

# Structured Credit Risk

This chapter focuses on a class of credit-risky securities called *securitizations* and *structured credit products*. These securities play an important role in contemporary finance, and had a major role in the subprime crisis of 2007 and after. As described in Chapter 1, these securities have been in existence for some time, and their issuance and trading volumes were quite large up until the onset of the crisis. They have also had a crucial impact on the development of the financial system, particularly on the formation of the market-based or “shadow banking system” of financial intermediation.

In this chapter, we look at structured products in more detail, with the goal of understanding both the challenges they present to risk management by traders and investors, and their impact on the financial system before and during the crisis. These products are complex, so we’ll employ an extended example to convey how they work. They are also issued in many variations, so the example will differ from any extant structured product, but capture the key features that recur across all variants. A grasp of structured credit products will also help readers understand the story, told in Chapters 12, 14 and 15, of the growth of leverage in the financial system and its role in the subprime crisis.

## 9.1 STRUCTURED CREDIT BASICS

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We begin by sketching the major types of securitizations and structured credit products, sometimes collectively called *portfolio credit products*. These are vehicles that create bonds or credit derivatives backed by a pool of loans or other claims. This broad definition can’t do justice to the bewildering variety of structured credit products, and the equally bewildering terminology associated with their construction.

First, let's put structured credit products into the context of other securities based on pooled loans. Not surprisingly, this hierarchy with respect to complexity of structure corresponds roughly to the historical development of structured products that we summarized in Chapter 1:

*Covered bonds* are issued mainly by European banks, mainly in Germany and Denmark. In a covered bond structure, mortgage loans are aggregated into a *cover pool*, by which a bond issue is secured. The cover pool stays on the balance sheet of the bank, rather than being sold off-balance-sheet, but is segregated from other assets of the bank in the event the bank defaults. The pool assets would be used to make the covered bond owners whole before they could be applied to repay general creditors of the bank. Because the underlying assets remain on the issuer's balance sheet, covered bonds are not considered full-fledged securitizations. Also, the principal and interest on the secured bond issue are paid out of the general cash flows of the issuer, rather than out of the cash flows generated by the cover pool. Finally, apart from the security of the cover pool, the covered bonds are backed by the issuer's obligation to pay.

*Mortgage pass-through securities* are true securitizations or structured products, since the cash flows paid out by the bonds, and the credit risk to which they are exposed, are more completely dependent on the cash flows and credit risks generated by the pool of underlying loans. Mortgage pass-throughs are backed by a pool of mortgage loans, removed from the mortgage originators' balance sheets, and administered by a *servicer*, who collects principal and interest from the underlying loans and distributes them to the bondholders. Most pass-throughs are *agency MBS*, issued under an explicit or implicit U.S. federal guarantee of the performance of the underlying loans, so there is little default risk. But the principal and interest on the bonds are "passed through" from the loans, so the cash flows depend not only on amortization, but also voluntary prepayments by the mortgagor. The bonds are repaid slowly over time, but at an uncertain pace, in contrast to bullet bonds, which receive full repayment of principal on one date. Bondholders are therefore exposed to prepayment risk.

*Collateralized mortgage obligations* were developed partly as a means of coping with prepayment risk, but also as a way to create both longer- and shorter-term bonds out of a pool of mortgage loans. Such loans amortize over time, creating cash flow streams that diminish over time. CMOs are "sliced," or tranches into bonds or

*tranches*, that are paid down on a specified schedule. The simplest structure is *sequential pay*, in which the tranches are ordered, with “Class A” receiving all principal repayments from the loan until it is retired, then “Class B,” and so on. The higher tranches in the sequence have less prepayment risk than a pass-through, while the lower ones bear more.

*Structured credit products* introduce one more innovation, namely the sequential distribution of credit losses. Structured products are backed by credit-risky loans or bonds. The tranching focuses on creating bonds that have different degrees of credit risk. As losses occur, the tranches are gradually written down. Junior tranches are written down first, and more senior tranches only begin to bear credit losses once the junior tranches have been written down to zero.

This basic credit tranching feature can be combined with other features to create, in some cases, extremely complex security structures. The bottom-up treatment of credit losses can be combined with the sequential payment technology introduced with CMOs. Cash flows and credit risk arising from certain constituents of the underlying asset pool may be directed to specific bonds.

Securitization is one approach to financing pools of loans and other receivables developed over the past two decades. An important alternative and complement to securitization are entities set up to issue *asset-backed commercial paper* (ABCP) against the receivables, or against securitization bonds themselves. We describe these in greater detail in Chapter 12.

A structured product can be thought of as a “robot” corporate entity with a balance sheet, but no other business. In fact, structured products are usually set up as *special purpose entities* (SPE) or *vehicles* (SPV), also known as a *trust*. This arrangement is intended to legally separate the assets and liabilities of the structured product from those of the original creditors and of the company that manages the payments. That is, it makes the SPE *bankruptcy remote*. This permits investors to focus on the credit quality of the loans themselves rather than that of the original lenders in assessing the credit quality of the securitization. The underlying debt instruments in the SPV are the robot entity’s assets, and the structured credit products built on it are its liabilities.

Securitizations are, depending on the type of underlying assets, often generically called *asset-* (ABS) or *mortgage-backed securities* (MBS), or *collateralized loan obligations* (CLOs). Securitizations that repackage other securitizations are called *collateralized debt obligations* (CDOs, issuing bonds against a collateral pool consisting of ABS, MBS, or CLOs), *collateralized*

*mortgage obligations* (CMOs), or *collateralized bond obligations* (CBOs). There even exist third-level securitizations, in which the collateral pool consists of CDO liabilities, which themselves consist of bonds backed by a collateral pool, called *CDO-squareds*.

There are several other dimensions along which we can classify the great variety of structured credit products:

*Underlying asset classes.* Every structured product is based on a set of underlying loans, receivables, or other claims. If you drill down far enough into a structured product, you will get to a set of relatively conventional debt instruments that constitute the *collateral* or *loan pool*. The collateral is typically composed of residential or commercial real estate loans, consumer debt such as credit cards balances and auto and student loans, and corporate bonds. But many other types of debt, and even nondebt assets such as recurring fee income, can also be packaged into securitizations. The credit quality and prepayment behavior of the underlying risks is, of course, critical in assessing the risks of the structured products built upon them.

*Type of structure.* Structured products are tools for redirecting the cash flows and credit losses generated by the underlying debt instruments. The latter each make contractually stipulated coupon or other payments. But rather than being made directly to debt holders, they are split up and channeled to the structured products in specified ways. A key dimension is tranching, the number and size of the bonds carved out of the liability side of the securitization. Another is how many levels of securitization are involved, that is, whether the collateral pool consists entirely of loans or liabilities of other securitizations.

*How much the pool changes over time.* We can distinguish here among three different approaches, tending to coincide with asset class. Each type of pool has its own risk management challenges:

*Static pools* are amortizing pools in which a fixed set of loans is placed in the trust. As the loans amortize, are repaid, or default, the deal, and the bonds it issues, gradually wind down. Static pools are common for such asset types as auto loans and residential mortgages, which generally themselves have a fixed and relatively long term at origination but pay down over time.

*Revolving pools* specify an overall level of assets that is to be maintained during a *revolving period*. As underlying loans are repaid, the size of the pool is maintained by introducing additional loans from the balance sheet of the originator. Revolving pools

are common for bonds backed by credit card debt, which is not issued in a fixed amount, but can within limits be drawn upon and repaid by the borrower at his own discretion and without notification. Once the revolving period ends, the loan pool becomes fixed, and the deal winds down gradually as debts are repaid or become delinquent and are charged off.

*Managed pools* are pools in which the manager of the structured product has discretion to remove individual loans from the pool, sell them, and replace them with others. Managed pools have typically been seen in CLOs. Managers of CLOs are hired in part for skill in identifying loans with higher spreads than warranted by their credit quality. They can, in theory, also see credit problems arising at an early stage, and trade out of loans they believe are more likely to default. There is a secondary market for syndicated loans that permits them to do so, at least in many cases. Also, syndicated loans are typically repaid in lump sum, well ahead of their legal final maturity, but with random timing, so a managed pool permits the manager to maintain the level of assets in the pool.

The number of debt instruments in pools depends on asset type and on the size of the securitization; some, for example CLO and commercial mortgage-backed securities (CMBS) pools, may contain around 100 different loans, each with an initial par value of several million dollars, while a large residential mortgage-backed security (RMBS) may have several tens of thousands of mortgage loans in its pool, with an average loan amount of \$200,000.

The assets of some structured products are not cash debt instruments, but rather credit derivatives, most frequently CDS. These are called *synthetic* securitizations, in contrast to *cash* or *cash-flow* securitizations. The set of underlying cash debt instruments on which a synthetic securitization is based generally consists of securitization liabilities rather than loans, and is called the *reference portfolio*.

Each structured product is defined by the cash flows thrown off by assets and the way they are distributed to the liabilities. Next, we examine the mechanisms by which they are distributed: the capital structure or tranching, the waterfall, and overcollateralization.

### **9.1.1 Capital Structure and Credit Losses in a Securitization**

Tranching refers to how the liabilities of the securitization SPV are split into a capital structure. Each type of bond or note within the capital structure has its own coupon or spread, and depending on its place in the capital

structure, its own priority or seniority with respect to losses. The general principle of tranching is that more senior tranches have priority, or the first right, to payments of principal and interest, while more junior tranches must be written down first when credit losses occur in the collateral pool. There may be many dozen, or only a small handful of tranches in a securitization, but they can be categorized into three groups:

*Equity.* The equity tranche is so called because it typically receives no fixed coupon payment, but is fully exposed to defaults in the collateral pool. It takes the form of a note with a specified notional value that is entitled to the residual cash flows after all the other obligations of the SPE have been satisfied. The notional value is typically small compared to the market value of the collateral; that is, it is a “thin” tranche.

*Junior debt* earns a relatively high fixed coupon or spread, but if the equity tranche is exhausted by defaults in the collateral pool, it is next in line to suffer default losses. Junior bonds are also called *mezzanine tranches* and are typically also thin.

*Senior debt* earns a relatively low fixed coupon or spread, but is protected by both the equity and mezzanine tranches from default losses. Senior bonds are typically the bulk of the liabilities in a securitization. This is a crucial feature of securitization economics, as we will see later. If the underlying collateral cannot be financed primarily by low-yielding senior debt, a securitization is generally not viable.

The capital structure is sometimes called the “capital stack,” with senior bonds at the “top of the stack.” Most securitizations also feature securities with different maturities but the same seniority, a technique similar to sequential-pay CMOs for coping with variation in the term to maturity and prepayment behavior of the underlying loans, while catering to the desire of different investors for bonds with different durations.

The example of the next few sections of this chapter features three tranches, a simple structure that can be summarized in this balance sheet:

Assets	Liabilities
Underlying debt instruments	Equity Mezzanine debt Senior debt

The boundary between two tranches, expressed as a percentage of the total of the liabilities, is called the *attachment point* of the more senior tranche and *detachment point* of the more junior tranche. The equity tranche only has a detachment point, and the most senior only has an attachment point.

The part of the capital structure below a bond tranche is called its *subordination* or *credit enhancement*. It is the fraction of the collateral pool that must be lost before the bond takes any loss. It is greater for more senior bonds in the structure. The credit enhancement may decline over time as the collateral experiences default losses, or increase as *excess spread*, the interest from the collateral that is not paid out to the liabilities or as fees and expenses, accumulates in the trust.

A securitization can be thought of as a mechanism for securing long-term financing for the collateral pool. To create this mechanism, the senior tranche must be a large portion of the capital structure, and it must have a low coupon compared to the collateral pool. In order to create such a liability, its credit risk must be low enough that it can be marketed. To this end, additional features can be introduced into the cash flow structure. The most important is *overcollateralization*; that is, selling a par amount of bonds that is smaller than the par amount of underlying collateral. Overcollateralization provides credit enhancement for all of the bond tranches of a securitization.

There are typically reserves within the capital structure that must be filled and kept at certain levels before junior and equity notes can receive money. These reserves can be filled from two sources: gradually, from the excess spread, or quickly via overcollateralization. These approaches are often used in combination. The latter is sometimes called *hard credit enhancement*, in contrast to the *soft credit enhancement* of excess spread, which accrues gradually over time and is not present at initiation of the securitization. Deals with revolving pools generally have an *early amortization trigger* that terminates the replenishment of the pool with fresh debt if a default trigger is breached.

