

9

NET PRESENT VALUE AND OTHER INVESTMENT CRITERIA

By 2006, the manufacture of large jet airplanes had shrunk to two major competitors, Boeing and Airbus. The competition between the two was stiff. In an effort to increase its market share, Boeing began development of the 787 Dreamliner.

Designed to carry 200 to 300 passengers, the

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Dreamliner was a radical departure from previous airplanes. The lightweight, one-piece, carbon fiber fuselage replaced about 1,200 sheets of aluminum and 40,000 rivets, reducing weight by 15 percent. Additionally, the new engines had larger fans that were expected to reduce fuel consumption by 20 percent. The estimated development cost of the Dreamliner? Over \$8 billion!

Boeing's development of the Dreamliner offers an example of a capital budgeting decision. A product introduction such as this one, with a price tag in the billions, is obviously a major undertaking, and the risks and rewards must be carefully weighed. In this chapter, we discuss the basic tools used in making such decisions.

This chapter introduces you to the practice of capital budgeting. Back in Chapter 1, we saw that increasing the value of the stock in a company is the goal of financial management. Thus, what we need to learn is how to tell whether a particular investment will achieve that. This chapter considers a variety of techniques that are actually used in practice. More important, it shows how many of these techniques can be misleading, and it explains why the net present value approach is the right one.

In Chapter 1, we identified the three key areas of concern to the financial manager. The first of these involved the question: What fixed assets should we buy? We called this the *capital budgeting decision*. In this chapter, we begin to deal with the issues that arise in answering this question.

The process of allocating or budgeting capital is usually more involved than just deciding whether to buy a particular fixed asset. We frequently face broader issues like whether we should launch a new product or enter a new market. Decisions such as these determine the nature of a firm's operations and products for years to come, primarily because fixed asset investments are generally long-lived and not easily reversed once they are made.

The most fundamental decision a business must make concerns its product line. What services will we offer or what will we sell? In what markets will we compete? What new products will we introduce? The answer to any of these questions will require that the firm commit its scarce and valuable capital to certain types of assets. As a result, all of these strategic issues fall under the general heading of capital budgeting. The process of capital budgeting could thus be given a more descriptive (not to mention impressive) name: *strategic asset allocation*.

For the reasons we have discussed, the capital budgeting question is probably the most important issue in corporate finance. How a firm chooses to finance its operations (the capital structure question) and how a firm manages its short-term operating activities (the working capital question) are certainly issues of concern, but the fixed assets define the business of the firm. Airlines, for example, are airlines because they operate airplanes, regardless of how they finance them.

Any firm possesses a huge number of possible investments. Each possible investment is an option available to the firm. Some options are valuable and some are not. The essence of successful financial management, of course, is learning to identify which are which. With this in mind, our goal in this chapter is to introduce you to the techniques used to analyze potential business ventures to decide which are worth undertaking.

We present and compare a number of different procedures used in practice. Our primary goal is to acquaint you with the advantages and disadvantages of the various approaches. As we will see, the most important concept in this area is the idea of net present value. We consider this next.

Net Present Value

9.1

In Chapter 1, we argued that the goal of financial management is to create value for the stockholders. The financial manager must thus examine a potential investment in light of its likely effect on the price of the firm's shares. In this section, we describe a widely used procedure for doing this: The net present value approach.

THE BASIC IDEA

An investment is worth undertaking if it creates value for its owners. In the most general sense, we create value by identifying an investment worth more in the marketplace than it costs us to acquire. How can something be worth more than it costs? It's a case of the whole being worth more than the cost of the parts.

For example, suppose you buy a run-down house for \$25,000 and spend another \$25,000 on painters, plumbers, and so on to get it fixed up. Your total investment is \$50,000. When the work is completed, you place the house back on the market and find that it's worth \$60,000. The market value (\$60,000) exceeds the cost (\$50,000) by \$10,000. What you have done here is to act as a manager and bring together some fixed assets (a house), some labor (plumbers, carpenters, and others), and some materials (carpeting, paint, and so on). The net result is that you have created \$10,000 in value. Put another way, this \$10,000 is the *value added* by management.

With our house example, it turned out *after the fact* that \$10,000 in value had been created. Things thus worked out nicely. The real challenge, of course, would have been to somehow identify *ahead of time* whether investing the necessary \$50,000 was a good idea in the first place. This is what capital budgeting is all about—namely, trying to determine whether a proposed investment or project will be worth more, once it is in place, than it costs.

For reasons that will be obvious in a moment, the difference between an investment's market value and its cost is called the **net present value** of the investment, abbreviated **NPV**. In other words, net present value is a measure of how much value is created or added today by undertaking an investment. Given our goal of creating value for the stockholders, the capital budgeting process can be viewed as a search for investments with positive net present values.

net present value (NPV)

The difference between an investment's market value and its cost.

With our run-down house, you can probably imagine how we would go about making the capital budgeting decision. We would first look at what comparable, fixed-up properties were selling for in the market. We would then get estimates of the cost of buying a particular property and bringing it to market. At this point, we would have an estimated total cost and an estimated market value. If the difference was positive, then this investment would be worth undertaking because it would have a positive estimated net present value. There is risk, of course, because there is no guarantee that our estimates will turn out to be correct.

As our example illustrates, investment decisions are greatly simplified when there is a market for assets similar to the investment we are considering. Capital budgeting becomes much more difficult when we cannot observe the market price for at least roughly comparable investments. The reason is that we then face the problem of estimating the value of an investment using only indirect market information. Unfortunately, this is precisely the situation the financial manager usually encounters. We examine this issue next.

ESTIMATING NET PRESENT VALUE

Imagine we are thinking of starting a business to produce and sell a new product—say organic fertilizer. We can estimate the start-up costs with reasonable accuracy because we know what we will need to buy to begin production. Would this be a good investment? Based on our discussion, you know that the answer depends on whether the value of the new business exceeds the cost of starting it. In other words, does this investment have a positive NPV?

This problem is much more difficult than our “fixer upper” house example because entire fertilizer companies are not routinely bought and sold in the marketplace, so it is essentially impossible to observe the market value of a similar investment. As a result, we must somehow estimate this value by other means.

Based on our work in Chapters 5 and 6, you may be able to guess how we will go about estimating the value of our fertilizer business. We will first try to estimate the future cash flows we expect the new business to produce. We will then apply our basic discounted cash flow procedure to estimate the present value of those cash flows. Once we have this estimate, we will then estimate NPV as the difference between the present value of the future cash flows and the cost of the investment. As we mentioned in Chapter 5, this procedure is often called **discounted cash flow (DCF) valuation**.

To see how we might go about estimating NPV, suppose we believe the cash revenues from our fertilizer business will be \$20,000 per year, assuming everything goes as expected. Cash costs (including taxes) will be \$14,000 per year. We will wind down the business in eight years. The plant, property, and equipment will be worth \$2,000 as salvage at that time. The project costs \$30,000 to launch. We use a 15 percent discount rate on new projects such as this one. Is this a good investment? If there are 1,000 shares of stock outstanding, what will be the effect on the price per share of taking this investment?

From a purely mechanical perspective, we need to calculate the present value of the future cash flows at 15 percent. The net cash inflow will be \$20,000 cash income less \$14,000 in costs per year for eight years. These cash flows are illustrated in Figure 9.1. As Figure 9.1 suggests, we effectively have an eight-year annuity of $\$20,000 - \$14,000 = \$6,000$ per year, along with a single lump sum inflow of \$2,000 in eight years. Calculating the present value of the future cash flows thus comes down to the same type of problem we considered in Chapter 6. The total present value is:

$$\begin{aligned} \text{Present value} &= \$6,000 \times [1 - (1/1.15^8)]/.15 + (2,000/1.15^8) \\ &= (\$6,000 \times 4.4873) + (2,000/3.0590) \\ &= \$26,924 + 654 \\ &= \$27,578 \end{aligned}$$

discounted cash flow (DCF) valuation

The process of valuing an investment by discounting its future cash flows.



Find out more about capital budgeting for small businesses at www.smallbusinesslearning.net.

Time (years)	0	1	2	3	4	5	6	7	8
Initial cost	-\$30								
Inflows		\$ 20	\$ 20	\$ 20	\$ 20	\$ 20	\$ 20	\$ 20	\$ 20
Outflows		-14	-14	-14	-14	-14	-14	-14	-14
Net inflow		\$ 6	\$ 6	\$ 6	\$ 6	\$ 6	\$ 6	\$ 6	\$ 6
Salvage									2
Net cash flow	-\$30	\$ 6	\$ 6	\$ 6	\$ 6	\$ 6	\$ 6	\$ 6	\$ 8

FIGURE 9.1

Project Cash Flows (\$000)

When we compare this to the \$30,000 estimated cost, we see that the NPV is:

$$\text{NPV} = -\$30,000 + 27,578 = -\$2,422$$

Therefore, this is *not* a good investment. Based on our estimates, taking it would *decrease* the total value of the stock by \$2,422. With 1,000 shares outstanding, our best estimate of the impact of taking this project is a loss of value of $\$2,422/1,000 = \2.42 per share.

Our fertilizer example illustrates how NPV estimates can be used to determine whether an investment is desirable. From our example, notice that if the NPV is negative, the effect on share value will be unfavorable. If the NPV were positive, the effect would be favorable. As a consequence, all we need to know about a particular proposal for the purpose of making an accept–reject decision is whether the NPV is positive or negative.

Given that the goal of financial management is to increase share value, our discussion in this section leads us to the *net present value rule*:

An investment should be accepted if the net present value is positive and rejected if it is negative.

In the unlikely event that the net present value turned out to be exactly zero, we would be indifferent between taking the investment and not taking it.

Two comments about our example are in order. First and foremost, it is not the rather mechanical process of discounting the cash flows that is important. Once we have the cash flows and the appropriate discount rate, the required calculations are fairly straightforward. The task of coming up with the cash flows and the discount rate is much more challenging. We will have much more to say about this in the next several chapters. For the remainder of this chapter, we take it as a given that we have estimates of the cash revenues and costs and, where needed, an appropriate discount rate.

The second thing to keep in mind about our example is that the $-\$2,422$ NPV is an estimate. Like any estimate, it can be high or low. The only way to find out the true NPV would be to place the investment up for sale and see what we could get for it. We generally won't be doing this, so it is important that our estimates be reliable. Once again, we will say more about this later. For the rest of this chapter, we will assume the estimates are accurate.

Using the NPV Rule

EXAMPLE 9.1

Suppose we are asked to decide whether a new consumer product should be launched. Based on projected sales and costs, we expect that the cash flows over the five-year life of the project will be \$2,000 in the first two years, \$4,000 in the next two, and \$5,000 in the last year. It will cost about \$10,000 to begin production. We use a 10 percent discount rate to evaluate new products. What should we do here?

(continued)

Given the cash flows and discount rate, we can calculate the total value of the product by discounting the cash flows back to the present:

$$\begin{aligned} \text{Present value} &= (\$2,000/1.1) + (2,000/1.1^2) + (4,000/1.1^3) \\ &\quad + (4,000/1.1^4) + (5,000/1.1^5) \\ &= \$1,818 + 1,653 + 3,005 + 2,732 + 3,105 \\ &= \$12,313 \end{aligned}$$

The present value of the expected cash flows is \$12,313, but the cost of getting those cash flows is only \$10,000, so the NPV is \$12,313 – 10,000 = \$2,313. This is positive; so, based on the net present value rule, we should take on the project.

As we have seen in this section, estimating NPV is one way of assessing the profitability of a proposed investment. It is certainly not the only way profitability is assessed, and we now turn to some alternatives. As we will see, when compared to NPV, each of the alternative ways of assessing profitability that we will examine is flawed in some key way; so NPV is the preferred approach in principle, if not always in practice.

