	6.952195	6.801692	6.515232
10	9.222185	8.110896	7.912718	7.721735	7.537626	7.360087	7.023582
11	10.071118	8.760477	8.528917	8.306414	8.092536	7.886875	7.498674
12	10.907505	9.385074	9.118581	8.863252	8.618518	8.383844	7.942686
13	11.731532	9.985648	9.682852	9.393573	9.117079	8.852683	8.357651
14	12.543382	10.563123	10.222825	9.898641	9.589648	9.294984	8.745468
15	13.343233	11.118387	10.739546	10.379658	10.037581	9.712249	9.107914
16	14.131264	11.652296	11.234015	10.837770	10.462162	10.105895	9.446649
17	14.907649	12.165669	11.707191	11.274066	10.864609	10.477260	9.763223
18	15.672561	12.659297	12.159992	11.689587	11.246074	10.827603	10.059087
19	16.426168	13.133939	12.593294	12.085321	11.607654	11.158116	10.335595
20	17.168639	13.590326	13.007936	12.462210	11.950382	11.469921	10.594014
21	17.900137	14.029160	13.404724	12.821153	12.275244	11.764077	10.835527
22	18.620824	14.451115	13.784425	13.163003	12.583170	12.041582	11.061240
23	19.330861	14.856842	14.147775	13.488574	12.875042	12.303379	11.272187
24	20.030405	15.246963	14.495478	13.798642	13.151699	12.550358	11.469334
25	20.719611	15.622080	14.828209	14.093945	13.413933	12.783356	11.653583
26	21.398632	15.982769	15.146611	14.375185	13.662495	13.003166	11.825779
27	22.067617	16.329586	15.451303	14.643034	13.898100	13.210534	11.986709
28	22.726717	16.663063	15.742874	14.898127	14.121422	13.406164	12.137111
29	23.376076	16.983715	16.021889	15.141074	14.333101	13.590721	12.277674
30	24.015838	17.292033	16.288889	15.372451	14.533745	13.764831	12.409041

n	8.0%	9.0%	10.0%	12.0%	14.0%	16.0%	18.0%
1	0.925926	0.917431	0.909091	0.892857	0.877193	0.862069	0.847458
2	1.783265	1.759111	1.735537	1.690051	1.646661	1.605232	1.565642
3	2.577097	2.531295	2.486852	2.401831	2.321632	2.245890	2.174273
4	3.312127	3.239720	3.169865	3.037349	2.913712	2.798181	2.690062
5	3.992710	3.889651	3.790787	3.604776	3.433081	3.274294	3.127171
6	4.622880	4.485919	4.355261	4.111407	3.888668	3.684736	3.497603
7	5.206370	5.032953	4.868419	4.563757	4.288305	4.038565	3.811528
8	5.746639	5.534819	5.334926	4.967640	4.638864	4.343591	4.077566
9	6.246888	5.995247	5.759024	5.328250	4.946372	4.606544	4.303022
10	6.710081	6.417658	6.144567	5.650223	5.216116	4.833227	4.494086
11	7.138964	6.805191	6.495061	5.937699	5.452733	5.028644	4.656005
12	7.536078	7.160725	6.813692	6.194374	5.660292	5.197107	4.793225
13	7.903776	7.486904	7.103356	6.423548	5.842362	5.342334	4.909513
14	8.244237	7.786150	7.366687	6.628168	6.002072	5.467529	5.008062
15	8.559479	8.060688	7.606080	6.810864	6.142168	5.575456	5.091578
16	8.851369	8.312558	7.823709	6.973986	6.265060	5.668497	5.162354
17	9.121638	8.543631	8.021553	7.119630	6.372859	5.748704	5.222334
18	9.371887	8.755625	8.201412	7.249670	6.467420	5.817848	5.273164
19	9.603599	8.950115	8.364920	7.365777	6.550369	5.877455	5.316241
20	9.818147	9.128546	8.513564	7.469444	6.623131	5.928841	5.352746
21	10.016803	9.292244	8.648694	7.562003	6.686957	5.973139	5.383683
22	10.200744	9.442425	8.771540	7.644646	6.742944	6.011326	5.409901
23	10.371059	9.580207	8.883218	7.718434	6.792056	6.044247	5.432120
24	10.528758	9.706612	8.984744	7.784316	6.835137	6.072627	5.450949
25	10.674776	9.822580	9.077040	7.843139	6.872927	6.097092	5.466906
26	10.809978	9.928972	9.160945	7.895660	6.906077	6.118183	5.480429
27	10.935165	10.026580	9.237223	7.942554	6.935155	6.136364	5.491889
28	11.051078	10.116128	9.306567	7.984423	6.960662	6.152038	5.501601
29	11.158406	10.198283	9.369606	8.021806	6.983037	6.165550	5.509831
30	11.257783	10.273654	9.426914	8.055184	7.002664	6.177198	5.516806