Typically, the collateral pool contains assets with different maturities, or that amortize over time. Loan maturities are uncertain because the loans can be prepaid prior to maturity, possibly after an initial *lockout period* has elapsed. The senior liabilities in particular are therefore generally amortized over time as the underlying loans amortize or mature; while they may have legal final maturity dates that are quite far in the future, their durations are uncertain and much shorter. Risk analysis therefore generally focuses on the *weighted average life* (WAL) of a securitization, the weighted average of the number of years each dollar of par value of the bond will remain outstanding before it is repaid or amortized. A WAL is associated with a

particular prepayment assumption, and standard assumptions are set for some asset classes by convention.

As noted above, the sequential-pay technology can be combined with credit tranching in securitizations. This creates multiple senior bonds with different WALs, to better adapt the maturity structure of the liabilities to that of the collateral pool. This feature is called *time tranching* to distinguish it from the seniority tranching related to credit priority in the capital structure. The example presented in the rest of this chapter abstracts from this important feature. Thus, in addition to the credit risk that is the focus of this chapter, securitizations also pose prepayment and *extension risk* arising from loans either prepaying faster or slower than anticipated, or being extended past their maturity in response to financial distress.

In any securitization, there is a possibility that at the maturity date, even if the coupons have been paid timely all along, there may not be enough principal left in the collateral pool to redeem the junior and/or senior debt at par unless loans can be refinanced. The bonds are therefore exposed to the *refinancing risk* of the loans in the collateral pool. If some principal cash flows are paid out to the equity note along the way, refinancing risk is greater. Time tranching of the senior bonds, and their gradual retirement through amortization, is one way securitizations cope with this risk.

The tranche structure of a securitization leads to a somewhat different definition of a default event from that pertaining to individual, corporate, and sovereign debt. Losses to the bonds in securitizations are determined by losses in the collateral pool together with the waterfall. Losses may be severe enough to cause some credit loss to a bond, but only a small one. For example, if a senior ABS bond has 20 percent credit enhancement, and the collateral pool has credit losses of 21 percent, the credit loss or writedown to the bond will be approximately  $\frac{1}{100-20}$  or 1.25 percent, since the bond is 80 percent of the balance sheet of the trust. The LGD of a securitization can therefore take on a very wide range, and is driven by the realization of defaults and recoveries in the collateral pool.

For a corporate or sovereign bond, default is a binary event; if interest and/or principal cannot be paid, bankruptcy or restructuring ensues. Corporate debt typically has a “hard” maturity date, while securitizations have a distant maturity date that is rarely the occasion for a default. For these reasons, default events in securitizations are often referred to as *material impairment* to distinguish them from defaults. A common definition of material impairment is either missed interest payments that go uncured for more than a few months, or a deterioration of collateral pool performance so severe that interest or principal payments are likely to stop in the future.

### 9.1.2 Waterfall

The *waterfall* refers to the rules about how the cash flows from the collateral are distributed to the various securities in the capital structure. The term “waterfall” arose because generally the capital structure is paid in sequence, “top down,” with the senior debt receiving all of its promised payments before any lower tranche receives any monies. In addition to the coupons and other payments promised to the bonds, there are fees and other costs to be paid, which typically take priority over coupons.

A typical structured credit product begins life with a certain amount of hard overcollateralization, since part of the capital structure is an equity note, and the debt tranches are less than 100 percent of the deal. Soft overcollateralization mechanisms may begin to pay down the senior debt over time with part of the collateral pool interest, or divert part of it into a reserve that provides additional credit enhancement for the senior tranches. That way, additional credit enhancement is built up at the beginning of the life of the product, when collateral cash flows are strongest. Typically, there is a detailed set of *overcollateralization triggers* that state the conditions under which excess spread is to be diverted into various reserves.

To clarify these concepts and introduce a few more, let’s develop our simple example. Imagine a CLO, the underlying assets of which are 100 identical leveraged loans, with a par value of \$1,000,000 each, and priced at par. The loans are floating rate obligations that pay a fixed spread of 3.5 percent over one-month Libor. We’ll assume there are no upfront, management, or trustee fees. The capital structure consists of equity, and a junior and a senior bond, as displayed in this schematic balance sheet:

Assets	Liabilities
Underlying debt instruments:	Equity note \$5 million Mezzanine debt \$10 million coupon: Libor+500 bps
\$100 million coupon: L+350 bps	Senior debt \$85 million coupon: Libor+50 bps

For the mezzanine debt in our example, the initial credit enhancement is equal to the initial size of the equity tranche. For the senior bond, it is equal to the sum of the equity and mezzanine tranches. There is initially no overcollateralization.

The junior bond has a much wider spread than that of the senior, and much less credit enhancement; the mezzanine attachment point is

5 percent, and the senior attachment point is 15 percent. We assume that, at these prices, the bonds will price at par when they are issued. In the further development of this example, we will explore the risk analysis that a potential investor might consider undertaking. The weighted average spread on the debt tranches is 97.4 basis points.

The loans in the collateral pool and the liabilities are assumed to have a maturity of five years. All coupons and loan interest payments are annual, and occur at year-end.

We assume the swap curve (“Libor”) is flat at 5 percent. If there are *no* defaults in the collateral pool, the annual cash flows are

	Libor+spread	×	Principal amount	=	Annual interest
Collateral	(0.050 + 0.0350)	×	100,000,000	=	\$8,500,000
Mezzanine	(0.050 + 0.0500)	×	10,000,000	=	\$1,000,000
Senior	(0.050 + 0.0050)	×	85,000,000	=	\$4,675,000

The excess spread if there are no defaults, the difference between the collateral cash flows coming into the trust and the tranche coupon payments flowing out, is \$2,825,000.

The assumption that all the loans and bonds have precisely the same maturity date is a great simplification in several respects. Although one of the major motivations of securitization is to obtain term financing of a pool of underlying loans, such perfect *maturity matching* is unusual in constructing a securitization. The problem of maturity transformation in financial markets is pervasive and important; we discuss it in Chapter 12.

The example so far has assumed no defaults. Of course, there may well be at least some defaults in a pool of 100 loans, even in a benign economic environment. If defaults occur at a constant rate, and defaulted collateral is not replaced, the annual number of defaults will fall over time as the pool shrinks due to defaults that have already occurred. The cumulative number of defaults will grow at a progressively slower rate. Suppose, for example, the default rate is expected to be 5 percent annually. The number of defaults in a pool of 100 loans is then likely to be an integer close to 5. After four years, if only 80 loans are still performing and we still expect 5 percent to default, the expected number of defaults is 4.

Regardless of whether the default rate is constant, default losses accumulate, so for any default rate, cash flows from any collateral pool will be larger early in the life of a structured credit product, from interest and amortization of surviving loans and recovery from defaulted loans, than later.

The example also illustrates a crucial characteristic of securitizations: the *timing* of defaults has an enormous influence on the returns to different

tranches. If the timing of defaults is uneven, the risk of inadequate principal at the end may be enhanced or dampened. If defaults are accelerating, the risk to the bond tranches will increase, and vice versa. Other things being equal, the equity tranche benefits relative to more senior debt tranches if defaults occur later in the life of the structured product deal.

### 9.1.3 Issuance Process

The process of creating a securitized credit product is best explained by describing some of the players in the cast of characters that bring it to market. As we do so, we note some of the conflicts of interest that pose risk management problems to investors.

**Loan Originator** The loan originator is the original lender who creates the debt obligations in the collateral pool. This is often a bank, for example, when the underlying collateral consists of bank loans or credit card receivables. But it can also be a specialty finance company or mortgage lender. If most of the loans have been originated by a single intermediary, the originator may be called the *sponsor* or *seller*.

**Underwriter** The *underwriter* or *arranger* is often, but not always, a large financial intermediary. Typically, the underwriter aggregates the underlying loans, designs the securitization structure and markets the liabilities. In this capacity, the underwriter is also the *issuer* of the securities. A somewhat technical legal term, *depositor*, is also used to describe the issuer.

During this aggregation phase, the underwriter bears *warehousing risk*, the risk that the deal will not be completed and the value of the accumulated collateral still on its balance sheet falls. Warehousing risk became important in the early days of the subprime crisis, as the market grew aware of the volumes of "hung loans" on intermediaries' balance sheets. Underwriting in the narrow sense is a "classical" broker-dealer function, namely, to hold the finished securitization liabilities until investors purchase them, and to take the risk that not all the securities can be sold at par.

**Rating Agencies** Rating agencies are engaged to assess the credit quality of the liabilities and assign ratings to them. An important part of this process is determining attachment points and credit subordination. In contrast to corporate bonds, in which rating agencies opine on creditworthiness, but have little influence over it, ratings of securitizations involve the agencies in decisions about structure.

As noted in Chapter 6, rating agencies are typically compensated by issuers, creating a potential conflict of interest between their desire to gain rating assignments and expand their business, and their duty to provide

an objective assessment. The potential conflict is exacerbated by the rating agencies' inherent role in determining the structure. The rating agency may tell the issuer how much enhancement is required, given the composition of the pool and other features of the deal, to gain an investment-grade rating for the top of the capital stack. These seniormost bonds have lower spreads and a wider investor audience, and are therefore uniquely important in the economics of securitizations. Or the issuer may guess at what the rating agency will require before submitting the deal to the agency for review. Either way, the rating agency has an incentive to require less enhancement, permitting the issuer to create a larger set of investment-grade tranches. Investors can cope with the potential conflict by either demanding a wider spread or carrying out their own credit review of the deal.

Ratings may be based solely on the credit quality of the pool and the liability structure. In many cases, however, bonds have higher ratings because of the provision of a guarantee, or *wrap*, by a third party. These guarantees, as noted in Chapter 6, are typically provided by *monoline insurance companies*. Monolines have high corporate ratings of their own and ample capital, and can use these to earn guarantee fees. Such guarantees were quite common until the subprime crisis caused large losses and widespread downgrades among monoline insurers.

**Servicers and Managers** The servicer collects principal and interest from the loans in the collateral pool and disburses principal and interest to the liability holders, as well as fees to the underwriter and itself. The servicer may be called upon to make advances to the securitization liabilities if loans in the trust are in arrears. Servicers may also be tasked with managing underlying loans in distress, determining, for example, whether they should be resolved by extending or refinancing the loan, or by foreclosing. Servicers are thereby often involved in conflicts of interest between themselves and bondholders, or between different classes of bondholders.

One example arises in CMBS. If one distressed loan is resolved by foreclosure, the senior bonds are unlikely to suffer a credit writedown, but rather will receive an earlier-than-anticipated repayment of principal, even if the property is sold at a loss. The junior bond, however, may suffer an immediate credit writedown. If, in contrast, the loan is extended, the junior bond avoids the immediate loss, and has at least a small positive probability of a recovery of value. The senior bond, in contrast, faces the risk that the loss on the property will be even greater, eroding the credit enhancement and increasing the riskiness of the bond. The servicer is obliged to maximize the total present value of the loan, but no matter what he does, he will take an action that is better aligned with the interests of some bonds than of others.

Managers of actively managed loan pools may also be involved in conflicts of interest. As is the case with bankers, investors delegate the task of

monitoring the credit quality of pools to the managers, and require mechanisms to align incentives. One such mechanism that has been applied to managed as well as static pools is to require the manager to own a first-loss portion of the deal. As we see in Chapter 15, this mechanism has been enshrined in the Dodd-Frank Act changes to financial regulatory policy. Such conflicts can be more severe for asset types, especially mortgages, in which servicing is not necessarily carried out by the loan originator. Third-party servicing also adds an entity whose soundness must be verified by investors in the bonds.

Among the economically minor players are the *trustee* and *custodian*, who are tasked with keeping records, verifying documentation, and moving cash flows among deal accounts and paying noteholders.

## **9.2 CREDIT SCENARIO ANALYSIS OF A SECURITIZATION**

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The next step in understanding how a securitization works is to put together the various elements we've just defined—collateral, the liability structure, and the waterfall—and see how the cash flows behave over time and in different default scenarios. We'll continue to use our three-tranche example to lay these issues out. We'll do this in two parts, first analyzing the cash flows prior to maturity, and then the cash flows in the final year of the illustrative securitization's life, which are very different.

Let's take as a base assumption an annual expected default rate of 2 percent. As we will see, the securitization is “designed” the securitization for that default rate, in the sense that if defaults prove to be much higher, the bond tranches may experience credit losses. If the default rate proves much lower, the equity tranche will be extremely valuable, and probably more valuable than the market requires to coax investors to hold the position at par.