SPREADSHEET STRATEGIES

Calculating NPVs with a Spreadsheet

Spreadsheets are commonly used to calculate NPVs. Examining the use of spreadsheets in this context also allows us to issue an important warning. Let's rework Example 9.1:

	A	B	C	D	E	F	G	H
1								
2	Using a spreadsheet to calculate net present values							
3								
4	From Example 9.1, the project's cost is \$10,000. The cash flows are \$2,000 per year for the first							
5	two years, \$4,000 per year for the next two, and \$5,000 in the last year. The discount rate is							
6	10 percent; what's the NPV?							
7								
8		Year	Cash Flow					
9		0	-\$10,000	Discount rate =		10%		
10		1	2,000					
11		2	2,000		NPV =	\$2,102.72	(wrong answer)	
12		3	4,000		NPV =	\$2,312.99	(right answer)	
13		4	4,000					
14		5	5,000					
15								
16	The formula entered in cell F11 is =NPV(F9, C9:C14). This gives the wrong answer because the							
17	NPV function actually calculates present values, not <i>net</i> present values.							
18								
19	The formula entered in cell F12 is =NPV(F9, C10:C14) + C9. This gives the right answer because the							
20	NPV function is used to calculate the present value of the cash flows and then the initial cost is							
21	subtracted to calculate the answer. Notice that we added cell C9 because it is already negative.							

You can get a freeware NPV calculator at www.wheatworks.com.

In our spreadsheet example, notice that we have provided two answers. By comparing the answers to that found in Example 9.1, we see that the first answer is wrong even though we used the spreadsheet's NPV formula. What happened is that the "NPV" function in our spreadsheet is actually a PV function; unfortunately, one of the original spreadsheet programs many years ago got the definition wrong, and subsequent spreadsheets have copied it! Our second answer shows how to use the formula properly.

The example here illustrates the danger of blindly using calculators or computers without understanding what is going on; we shudder to think of how many capital budgeting decisions in the real world are based on incorrect use of this particular function. We will see another example of something that can go wrong with a spreadsheet later in the chapter.

Concept Questions

9.1a What is the net present value rule?

9.1b If we say an investment has an NPV of \$1,000, what exactly do we mean?

The Payback Rule

9.2

It is common in practice to talk of the payback on a proposed investment. Loosely, the *payback* is the length of time it takes to recover our initial investment or “get our bait back.” Because this idea is widely understood and used, we will examine it in some detail.

DEFINING THE RULE

We can illustrate how to calculate a payback with an example. Figure 9.2 shows the cash flows from a proposed investment. How many years do we have to wait until the accumulated cash flows from this investment equal or exceed the cost of the investment? As Figure 9.2 indicates, the initial investment is \$50,000. After the first year, the firm has recovered \$30,000, leaving \$20,000. The cash flow in the second year is exactly \$20,000, so this investment “pays for itself” in exactly two years. Put another way, the **payback period** is two years. If we require a payback of, say, three years or less, then this investment is acceptable. This illustrates the *payback period rule*:

Based on the payback rule, an investment is acceptable if its calculated payback period is less than some prespecified number of years.

payback period

The amount of time required for an investment to generate cash flows sufficient to recover its initial cost.

In our example, the payback works out to be exactly two years. This won’t usually happen, of course. When the numbers don’t work out exactly, it is customary to work with fractional years. For example, suppose the initial investment is \$60,000, and the cash flows are \$20,000 in the first year and \$90,000 in the second. The cash flows over the first two years are \$110,000, so the project obviously pays back sometime in the second year. After the first year, the project has paid back \$20,000, leaving \$40,000 to be recovered. To figure

Calculating Payback

EXAMPLE 9.2

Here are the projected cash flows from a proposed investment:

Year	Cash Flow
1	\$100
2	200
3	500

This project costs \$500. What is the payback period for this investment?

The initial cost is \$500. After the first two years, the cash flows total \$300. After the third year, the total cash flow is \$800, so the project pays back sometime between the end of year 2 and the end of year 3. Because the accumulated cash flows for the first two years are \$300, we need to recover \$200 in the third year. The third-year cash flow is \$500, so we will have to wait $\$200/\$500 = .4$ year to do this. The payback period is thus 2.4 years, or about two years and five months.

FIGURE 9.2

Net Project Cash Flows

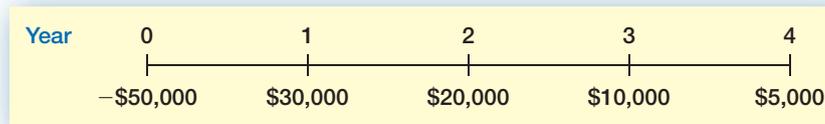


TABLE 9.1

Expected Cash Flows
for Projects A
through E

Year	A	B	C	D	E
0	-\$100	-\$200	-\$200	-\$200	-\$ 50
1	30	40	40	100	100
2	40	20	20	100	-50,000,000
3	50	10	10	-200	
4	60		130	200	

out the fractional year, note that this \$40,000 is $\$40,000/\$90,000 = 4/9$ of the second year's cash flow. Assuming that the \$90,000 cash flow is received uniformly throughout the year, the payback would be $1\frac{4}{9}$ years.

Now that we know how to calculate the payback period on an investment, using the payback period rule for making decisions is straightforward. A particular cutoff time is selected—say, two years—and all investment projects that have payback periods of two years or less are accepted, whereas any that pay off in more than two years are rejected.

Table 9.1 illustrates cash flows for five different projects. The figures shown as the Year 0 cash flows are the costs of the investments. We examine these to indicate some peculiarities that can, in principle, arise with payback periods.

The payback for the first project, A, is easily calculated. The sum of the cash flows for the first two years is \$70, leaving us with $\$100 - 70 = \30 to go. Because the cash flow in the third year is \$50, the payback occurs sometime in that year. When we compare the \$30 we need to the \$50 that will be coming in, we get $\$30/\$50 = .6$; so, payback will occur 60 percent of the way into the year. The payback period is thus 2.6 years.

Project B's payback is also easy to calculate: It *never* pays back because the cash flows never total up to the original investment. Project C has a payback of exactly four years because it supplies the \$130 that B is missing in year 4. Project D is a little strange. Because of the negative cash flow in year 3, you can easily verify that it has two different payback periods, two years and four years. Which of these is correct? Both of them; the way the payback period is calculated doesn't guarantee a single answer. Finally, Project E is obviously unrealistic, but it does pay back in six months, thereby illustrating the point that a rapid payback does not guarantee a good investment.

ANALYZING THE RULE

When compared to the NPV rule, the payback period rule has some rather severe shortcomings. First, we calculate the payback period by simply adding up the future cash flows. There is no discounting involved, so the time value of money is completely ignored. The payback rule also fails to consider any risk differences. The payback would be calculated the same way for both very risky and very safe projects.

Perhaps the biggest problem with the payback period rule is coming up with the right cutoff period: We don't really have an objective basis for choosing a particular number. Put another way, there is no economic rationale for looking at payback in the first place, so we have no guide for how to pick the cutoff. As a result, we end up using a number that is arbitrarily chosen.

Suppose we have somehow decided on an appropriate payback period of two years or less. As we have seen, the payback period rule ignores the time value of money for the first

Year	Long	Short
0	-\$250	-\$250
1	100	100
2	100	200
3	100	0
4	100	0

TABLE 9.2Investment Projected
Cash Flows

two years. More seriously, cash flows after the second year are ignored entirely. To see this, consider the two investments, Long and Short, in Table 9.2. Both projects cost \$250. Based on our discussion, the payback on Long is $2 + (\$50/100) = 2.5$ years, and the payback on Short is $1 + (\$150/200) = 1.75$ years. With a cutoff of two years, Short is acceptable and Long is not.

Is the payback period rule guiding us to the right decisions? Maybe not. Suppose we require a 15 percent return on this type of investment. We can calculate the NPV for these two investments as:

$$\text{NPV}(\text{Short}) = -\$250 + (100/1.15) + (200/1.15^2) = -\$11.81$$

$$\text{NPV}(\text{Long}) = -\$250 + (100 \times \{[1 - (1/1.15^4)]/.15\}) = \$35.50$$

Now we have a problem. The NPV of the shorter-term investment is actually negative, meaning that taking it diminishes the value of the shareholders' equity. The opposite is true for the longer-term investment—it increases share value.

Our example illustrates two primary shortcomings of the payback period rule. First, by ignoring time value, we may be led to take investments (like Short) that actually are worth less than they cost. Second, by ignoring cash flows beyond the cutoff, we may be led to reject profitable long-term investments (like Long). More generally, using a payback period rule will tend to bias us toward shorter-term investments.

REDEEMING QUALITIES OF THE RULE

Despite its shortcomings, the payback period rule is often used by large and sophisticated companies when they are making relatively minor decisions. There are several reasons for this. The primary reason is that many decisions simply do not warrant detailed analysis because the cost of the analysis would exceed the possible loss from a mistake. As a practical matter, it can be said that an investment that pays back rapidly and has benefits extending beyond the cutoff period probably has a positive NPV.

Small investment decisions are made by the hundreds every day in large organizations. Moreover, they are made at all levels. As a result, it would not be uncommon for a corporation to require, for example, a two-year payback on all investments of less than \$10,000. Investments larger than this would be subjected to greater scrutiny. The requirement of a two-year payback is not perfect for reasons we have seen, but it does exercise some control over expenditures and thus limits possible losses.

In addition to its simplicity, the payback rule has two other positive features. First, because it is biased toward short-term projects, it is biased toward liquidity. In other words, a payback rule tends to favor investments that free up cash for other uses quickly. This could be important for a small business; it would be less so for a large corporation. Second, the cash flows that are expected to occur later in a project's life are probably more uncertain. Arguably, a payback period rule adjusts for the extra riskiness of later cash flows, but it does so in a rather draconian fashion—by ignoring them altogether.

We should note here that some of the apparent simplicity of the payback rule is an illusion. The reason is that we still must come up with the cash flows first, and, as we discussed

earlier, this is not at all easy to do. Thus, it would probably be more accurate to say that the *concept* of a payback period is both intuitive and easy to understand.

SUMMARY OF THE RULE

To summarize, the payback period is a kind of “break-even” measure. Because time value is ignored, you can think of the payback period as the length of time it takes to break even in an accounting sense, but not in an economic sense. The biggest drawback to the payback period rule is that it doesn’t ask the right question. The relevant issue is the impact an investment will have on the value of the stock, not how long it takes to recover the initial investment.

Nevertheless, because it is so simple, companies often use it as a screen for dealing with the myriad minor investment decisions they have to make. There is certainly nothing wrong with this practice. As with any simple rule of thumb, there will be some errors in using it; but it wouldn’t have survived all this time if it weren’t useful. Now that you understand the rule, you can be on the alert for circumstances under which it might lead to problems. To help you remember, the following table lists the pros and cons of the payback period rule:

Advantages and Disadvantages of the Payback Period Rule	
Advantages	Disadvantages
1. Easy to understand.	1. Ignores the time value of money.
2. Adjusts for uncertainty of later cash flows.	2. Requires an arbitrary cutoff point.
3. Biased toward liquidity.	3. Ignores cash flows beyond the cutoff date.
	4. Biased against long-term projects, such as research and development, and new projects.

Concept Questions

9.2a In words, what is the payback period? The payback period rule?

9.2b Why do we say that the payback period is, in a sense, an accounting break-even measure?

9.3 The Discounted Payback

We saw that one shortcoming of the payback period rule was that it ignored time value. A variation of the payback period, the discounted payback period, fixes this particular problem. The **discounted payback period** is the length of time until the sum of the discounted cash flows is equal to the initial investment. The *discounted payback rule* would be:

discounted payback period

The length of time required for an investment’s discounted cash flows to equal its initial cost.

Based on the discounted payback rule, an investment is acceptable if its discounted payback is less than some prespecified number of years.