Table 5 PRESENT VALUE OF ANNUITY DUE: $P_{d_{n,i}} = \dfrac{1-\dfrac{1}{(1+i)^{n-1}}}{i}+1$

n	1.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	1.985222	1.961538	1.956938	1.952381	1.947867	1.943396	1.934579
3	2.955883	2.886095	2.872668	2.859410	2.846320	2.833393	2.808018
4	3.912200	3.775091	3.748964	3.723248	3.697933	3.673012	3.624316
5	4.854385	4.629895	4.587526	4.545951	4.505150	4.465106	4.387211
6	5.782645	5.451822	5.389977	5.329477	5.270284	5.212364	5.100197
7	6.697187	6.242137	6.157872	6.075692	5.995530	5.917324	5.766540
8	7.598214	7.002055	6.892701	6.786373	6.682967	6.582381	6.389289
9	8.485925	7.732745	7.595886	7.463213	7.334566	7.209794	6.971299
10	9.360517	8.435332	8.268790	8.107822	7.952195	7.801692	7.515232
11	10.222185	9.110896	8.912718	8.721735	8.537626	8.360087	8.023582
12	11.071118	9.760477	9.528917	9.306414	9.092536	8.886875	8.498674
13	11.907505	10.385074	10.118581	9.863252	9.618518	9.383844	8.942686
14	12.731532	10.985648	10.682852	10.393573	10.117079	9.852683	9.357651
15	13.543382	11.563123	11.222825	10.898641	10.589648	10.294984	9.745468
16	14.343233	12.118387	11.739546	11.379658	11.037581	10.712249	10.107914
17	15.131264	12.652296	12.234015	11.837770	11.462162	11.105895	10.446649
18	15.907649	13.165669	12.707191	12.274066	11.864609	11.477260	10.763223
19	16.672561	13.659297	13.159992	12.689587	12.246074	11.827603	11.059087
20	17.426168	14.133939	13.593294	13.085321	12.607654	12.158116	11.335595
21	18.168639	14.590326	14.007936	13.462210	12.950382	12.469921	11.594014
22	18.900137	15.029160	14.404724	13.821153	13.275244	12.764077	11.835527
23	19.620824	15.451115	14.784425	14.163003	13.583170	13.041582	12.061240
24	20.330861	15.856842	15.147775	14.488574	13.875042	13.303379	12.272187
25	21.030405	16.246963	15.495478	14.798642	14.151699	13.550358	12.469334
26	21.719611	16.622080	15.828209	15.093945	14.413933	13.783356	12.653583
27	22.398632	16.982769	16.146611	15.375185	14.662495	14.003166	12.825779
28	23.067617	17.329586	16.451303	15.643034	14.898100	14.210534	12.986709
29	23.726717	17.663063	16.742874	15.898127	15.121422	14.406164	13.137111
30	24.376076	17.983715	17.021889	16.141074	15.333101	14.590721	13.277674

n	8.0%	9.0%	10.0%	12.0%	14.0%	16.0%	18.0%
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	1.925926	1.917431	1.909091	1.892857	1.877193	1.862069	1.847458
3	2.783265	2.759111	2.735537	2.690051	2.646661	2.605232	2.565642
4	3.577097	3.531295	3.486852	3.401831	3.321632	3.245890	3.174273
5	4.312127	4.239720	4.169865	4.037349	3.913712	3.798181	3.690062
6	4.992710	4.889651	4.790787	4.604776	4.433081	4.274294	4.127171
7	5.622880	5.485919	5.355261	5.111407	4.888668	4.684736	4.497603
8	6.206370	6.032953	5.868419	5.563757	5.288305	5.038565	4.811528
9	6.746639	6.534819	6.334926	5.967640	5.638864	5.343591	5.077566
10	7.246888	6.995247	6.759024	6.328250	5.946372	5.606544	5.303022
11	7.710081	7.417658	7.144567	6.650223	6.216116	5.833227	5.494086
12	8.138964	7.805191	7.495061	6.937699	6.452733	6.028644	5.656005
13	8.536078	8.160725	7.813692	7.194374	6.660292	6.197107	5.793225
14	8.903776	8.486904	8.103356	7.423548	6.842362	6.342334	5.909513
15	9.244237	8.786150	8.366687	7.628168	7.002072	6.467529	6.008062
16	9.559479	9.060688	-8.606080	7.810864	7.142168	6.575456	6.091578
17	9.851369	9.312558	8.823709	7.973986	7.265060	6.668497	6.162354
18	10.121638	9.543631	9.021553	8.119630	7.372859	6.748704	6.222334
19	10.371887	9.755625	9.201412	8.249670	7.467420	6.817848	6.273164
20	10.603599	9.950115	9.364920	8.365777	7.550369	6.877455	6.316241
21	10.818147	10.128546	9.513564	8.469444	7.623131	6.928841	6.352746
22	11.016803	10.292244	9.648694	8.562003	7.686957	6.973139	6.383683
23	11.200744	10.442425	9.771540	8.644646	7.742944	7.011326	6.409901
24	11.371059	10.580207	9.883218	8.718434	7.792056	7.044247	6.432120
25	11.528758	10.706612	9.984744	8.784316	7.835137	7.072627	6.450949
26	11.674776	10.822580	10.077040	8.843139	7.872927	7.097092	6.466906
27	11.809978	10.928972	10.160945	8.895660	7.906077	7.118183	6.480429
28	11.935165	11.026580	10.237223	8.942554	7.935155	7.136364	6.491889
29	12.051078	11.116128	10.306567	8.984423	7.960662	7.152038	6.501601
30	12.158406	11.198283	10.369606	9.021806	7.983037	7.165550	6.509831