### **9.2.1 Tracking the Interim Cash Flows**

Let's introduce a simple overcollateralization mechanism into our example. Instead of letting all the excess spread flow to the equity note, we divert up to \$1,750,000 per year to a reserve account, which we will call the “overcollateralization account,” where it will earn the financing/money market rate of 5 percent. This is a bit of a misnomer, since the funds in the account represent soft rather than hard credit enhancement. If excess spread is less than \$1,750,000, that smaller amount is diverted to the overcollateralization account. If excess spread is greater than \$1,750,000, the amount that exceeds \$1,750,000 is paid out to the equity.

The funds in the overcollateralization account will be used to pay interest on the bonds if there is not enough interest flowing from the loans in the collateral pool during that period. Any remaining funds in the account will be released to the equity tranche only at maturity. It is not a robust mechanism for protecting the senior bonds, but at least has the virtue that, unless defaults are very high early in the deal's life, the overcollateralization account is likely to accumulate funds while cumulative defaults are low.

We assume that the loans in the collateral pay no interest if they have defaulted any time during the prior year. There is no partial interest; interest is paid at the end of the year by surviving loans only.

We also have to make an assumption about recovery value if a loan defaults. We will assume that in the event of default, the recovery rate is 40 percent, and that the recovery amount is paid into the overcollateralization account, where it is also invested at the financing/money market rate. We have to treat recovery this way in order to protect the senior bond; if the recovery amounts flowed through the waterfall, the equity would perversely benefit from defaults. In a typical real-world securitization, the recovery would flow to the senior bonds, and eventually the mezzanine bond tranche, until they are paid off. Time-tranching would endeavor to have recoveries that occur early in the life of the deal flow to short-duration bonds and later recoveries to long-duration bonds. To keep our example simple, we “escrow” the recovery and defer writedowns until the maturity of the securitization.

We need some notation to help us track cash flows in more detail for different default scenarios. We'll assign these symbols to the cash flows and account values:

$N$	Number of loans in initial collateral pool; here $N = 100$
$d_t$	Number of defaults in the course of year $t$
$L_t$	Aggregate loan interest received by the trust at the end of year $t$
$B$	Bond coupon interest due to both the junior and senior bonds (a constant for all $t$ ; here \$5,675,000).
$K$	Maximum amount diverted annually from excess spread into the overcollateralization account; here \$1,750,000
$OC_t$	Amount actually diverted from excess spread into the overcollateralization account at the end of year $t$
$R_t$	Recovery amount deposited into the overcollateralization account at the end of year $t$
$r$	Money market or swap rate, assumed to be constant over time and for all maturities; here $r = 0.05$

Once we take defaults into account, the loan interest flowing from the surviving collateral at the end of year  $t$  is

$$L_t = (0.050 + 0.035) \times \left( N - \sum_{\tau=1}^t d_\tau \right) \times 1000000 \quad t = 1, \dots, T - 1$$

Let's tabulate the interim cash flows for three scenarios, with default rates of 1.5, 5.25, and 9.0 percent annually. As noted, the cash flows during the first four years of our five-year securitization are different from the terminal cash flows, so we tabulate them separately a bit further on.

Interest equal to \$5,675,000 is due to the bondholders. The excess spread is  $L_t - B$ . The excess spread will turn negative if defaults have been high. In that case, bond interest can't be paid out of the collateral cash flow, but must come in whole or in part out of the overcollateralization account.

The amount diverted from the excess spread to the overcollateralization account is

$$\max[\min(L_t - B, K), 0] \quad t = 1, \dots, T - 1$$

If the excess spread is negative, any bond interest shortfall will be paid out of the overcollateralization account. Also, additional funds equal to

$$R_t = 0.4d_t \times 1,000,000 \quad t = 1, \dots, T - 1$$

will flow into the overcollateralization account from default recovery. Thus the value of the overcollateralization account at the end of year  $t$ , including the cash flows from recovery and interest paid on the value of the account at the end of the prior year, is

$$R_t + OC_t + \sum_{\tau=1}^{t-1} (1+r)^{t-\tau} OC_\tau \quad t = 1, \dots, T - 1$$

This value is not fully determined until we know  $OC_t$ . And as simple as this securitization structure is, there are a few tests that the custodian must go through to determine the overcollateralization cash flow. These rules can be thought of as a two-step decision tree, each step having two branches. The test is carried out at the end of each year. In the first step, the custodian tests whether the excess spread is positive; that is, is  $L_t - B > 0$ ?

- If  $L_t - B \geq 0$ , the next test determines whether the excess spread is great enough to cover  $K$ ; that is, is  $L_t - B \geq K$ ?

- If  $L_t - B \geq K$ , then  $K$  flows into the overcollateralization account, and there may be some excess spread left over for the equity, unless  $L_t - B = K$ .
- If  $L_t - B < K$ , then the entire amount  $L_t - B$  flows into the overcollateralization account, and there is no excess spread left over for the equity. If  $L_t - B = 0$ , then there is exactly enough excess spread to cover bond payments and nothing flows into the overcollateralization account.
- If the excess spread is negative ( $L_t - B < 0$ ), the custodian tests whether there are enough funds in the overcollateralization account, plus proceeds from recovery on defaults over the past year, to cover the shortfall. The funds in the overcollateralization account from prior years amount to  $\sum_{\tau=1}^{t-1} (1+r)^{t-\tau} OC_{\tau}$  and current year recoveries are  $R_t$ , so the test is

$$\sum_{\tau=1}^{t-1} (1+r)^{t-\tau} OC_{\tau} + R_t \geq B - L_t$$

- If the shortfall can be covered, then the entire amount  $B - L_t$  flows out of the overcollateralization account.
- If not, that is, if

$$\sum_{\tau=1}^{t-1} (1+r)^{t-\tau} OC_{\tau} + R_t < B - L_t$$

then  $\sum_{\tau=1}^{t-1} (1+r)^{t-\tau} OC_{\tau} + R_t$  flows out of the overcollateralization account, leaving it entirely depleted.

The amount to be diverted can be written

$$OC_t = \left\{ \begin{array}{l} \min(L_t - B, K) \\ \max[L_t - B, -(\sum_{\tau=1}^{t-1} (1+r)^{t-\tau} OC_{\tau} + R_t)] \end{array} \right\} \quad \text{for} \quad \left\{ \begin{array}{l} L_t \geq B \\ L_t < B \end{array} \right\}$$

Once we know how much excess spread, if any, flows into the overcollateralization account at the end of year  $t$ , we can determine how much cash flows to the equity noteholders at the end of year  $t$ . The equity cash flow is

$$\max(L_t - B - OC_t, 0) \quad t = 1, \dots, T - 1$$

Obviously, there is no cash flow to the equity prior to maturity unless there is positive excess spread.

The results for our example can be presented in a cash flow table, presented as Table 9.1, that shows the cash flows in detail, as specified by

**TABLE 9.1** Interim Cash Flow Table for the CLO

(1) <i>t</i>	(2) Def	(3) Cum	(4) Srv	(5) Loan int	(6) Exc spr	(7) OC	(8) Recov	(9) OC+Recov	(10) Eq flow	(11) Results	(12) OC a/c
<i>Default rate 2.0 percent</i>											
1	2	2	98	8,330,000	2,655,000	1,750,000	800,000	2,550,000	905,000	Y	2,550,000
2	2	4	96	8,160,000	2,485,000	1,750,000	800,000	2,550,000	735,000	Y	5,227,500
3	2	6	94	7,990,000	2,315,000	1,750,000	800,000	2,550,000	565,000	Y	8,038,875
4	2	8	92	7,820,000	2,145,000	1,750,000	800,000	2,550,000	395,000	Y	10,990,819
<i>Default rate 7.5 percent</i>											
1	8	8	92	7,820,000	2,145,000	1,750,000	3,200,000	4,950,000	395,000	Y	4,950,000
2	7	15	85	7,225,000	1,550,000	1,550,000	2,800,000	4,350,000	0	Y	9,547,500
3	6	21	79	6,715,000	1,040,000	1,040,000	2,400,000	3,440,000	0	Y	13,464,875
4	6	27	73	6,205,000	530,000	530,000	2,400,000	2,930,000	0	Y	17,068,119
<i>Default rate 10.0 percent</i>											
1	10	10	90	7,650,000	1,975,000	1,750,000	4,000,000	5,750,000	225,000	Y	5,750,000
2	9	19	81	6,885,000	1,210,000	1,210,000	3,600,000	4,810,000	0	Y	10,847,500
3	8	27	73	6,205,000	530,000	530,000	3,200,000	3,730,000	0	Y	15,119,875
4	7	34	66	5,610,000	-65,000	-65,000	2,800,000	2,735,000	0	Y	18,610,869

Key to columns:

- |   |   |
|---|---|
| (1) Year index  | (7) Overcollateralization increment   |
| (2) Number of defaults during year <i>t</i>   | (8) Recovery amount $R_t$   |
| (3) Cumulative number of defaults $\sum_{\tau=1}^t d_\tau$<br>at the end of year <i>t</i> | (9) Aggregate flow into overcollateralization account $OC_t + R_t$  |
| (4) Number of surviving loans $N - \sum_{\tau=1}^t d_\tau$<br>at the end of year <i>t</i> | (10) Interim cash flow to the equity at the end of year <i>t</i> .  |
| (5) Loan interest   | (11) Results of a test to see if interest on the bonds can be paid<br>in full at the end of year <i>t</i> . |
| (6) Excess spread   | (12) The value of the overcollateralization account at time <i>t</i> .                                      |

the waterfall, in each period. There is a panel in the cash flow table for each default scenario.

We can now summarize the results. The excess spread declines over time in all scenarios as defaults pile up, as one would expect. For the high-default scenarios, the loan interest in later years is not sufficient to cover the bond interest and the excess spread turns negative.

The overcollateralization amount is capped at \$1,750,000, and when the default rate is 2.0 percent that amount can be paid into the overcollateralization account in full every year. For higher default rates, the cap kicks in early on. For the highest default rate, in which the excess spread in the later years turns negative, not only is no additional overcollateralization diverted away from the equity, but rather funds must be paid out of the overcollateralization account to cover the bond interest.

The most dramatic differences between the default scenarios are in the equity cash flows, the last cash flows to be determined. For the lowest default rate, the equity continues to receive at least some cash almost throughout the life of the securitization. In the higher default scenarios, interim cash flows to the equity terminate much earlier.

Because the recovery amounts are held back rather than used to partially redeem the bonds prior to maturity, and because, in addition, even in a very high default scenario, there are enough funds available to pay the coupons on the bond tranches, the bonds cannot “break” before maturity. In real-world securitizations, trust agreements are written so that in an extreme scenario, the securitization can be unwound early, thus protecting the bond tranches from further loss.

### 9.2.2 Tracking the Final-Year Cash Flows

To complete the cash flow analysis, we need to examine the final-year payment streams. Our securitization has an anticipated maturity of five years, and we have tabulated cash flows for the first four. Next, we examine the terminal, year 5, cash flows. There are four sources of funds at the end of year 5:

1. Loan interest from the surviving loans paid at the end of year 5, equal to

$$\left(N - \sum_{t=1}^T d_t\right) \times (0.05 + 0.035) \times 1,000,000$$

2. Proceeds from redemptions at par of the surviving loans:

$$\left(N - \sum_{t=1}^T d_t\right) \times 1,000,000$$

3. The recovery from loans defaulting in year 5:

$$R_T = 0.4 \times d_T \times 1,000,000$$

4. The value of the overcollateralization account at the end of year 5, equal to  $1 + r$  times the value displayed, for each default rate, in the last row of the last column of Table 9.1:

$$\sum_{\tau=1}^T (1+r)^{t-\tau} OC_{\tau}$$

There is no longer any need to divert funds to overcollateralization, so all funds are to be used to pay the final coupon and redemption proceeds to the bondholders, in order of priority and to the extent possible. There is also no longer any need to carry out an overcollateralization test.

Next, we add all the terminal cash flows and compare their sum with the amount due to the bondholders. If too many loans have defaulted, then one or both bonds may not receive its stipulated final payments in full. The terminal available funds are:

$$F = \sum_{t=1}^{T-1} (1+r)^{t-\tau} OC_{\tau} + \left[ \left( N - \sum_{t=1}^T d_t \right) 1.085 + 0.4d_T \right] \times 1,000,000$$

If this amount is greater than the \$100,675,000 due to the bondholders, the equity note receives a final payment. If it is less, at least one of the bonds will default. The custodian therefore must perform a sequence of two shortfall tests. The first tests if the senior note can be paid in full:

$$F \begin{cases} \geq \\ < \end{cases} 89,675,000$$

If this test is passed, the senior bond is money good. If not, we subtract the shortfall from its par value. The senior bond value then experiences a credit loss or writedown of  $\$89,675,000 - F$ . We can express the loss as  $\max(89,675,000 - F, 0)$ .