To see how we might calculate the discounted payback period, suppose we require a 12.5 percent return on new investments. We have an investment that costs \$300 and

Year	Cash Flow		Accumulated Cash Flow	
	Undiscounted	Discounted	Undiscounted	Discounted
1	\$100	\$89	\$100	\$ 89
2	100	79	200	168
3	100	70	300	238
4	100	62	400	300
5	100	55	500	355

TABLE 9.3

Ordinary and Discounted Payback

has cash flows of \$100 per year for five years. To get the discounted payback, we have to discount each cash flow at 12.5 percent and then start adding them. We do this in Table 9.3. In Table 9.3, we have both the discounted and the undiscounted cash flows. Looking at the accumulated cash flows, we see that the regular payback is exactly three years (look for the highlighted figure in year 3). The discounted cash flows total \$300 only after four years, however, so the discounted payback is four years, as shown.¹

How do we interpret the discounted payback? Recall that the ordinary payback is the time it takes to break even in an accounting sense. Because it includes the time value of money, the discounted payback is the time it takes to break even in an economic or financial sense. Loosely speaking, in our example, we get our money back, along with the interest we could have earned elsewhere, in four years.

Figure 9.3 illustrates this idea by comparing the *future* value at 12.5 percent of the \$300 investment to the *future* value of the \$100 annual cash flows at 12.5 percent. Notice that the two lines cross at exactly four years. This tells us that the value of the project's cash flows catches up and then passes the original investment in four years.

Table 9.3 and Figure 9.3 illustrate another interesting feature of the discounted payback period. If a project ever pays back on a discounted basis, then it must have a positive NPV.² This is true because, by definition, the NPV is zero when the sum of the discounted cash flows equals the initial investment. For example, the present value of all the cash flows in Table 9.3 is \$355. The cost of the project was \$300, so the NPV is obviously \$55. This \$55 is the value of the cash flow that occurs *after* the discounted payback (see the last line in Table 9.3). In general, if we use a discounted payback rule, we won't accidentally take any projects with a negative estimated NPV.

Based on our example, the discounted payback would seem to have much to recommend it. You may be surprised to find out that it is rarely used in practice. Why? Probably because it really isn't any simpler to use than NPV. To calculate a discounted payback, you have to discount cash flows, add them up, and compare them to the cost, just as you do with NPV. So, unlike an ordinary payback, the discounted payback is not especially simple to calculate.

A discounted payback period rule has a couple of other significant drawbacks. The biggest one is that the cutoff still has to be arbitrarily set, and cash flows beyond that point are ignored.³ As a result, a project with a positive NPV may be found unacceptable because

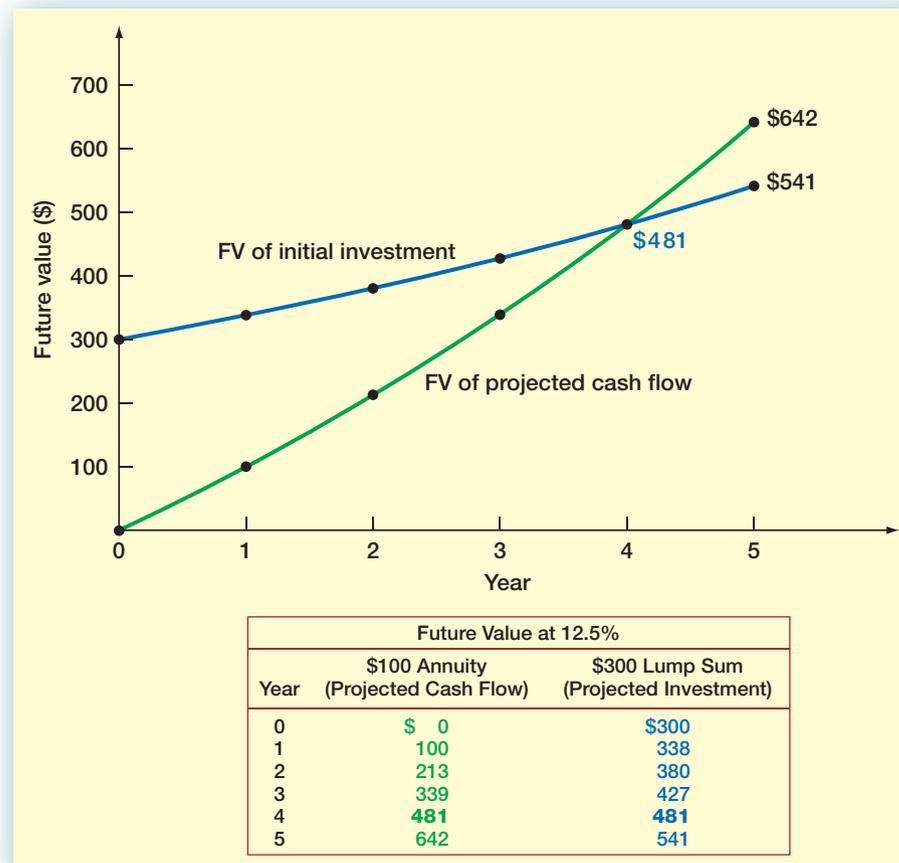
¹In this case, the discounted payback is an even number of years. This won't ordinarily happen, of course. However, calculating a fractional year for the discounted payback period is more involved than it is for the ordinary payback, and it is not commonly done.

²This argument assumes the cash flows, other than the first, are all positive. If they are not, then these statements are not necessarily correct. Also, there may be more than one discounted payback.

³If the cutoff were forever, then the discounted payback rule would be the same as the NPV rule. It would also be the same as the profitability index rule considered in a later section.

FIGURE 9.3

Future Value of Project Cash Flows



the cutoff is too short. Also, just because one project has a shorter discounted payback than another does not mean it has a larger NPV.

All things considered, the discounted payback is a compromise between a regular payback and NPV that lacks the simplicity of the first and the conceptual rigor of the second. Nonetheless, if we need to assess the time it will take to recover the investment required by a project, then the discounted payback is better than the ordinary payback because it considers time value. In other words, the discounted payback recognizes that we could have invested the money elsewhere and earned a return on it. The ordinary payback does not take this into account. The advantages and disadvantages of the discounted payback rule are summarized in the following table:

Advantages and Disadvantages of the Discounted Payback Period Rule	
Advantages	Disadvantages
1. Includes time value of money.	1. May reject positive NPV investments.
2. Easy to understand.	2. Requires an arbitrary cutoff point.
3. Does not accept negative estimated NPV investments.	3. Ignores cash flows beyond the cutoff date.
4. Biased toward liquidity.	4. Biased against long-term projects, such as research and development, and new projects.

Calculating Discounted Payback

EXAMPLE 9.3

Consider an investment that costs \$400 and pays \$100 per year forever. We use a 20 percent discount rate on this type of investment. What is the ordinary payback? What is the discounted payback? What is the NPV?

The NPV and ordinary payback are easy to calculate in this case because the investment is a perpetuity. The present value of the cash flows is $\$100/.2 = \500 , so the NPV is $\$500 - 400 = \100 . The ordinary payback is obviously four years.

To get the discounted payback, we need to find the number of years such that a \$100 annuity has a present value of \$400 at 20 percent. In other words, the present value annuity factor is $\$400/100 = 4$, and the interest rate is 20 percent per period; so what's the number of periods? If we solve for the number of periods, we find that the answer is a little less than nine years, so this is the discounted payback.

Concept Questions

9.3a In words, what is the discounted payback period? Why do we say it is, in a sense, a financial or economic break-even measure?

9.3b What advantage(s) does the discounted payback have over the ordinary payback?

The Average Accounting Return

9.4

Another attractive, but flawed, approach to making capital budgeting decisions involves the **average accounting return (AAR)**. There are many different definitions of the AAR. However, in one form or another, the AAR is always defined as:

$$\frac{\text{Some measure of average accounting profit}}{\text{Some measure of average accounting value}}$$

The specific definition we will use is:

$$\frac{\text{Average net income}}{\text{Average book value}}$$

To see how we might calculate this number, suppose we are deciding whether to open a store in a new shopping mall. The required investment in improvements is \$500,000. The store would have a five-year life because everything reverts to the mall owners after that time. The required investment would be 100 percent depreciated (straight-line) over five years, so the depreciation would be $\$500,000/5 = \$100,000$ per year. The tax rate is 25 percent. Table 9.4 contains the projected revenues and expenses. Net income in each year, based on these figures, is also shown.

To calculate the average book value for this investment, we note that we started out with a book value of \$500,000 (the initial cost) and ended up at \$0. The average book value during the life of the investment is thus $(\$500,000 + 0)/2 = \$250,000$. As long as we use straight-line depreciation, the average investment will always be one-half of the initial investment.⁴

average accounting return (AAR)

An investment's average net income divided by its average book value.

⁴We could, of course, calculate the average of the six book values directly. In thousands, we would have $(\$500 + 400 + 300 + 200 + 100 + 0)/6 = \250 .

TABLE 9.4

Projected Yearly Revenue and Costs for Average Accounting Return

	Year 1	Year 2	Year 3	Year 4	Year 5
Revenue	\$433,333	\$450,000	\$266,667	\$200,000	\$133,333
Expenses	\$200,000	\$150,000	\$100,000	\$100,000	\$100,000
Earnings before Depreciation	\$233,333	\$300,000	\$166,667	\$100,000	\$ 33,333
Depreciation	\$100,000	\$100,000	\$100,000	\$100,000	\$100,000
Earnings before Taxes	\$133,333	\$200,000	\$ 66,667	\$ 0	-\$ 66,667
Taxes (25%)	33,333	50,000	16,667	0	- 16,667
Net Income	\$100,000	\$150,000	\$ 50,000	\$ 0	-\$ 50,000

$$\text{Average net income} = \frac{\$100,000 + 150,000 + 50,000 + 0 - 50,000}{5} = \$50,000$$

$$\text{Average book value} = \frac{\$500,000 + 0}{2} = \$250,000$$

Looking at Table 9.4, we see that net income is \$100,000 in the first year, \$150,000 in the second year, \$50,000 in the third year, \$0 in Year 4, and -\$50,000 in Year 5. The average net income, then, is:

$$[\$100,000 + 150,000 + 50,000 + 0 + (-50,000)]/5 = \$50,000$$

The average accounting return is:

$$\text{AAR} = \frac{\text{Average net income}}{\text{Average book value}} = \frac{\$50,000}{\$250,000} = 20\%$$

If the firm has a target AAR of less than 20 percent, then this investment is acceptable; otherwise it is not. The *average accounting return rule* is thus:

Based on the average accounting return rule, a project is acceptable if its average accounting return exceeds a target average accounting return.

As we will now see, the use of this rule has a number of problems.

You should recognize the chief drawback to the AAR immediately. Above all else, the AAR is not a rate of return in any meaningful economic sense. Instead, it is the ratio of two accounting numbers, and it is not comparable to the returns offered, for example, in financial markets.⁵

One of the reasons the AAR is not a true rate of return is that it ignores time value. When we average figures that occur at different times, we are treating the near future and the more distant future in the same way. There was no discounting involved when we computed the average net income, for example.

The second problem with the AAR is similar to the problem we had with the payback period rule concerning the lack of an objective cutoff period. Because a calculated AAR is really not comparable to a market return, the target AAR must somehow be specified. There is no generally agreed-upon way to do this. One way of doing it is to calculate the AAR for the firm as a whole and use this as a benchmark, but there are lots of other ways as well.

⁵The AAR is closely related to the return on assets (ROA) discussed in Chapter 3. In practice, the AAR is sometimes computed by first calculating the ROA for each year and then averaging the results. This produces a number that is similar, but not identical, to the one we computed.

The third, and perhaps worst, flaw in the AAR is that it doesn't even look at the right things. Instead of cash flow and market value, it uses net income and book value. These are both poor substitutes. As a result, an AAR doesn't tell us what the effect on share price will be of taking an investment, so it doesn't tell us what we really want to know.