Since the senior bond must be paid first, the default test for the junior bond is

$$F - 89,675,000 \begin{cases} \geq \\ < \end{cases} 11,000,000$$

which is the amount due the mezzanine note holders. If there is a shortfall, the credit loss of the mezzanine is  $\max[11,000,000 - (F - 89,675,000), 0]$ .

The credit risk to the bonds is of a shortfall of interest and, potentially, even principal. What about the equity? The equity is not “owed” anything, so is there a meaningful measure of its credit risk? One approach is to compute the equity tranche’s *internal rate of return* (IRR) in different scenarios. Credit losses in excess of expectations will bring the rate of return down, possibly below zero, if not even the par value the equity investor advanced is recovered over time. The equity investor will typically have a target rate of return, or *hurdle rate*, representing an appropriate compensation for risk, given the possible alternative uses of capital. Even if the rate of return is non-negative, it may fall below this hurdle rate and represent a loss. We could use a posited hurdle rate to discount cash flows and arrive at an equity dollar price. While the results would be somewhat dependent on the choice of hurdle rate, we can speak of the equity’s value more or less interchangeably in terms of price or IRR.

To compute the equity IRR, we first need to assemble all the cash flows to the equity tranche. The initial outlay for the equity tranche is \$5,000,000. If the equity tranche owner is both the originator of the underlying loans and the sponsor of the securitization, this amount represents the difference between the amount lent and the amount funded at term via the bond tranches. If the equity tranche owner is a different party, we assume that party bought the equity “at par.” Recall that we’ve assumed that the bond and underlying loan interest rates are market-clearing, equilibrium rates. We similarly assume the equity has a market-clearing expected return at par.

We saw earlier that the interim cash flows to the equity, that is, those in the first 4 years, are  $\max(L_t - B - OC_t, 0)$ ,  $t = 1, \dots, 4$ . The terminal cash flow to the equity is  $\max(F - 100,675,000, 0)$ , since the bond tranches have a prior claim to any available funds in the final period. Thus the IRR is the value of  $x$  that satisfies

$$0 = -5,000,000 + \sum_{t=1}^{T-1} (1+x)^{-t} \max(L_t - B - OC_t, 0) \\ + (1+x)^{-T} \max(F - 100,675,000, 0)$$

To complete the scenario analysis, we display these values for the three default scenarios in Table 9.2. The first three rows of data display the final-year default count, and the cumulative number of defaulted and surviving loans. The next five rows of data show the terminal available funds and how they are generated—loan interest, redemption proceeds, and recovery. The main driver is, not surprisingly, redemption proceeds from surviving loans. The next row of data is the amount owed to the bondholders at time  $T$ , the same, of course, in all default scenarios.

We can see that in the low default scenario, the bonds will be paid in full and the equity tranche will get a large final payment. At higher default

**TABLE 9.2** Terminal Cash Flows of the CDO

Default rate	2.0	7.5	10.0
<i>Time T default counts:</i>			
Final period current default count	2	5	7
Final period cumulative default count	10	32	41
Final period surviving loan count	90	68	59
<i>Available funds at time T:</i>			
Final period loan interest	7,650,000	5,780,000	5,015,000
Loan redemption proceeds	90,000,000	68,000,000	59,000,000
Final period recovery amount	800,000	2,000,000	2,800,000
Ending balance of overcollateralization account	11,540,360	17,921,525	19,541,412
Total terminal available funds	109,990,360	93,701,525	86,356,412
Owed to bond tranches	100,675,000	100,675,000	100,675,000
<i>Equity returns:</i>			
Equity terminal cash flow	9,315,360	0	0
Equity internal rate of return (%)	23.0	-92.1	-95.5
<i>Bond writedowns:</i>			
Total terminal bond shortfall	0	6,973,475	14,318,588
Terminal mezzanine shortfall	0	6,973,475	11,000,000
Terminal senior shortfall	0	0	3,318,588

rates, the equity receives no residual payment, and one or both of the bonds cannot be paid in full.

For the expected default rate of 2 percent, the equity IRR is 23 percent. At high default rates, the IRR approaches minus 100 percent. At a default rate of 10 percent, for example, the equity receives an early payment out of excess spread, but nothing subsequently, so the equity tranche owner is “out” nearly the entire \$5,000,000 initial investment.

The final rows of the table show credit losses, if any, on the bond tranches. At a default rate of 2 percent, both bonds are repaid in full. At a default rate of 7.5 percent, the junior bond loses its final coupon payment and a portion of principal. At a default rate of 10 percent, even the senior bond cannot be paid off in full, but loses part of its final coupon payment.

The table shows extreme loss levels that will “break” the bonds and essentially wipe out the equity tranche. We have focused here on explaining how to take account of the structure and waterfall of the securitization in determining losses, while making broad-brush assumptions about the performance of the collateral pool. Another equally important task in scenario analysis is to determine what are reasonable scenarios about pool losses. How we interpret the results for a 10 percent collateral pool default rate depends on how likely we consider that outcome to be. As explained in

Chapter 10, it is difficult to estimate the probabilities of such extreme events precisely. But we can make sound judgements about whether they are highly unlikely but possible, or close to impossible.

For structured credit products, such judgments are based on two assessments. The first is a credit risk assessment of the underlying loans, to determine how the distribution of defaults will vary under different economic conditions, requiring expertise and models pertinent to the type of credit in the pool, say, consumer credit or commercial real estate loans. The second is a judgement about how adverse an economic environment to take into account. The latter is based on both economic analysis and the risk appetite of the investor. We discuss the issues arising in designing appropriate stress tests in Chapter 13.

### **9.3 MEASURING STRUCTURED CREDIT RISK VIA SIMULATION**

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Up until now, we have analyzed losses in the securitization for specific default scenarios. But this approach ignores default correlation, that is, the propensity of defaults to coincide. Once we take default correlation into account, we can estimate the entire probability distribution of losses for each tranche into account. The loss distributions provide us with insights into valuation as well as risk.

Chapter 8 introduced two approaches to taking account of default correlation in a credit portfolio, one based on the single-factor model and the other on simulation via a copula model. We'll apply the simulation/copula approach to the loan portfolio that constitutes the securitization trust's collateral pool. While in Chapter 8, we applied the simulation approach to a portfolio of two credit-risky securities, here we apply it to a case in which the underlying collateral contains many loans. This simulation-based analysis of the risk of a securitization, by taking into account the default correlation, unlocks the entire distribution of outcomes, not just particular outcomes.

#### **9.3.1 The Simulation Procedure and the Role of Correlation**

The simulation process can be summarized in these steps:

*Estimate parameters.* First we need to determine the parameters for the valuation, in particular, the default probabilities or default distribution of each individual security in the collateral pool, and the correlation used to tie the individual default distributions together.

*Generate default time simulations.* Using the estimated parameters and the copula approach, we simulate the default times of each security (here, the underlying loans) in the collateral pool. With the default times in hand, we can next identify, for each simulation thread and each security, whether it defaults within the life of the securitization, and if so, in what period.

*Compute the credit losses.* The default times can be used to generate a sequence of cash flows from the collateral pool in each period, for each simulation thread. This part of the procedure is the same as the cash flow analysis of the previous section. The difference is only that in the simulation approach, the number of defaults each period is dictated by the results of the simulation rather than assumed. The securitization capital structure and waterfall allocate the cash flows over time, for each simulation thread, to the securitization tranches. For each simulation thread, the credit loss, if any, to each liability and the residual cash flow, if any, to the equity tranche can then be computed. This gives us the entire distribution of losses for the bonds and of IRRs for the equity. The distributions can be used to compute credit statistics such as credit VaR for each tranche.

The default probability parameters can, as usual, be estimated in two ways, either as a physical or, if comparable spread data is available, as a risk-neutral probability. We have  $N$  (in our example, 100) pieces of collateral in the pool, so we need up to  $N$  distinct default probabilities  $\pi_n$ ,  $n = 1, \dots, N$ . If we want to use time-varying hazard rates, we also need a term structure of default probabilities. For our securitization example, we assume, as in Chapter 8, that we have obtained a one-year physical default probability  $\pi_n$  from an internal or external rating. We convert this to a hazard rate using the formula

$$\pi_n = 1 - e^{-\lambda_n} \quad \Leftrightarrow \quad \lambda_n = -\log(1 - \pi_n) \quad n = 1, \dots, N$$

We'll assume each loan has the same probability of default, so  $\pi_n = \pi$ ,  $n = 1, \dots, 100$ .

The correlations  $\rho_{m,n}$ ,  $m, n = 1, \dots, N$  between the elements of the collateral pool are more difficult to obtain, since the copula correlation, as we have seen, is not a natural or intuitive quantity, and there is not much market or financial data with which to estimate it. We'll put only one restriction on the correlation assumption: that  $\rho_{m,n} \geq 0$ ,  $m, n = 1, \dots, N$ .

In our example we assume the correlations are pairwise constant, so  $\rho_{mm} = \rho$ ,  $m, n = 1, \dots, 100$ . We will want to see the effects of different

assumptions about default probability and correlation, so, for both the default probability and correlation parameter, we'll compare results for different pairs of  $\pi$  and  $\rho$ . Our posited loan default probabilities range from  $\pi = 0.075$  to  $\pi = 0.975$ , in increments of 0.075, and we apply correlation parameters between  $\rho = 0$  and  $\rho = 0.9$ , in increments of 0.3. This gives us a total of 52 pairs of default probability and correlation parameter settings to study.

Once we have the parameters, we can begin to simulate. Since we are dealing with an  $N$ -security portfolio, each simulation thread must have  $N$  elements. Let  $I$  be the number of simulations we propose to do. In our example, we set the number of simulations at  $I = 1,000$ . The first step is to generate a set of  $I$  draws from an  $N$ -dimensional joint standard normal distribution. This is an  $N$ -dimensional random variable in which each element is normally distributed with a mean of zero and a standard deviation equal to unity, and in which each pair of elements  $m$  and  $n$  has a correlation coefficient of  $\rho_{m,n}$ .

The result of this step is a matrix

$$\tilde{\mathbf{z}} = \begin{pmatrix} \tilde{z}_{11} & \tilde{z}_{12} & \cdots & \tilde{z}_{1N} \\ \tilde{z}_{21} & \tilde{z}_{22} & \cdots & \tilde{z}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{z}_{I1} & \tilde{z}_{I2} & \cdots & \tilde{z}_{IN} \end{pmatrix}$$

Each row of  $\tilde{\mathbf{z}}$  is one simulation thread of an  $N$ -dimensional standard normal variate with a covariance matrix equal to

$$\Sigma = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{12} & 1 & \cdots & \rho_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1N} & \rho_{2N} & \cdots & 1 \end{pmatrix}$$

For example, suppose we take the correlations coefficient to be a constant  $\rho = 0.30$ . We can generate 1,000 correlated normals. The result of this step is a matrix

$$\tilde{\mathbf{z}} = \begin{pmatrix} \tilde{z}_{1,1} & \cdots & \tilde{z}_{1,100} \\ \vdots & \ddots & \vdots \\ \tilde{z}_{1000,1} & \cdots & \tilde{z}_{1000,100} \end{pmatrix}$$

with each row representing one simulation thread of a 100-dimensional standard normal variate with a mean of zero and a covariance matrix equal to

$$\begin{pmatrix} 1 & \cdots & \rho \\ \vdots & \ddots & \vdots \\ \rho & \cdots & 1 \end{pmatrix} = \begin{pmatrix} 1 & \cdots & 0.3 \\ \vdots & \ddots & \vdots \\ 0.3 & \cdots & 1 \end{pmatrix}$$

In the implementation used in the example, the upper left  $4 \times 4$  submatrix of the matrix  $\tilde{z}$  is

$$\begin{pmatrix} -1.2625 & -0.3968 & -0.4285 & -1.0258 & \cdots \\ -0.3778 & -0.1544 & -1.5535 & -0.4684 & \cdots \\ 0.2319 & -0.1779 & -0.4377 & -0.5282 & \cdots \\ -0.6915 & -0.5754 & -0.3939 & -0.1683 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

The actual numerical values would depend on the random number generation technique being used and random variation in the simulation results, as outlined in Appendix A.5. For our example, we generate four such matrices  $\tilde{z}$ , one for each of our correlation assumptions.

The next step is to map each element of  $\tilde{z}$  to a default time  $\tilde{t}_{ni}$ ,  $n = 1, \dots, N$ ,  $i = 1, \dots, I$ . If we have one hazard rate  $\lambda_n$  for each security, we carry out the mapping via the formula

$$\tilde{t}_{ni} = -\frac{\log[1 - \Phi(\tilde{z}_{ni})]}{\lambda_n} \quad n = 1, \dots, N; i = 1, \dots, I$$

giving us a matrix  $\tilde{t}$  of default times. We can now count off the number of defaults, and the cumulative number, occurring in each period within the term of the securitization liabilities. Note that we need a distinct matrix of simulated standard normals for each correlation assumption, but not for each default probability assumption.

In our example, we use a constant hazard rate across loans:

$$\tilde{t}_{ni} = -\frac{\log[1 - \Phi(\tilde{z}_{ni})]}{\lambda} \quad n = 1, \dots, 100; i = 1, \dots, 1000$$

For  $\pi = 0.0225$ , for example, we have  $\lambda = -\log(1 - \pi) = -\log(0.9775) = 0.022757$ . Together with the assumption  $\rho = 0.30$ , this results in another  $1,000 \times 100$  matrix. In our simulation example, it has upper left  $4 \times 4$  submatrix

$$\begin{pmatrix} 4.7951 & 18.6433 & 17.8702 & 7.2705 & \dots \\ 19.1194 & 25.3727 & 2.7262 & 16.9309 & \dots \\ 39.3599 & 24.6542 & 17.6515 & 15.5915 & \dots \\ 12.3274 & 14.5882 & 18.7161 & 24.9461 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

We generate as many such matrices of simulated default times as we have pairs of default time-correlation parameter assumptions, namely 52. For example, focusing on element (1,1) of the submatrix of correlated normal simulations of the previous page, we compute the corresponding element (1,1) of the matrix of default times as

$$-\frac{\log[1 - \Phi(-1.2625)]}{-\log(1 - 0.0225)} = 4.7951$$

Note that there are two defaults (default times less than 5.0) within the five-year term of the securitization for these first four loans in the first four threads of the simulation for the parameter pair  $\pi = 0.0225, \rho = 0.30$ . Again, we have 52 such matrices, one for each parameter pair, each of dimension  $1,000 \times 100$ .

This completes the process of generating simulations of default times. The next step is to turn the simulated default times  $\tilde{t}$  into vectors of year-by-year defaults and cumulative defaults, similar to columns (2) and (3) of the cash flow Table 9.1, and the row of final year defaults in Table 9.2. To do this, we count, for each of the 1,000 rows, how many of the 100 simulated default times fall into each of the intervals  $(t - 1, t], t = 1, \dots, T$ . The result in our example is a set of 1,000 vectors of length  $T = 5$ , each containing the number of defaults occurring in each  $\rho$  of the five years of the CLO. The cumulative sum of each of these vectors is the cumulative default count, also a five-element vector.

The full first row of  $\tilde{t}$  for the parameter pair  $\pi = 0.0225, \rho = 0.30$ , for example, is

4.80	18.64	17.87	7.27	3.86	18.39	3.89	5.85	11.37	25.80
22.39	5.35	17.60	20.62	0.84	4.27	39.38	11.22	30.37	3.44
6.70	10.21	29.41	26.93	8.79	36.20	24.55	48.12	2.48	0.55
11.89	4.55	12.81	69.02	24.22	7.99	16.70	4.94	12.36	7.48
2.55	8.12	4.75	91.37	32.10	35.34	25.53	0.39	3.55	10.55
1.83	2.80	0.79	1.26	5.72	2.69	1.12	0.91	3.94	32.04
2.69	2.94	12.66	9.80	2.40	40.70	7.47	0.46	15.31	16.72
5.31	5.85	0.14	5.89	25.30	9.80	13.96	8.73	5.73	48.27
26.22	7.39	5.25	3.13	0.68	4.51	1.88	3.31	39.46	8.38
42.29	0.73	4.53	11.38	15.70	0.99	0.91	22.43	1.94	12.41

It gives us the simulated default times in the first simulation thread of each of the 100 pieces of collateral. The associated current default count vector is

$$(11, 5, 7, 7, 7)$$

since there are 11 elements in the first row of  $\tilde{\mathbf{t}}$  that are less than or equal to 1, 4 elements in the range (1, 2), and so on. The corresponding cumulative default count vector is

$$(11, 16, 23, 30, 37)$$

Thus, in that first simulation thread, there is a cumulative total of 37 defaults by the end of year 5. (This is, incidentally, one of the grimmer simulation threads for this parameter pair.) We generate 1,000 such cumulative default count vectors, one for each simulation thread, for this parameter pair.

We want to see the effects of different assumptions, so we repeat this procedure for all 52 pairs of default probabilities  $\pi = 0.0075, 0.0150, \dots, 0.0975$  and correlations  $\rho = 0.00, 0.30, 0.60, 0.90$ . One of the advantages of this approach is that, if we want to see the effects of changing distributional parameters, or characteristics of the collateral or liability structure, such as the recovery rate or the interest paid by the collateral, we don't have to do a fresh set of simulations. We only change the way the simulations are processed. We would need to do new simulations only if we want to increase the number of threads  $I$  for greater simulation accuracy, or we change the number of loans in the collateral pool, or we introduce new correlation settings not included in the set  $\{0.00, 0.30, 0.60, 0.90\}$ .

The final step is to pass these loan-loss results, scenario by scenario, through the waterfall. To accomplish this, we repeat, for each simulation, the process we laid out for scenario analysis. For each simulation, we use the current and cumulative default count vectors to generate the cash flows, distribute them through the waterfall, and tabulate the cash flows for each security.

### 9.3.2 Means of the Distributions

We can now describe the distributions of the results. We'll begin with the means.

The results for the equity tranche are displayed in the next table. Each value is the *mean* IRR over all the simulations for the parameter pair displayed in the row and column headers. For low default rates, the mean equity IRRs are over 30 percent per annum, while for high default rates

and low correlations, the equity tranche is effectively wiped out in many simulation threads.

<i>Equity IRR (percent)</i>				
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$
0.0075	33.9	32.4	30.8	32.2
0.0225	20.6	13.3	14.2	19.8
0.0375	-2.8	-8.1	-0.9	10.5
0.0525	-46.9	-26.3	-13.8	1.5
0.0675	-79.3	-41.2	-24.0	-6.5
0.0825	-89.7	-53.5	-33.3	-13.8
0.0975	-93.8	-63.1	-41.1	-20.3

In order to compute risk statistics such as VaR, we use dollar values rather than IRRs. To do so, we make a somewhat arbitrary parameter assignment, namely, that the equity hurdle rate is 25 percent. Some assumption on hurdle rates is required in order to identify the IRR at which a loss occurs, and is similar to our setting the market-clearing bond coupons as part of the example. This hurdle rate more or less prices the equity tranche at its par value of \$5,000,000 for  $\pi = 2.25$  percent and  $\rho = 0.30$ . We use this hurdle rate to discount to the present the future cash flows to the equity tranche in each simulation scenario. The sum of these present values is the equity value in that scenario. A present value is computed for each simulation as:

$$\sum_{t=1}^{T-1} (1.25)^{-t} \max(L_t - B - OC_t, 0) + (1.25)^{-T} \max(F - 100675000, 0)$$

Averaging these present values over all 1,000 simulations gives us the estimated equity value for each  $(\pi, \rho)$  pair. Table 9.3 tabulates the means of the simulated equity values and the bond credit writedowns. We display them graphically in Figure 9.1. Each result is the mean over the 1,000 simulations of the IRR or credit loss. The bond writedowns are expressed as a percent of the par value of the bond, rather than in millions of dollars to make comparison of the results for the mezzanine and senior bonds more meaningful.

The means of the mezzanine and senior bond writedowns don't "add up," even though the results add up simulation by simulation. Consider, for example, the parameter pair  $\pi = 0.0225$  and  $\rho = 0.30$ . There are small losses for both the senior and junior bonds. How can there be losses to the senior at all, if the junior losses are small? The reason is that the senior loss

**TABLE 9.3** Mean Equity Values and Bond Credit Losses

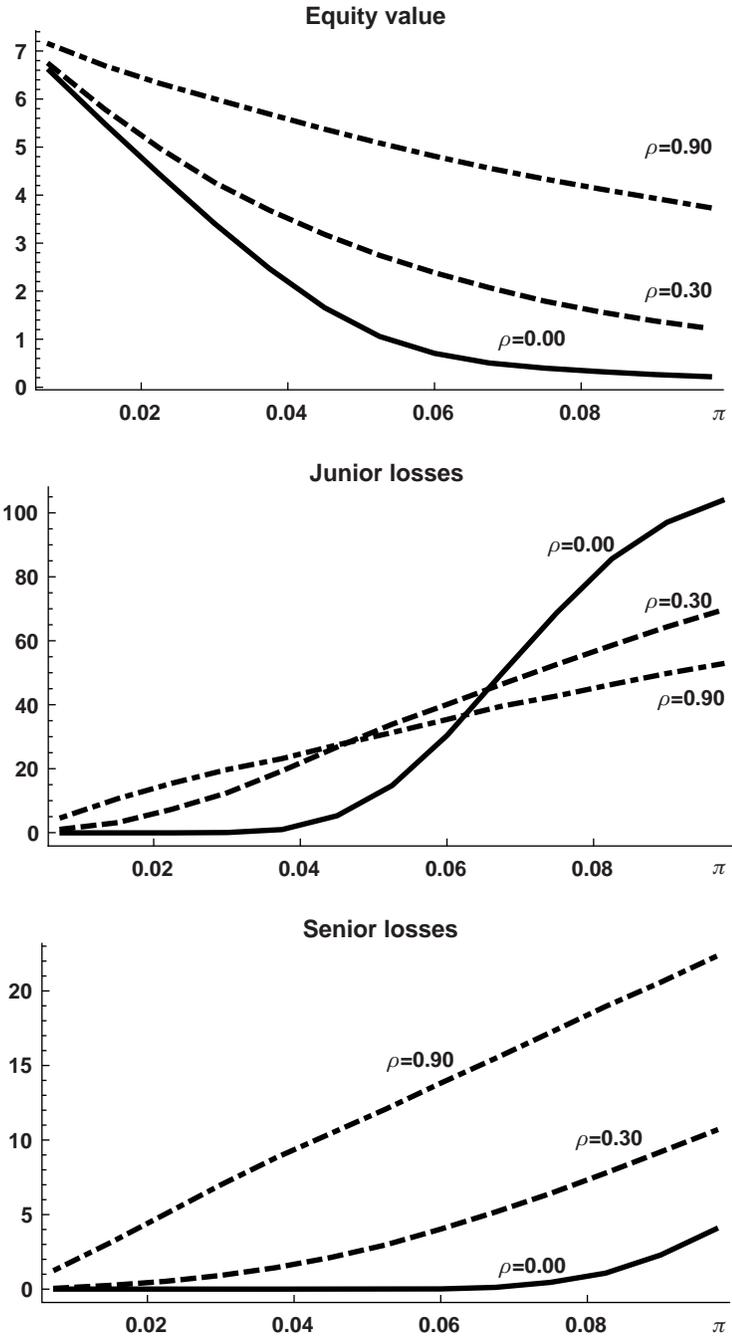
<i>Equity value (\$ million)</i>					
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$	
0.0075	6.59	6.72	6.85	7.14	
0.0225	4.44	4.98	5.61	6.33	
0.0375	2.47	3.69	4.64	5.69	
0.0525	1.06	2.75	3.90	5.08	
0.0675	0.51	2.07	3.32	4.56	
0.0825	0.33	1.57	2.84	4.13	
0.0975	0.22	1.23	2.44	3.74	
<i>Mezzanine bond writedown (percent of tranche par value)</i>					
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$	
0.0075	0.00	1.11	3.36	4.84	
0.0225	0.00	7.35	12.82	15.49	
0.0375	1.03	19.30	23.97	23.14	
0.0525	14.81	33.90	33.75	31.32	
0.0675	49.86	46.45	43.82	39.64	
0.0825	85.74	58.60	51.54	46.40	
0.0975	103.92	69.58	58.68	52.87	
<i>Senior bond writedown (percent of tranche par value)</i>					
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$	
0.0075	0.00	0.05	0.41	1.31	
0.0225	0.00	0.52	2.14	5.05	
0.0375	0.00	1.44	4.36	8.81	
0.0525	0.00	2.96	6.96	12.08	
0.0675	0.12	5.17	9.71	15.49	
0.0825	1.07	7.78	12.75	18.96	
0.0975	4.02	10.64	15.92	22.29	

of 0.05 percent of par stems from six simulation threads out of the 1,000 in which, of course, the junior tranche is entirely wiped out. However, there are only 19 threads in which the junior tranche experiences a loss at all, so the average loss for the parameter pair is low.