Does the AAR have any redeeming features? About the only one is that it almost always can be computed. The reason is that accounting information will almost always be available, both for the project under consideration and for the firm as a whole. We hasten to add that once the accounting information is available, we can always convert it to cash flows, so even this is not a particularly important fact. The AAR is summarized in the following table:

Advantages and Disadvantages of the Average Accounting Return	
Advantages	Disadvantages
1. Easy to calculate. 2. Needed information will usually be available.	1. Not a true rate of return; time value of money is ignored. 2. Uses an arbitrary benchmark cutoff rate. 3. Based on accounting (book) values, not cash flows and market values.

Concept Questions

- 9.4a What is an average accounting rate of return (AAR)?
- 9.4b What are the weaknesses of the AAR rule?

The Internal Rate of Return

We now come to the most important alternative to NPV, the **internal rate of return**, universally known as the **IRR**. As we will see, the IRR is closely related to NPV. With the IRR, we try to find a single rate of return that summarizes the merits of a project. Furthermore, we want this rate to be an "internal" rate in the sense that it depends only on the cash flows of a particular investment, not on rates offered elsewhere.

To illustrate the idea behind the IRR, consider a project that costs \$100 today and pays \$110 in one year. Suppose you were asked, "What is the return on this investment?" What would you say? It seems both natural and obvious to say that the return is 10 percent because, for every dollar we put in, we get \$1.10 back. In fact, as we will see in a moment, 10 percent is the internal rate of return, or IRR, on this investment.

Is this project with its 10 percent IRR a good investment? Once again, it would seem apparent that this is a good investment only if our required return is less than 10 percent. This intuition is also correct and illustrates the *IRR rule*:

Based on the IRR rule, an investment is acceptable if the IRR exceeds the required return. It should be rejected otherwise.

Imagine that we want to calculate the NPV for our simple investment. At a discount rate of R , the NPV is:

$$NPV = -\$100 + [110/(1 + R)]$$

9.5

internal rate of return (IRR)

The discount rate that makes the NPV of an investment zero.

Now, suppose we don't know the discount rate. This presents a problem, but we can still ask how high the discount rate would have to be before this project was deemed unacceptable. We know that we are indifferent between taking and not taking this investment when its NPV is just equal to zero. In other words, this investment is *economically* a break-even proposition when the NPV is zero because value is neither created nor destroyed. To find the break-even discount rate, we set NPV equal to zero and solve for R :

$$\text{NPV} = 0 = -\$100 + [110/(1 + R)]$$

$$\$100 = \$110/(1 + R)$$

$$1 + R = \$110/\$100 = 1.1$$

$$R = 10\%$$

This 10 percent is what we already have called the return on this investment. What we have now illustrated is that the internal rate of return on an investment (or just "return" for short) is the discount rate that makes the NPV equal to zero. This is an important observation, so it bears repeating:

The IRR on an investment is the required return that results in a zero NPV when it is used as the discount rate.

The fact that the IRR is simply the discount rate that makes the NPV equal to zero is important because it tells us how to calculate the returns on more complicated investments. As we have seen, finding the IRR turns out to be relatively easy for a single-period investment. However, suppose you were now looking at an investment with the cash flows shown in Figure 9.4. As illustrated, this investment costs \$100 and has a cash flow of \$60 per year for two years, so it's only slightly more complicated than our single-period example. However, if you were asked for the return on this investment, what would you say? There doesn't seem to be any obvious answer (at least not to us). However, based on what we now know, we can set the NPV equal to zero and solve for the discount rate:

$$\text{NPV} = 0 = -\$100 + [60/(1 + \text{IRR})] + [60/(1 + \text{IRR})^2]$$

Unfortunately, the only way to find the IRR in general is by trial and error, either by hand or by calculator. This is precisely the same problem that came up in Chapter 5 when we found the unknown rate for an annuity and in Chapter 7 when we found the yield to maturity on a bond. In fact, we now see that in both of those cases, we were finding an IRR.

In this particular case, the cash flows form a two-period, \$60 annuity. To find the unknown rate, we can try some different rates until we get the answer. If we were to start with a 0 percent rate, the NPV would obviously be $\$120 - 100 = \20 . At a 10 percent discount rate, we would have:

$$\text{NPV} = -\$100 + (60/1.1) + (60/1.1^2) = \$4.13$$

FIGURE 9.4

Project Cash Flows



Discount Rate	NPV
0%	\$20.00
5%	11.56
10%	4.13
15%	- 2.46
20%	- 8.33

TABLE 9.5

NPV at Different Discount Rates

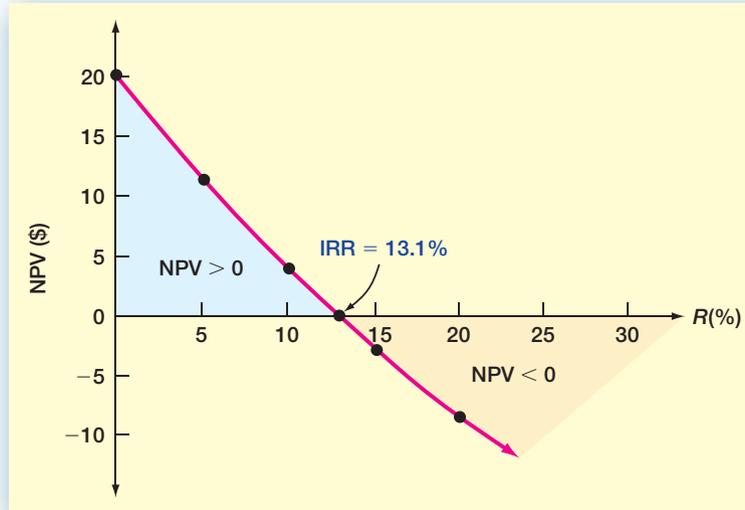


FIGURE 9.5

An NPV Profile

Now, we're getting close. We can summarize these and some other possibilities as shown in Table 9.5. From our calculations, the NPV appears to be zero with a discount rate between 10 percent and 15 percent, so the IRR is somewhere in that range. With a little more effort, we can find that the IRR is about 13.1 percent.⁶ So, if our required return were less than 13.1 percent, we would take this investment. If our required return exceeded 13.1 percent, we would reject it.

By now, you have probably noticed that the IRR rule and the NPV rule appear to be quite similar. In fact, the IRR is sometimes simply called the *discounted cash flow*, or *DCF*, *return*. The easiest way to illustrate the relationship between NPV and IRR is to plot the numbers we calculated for Table 9.5. We put the different NPVs on the vertical axis, or *y*-axis, and the discount rates on the horizontal axis, or *x*-axis. If we had a very large number of points, the resulting picture would be a smooth curve called a **net present value profile**. Figure 9.5 illustrates the NPV profile for this project. Beginning with a 0 percent discount rate, we have \$20 plotted directly on the *y*-axis. As the discount rate increases, the NPV declines smoothly. Where will the curve cut through the *x*-axis? This will occur where the NPV is just equal to zero, so it will happen right at the IRR of 13.1 percent.

In our example, the NPV rule and the IRR rule lead to identical accept–reject decisions. We will accept an investment using the IRR rule if the required return is less than 13.1 percent. As Figure 9.5 illustrates, however, the NPV is positive at any discount rate less than 13.1 percent, so we would accept the investment using the NPV rule as well. The two rules give equivalent results in this case.

net present value profile

A graphical representation of the relationship between an investment's NPVs and various discount rates.

⁶With a lot more effort (or a personal computer), we can find that the IRR is approximately (to 9 decimal places) 13.066238629 percent—not that anybody would ever want this many decimal places!

EXAMPLE 9.4

Calculating the IRR

A project has a total up-front cost of \$435.44. The cash flows are \$100 in the first year, \$200 in the second year, and \$300 in the third year. What's the IRR? If we require an 18 percent return, should we take this investment?

We'll describe the NPV profile and find the IRR by calculating some NPVs at different discount rates. You should check our answers for practice. Beginning with 0 percent, we have:

Discount Rate	NPV
0%	\$164.56
5%	100.36
10%	46.15
15%	0.00
20%	– 39.61

The NPV is zero at 15 percent, so 15 percent is the IRR. If we require an 18 percent return, then we should not take the investment. The reason is that the NPV is negative at 18 percent (verify that it is $-\$24.47$). The IRR rule tells us the same thing in this case. We shouldn't take this investment because its 15 percent return is below our required 18 percent return.

At this point, you may be wondering if the IRR and NPV rules always lead to identical decisions. The answer is yes, as long as two very important conditions are met. First, the project's cash flows must be *conventional*, meaning that the first cash flow (the initial investment) is negative and all the rest are positive. Second, the project must be *independent*, meaning that the decision to accept or reject this project does not affect the decision to accept or reject any other. The first of these conditions is typically met, but the second often is not. In any case, when one or both of these conditions are not met, problems can arise. We discuss some of these next.

SPREADSHEET STRATEGIES

Calculating IRRs with a Spreadsheet

Because IRRs are so tedious to calculate by hand, financial calculators and especially spreadsheets are generally used. The procedures used by various financial calculators are too different for us to illustrate here, so we will focus on using a spreadsheet (financial calculators are covered in Appendix D). As the following example illustrates, using a spreadsheet is easy.

	A	B	C	D	E	F	G	H
1								
2	Using a spreadsheet to calculate internal rates of return							
3								
4	Suppose we have a four-year project that costs \$500. The cash flows over the four-year life will be							
5	\$100, \$200, \$300, and \$400. What is the IRR?							
6								
7		Year	Cash Flow					
8		0	-\$500					
9		1	100		IRR =	27.3%		
10		2	200					
11		3	300					
12		4	400					
13								
14								
15	The formula entered in cell F9 is =IRR(C8:C12). Notice that the year 0 cash flow has a negative							
16	sign representing the initial cost of the project.							
17								

PROBLEMS WITH THE IRR

The problems with the IRR come about when the cash flows are not conventional or when we are trying to compare two or more investments to see which is best. In the first case, surprisingly, the simple question: What's the return? can become difficult to answer. In the second case, the IRR can be a misleading guide.

Nonconventional Cash Flows Suppose we have a strip-mining project that requires a \$60 investment. Our cash flow in the first year will be \$155. In the second year, the mine will be depleted, but we will have to spend \$100 to restore the terrain. As Figure 9.6 illustrates, both the first and third cash flows are negative.

To find the IRR on this project, we can calculate the NPV at various rates:

Discount Rate	NPV
0%	-\$5.00
10%	- 1.74
20%	- 0.28
30%	0.06
40%	- 0.31

The NPV appears to be behaving in a peculiar fashion here. First, as the discount rate increases from 0 percent to 30 percent, the NPV starts out negative and becomes positive. This seems backward because the NPV is rising as the discount rate rises. It then starts getting smaller and becomes negative again. What's the IRR? To find out, we draw the NPV profile as shown in Figure 9.7.

In Figure 9.7, notice that the NPV is zero when the discount rate is 25 percent, so this is the IRR. Or is it? The NPV is also zero at $33\frac{1}{3}$ percent. Which of these is correct? The answer is both or neither; more precisely, there is no unambiguously correct answer. This is the **multiple rates of return** problem. Many financial computer packages (including a best-seller for personal computers) aren't aware of this problem and just report the first IRR that is found. Others report only the smallest positive IRR, even though this answer is no better than any other.

In our current example, the IRR rule breaks down completely. Suppose our required return is 10 percent. Should we take this investment? Both IRRs are greater than 10 percent, so, by the IRR rule, maybe we should. However, as Figure 9.7 shows, the NPV is negative at any discount rate less than 25 percent, so this is not a good investment. When should we take it? Looking at Figure 9.7 one last time, we see that the NPV is positive only if our required return is between **25 percent** and **$33\frac{1}{3}$ percent**.

Nonconventional cash flows can occur in a variety of ways. For example, Northeast Utilities, owner of the Connecticut-located Millstone nuclear power plant, had to shut down the plant's three reactors in November 1995. The reactors were expected to be back online in January 1997. By some estimates, the cost of the shutdown would run about \$334 million. In fact, all nuclear plants eventually have to be shut down forever, and the costs associated with decommissioning a plant are enormous, creating large negative cash flows at the end of the project's life.