There are several important patterns in the results we see in the example, particularly with respect to the interaction between correlation and default probability:

*Increases in the default rate* increase bond losses and decrease the equity IRR for all correlation assumptions. In other words, for any given correlation, an increase in the default rate will hurt all of the tranches. This is an unsurprising result, in contrast to the next two.

*Increases in correlation* can have a very different effect, depending on the level of defaults. At low default rates, the impact of an increase



**FIGURE 9.1** Values of CLO Tranches  
Equity value and bond losses in millions of \$ as a function of default probabilities for different constant pairwise correlations. The equity is valued using a discount factor of 25 percent per annum. Bond losses are in percent of par value.

in correlation is relatively low. But when default rates are relatively high, an increase in correlation can materially *increase* the IRR of the equity tranche, but also *increase* the losses to the senior bond tranche. In other words, the equity benefits from high correlation, while the senior bond is hurt by it. We will discuss this important result in more detail in a moment.

The effect on the mezzanine bond is more complicated. At low default rates, an increase in correlation increases losses on the mezzanine bond, but decreases losses for high default rates. In other words, the mezzanine bond behaves more like a senior bond at low default rates, when it is unlikely that losses will approach its attachment point and the bond will be broken, and behaves more like the equity tranche when default rates are high and a breach of the attachment point appears likelier.

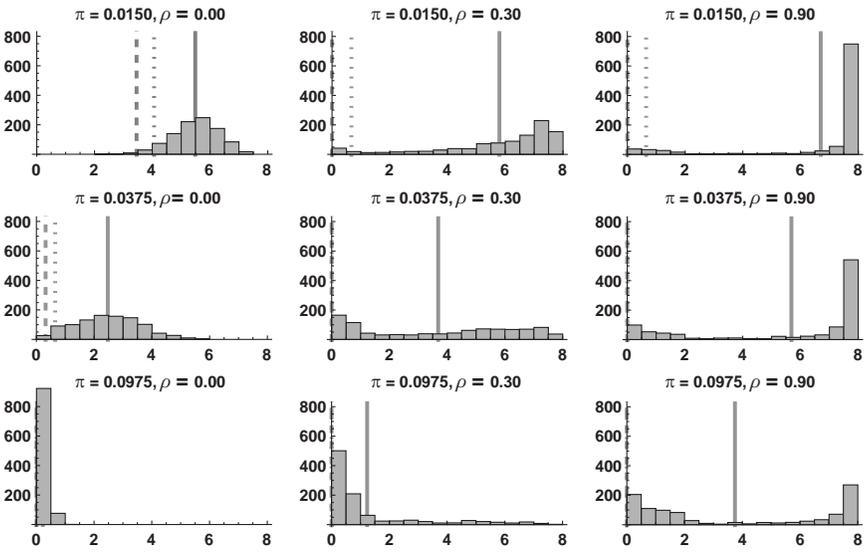
*Convexity.* At low correlations, the equity value is substantially *positively convex* in default rates. That is, the equity tranche loses value rapidly as default rates increase from a low level. But as default rates increase, the responsiveness of the equity value to further increases in the default rate drops off. In other words, you can't beat a dead horse: If you are long the equity tranche, once you've lost most of your investment due to increases in default rates, you will lose a bit less from the next increase in default rates.

For low correlations, the senior bond tranche has *negative convexity* in default rates; its losses accelerate as defaults rise. The mezzanine tranche, again, is ambiguous. It has negative convexity for low default rates, but is positively convex for high default rates. At high correlations, all the tranches are less convex; that is, they respond more nearly linearly to changes in default rates.

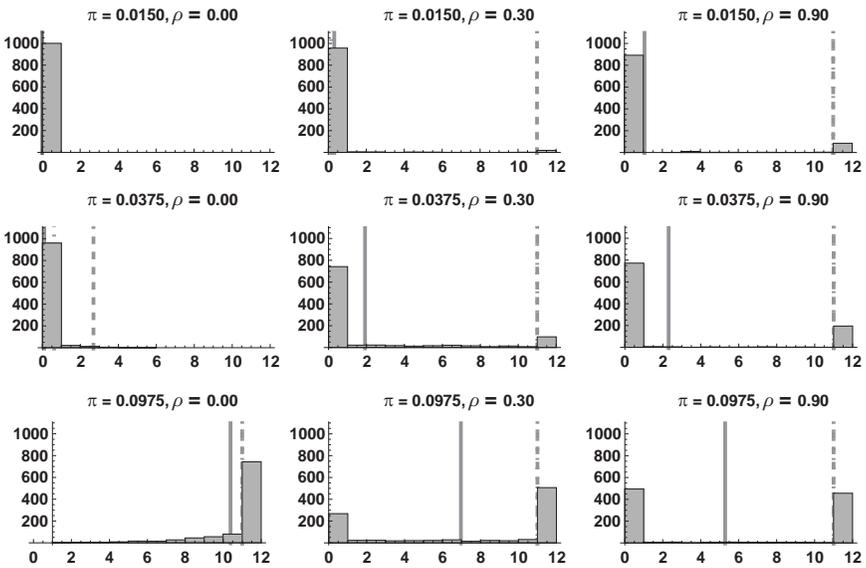
### 9.3.3 Distribution of Losses and Credit VaR

Table 9.3 and Figure 9.1 display the means over all the simulations for each parameter pair. We can gain additional insights into the risk characteristics of each tranche by examining the entire distribution of outcomes for different parameter pairs; the patterns we see differ across tranches.

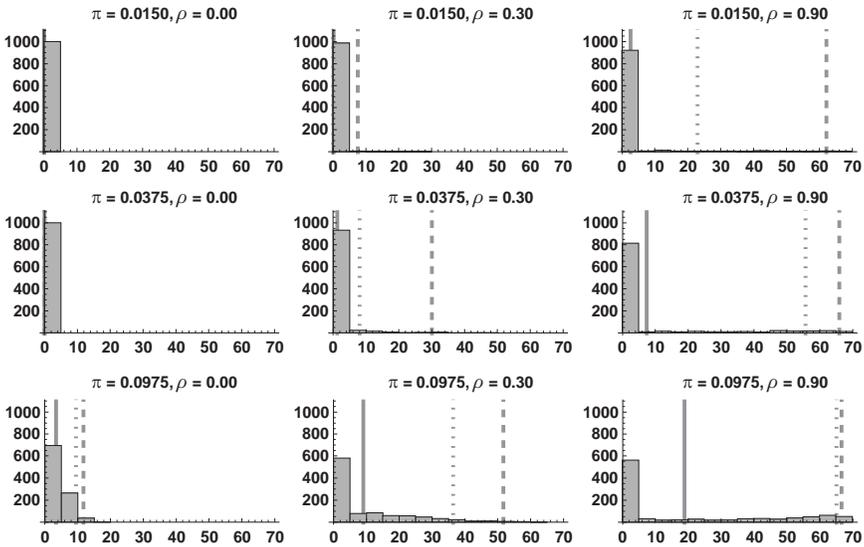
**Characteristics of the Distributions** Figures 9.2 through 9.4 present histograms of all 1,000 simulated values of each of the three CLO tranches for a subset of our 52  $(\pi, \rho)$  assumption pairs. Each histogram is labeled by its  $(\pi, \rho)$  assumption. The expected value of the tranche for the  $(\pi, \rho)$



**FIGURE 9.2** Distribution of Simulated Equity Tranche Values  
 Histograms of simulated values of equity tranche, in millions of \$. Each histogram is labeled by its default probability and correlation assumption. Values are computed using a discounting rate of 25 percent. The solid grid line marks the mean value over the 1,000 simulations. The dashed and dotted grid lines mark the 0.01 and 0.05 quantile values.



**FIGURE 9.3** Distribution of Simulated Mezzanine Bond Tranche Losses  
 Histograms of simulated losses of mezzanine bond, in millions of \$. Each histogram is labeled by its default probability and correlation assumption. The solid grid line marks the mean loss over the 1,000 simulations. The dashed and dotted grid lines mark the 0.99 and 0.95 quantiles of the loss.



**FIGURE 9.4** Distribution of Simulated Senior Bond Tranche Losses  
 Histograms of simulated losses of senior bonds, in millions of \$. Each histogram is labeled by its default probability and correlation assumption. The solid grid line marks the mean loss over the 1,000 simulations. The dashed and dotted grid lines mark the 0.99 and 0.95 quantiles of the loss.

assumption is marked by a solid grid line. The 0.01-(0.05)-quantile of the value distribution is marked by a dashed (dotted) grid line.

The distribution plots help us more fully understand the behavior of the mean values or writedowns of the different tranches. Before we look at each tranche in detail, let’s recall how correlation affects the pattern of defaults. When correlation is high, defaults tend to arrive in clusters. Averaged over all of the simulations, the number of defaults will be approximately equal to the default probability. But the defaults will not be evenly spread over the simulation. Some simulations will experience unusually many and some unusually few defaults for any default probability. The higher the correlation, the more such extreme simulation results there will be.

*Equity tranche.* The results for the equity tranche (Figure 9.2) are plotted as dollar values, with the cash flows discounted at our stipulated IRR of 25 percent. The most obvious feature of the histograms is that for low correlations, the simulated values form a bell curve. The center of the bell curve is higher for lower default probabilities. For high enough default rates, the bell curve is squeezed up against the lower bound of zero value, as it is wiped out in most scenarios

before it can receive much cash flow. In a low correlation environment, the equity note value is close to what you would expect based on the default probability. It is high for a low default rate and vice versa.

The surprising results are for high correlations. The distribution is U-shaped; extreme outcomes, good or bad, for the equity value are more likely than for low correlations. In a scenario with unusually many defaults, given the default probability, the equity is more likely to be wiped out, while in low-default scenarios, the equity will keep receiving cash flows for a surprisingly long time.

The equity note thus behaves like a lottery ticket in a high-correlation environment. If defaults are low, the high correlation induces a higher probability of a high-default state, reducing the equity note's value. If defaults are high, the high correlation induces a higher probability of a low-default state, raising the equity note's value.

If, in contrast, the correlation is low, some defaults will occur in almost every simulation thread. Since the equity tranche takes the first loss, this means that at least some equity losses are highly likely even in a relatively low-default environment. Therefore low correlation is bad for the equity. Correlation is more decisive for equity risk and value than default probability.

This behavior is related to the convexity of the mean equity values we see in Figure 9.1. At a low correlation, equity values have fewer extremes and are bunched closer to what you would expect based on the default probability, much like the results we obtained using default scenario analysis above. But there is only so much damage you can do to the equity tranche; as it gets closer to being wiped out, a higher default rate has less incremental impact.

*Bond tranches.* The bond tranche distributions (Figures 9.3 and 9.4) are plotted as dollar amounts of credit losses. They appear quite different from those of the equity; even for low correlations, they are not usually bell-curve-shaped. Rather, in contrast to the equity, the credit subordination concentrates the simulated losses close to zero, but with a long tail of loss scenarios. For the senior bond, this is particularly clear, as almost all simulation outcomes show a zero or small loss, unless both default probability and correlation are quite high.

But the distributions have one characteristic in common with the equity. The distribution of simulated loss tends towards a U shape for higher default probabilities and higher correlations. This tendency is much stronger for the mezzanine, which, as we have

seen, behaves like the equity at high correlations and default probabilities.

For low correlations and high default probabilities, finally, we see an important contrast between the mezzanine and senior bonds. The mezzanine is a relatively thin tranche, so a small increase in default rates shifts the center of gravity of the distribution from par to a total loss. We can see this clearly by comparing the histograms for  $(\pi = 0.0375, \rho = 0.00)$  with that for  $(\pi = 0.0975, \rho = 0.00)$ .

**Credit VaR of the Tranches** We can, finally, compute the credit VaR. To do so, we need to sort the simulated values for each tranche by size. For the equity tranche, we measure the credit VaR at a confidence level of 99 (or 95) percent as the difference between the 10th (or 50th) lowest sorted simulation value and the par value of \$5,000,000. The latter value, as noted, is close to the mean present value of the cash flows with  $\pi = 2.25$  percent and  $\rho = 0.30$ . For the bonds, we measure the VaR as the difference between the expected loss and the 10th (or 50th) highest loss in the simulations.

The results at a 99 percent confidence level are displayed in Table 9.4. To make it easier to interpret the table, we have also marked with grid lines the 99th (dashed) and 95th (dotted) percentiles in Figures 9.2 through 9.4. The 99-percent credit VaR can then be read graphically as the horizontal distance between the dashed and solid grid lines. To summarize the results:

*Equity tranche.* The equity VaR actually falls for higher default probabilities and correlations, because the expected loss is so high at those parameter levels. Although the mean values of the equity tranche increase with correlation, so also does its risk.

*Mezzanine bond.* The junior bond again shows risk characteristics similar to those of the equity at higher default rates and correlation and to those of the senior bond for lower ones.

The mezzanine, like the equity tranche, is thin. One consequence is that, particularly for higher  $(\pi, \rho)$  pairs, the credit VaRs at the 95 and 99 percent confidence levels are very close together. This means that, conditional on the bond suffering a loss at all, the loss is likely to be very large relative to its par value.

*Senior bond.* We see once again that correlation is bad for the senior bond. At high correlations, the 99 percent credit VaR of the senior bond is on the order of one-half the par value, while if defaults are uncorrelated, the bond is virtually risk-free even at high default probabilities.

**TABLE 9.4** CLO Tranche Credit VaR at a 99 Percent Confidence Level

<i>Equity VaR (\$ million)</i>				
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$
0.0075	1.62	6.33	6.85	7.14
0.0225	2.53	4.98	5.61	6.33
0.0375	2.16	3.69	4.64	5.69
0.0525	0.95	2.75	3.90	5.08
0.0675	0.51	2.07	3.32	4.56
0.0825	0.33	1.57	2.84	4.13
0.0975	0.22	1.23	2.44	3.74
<i>Mezzanine bond VaR (\$ million)</i>				
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$
0.0075	0.00	3.79	10.66	10.52
0.0225	0.00	10.26	9.72	9.45
0.0375	2.59	9.07	8.60	8.69
0.0525	7.09	7.61	7.63	7.87
0.0675	6.01	6.36	6.62	7.04
0.0825	2.43	5.14	5.85	6.36
0.0975	0.61	4.04	5.13	5.71
<i>Senior bond VaR (\$ million)</i>				
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$
0.0075	0.00	-0.04	11.23	48.30
0.0225	0.00	17.77	41.43	59.99
0.0375	0.00	28.82	49.76	58.61
0.0525	0.00	35.74	52.66	56.23
0.0675	2.85	39.89	52.75	53.57
0.0825	7.03	41.61	51.88	50.62
0.0975	8.33	42.60	50.25	47.79

For a correlation of 0.90, the risk of the senior bond at a 99 percent confidence level varies surprisingly little with default probability. The reason is that at a high correlation, clusters of defaults in a handful of simulations guarantee that at least 1 percent of the simulations will show extremely high losses.

Note that there is one entry, for the senior bond with  $(\pi, \rho) = (0.0075, 0.30)$ , for which the VaR is negative. This odd result is an artifact of the simulation procedure, and provides an illustration of the difficulties of simulation for a credit portfolio. For this assumption pair, almost all the simulation results value the senior bond at par, *including* the 10th ordered simulation result. There are, however, seven threads in which the senior bond has a loss.

So the expected loss is in this odd case actually higher than the 0.01-quantile loss, and the VaR scenario is a gain.

The anomaly would disappear if we measured VaR at the 99.5 or 99.9 percent confidence level. However, the higher the confidence level, the more simulations we have to perform to be reasonably sure the results are not distorted by simulation error.

### 9.3.4 Default Sensitivities of the Tranches

The analysis thus far has shown that the securitization tranches have very different sensitivities to default rates. Equity values always fall and bond losses always rise as default probabilities rise, but the response varies for different default correlations and as default rates change. This has important implications for risk management of tranche exposures. In this section, we examine these default sensitivities more closely.

To do so, we develop a measure of the responsiveness of equity value or bond loss to small changes in default probabilities. The “default01” measures the impact of an increase of 1 basis point in the default probability. It is analogous to the DV01 we studied in Chapter 4 and the spread01 we studied in Chapter 7 and is calculated numerically in a similar way.

To compute the default01, we increase and decrease default probability 10bps and revalue each tranche at these new values of  $\pi$ . This requires repeating, twice, the entire valuation procedure from the point onward at which we generate simulated default times. We can reuse our correlated normal simulations  $\tilde{z}$ . In fact, we should, in order to avoid a change of random seed and the attendant introduction of additional simulation noise. But we have to recompute  $\tilde{t}$ , the list of vectors of default counts for each simulation, and all the subsequent cash flow analysis, valuation, and computation of losses. The default01 sensitivity of each tranche is then computed as

$$\frac{1}{20}[(\text{mean value/loss for } \pi + 0.0010) - (\text{mean value/loss for } \pi - 0.0010)]$$

We compute this default01 for each combination of  $\pi$  and  $\rho$ . The results are displayed in Table 9.5 and Figure 9.5. Each default01 is expressed as a positive number and expresses the decline in value or increase in loss resulting from a 1-basis point rise in default probability.

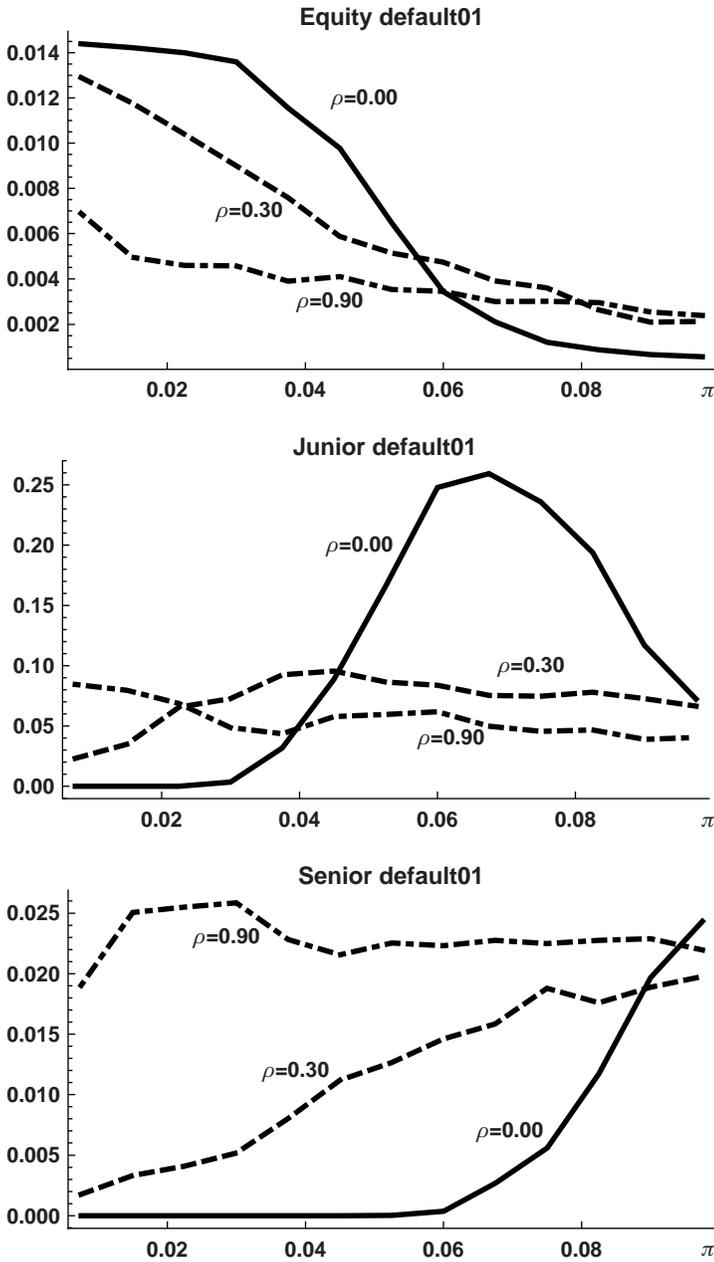
For all tranches, in all cases, default01 is positive, as expected, regardless of the initial value of  $\pi$  and  $\rho$ , since equity and bond values decrease monotonically as the default probability rises. The default01 sensitivity converges to zero for all the tranches for very high default rates (though we are not displaying high enough default probabilities to see this for the senior

**TABLE 9.5** CLO Tranche Default Sensitivities

<i>Equity loss (\$ million per bp)</i>				
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$
0.0075	0.0144	0.0129	0.0094	0.0069
0.0225	0.0140	0.0104	0.0076	0.0046
0.0375	0.0116	0.0076	0.0056	0.0039
0.0525	0.0065	0.0052	0.0038	0.0035
0.0675	0.0021	0.0039	0.0036	0.0030
0.0825	0.0009	0.0026	0.0032	0.0030
0.0975	0.0006	0.0021	0.0025	0.0024
<i>Mezzanine bond loss (Percent of par per bp)</i>				
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$
0.0075	0.0000	0.0231	0.0572	0.0843
0.0225	0.0000	0.0655	0.0658	0.0687
0.0375	0.0317	0.0924	0.0659	0.0436
0.0525	0.1660	0.0863	0.0658	0.0595
0.0675	0.2593	0.0753	0.0605	0.0498
0.0825	0.1939	0.0778	0.0478	0.0465
0.0975	0.0723	0.0664	0.0458	0.0403
<i>Senior bond loss (Percent of par per bp)</i>				
$\pi$	$\rho = 0.00$	$\rho = 0.30$	$\rho = 0.60$	$\rho = 0.90$
0.0075	0.0000	0.0018	0.0084	0.0190
0.0225	0.0000	0.0041	0.0140	0.0255
0.0375	0.0000	0.0081	0.0152	0.0229
0.0525	0.0000	0.0127	0.0170	0.0226
0.0675	0.0027	0.0159	0.0194	0.0228
0.0825	0.0117	0.0176	0.0220	0.0228
0.0975	0.0243	0.0198	0.0217	0.0220

bond). Once losses are extremely high, the incremental impact of additional defaults is low.

The default01 varies most as a function of default probability when correlation is low. With  $\rho = 0$ , the default01 changes sharply in a certain range of default probabilities, and then tapers off as the tranche losses become very large. The differences in the patterns for the different tranches are related to the locations of their attachment points. For each tranche, the range of greatest sensitivity to an increase in defaults, that is, the largest-magnitude default01, begins at a default rate that brings losses in the collateral pool near that tranche's attachment point. Thus the peak default01 is at a default probability of zero for the equity tranche, and occurs at a lower default rate for the mezzanine than for the senior tranche because it has a lower attachment point. As we see in Chapter 11, this introduces additional risk when



**FIGURE 9.5** Default Sensitivities of CLO Tranches  
 Default01 as a function of default probability for different constant pairwise correlations. Equity in \$ million per bp, bonds in percent of par per bp.

structured credit exposures are put on in a low-correlation environment, or correlation is underestimated. Underestimation of default correlation in structured credit products was an important factor in the origins of the subprime crisis.

Note that some of the default plots are not smooth curves, providing us with two related insights. The first is about the difficulty or “expense” of estimating the value and risk of credit portfolios using simulation methods. The number of defaults at each  $t$  in each simulation thread must be an integer. Even with 100 loans in the collateral pool, the distribution of value is so skewed and fat-tailed that simulation noise amounting to one or two defaults can make a material difference in the average of the simulation results. The curves could be smoothed out further by substantially increasing the number of simulations used in the estimation procedure. This would be costly in computing time and storage of interim results.

We have enough simulations that the fair value plots are reasonably smooth, but not so all the default plots. The lumpiness shows up particularly in the plots of the senior bond default plots and those for higher correlations for all tranches. The reason is intuitive. At higher correlations, the defaults tend to come in clusters, amplifying the lumpiness. A chance variation in the number of default clusters in a few simulation threads can materially change the average over all the threads.

The second insight is that because of the fat-tailed distribution of losses, it is difficult to diversify a credit portfolio and reduce idiosyncratic risk. Even in a portfolio of 100 credits, defaults remain “lumpy” events.

### 9.3.5 Summary of Tranche Risks

We’ve now examined the risk of the securitization liabilities in several different ways: mean values, the distribution of values and credit VaR, and the sensitivities of the values to changes in default behavior, all measured for varying default probabilities and correlations. As in the scenario analysis of the previous section, we’ve focused on the securitization’s liability structure and waterfall, and less on the equally crucial credit analysis of the underlying loans.

On the basis of the example, we can make a few generalizations about structured credit product risk. In Chapter 14, we see that neglect of these risks played an important role during the subprime crisis, in its propagation and in the losses suffered by individual institutions.

*Systematic risk.* Structured credit products can have a great deal of systematic risk, even when the collateral pools are well-diversified. In our example, the systematic risk shows up in the equity values and bond losses when default correlation is high. High default correlation is one way of expressing high systematic risk, since it means

that there is a low but material probability of a state of the world in which an unusually large number of defaults occurs.