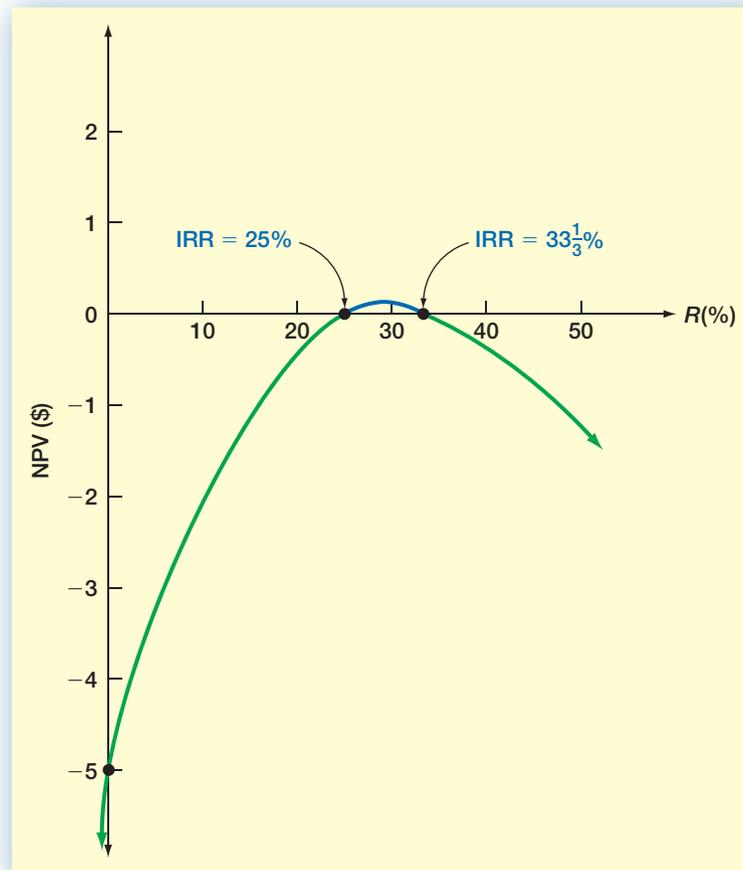
multiple rates of return

The possibility that more than one discount rate will make the NPV of an investment zero.

FIGURE 9.6

Project Cash Flows

FIGURE 9.7
NPV Profile



The moral of the story is that when the cash flows aren't conventional, strange things can start to happen to the IRR. This is not anything to get upset about, however, because the NPV rule, as always, works just fine. This illustrates the fact that, oddly enough, the obvious question—What's the rate of return?—may not always have a good answer.

EXAMPLE 9.5

What's the IRR?

You are looking at an investment that requires you to invest \$51 today. You'll get \$100 in one year, but you must pay out \$50 in two years. What is the IRR on this investment?

You're on the alert now for the nonconventional cash flow problem, so you probably wouldn't be surprised to see more than one IRR. However, if you start looking for an IRR by trial and error, it will take you a long time. The reason is that there is no IRR. The NPV is negative at every discount rate, so we shouldn't take this investment under any circumstances. What's the return on this investment? Your guess is as good as ours.

“I Think; Therefore, I Know How Many IRRs There Can Be.”**EXAMPLE 9.6**

We've seen that it's possible to get more than one IRR. If you wanted to make sure that you had found all of the possible IRRs, how could you do it? The answer comes from the great mathematician, philosopher, and financial analyst Descartes (of “I think; therefore I am” fame). Descartes' Rule of Sign says that the maximum number of IRRs that there can be is equal to the number of times that the cash flows change sign from positive to negative and/or negative to positive.⁷

In our example with the 25 percent and 33 $\frac{1}{3}$ percent IRRs, could there be yet another IRR? The cash flows flip from negative to positive, then back to negative, for a total of two sign changes. Therefore, according to Descartes' rule, the maximum number of IRRs is two and we don't need to look for any more. Note that the actual number of IRRs can be less than the maximum (see Example 9.5).

Mutually Exclusive Investments Even if there is a single IRR, another problem can arise concerning **mutually exclusive investment decisions**. If two investments, X and Y, are mutually exclusive, then taking one of them means that we cannot take the other. Two projects that are not mutually exclusive are said to be independent. For example, if we own one corner lot, then we can build a gas station or an apartment building, but not both. These are mutually exclusive alternatives.

mutually exclusive investment decisions

A situation in which taking one investment prevents the taking of another.

Thus far, we have asked whether a given investment is worth undertaking. However, a related question comes up often: Given two or more mutually exclusive investments, which one is the best? The answer is simple enough: The best one is the one with the largest NPV. Can we also say that the best one has the highest return? As we show, the answer is no.

To illustrate the problem with the IRR rule and mutually exclusive investments, consider the following cash flows from two mutually exclusive investments:

Year	Investment A	Investment B
0	-\$100	-\$100
1	50	20
2	40	40
3	40	50
4	30	60

The IRR for A is 24 percent, and the IRR for B is 21 percent. Because these investments are mutually exclusive, we can take only one of them. Simple intuition suggests that investment A is better because of its higher return. Unfortunately, simple intuition is not always correct.

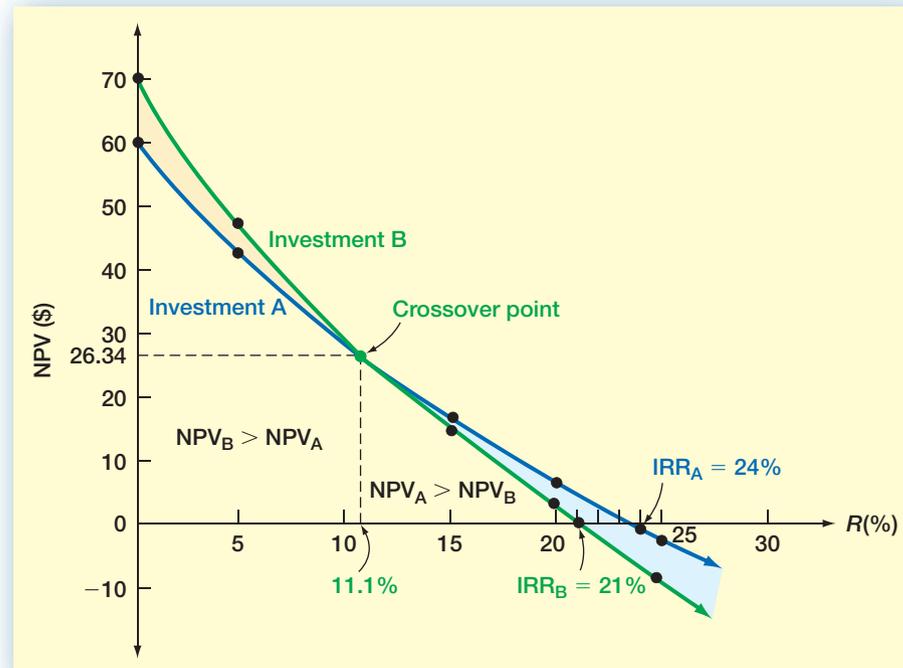
To see why investment A is not necessarily the better of the two investments, we've calculated the NPV of these investments for different required returns:

Discount Rate	NPV(A)	NPV(B)
0%	\$60.00	\$70.00
5	43.13	47.88
10	29.06	29.79
15	17.18	14.82
20	7.06	2.31
25	- 1.63	- 8.22

⁷To be more precise, the number of IRRs that are bigger than -100 percent is equal to the number of sign changes, or it differs from the number of sign changes by an even number. Thus, for example, if there are five sign changes, there are five IRRs, three IRRs, or one IRR. If there are two sign changes, there are either two IRRs or no IRRs.

FIGURE 9.8

NPV Profiles for Mutually Exclusive Investments



The IRR for A (24 percent) is larger than the IRR for B (21 percent). However, if you compare the NPVs, you'll see that which investment has the higher NPV depends on our required return. B has greater total cash flow, but it pays back more slowly than A. As a result, it has a higher NPV at lower discount rates.

In our example, the NPV and IRR rankings conflict for some discount rates. If our required return is 10 percent, for instance, then B has the higher NPV and is thus the better of the two even though A has the higher return. If our required return is 15 percent, then there is no ranking conflict: A is better.

The conflict between the IRR and NPV for mutually exclusive investments can be illustrated by plotting the investments' NPV profiles as we have done in Figure 9.8. In Figure 9.8, notice that the NPV profiles cross at about 11 percent. Notice also that at any discount rate less than 11 percent, the NPV for B is higher. In this range, taking B benefits us more than taking A, even though A's IRR is higher. At any rate greater than 11 percent, investment A has the greater NPV.

This example illustrates that when we have mutually exclusive projects, we shouldn't rank them based on their returns. More generally, anytime we are comparing investments to determine which is best, looking at IRRs can be misleading. Instead, we need to look at the relative NPVs to avoid the possibility of choosing incorrectly. Remember, we're ultimately interested in creating value for the shareholders, so the option with the higher NPV is preferred, regardless of the relative returns.

If this seems counterintuitive, think of it this way. Suppose you have two investments. One has a 10 percent return and makes you \$100 richer immediately. The other has a 20 percent return and makes you \$50 richer immediately. Which one do you like better? We would rather have \$100 than \$50, regardless of the returns, so we like the first one better.

Calculating the Crossover Rate

EXAMPLE 9.7

In Figure 9.8, the NPV profiles cross at about 11 percent. How can we determine just what this crossover point is? The *crossover rate*, by definition, is the discount rate that makes the NPVs of two projects equal. To illustrate, suppose we have the following two mutually exclusive investments:

Year	Investment A	Investment B
0	-\$400	-\$500
1	250	320
2	280	340

What's the crossover rate?

To find the crossover, first consider moving out of investment A and into investment B. If you make the move, you'll have to invest an extra \$100 (= \$500 - 400). For this \$100 investment, you'll get an extra \$70 (= \$320 - 250) in the first year and an extra \$60 (= \$340 - 280) in the second year. Is this a good move? In other words, is it worth investing the extra \$100?

Based on our discussion, the NPV of the switch, $NPV(B - A)$, is:

$$NPV(B - A) = -\$100 + [70/(1 + R)] + [60/(1 + R)^2]$$

We can calculate the return on this investment by setting the NPV equal to zero and solving for the IRR:

$$NPV(B - A) = 0 = -\$100 + [70/(1 + R)] + [60/(1 + R)^2]$$

If you go through this calculation, you will find the IRR is exactly 20 percent. What this tells us is that at a 20 percent discount rate, we are indifferent between the two investments because the NPV of the difference in their cash flows is zero. As a consequence, the two investments have the same value, so this 20 percent is the crossover rate. Check to see that the NPV at 20 percent is \$2.78 for both investments.

In general, you can find the crossover rate by taking the difference in the cash flows and calculating the IRR using the difference. It doesn't make any difference which one you subtract from which. To see this, find the IRR for $(A - B)$; you'll see it's the same number. Also, for practice, you might want to find the exact crossover in Figure 9.8. (*Hint: It's 11.0704 percent.*)

REDEEMING QUALITIES OF THE IRR

Despite its flaws, the IRR is very popular in practice—more so than even the NPV. It probably survives because it fills a need that the NPV does not. In analyzing investments, people in general, and financial analysts in particular, seem to prefer talking about rates of return rather than dollar values.

In a similar vein, the IRR also appears to provide a simple way of communicating information about a proposal. One manager might say to another, "Remodeling the clerical wing has a 20 percent return." This may somehow seem simpler than saying, "At a 10 percent discount rate, the net present value is \$4,000."

Finally, under certain circumstances, the IRR may have a practical advantage over the NPV. We can't estimate the NPV unless we know the appropriate discount rate, but we can still estimate the IRR. Suppose we didn't know the required return on an investment, but we found, for example, that it had a 40 percent return. We would probably be inclined to take it because it would be unlikely that the required return would be that high. The

advantages and disadvantages of the IRR are summarized as follows:

Advantages and Disadvantages of the Internal Rate of Return	
Advantages	Disadvantages
1. Closely related to NPV, often leading to identical decisions. 2. Easy to understand and communicate.	1. May result in multiple answers or not deal with nonconventional cash flows. 2. May lead to incorrect decisions in comparisons of mutually exclusive investments.

THE MODIFIED INTERNAL RATE OF RETURN (MIRR)

To address some of the problems that can crop up with the standard IRR, it is often proposed that a modified version be used. As we will see, there are several different ways of calculating a modified IRR, or MIRR, but the basic idea is to modify the cash flows first and then calculate an IRR using the modified cash flows.

To illustrate, let's go back to the cash flows in Figure 9.6: $-\$60$, $+\$155$, and $-\$100$. As we saw, there are two IRRs, 25 percent and $33\frac{1}{3}$ percent. We next illustrate three different MIRRs, all of which have the property that only one answer will result, thereby eliminating the multiple IRR problem.

Method #1: The Discounting Approach With the discounting approach, the idea is to discount all negative cash flows back to the present at the required return and add them to the initial cost. Then, calculate the IRR. Because only the first modified cash flow is negative, there will be only one IRR. The discount rate used might be the required return, or it might be some other externally supplied rate. We will use the project's required return.

If the required return on the project is 20 percent, then the modified cash flows look like this:

$$\text{Time 0: } -\$60 + \frac{-\$100}{1.20^2} = -\$129.44$$

$$\text{Time 1: } +\$155$$

$$\text{Time 2: } +\$0$$

If you calculate the MIRR now, you should get 19.71 percent.

Method #2: The Reinvestment Approach With the reinvestment approach, we compound *all* cash flows (positive and negative) except the first out to the end of the project's life and then calculate the IRR. In a sense, we are "reinvesting" the cash flows and not taking them out of the project until the very end. The rate we use could be the required return on the project, or it could be a separately specified "reinvestment rate." We will use the project's required return. When we do, here are the modified cash flows:

$$\text{Time 0: } -\$60$$

$$\text{Time 1: } +0$$

$$\text{Time 2: } -\$100 + (\$155 \times 1.2) = \$86$$

The MIRR on this set of cash flows is 19.72 percent, or a little higher than we got using the discounting approach.

Method #3: The Combination Approach As the name suggests, the combination approach blends our first two methods. Negative cash flows are discounted back to the present, and positive cash flows are compounded to the end of the project. In practice, different discount or compounding rates might be used, but we will again stick with the project's required return.

With the combination approach, the modified cash flows are as follows:

$$\text{Time 0: } -\$60 + \frac{-\$100}{1.20^2} = -\$129.44$$

$$\text{Time 1: } +0$$

$$\text{Time 2: } \$155 \times 1.2 = \$186$$

See if you don't agree that the MIRR is 19.87 percent, the highest of the three.

MIRR or IRR: Which Is Better? MIRRs are controversial. At one extreme are those who claim that MIRRs are superior to IRRs, period. For example, by design, they clearly don't suffer from the multiple rate of return problem.

At the other end, detractors say that MIRR should stand for "meaningless internal rate of return." As our example makes clear, one problem with MIRRs is that there are different ways of calculating them, and there is no clear reason to say one of our three methods is better than any other. The differences are small with our simple cash flows, but they could be much larger for a more complex project. Further, it's not clear how to interpret an MIRR. It may look like a rate of return; but it's a rate of return on a modified set of cash flows, not the project's actual cash flows.

We're not going to take sides. However, notice that calculating an MIRR requires discounting, compounding, or both, which leads to two obvious observations. First, if we have the relevant discount rate, why not calculate the NPV and be done with it? Second, because an MIRR depends on an externally supplied discount (or compounding) rate, the answer you get is not truly an "internal" rate of return, which, by definition, depends on only the project's cash flows.

We *will* take a stand on one issue that frequently comes up in this context. The value of a project does not depend on what the firm does with the cash flows generated by that project. A firm might use a project's cash flows to fund other projects, to pay dividends, or to buy an executive jet. It doesn't matter: How the cash flows are spent in the future does not affect their value today. As a result, there is generally no need to consider reinvestment of interim cash flows.

Concept Questions

- 9.5a** Under what circumstances will the IRR and NPV rules lead to the same accept-reject decisions? When might they conflict?
- 9.5b** Is it generally true that an advantage of the IRR rule over the NPV rule is that we don't need to know the required return to use the IRR rule?

The Profitability Index

Another tool used to evaluate projects is called the **profitability index (PI)** or benefit-cost ratio. This index is defined as the present value of the future cash flows divided by the initial investment. So, if a project costs \$200 and the present value of its future cash flows is \$220, the profitability index value would be $\$220/\$200 = 1.1$. Notice that the NPV for this investment is \$20, so it is a desirable investment.

More generally, if a project has a positive NPV, then the present value of the future cash flows must be bigger than the initial investment. The profitability index would thus be bigger than 1 for a positive NPV investment and less than 1 for a negative NPV investment.

How do we interpret the profitability index? In our example, the PI was 1.1. This tells us that, per dollar invested, \$1.10 in value or \$.10 in NPV results. The profitability index

9.6

profitability index (PI)

The present value of an investment's future cash flows divided by its initial cost. Also called the *benefit-cost ratio*.

thus measures “bang for the buck”—that is, the value created per dollar invested. For this reason, it is often proposed as a measure of performance for government or other not-for-profit investments. Also, when capital is scarce, it may make sense to allocate it to projects with the highest PIs. We will return to this issue in a later chapter.

The PI is obviously similar to the NPV. However, consider an investment that costs \$5 and has a \$10 present value and an investment that costs \$100 with a \$150 present value. The first of these investments has an NPV of \$5 and a PI of 2. The second has an NPV of \$50 and a PI of 1.5. If these are mutually exclusive investments, then the second one is preferred even though it has a lower PI. This ranking problem is similar to the IRR ranking problem we saw in the previous section. In all, there seems to be little reason to rely on the PI instead of the NPV. Our discussion of the PI is summarized as follows:

Advantages and Disadvantages of the Profitability Index	
Advantages	Disadvantages
<ol style="list-style-type: none"> 1. Closely related to NPV, generally leading to identical decisions. 2. Easy to understand and communicate. 3. May be useful when available investment funds are limited. 	<ol style="list-style-type: none"> 1. May lead to incorrect decisions in comparisons of mutually exclusive investments.

Concept Questions

9.6a What does the profitability index measure?

9.6b How would you state the profitability index rule?

9.7 The Practice of Capital Budgeting

Given that NPV seems to be telling us directly what we want to know, you might be wondering why there are so many other procedures and why alternative procedures are commonly used. Recall that we are trying to make an investment decision and that we are frequently operating under considerable uncertainty about the future. We can only *estimate* the NPV of an investment in this case. The resulting estimate can be very “soft,” meaning that the true NPV might be quite different.

Because the true NPV is unknown, the astute financial manager seeks clues to help in assessing whether the estimated NPV is reliable. For this reason, firms would typically use multiple criteria for evaluating a proposal. For example, suppose we have an investment with a positive estimated NPV. Based on our experience with other projects, this one appears to have a short payback and a very high AAR. In this case, the different indicators seem to agree that it’s “all systems go.” Put another way, the payback and the AAR are consistent with the conclusion that the NPV is positive.

On the other hand, suppose we had a positive estimated NPV, a long payback, and a low AAR. This could still be a good investment, but it looks like we need to be much more careful in making the decision because we are getting conflicting signals. If the estimated NPV is based on projections in which we have little confidence, then further analysis is probably in order. We will consider how to evaluate NPV estimates in more detail in the next two chapters.

Capital expenditures by individual corporations can add up to enormous sums for the economy as a whole. For example, ExxonMobil announced plans to increase its capital spending by about 35 percent in 2006, to \$11 billion. Auto manufacturer Toyota announced

TABLE 9.6 Capital Budgeting Techniques in Practice

A. Historical Comparison of the Primary Use of Various Capital Budgeting Techniques							
	1959	1964	1970	1975	1977	1979	1981
Payback period	34%	24%	12%	15%	9%	10%	5.0%
Average accounting return (AAR)	34	30	26	10	25	14	10.7
Internal rate of return (IRR)	19	38	57	37	54	60	65.3
Net present value (NPV)	—	—	—	26	10	14	16.5
IRR or NPV	19	38	57	63	64	74	81.8
B. Percentage of CFOs Who Always or Almost Always Used a Given Technique in 1999							
Capital Budgeting Technique	Percentage Always or Almost Always Using	Average Score [Scale is 4 (always) to 0 (never).]					
		Overall	Large Firms	Small Firms			
Internal rate of return	76%	3.09	3.41	2.87			
Net present value	75	3.08	3.42	2.83			
Payback period	57	2.53	2.25	2.72			
Discounted payback period	29	1.56	1.55	1.58			
Accounting rate of return	20	1.34	1.25	1.41			
Profitability index	12	0.83	0.75	0.88			

SOURCES: J.R. Graham and C.R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics*, May–June 2001, pp. 187–244; J.S. Moore and A.K. Reichert, "An Analysis of the Financial Management Techniques Currently Employed by Large U.S. Corporations," *Journal of Business Finance and Accounting*, Winter 1983, pp. 623–45; M.T. Stanley and S.R. Block, "A Survey of Multinational Capital Budgeting," *The Financial Review*, March 1984, pp. 36–51.

that it would spend about \$14 billion during the year, and Shell Oil said it would spend \$21 billion during 2006 alone on capital investment.

Increases in capital spending are often an industrywide occurrence. For example, in 2006, the worldwide semiconductor industry was expected to boost capital spending by 10 percent to \$50.4 billion, the second largest one-year total in the industry's history. The biggest year was 2000, with \$60.3 billion spent.

According to information released by the Census Bureau in 2006, capital investment for the economy as a whole was \$1.05 trillion in 2004, \$975 billion in 2003, and \$953 billion in 2002. The total for the three years therefore was just under \$3 trillion. Given the sums at stake, it is not too surprising that careful analysis of capital expenditures is something at which successful businesses seek to become adept.

There have been a number of surveys conducted asking firms what types of investment criteria they actually use. Table 9.6 summarizes the results of several of these. Panel A of the table is a historical comparison looking at the primary capital budgeting techniques used by large firms through time. In 1959, only 19 percent of the firms surveyed used either IRR or NPV, and 68 percent used either payback periods or accounting returns. It is clear that by the 1980s, IRR and NPV had become the dominant criteria.

Panel B of Table 9.6 summarizes the results of a 1999 survey of chief financial officers (CFOs) at both large and small firms in the United States. A total of 392 CFOs responded. What is shown is the percentage of CFOs who always or almost always used the various capital budgeting techniques we described in this chapter. Not surprisingly, IRR and NPV were the two most widely used techniques, particularly at larger firms. However, over half of the respondents always, or almost always, used the payback criterion as well. In fact, among smaller firms, payback was used just about as much as NPV and IRR. Less commonly used were discounted payback, accounting rates of return, and the profitability index. For future reference, the various criteria we have discussed are summarized in Table 9.7.

TABLE 9.7

Summary of Investment Criteria

I. Discounted Cash Flow Criteria

- A. *Net present value (NPV)*: The NPV of an investment is the difference between its market value and its cost. The NPV rule is to take a project if its NPV is positive. NPV is frequently estimated by calculating the present value of the future cash flows (to estimate market value) and then subtracting the cost. NPV has no serious flaws; it is the preferred decision criterion.
- B. *Internal rate of return (IRR)*: The IRR is the discount rate that makes the estimated NPV of an investment equal to zero; it is sometimes called the *discounted cash flow (DCF) return*. The IRR rule is to take a project when its IRR exceeds the required return. IRR is closely related to NPV, and it leads to exactly the same decisions as NPV for conventional, independent projects. When project cash flows are not conventional, there may be no IRR or there may be more than one. More seriously, the IRR cannot be used to rank mutually exclusive projects; the project with the highest IRR is not necessarily the preferred investment.
- C. *Modified internal rate of return (MIRR)*: The MIRR is a modification to the IRR. A project's cash flows are modified by (1) discounting the negative cash flows back to the present; (2) compounding all cash flows to the end of the project's life; or (3) combining (1) and (2). An IRR is then computed on the modified cash flows. MIRR's are guaranteed to avoid the multiple rate of return problem, but it is unclear how to interpret them; and they are not truly "internal" because they depend on externally supplied discounting or compounding rates.
- D. *Profitability index (PI)*: The PI, also called the *benefit–cost ratio*, is the ratio of present value to cost. The PI rule is to take an investment if the index exceeds 1. The PI measures the present value of an investment per dollar invested. It is quite similar to NPV; but, like IRR, it cannot be used to rank mutually exclusive projects. However, it is sometimes used to rank projects when a firm has more positive NPV investments than it can currently finance.

II. Payback Criteria

- A. *Payback period*: The payback period is the length of time until the sum of an investment's cash flows equals its cost. The payback period rule is to take a project if its payback is *less* than some cutoff. The payback period is a flawed criterion, primarily because it ignores risk, the time value of money, and cash flows beyond the cutoff point.
- B. *Discounted payback period*: The discounted payback period is the length of time until the sum of an investment's discounted cash flows equals its cost. The discounted payback period rule is to take an investment if the discounted payback is *less* than some cutoff. The discounted payback rule is flawed, primarily because it ignores cash flows after the cutoff.

III. Accounting Criterion

- A. *Average accounting return (AAR)*: The AAR is a measure of accounting profit relative to book value. It is *not* related to the IRR, but it is similar to the accounting return on assets (ROA) measure in Chapter 3. The AAR rule is to take an investment if its AAR exceeds a benchmark AAR. The AAR is seriously flawed for a variety of reasons, and it has little to recommend it.

Concept Questions

9.7a What are the most commonly used capital budgeting procedures?

9.7b If NPV is conceptually the best procedure for capital budgeting, why do you think multiple measures are used in practice?

Summary and Conclusions

9.8

This chapter has covered the different criteria used to evaluate proposed investments. The seven criteria, in the order we discussed them, are these:

1. Net present value (NPV).
2. Payback period.
3. Discounted payback period.
4. Average accounting return (AAR).
5. Internal rate of return (IRR).
6. Modified internal rate of return (MIRR).
7. Profitability index (PI).

We illustrated how to calculate each of these and discussed the interpretation of the results. We also described the advantages and disadvantages of each of them. Ultimately a good capital budgeting criterion must tell us two things. First, is a particular project a good investment? Second, if we have more than one good project, but we can take only one of them, which one should we take? The main point of this chapter is that only the NPV criterion can always provide the correct answer to both questions.

For this reason, NPV is one of the two or three most important concepts in finance, and we will refer to it many times in the chapters ahead. When we do, keep two things in mind: (1) NPV is always just the difference between the market value of an asset or project and its cost, and (2) the financial manager acts in the shareholders' best interests by identifying and taking positive NPV projects.

Finally, we noted that NPVs can't normally be observed in the market; instead, they must be estimated. Because there is always the possibility of a poor estimate, financial managers use multiple criteria for examining projects. The other criteria provide additional information about whether a project truly has a positive NPV.

CHAPTER REVIEW AND SELF-TEST PROBLEMS

- 9.1 Investment Criteria** This problem will give you some practice calculating NPVs and paybacks. A proposed overseas expansion has the following cash flows:

Year	Cash Flow
0	-\$200
1	50
2	60
3	70
4	200

Calculate the payback, the discounted payback, and the NPV at a required return of 10 percent.

- 9.2 Mutually Exclusive Investments** Consider the following two mutually exclusive investments. Calculate the IRR for each and the crossover rate. Under what circumstances will the IRR and NPV criteria rank the two projects differently?

Year	Investment A	Investment B
0	-\$75	-\$75
1	20	60
2	40	50
3	70	15

- 9.3 Average Accounting Return** You are looking at a three-year project with a projected net income of \$2,000 in year 1, \$4,000 in year 2, and \$6,000 in year 3. The cost is \$12,000, which will be depreciated straight-line to zero over the three-year life of the project. What is the average accounting return (AAR)?

ANSWERS TO CHAPTER REVIEW AND SELF-TEST PROBLEMS

- 9.1** In the following table, we have listed the cash flow, cumulative cash flow, discounted cash flow (at 10 percent), and cumulative discounted cash flow for the proposed project.

Year	Cash Flow		Accumulated Cash Flow	
	Undiscounted	Discounted	Undiscounted	Discounted
1	\$ 50	\$ 45.45	\$ 50	\$ 45.45
2	60	49.59	110	95.04
3	70	52.59	180	147.63
4	200	136.60	380	284.23

Recall that the initial investment was \$200. When we compare this to accumulated undiscounted cash flows, we see that payback occurs between years 3 and 4. The cash flows for the first three years are \$180 total, so, going into the fourth year, we are short by \$20. The total cash flow in year 4 is \$200, so the payback is $3 + (\$20/\$200) = 3.10$ years.

Looking at the accumulated discounted cash flows, we see that the discounted payback occurs between years 3 and 4. The sum of the discounted cash flows is \$284.23, so the NPV is \$84.23. Notice that this is the present value of the cash flows that occur after the discounted payback.

- 9.2** To calculate the IRR, we might try some guesses, as in the following table:

Discount Rate	NPV(A)	NPV(B)
0%	\$55.00	\$50.00
10	28.83	32.14
20	9.95	18.40
30	- 4.09	7.57
40	-14.80	- 1.17

Several things are immediately apparent from our guesses. First, the IRR on A must be between 20 percent and 30 percent (why?). With some more effort, we find that it's 26.79 percent. For B, the IRR must be a little less than 40 percent (again, why?); it works out to be 38.54 percent. Also, notice that at rates between 0 percent and 10 percent, the NPVs are very close, indicating that the crossover is in that vicinity.

To find the crossover exactly, we can compute the IRR on the difference in the cash flows. If we take the cash flows from A minus the cash flows from B, the resulting cash flows are:

Year	A – B
0	\$ 0
1	– 40
2	– 10
3	55

These cash flows look a little odd; but the sign changes only once, so we can find an IRR. With some trial and error, you'll see that the NPV is zero at a discount rate of 5.42 percent, so this is the crossover rate.

The IRR for B is higher. However, as we've seen, A has the larger NPV for any discount rate less than 5.42 percent, so the NPV and IRR rankings will conflict in that range. Remember, if there's a conflict, we will go with the higher NPV. Our decision rule is thus simple: Take A if the required return is less than 5.42 percent, take B if the required return is between 5.42 percent and 38.54 percent (the IRR on B), and take neither if the required return is more than 38.54 percent.

- 9.3** Here we need to calculate the ratio of average net income to average book value to get the AAR. Average net income is:

$$\text{Average net income} = (\$2,000 + 4,000 + 6,000)/3 = \$4,000$$

Average book value is:

$$\text{Average book value} = \$12,000/2 = \$6,000$$

So the average accounting return is:

$$\text{AAR} = \$4,000/\$6,000 = 66.67\%$$

This is an impressive return. Remember, however, that it isn't really a rate of return like an interest rate or an IRR, so the size doesn't tell us a lot. In particular, our money is probably not going to grow at a rate of 66.67 percent per year, sorry to say.

CONCEPTS REVIEW AND CRITICAL THINKING QUESTIONS

- 1. Payback Period and Net Present Value** If a project with conventional cash flows has a payback period less than the project's life, can you definitively state the algebraic sign of the NPV? Why or why not? If you know that the discounted payback period is less than the project's life, what can you say about the NPV? Explain.
- 2. Net Present Value** Suppose a project has conventional cash flows and a positive NPV. What do you know about its payback? Its discounted payback? Its profitability index? Its IRR? Explain.
- 3. Payback Period** Concerning payback:
 - a.** Describe how the payback period is calculated, and describe the information this measure provides about a sequence of cash flows. What is the payback criterion decision rule?
 - b.** What are the problems associated with using the payback period to evaluate cash flows?

- c. What are the advantages of using the payback period to evaluate cash flows? Are there any circumstances under which using payback might be appropriate? Explain.
4. **Discounted Payback** Concerning discounted payback:
- Describe how the discounted payback period is calculated, and describe the information this measure provides about a sequence of cash flows. What is the discounted payback criterion decision rule?
 - What are the problems associated with using the discounted payback period to evaluate cash flows?
 - What conceptual advantage does the discounted payback method have over the regular payback method? Can the discounted payback ever be longer than the regular payback? Explain.
5. **Average Accounting Return** Concerning AAR:
- Describe how the average accounting return is usually calculated, and describe the information this measure provides about a sequence of cash flows. What is the AAR criterion decision rule?
 - What are the problems associated with using the AAR to evaluate a project's cash flows? What underlying feature of AAR is most troubling to you from a financial perspective? Does the AAR have any redeeming qualities?
6. **Net Present Value** Concerning NPV:
- Describe how NPV is calculated, and describe the information this measure provides about a sequence of cash flows. What is the NPV criterion decision rule?
 - Why is NPV considered a superior method of evaluating the cash flows from a project? Suppose the NPV for a project's cash flows is computed to be \$2,500. What does this number represent with respect to the firm's shareholders?
7. **Internal Rate of Return** Concerning IRR:
- Describe how the IRR is calculated, and describe the information this measure provides about a sequence of cash flows. What is the IRR criterion decision rule?
 - What is the relationship between IRR and NPV? Are there any situations in which you might prefer one method over the other? Explain.
 - Despite its shortcomings in some situations, why do most financial managers use IRR along with NPV when evaluating projects? Can you think of a situation in which IRR might be a more appropriate measure to use than NPV? Explain.
8. **Profitability Index** Concerning the profitability index:
- Describe how the profitability index is calculated, and describe the information this measure provides about a sequence of cash flows. What is the profitability index decision rule?
 - What is the relationship between the profitability index and NPV? Are there any situations in which you might prefer one method over the other? Explain.
9. **Payback and Internal Rate of Return** A project has perpetual cash flows of C per period, a cost of I , and a required return of R . What is the relationship between the project's payback and its IRR? What implications does your answer have for long-lived projects with relatively constant cash flows?
10. **International Investment Projects** In November 2004, automobile manufacturer Honda announced plans to build an automatic transmission plant in Georgia and expand its transmission plant in Ohio. Honda apparently felt that it would be better able to compete and create value with U.S.-based facilities. Other companies such as Fuji

Film and Swiss chemical company Lonza have reached similar conclusions and taken similar actions. What are some of the reasons that foreign manufacturers of products as diverse as automobiles, film, and chemicals might arrive at this same conclusion?

11. **Capital Budgeting Problems** What difficulties might come up in actual applications of the various criteria we discussed in this chapter? Which one would be the easiest to implement in actual applications? The most difficult?
12. **Capital Budgeting in Not-for-Profit Entities** Are the capital budgeting criteria we discussed applicable to not-for-profit corporations? How should such entities make capital budgeting decisions? What about the U.S. government? Should it evaluate spending proposals using these techniques?
13. **Modified Internal Rate of Return** One of the less flattering interpretations of the acronym MIRR is “meaningless internal rate of return.” Why do you think this term is applied to MIRR?
14. **Net Present Value** It is sometimes stated that “the net present value approach assumes reinvestment of the intermediate cash flows at the required return.” Is this claim correct? To answer, suppose you calculate the NPV of a project in the usual way. Next, suppose you do the following:
 - a. Calculate the future value (as of the end of the project) of all the cash flows other than the initial outlay assuming they are reinvested at the required return, producing a single future value figure for the project.
 - b. Calculate the NPV of the project using the single future value calculated in the previous step and the initial outlay. It is easy to verify that you will get the same NPV as in your original calculation only if you use the required return as the reinvestment rate in the previous step.
15. **Internal Rate of Return** It is sometimes stated that “the internal rate of return approach assumes reinvestment of the intermediate cash flows at the internal rate of return.” Is this claim correct? To answer, suppose you calculate the IRR of a project in the usual way. Next, suppose you do the following:
 - a. Calculate the future value (as of the end of the project) of all the cash flows other than the initial outlay assuming they are reinvested at the IRR, producing a single future value figure for the project.
 - b. Calculate the IRR of the project using the single future value calculated in the previous step and the initial outlay. It is easy to verify that you will get the same IRR as in your original calculation only if you use the IRR as the reinvestment rate in the previous step.



QUESTIONS AND PROBLEMS

1. **Calculating Payback** What is the payback period for the following set of cash flows?

BASIC

(Questions 1–19)

Year	Cash Flow
0	-\$4,800
1	1,500
2	2,600
3	2,900
4	1,700

2. **Calculating Payback** An investment project provides cash inflows of \$860 per year for eight years. What is the project payback period if the initial cost is \$3,000? What if the initial cost is \$5,000? What if it is \$7,000?
3. **Calculating Payback** Old Country, Inc., imposes a payback cutoff of three years for its international investment projects. If the company has the following two projects available, should it accept either of them?



Year	Cash Flow (A)	Cash Flow (B)
0	-\$50,000	-\$ 70,000
1	35,000	15,000
2	21,000	22,000
3	10,000	31,000
4	5,000	240,000

4. **Calculating Discounted Payback** An investment project has annual cash inflows of \$6,500, \$7,000, \$7,500, and \$8,000, and a discount rate of 14 percent. What is the discounted payback period for these cash flows if the initial cost is \$8,000? What if the initial cost is \$13,000? What if it is \$18,000?
5. **Calculating Discounted Payback** An investment project costs \$15,000 and has annual cash flows of \$3,700 for six years. What is the discounted payback period if the discount rate is zero percent? What if the discount rate is 5 percent? If it is 15 percent?
6. **Calculating AAR** You're trying to determine whether to expand your business by building a new manufacturing plant. The plant has an installation cost of \$18 million, which will be depreciated straight-line to zero over its four-year life. If the plant has projected net income of \$1,632,000, \$2,106,500, \$1,941,700, and \$1,298,000 over these four years, what is the project's average accounting return (AAR)?
7. **Calculating IRR** A firm evaluates all of its projects by applying the IRR rule. If the required return is 18 percent, should the firm accept the following project?



Year	Cash Flow
0	-\$30,000
1	13,000
2	19,000
3	12,000

8. **Calculating NPV** For the cash flows in the previous problem, suppose the firm uses the NPV decision rule. At a required return of 11 percent, should the firm accept this project? What if the required return was 30 percent?
9. **Calculating NPV and IRR** A project that provides annual cash flows of \$24,000 for nine years costs \$110,000 today. Is this a good project if the required return is 8 percent? What if it's 20 percent? At what discount rate would you be indifferent between accepting the project and rejecting it?
10. **Calculating IRR** What is the IRR of the following set of cash flows?



Year	Cash Flow
0	-\$18,000
1	9,800
2	7,500
3	7,300

11. **Calculating NPV** For the cash flows in the previous problem, what is the NPV at a discount rate of zero percent? What if the discount rate is 10 percent? If it is 20 percent? If it is 30 percent?
12. **NPV versus IRR** Bumble’s Bees, Inc., has identified the following two mutually exclusive projects:



Year	Cash Flow (A)	Cash Flow (B)
0	-\$37,000	-\$37,000
1	19,000	6,000
2	14,500	12,500
3	12,000	19,000
4	9,000	23,000

- a. What is the IRR for each of these projects? Using the IRR decision rule, which project should the company accept? Is this decision necessarily correct?
- b. If the required return is 11 percent, what is the NPV for each of these projects? Which project will the company choose if it applies the NPV decision rule?
- c. Over what range of discount rates would the company choose project A? Project B? At what discount rate would the company be indifferent between these two projects? Explain.
13. **NPV versus IRR** Consider the following two mutually exclusive projects:

Year	Cash Flow (X)	Cash Flow (Y)
0	-\$10,000	-\$10,000
1	5,400	4,500
2	3,400	3,600
3	4,500	5,400

- Sketch the NPV profiles for X and Y over a range of discount rates from zero to 25 percent. What is the crossover rate for these two projects?
14. **Problems with IRR** Sweet Petroleum, Inc., is trying to evaluate a generation project with the following cash flows:

Year	Cash Flow
0	-\$27,000,000
1	46,000,000
2	-6,000,000

- a. If the company requires a 10 percent return on its investments, should it accept this project? Why?
- b. Compute the IRR for this project. How many IRRs are there? Using the IRR decision rule, should the company accept the project? What’s going on here?
15. **Calculating Profitability Index** What is the profitability index for the following set of cash flows if the relevant discount rate is 10 percent? What if the discount rate is 15 percent? If it is 22 percent?



Year	Cash Flow
0	-\$12,000
1	6,200
2	5,600
3	3,900

- 16. Problems with Profitability Index** The Shine On Computer Corporation is trying to choose between the following two mutually exclusive design projects:

Year	Cash Flow (I)	Cash Flow (II)
0	-\$40,000	-\$12,000
1	18,000	6,100
2	18,000	6,100
3	18,000	6,100

- If the required return is 10 percent and the company applies the profitability index decision rule, which project should the firm accept?
 - If the company applies the NPV decision rule, which project should it take?
 - Explain why your answers in (a) and (b) are different.
- 17. Comparing Investment Criteria** Consider the following two mutually exclusive projects:

Year	Cash Flow (A)	Cash Flow (B)
0	-\$350,000	-\$35,000
1	25,000	17,000
2	70,000	11,000
3	70,000	17,000
4	430,000	11,000

Whichever project you choose, if any, you require a 15 percent return on your investment.

- If you apply the payback criterion, which investment will you choose? Why?
 - If you apply the discounted payback criterion, which investment will you choose? Why?
 - If you apply the NPV criterion, which investment will you choose? Why?
 - If you apply the IRR criterion, which investment will you choose? Why?
 - If you apply the profitability index criterion, which investment will you choose? Why?
 - Based on your answers in (a) through (e), which project will you finally choose? Why?
- 18. NPV and Discount Rates** An investment has an installed cost of \$724,860. The cash flows over the four-year life of the investment are projected to be \$324,186, \$375,085, \$354,302, and \$205,680. If the discount rate is zero, what is the NPV? If the discount rate is infinite, what is the NPV? At what discount rate is the NPV just equal to zero? Sketch the NPV profile for this investment based on these three points.
- 19. MIRR** Slow Ride Corp. is evaluating a project with the following cash flows:

Year	Cash Flow
0	-\$12,000
1	5,800
2	6,500
3	6,200
4	5,100
5	- 4,300

The company uses a 10 percent interest rate on all of its projects. Calculate the MIRR of the project using all three methods.

20. **NPV and the Profitability Index** If we define the NPV index as the ratio of NPV to cost, what is the relationship between this index and the profitability index?
21. **Cash Flow Intuition** A project has an initial cost of I , has a required return of R , and pays C annually for N years.
- Find C in terms of I and N such that the project has a payback period just equal to its life.
 - Find C in terms of I , N , and R such that this is a profitable project according to the NPV decision rule.
 - Find C in terms of I , N , and R such that the project has a benefit-cost ratio of 2.
22. **MIRR** Suppose the company in Problem 19 uses an 11 percent discount rate and an 8 percent reinvestment rate on all of its projects. Calculate the MIRR of the project using all three methods using these interest rates.
23. **Payback and NPV** An investment under consideration has a payback of seven years and a cost of \$537,000. If the required return is 12 percent, what is the worst-case NPV? The best-case NPV? Explain. Assume the cash flows are conventional.
24. **Multiple IRRs** This problem is useful for testing the ability of financial calculators and computer software. Consider the following cash flows. How many different IRRs are there? (*Hint*: Search between 20 percent and 70 percent.) When should we take this project?

Year	Cash Flow
0	-\$ 756
1	4,293
2	-9,105
3	8,550
4	-3,000

25. **NPV Valuation** The Yurdone Corporation wants to set up a private cemetery business. According to the CFO, Barry M. Deep, business is “looking up.” As a result, the cemetery project will provide a net cash inflow of \$60,000 for the firm during the first year, and the cash flows are projected to grow at a rate of 6 percent per year forever. The project requires an initial investment of \$925,000.
- If Yurdone requires a 13 percent return on such undertakings, should the cemetery business be started?
 - The company is somewhat unsure about the assumption of a 6 percent growth rate in its cash flows. At what constant growth rate would the company just break even if it still required a 13 percent return on investment?
26. **Problems with IRR** A project has the following cash flows:

Year	Cash Flow
0	\$64,000
1	-30,000
2	-48,000

What is the IRR for this project? If the required return is 12 percent, should the firm accept the project? What is the NPV of this project? What is the NPV of the project

INTERMEDIATE

(Questions 20–22)

CHALLENGE

(Questions 23–28)

if the required return is 0 percent? 24 percent? What is going on here? Sketch the NPV profile to help you with your answer.

27. **Problems with IRR** McKeekin Corp. has a project with the following cash flows:

Year	Cash Flow
0	\$20,000
1	−26,000
2	13,000

What is the IRR of the project? What is happening here?

28. **NPV and IRR** Anderson International Limited is evaluating a project in Erewhon. The project will create the following cash flows:

Year	Cash Flow
0	−\$450,000
1	165,000
2	190,000
3	205,000
4	183,000

All cash flows will occur in Erewhon and are expressed in dollars. In an attempt to improve its economy, the Erewhonian government has declared that all cash flows created by a foreign company are “blocked” and must be reinvested with the government for one year. The reinvestment rate for these funds is 4 percent. If Anderson uses an 11 percent required return on this project, what are the NPV and IRR of the project? Is the IRR you calculated the MIRR of the project? Why or why not?

WEB EXERCISES



- 9.1 **Net Present Value** You have a project that has an initial cash outflow of −\$20,000 and cash inflows of \$6,000, \$5,000, \$4,000 and \$3,000, respectively, for the next four years. Go to www.datadynamica.com, and follow the “Online IRR NPV Calculator” link. Enter the cash flows. If the required return is 12 percent, what is the IRR of the project? The NPV?
- 9.2 **Internal Rate of Return** Using the online calculator from the previous problem, find the IRR for a project with cash flows of −\$500, \$1,200, and −\$400. What is going on here?

MINICASE

Bullock Gold Mining

Seth Bullock, the owner of Bullock Gold Mining, is evaluating a new gold mine in South Dakota. Dan Dority, the company’s geologist, has just finished his analysis of the mine site. He has estimated that the mine would be productive for eight years, after which the gold would be completely mined. Dan has taken an estimate of the gold deposits to Alma Garrett, the company’s financial officer. Alma has been asked by Seth to perform an analysis of the new mine and present her rec-

ommendation on whether the company should open the new mine.

Alma has used the estimates provided by Dan to determine the revenues that could be expected from the mine. She has also projected the expense of opening the mine and the annual operating expenses. If the company opens the mine, it will cost \$500 million today, and it will have a cash outflow of \$80 million nine years from today in costs associated with closing

Year	Cash Flow
0	-\$500,000,000
1	60,000,000
2	90,000,000
3	170,000,000
4	230,000,000
5	205,000,000
6	140,000,000
7	110,000,000
8	70,000,000
9	-80,000,000

the mine and reclaiming the area surrounding it. The expected cash flows each year from the mine are shown in the table. Bullock Mining has a 12 percent required return on all of its gold mines.

1. Construct a spreadsheet to calculate the payback period, internal rate of return, modified internal rate of return, and net present value of the proposed mine.
2. Based on your analysis, should the company open the mine?
3. Bonus question: Most spreadsheets do not have a built-in formula to calculate the payback period. Write a VBA script that calculates the payback period for a project.