Most notably, even if the collateral is well-diversified, the senior bond has a risk of loss, and potentially a large loss, if correlation is high. While its expected loss may be lower than that of the underlying loan pool, the tail of the loss and the credit VaR are high, as seen in the rightmost column of plots in Figure 9.4. In other words, they are very exposed to systematic risk. The degree of exposure depends heavily on the credit quality of the underlying collateral and the credit enhancement.

*Tranche thinness.* Another way in which the senior bond's exposure to systematic risk is revealed is in the declining difference between the senior bond's credit VaRs at the 99 and 95 percent confidence levels as default probabilities rise for high default correlations. For the mezzanine bond, the difference between credit VaR at the 99 and 95 percent confidence levels is small for most values of  $\pi$  and  $\rho$ , as seen in Figure 9.3. The reason is that tranche is relatively thin. The consequence of tranche thinness is that, conditional on the tranche suffering a loss at all, the size of the loss is likely to be large.

*Granularity* can significantly diminish securitization risks. In Chapter 8, we saw that a portfolio of large loans has greater risk than a portfolio with equal par value of smaller loans, each of which has the same default probability, recovery rate, and default correlation to other loans. Similarly, "lumpy" pools of collateral have greater risk of extreme outliers than granular ones. A securitization with a more granular collateral pool can have a somewhat larger senior tranche with no increase in credit VaR. A good example of securitizations that are not typically granular are the many CMBS deals in which the pool consists of relative few mortgage loans on large properties, or so-called *fusion* deals in which a fairly granular pool of smaller loans is combined with a few large loans. When the asset pool is not granular, and/or correlation is high, the securitization is said to have high *concentration risk*.

## 9.4 STANDARD TRANCHES AND IMPLIED CREDIT CORRELATION

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Structured credit products are claims on cash flows of credit portfolios. Their prices therefore contain information about how the market values certain characteristics of those portfolios, among them default correlation. In the previous section, we have seen how to use an estimate of the default

correlation to estimate model values of securitization tranches. Next, we see how we can reverse engineer the modeling process, applying the model to observed market prices of structured credit products to estimate a default correlation. The correlation obtained in this way is called an *implied credit* or *implied default correlation*. It is a risk-neutral parameter that we can estimate whenever we observe prices of portfolio credit products. In Chapter 11, we will discuss an example of the risk management challenges presented by implied correlation. In Chapter 14 (see especially Figure 14.21), we discuss the use of implied credit correlation as an indicator of market sentiment regarding systemic risk.

#### 9.4.1 Credit Index Default Swaps and Standard Tranches

We begin by introducing an important class of securitized credit products that trades in relatively liquid markets. In Chapter 7, we studied CDS, the basic credit derivative, and earlier in this chapter we noted that CDS are often building blocks in synthetic structured products. *Credit index default swaps* or CDS indexes are a variant of CDS in which the underlying security is a portfolio of CDS on individual companies, rather than a single company's debt obligations. Two groups of CDS indexes are particularly frequently traded:

*CDX* (or *CDX.NA*) are index CDS on North American companies.

*iTraxx* are index CDS on European and Asian companies.

Both groups are managed by Markit, a company specializing in credit-derivatives pricing and administration. There are, in addition, customized credit index default swaps on sets of companies chosen by a client or financial intermediary.

CDX and iTraxx come in series, initiated semiannually, and indexed by a series number. For example, series CDX.NA.IG.10 was introduced in March 2008. Each series has a number of index products, which can be classified by

*Maturity.* The standard maturities, as with single-name CDS, are 1, 3, 5, 7, and 10 years. The maturity dates are fixed calendar dates.

*Credit quality.* In addition to the investment grade CDX.NA.IG, there is a high-yield group (CDX.NA.HY), and subsets of IG and HY that focus on narrower ranges of credit quality.

We'll focus on investment grade CDX (CDX.NA.IG); the iTraxx are analogous. Each IG series has an underlying basket consisting of equal notional

amounts of CDS on 125 investment-grade companies. Thus a notional amount \$125,000,000 of the CDX contains \$1,000,000 notional of CDS on each of the 125 names. The list of 125 names changes from series to series as firms lose or obtain investment-grade ratings, and merge with or spin off from other firms.

Cash flows and defaults are treated similarly to those of a single-name CDS credit event. Given the CDS spread premiums on the individual names in the index, there is a fair market CDS spread premium on the CDX. One difference from single-name CDS is that the spread premium is fixed on the initiation date, so subsequent trading in CDX of a particular series generally involves the exchange of net present value. The buyer of CDX protection pays the fixed spread premium to the seller. If credit spreads have widened since the initiation date, the buyer of protection on CDX will pay an amount to the seller of protection.

In the event of default of a constituent of the CDX, the company is removed from the index. In general, there is cash settlement, with an auction to determine the value of recovery on the defaulting company's debt. The dollar amount of spread premium and the notional amount of the CDX contract are reduced by 0.8 percent (since there are 125 equally weighted constituents), and the CDX protection seller pays 0.8 percent of the notional, minus the recovery amount, to the protection buyer.

The constituents of a CDX series can be used as the reference portfolio of a synthetic CDO. The resulting CDO is then economically similar to a cash CLO or CDO with a collateral pool consisting of equal par amounts of bonds issued by the 125 firms in the index. There is a standard capital structure for such synthetic CDOs based on CDX:

Assets	Liabilities
\$1 million notional long protection on each constituent of CDX.NA.IG, total \$125 million notional	Equity 0–3% Junior mezzanine 3–7% Senior mezzanine 7–10% Senior 10–15% Super senior 15–30%

The liabilities in this structure are called the *standard tranches*. They are fairly liquid and widely traded, in contrast to *bespoke tranches*, generally issued “to order” for a client that wishes to hedge or take on exposures to a specific set of credits, with a specific maturity, and at a specific point in the capital structure. Similar products exist for the iTraxx, with a somewhat different tranching. The fair market value or spread of each tranche is tied to those of the constituent CDS by arbitrage.

The equity tranche, which is exposed to the first loss, is completely wiped out when the loss reaches 3 percent of the notional value. The weight of each constituent is the same, and if we assume default recovery is the same 40 percent for each constituent, then 9 or 10 defaults will suffice: 9 defaults will leave just a small sliver of the equity tranche, and 10 defaults will entirely wipe it out and begin to eat into the junior mezzanine tranche.

### 9.4.2 Implied Correlation

The values, sensitivities, and other risk characteristics of a standard tranche can be computed using the copula techniques described in this and Chapter 8, but with one important difference. In the previous section, the key inputs to the valuation were estimates of default probabilities and default correlation. But the constituents of the collateral pools of the standard tranches are the 125 single-name CDS in the IG index, relatively liquid products whose spreads can be observed daily, or even at higher frequency. Rather than using the default probabilities of the underlying firms to value the constituents of the IG index, as in our CLO example in this chapter, we use their market CDS spreads, as in Chapter 7, to obtain risk-neutral default probabilities. In many cases, there is not only an observation of the most liquid five-year CDS, but of spreads on other CDS along the term structure. There may also be a risk-neutral estimate of the recovery rate from recovery swaps. CDS indexes and their standard tranches are therefore typically valued, and their risks analyzed, using risk-neutral estimates of default probabilities.

The remaining key input into the valuation, using the copula technique of this and the last chapter, is the constant pairwise correlation. While the copula correlation is not observable, it can be inferred from the market values of the tranches themselves, once the risk-neutral probabilities implied by the single-name CDS are accounted for. Not only the underlying CDS, but the tranches themselves, are relatively liquid products for which daily market prices can be observed. Given these market prices, and the risk-neutral default curves, a risk-neutral *implied correlation* can be computed for each tranche. Typically, the correlation computed in this fashion is called a *base correlation*, since it is associated with the attachment point of a specific tranche. Correlations generally vary by tranche, a phenomenon called *correlation skew*.

Since the implied correlation is computed using risk-neutral parameter inputs, the calculation uses risk-free rates rather than the fair market discount rates of the tranches. To compute the equity base correlation, we require the market equity tranche price (or compute it from the points up-front and running spread), and the spreads of the constituent CDS. Next, we compute the risk-neutral default probabilities of each of the underlying

125 CDS. Given these default probabilities, and a copula correlation, we can simulate the cash flows to the equity tranche. There will be one unique correlation for which the present value of the cash flows matches the market price of the equity tranche. That unique value is the implied correlation.

The CLO example of the previous section can be used to illustrate these computations. Suppose the observed market price of the equity is \$5 million, and that we obtain a CDS-based risk-neutral default probability of the underlying loans equal to 2 percent. In the top panel of Table 9.3, we can see that a constant pairwise correlation of 0.3 “matches” the equity price to the default probability. If we were to observe the equity price rising to \$5.6 million, with no change in the risk-neutral default probability, we would conclude that the implied correlation had risen to 0.6, reflecting an increase in the market’s assessment of the systematic risk of the underlying loans.

Implied credit correlation is as much a market-risk as a credit-risk concept. The value of each tranche has a distinct risk-neutral partial spread<sup>01</sup>, rather than a default<sup>01</sup>, that is, sensitivities to each of the constituents of the IG 125. The spread<sup>01</sup> measures a market, rather than a credit risk, though it will be influenced by changing market assessments of each firm’s creditworthiness. Each of these sensitivities is a function, *inter alia*, of the implied correlation. Conversely, the implied correlation varies in its own right, as well as with the constituent and index credit spreads. For the cash CLO example in this chapter, changes in default rates and correlation result in changes in expected cash flows and credit losses to the CLO tranches, that is, changes in fundamental value. For the standard tranches, changes in risk-neutral probabilities and correlations bring about mark-to-market changes in tranche values. Chapter 11 explores the correlation and other market risks of synthetic CDOs in more detail.

### 9.4.3 Summary of Default Correlation Concepts

In discussing credit risk, we have used the term “correlation” in several different ways. This is a potential source of confusion, so let’s review and summarize these correlation concepts:

*Default correlation* is the correlation concept most directly related to portfolio credit risk. We formally defined the default correlation of two firms over a given future time period in Section 8.1 as the correlation coefficient of the two random variables describing the firms’ default behavior over a given time period.

*Asset return correlation* is the correlation of logarithmic changes in two firms’ asset values. In practice, portfolio credit risk measurement of

corporate obligations often relies on asset return correlations. Although this is in a sense the “wrong” correlation concept, since it isn’t default correlation, it can be appropriate in the right type of model. For example, in a Merton-type credit risk model, the occurrence of default is a function of the firm’s asset value. The asset return correlation in a factor model is driven by each firm’s factor loading.

*Equity return correlation* is the correlation of logarithmic changes in the market value of two firms’ equity prices. The asset correlation is not directly unobservable, so in practice, asset correlations are often proxied by equity correlations.

*Copula correlations* are the values entered into the off-diagonal cells of the correlation matrix of the distribution used in the copula approach to measuring credit portfolio risk. Unlike the other correlation concepts, the copula correlations have no direct economic interpretation. They depend on which family of statistical distributions is used in the copula-based risk estimate. However, the correlation of a Gaussian copula is identical to the correlation of a Gaussian single-factor factor models.

The normal copula has become something of a standard in credit risk. The values of certain types of securities, such as the standard CDS index equity tranches, as we just noted, depend as heavily on default correlation as on the levels of the spreads in the index. The values of these securities can therefore be expressed in terms of the implied correlation.

*Spread correlation* is the correlation of changes, generally in basis points, in the spreads on two firms’ comparable debt obligations. It is a mark-to-market rather than credit risk concept.

*Implied credit correlation* is an estimate of the copula correlation derived from market prices. It is not a distinct “theoretical” concept from the copula correlation, but is arrived at differently. Rather than estimating or guessing at it, we infer it from market prices. Like spread correlation, it is a market, rather than credit risk concept.

## **9.5 ISSUER AND INVESTOR MOTIVATIONS FOR STRUCTURED CREDIT**

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To better understand why securitizations are created, we need to identify the incentives of the loan originators, who sell the underlying loans into the trust in order create a securitization, and of the investors, who buy the equity

and bonds. These motivations are also key to understanding the regulatory issues raised by securitization and the role it played in the subprime crisis, themes we return to in Chapters 12, 14 and 15.

### 9.5.1 Incentives of Issuers

An important motive for securitization is that it provides a technology for maturity matching, that is, for providing term funding for the underlying loans.<sup>1</sup> There are two aspects to this motive: first, whether lower cost of funding can be achieved via securitization, and, second, whether, in the absence of securitization, the loan originator would have to sell the loans into the secondary market or would be able to retain them on his balance sheet. The securitization “exit” is attractive for lenders only if the cost of funding via securitization is lower than the next-best alternative. If the loans are retained, the loan originator may be able to fund the loans via unsecured borrowing. But doing so is generally costlier than secured borrowing via securitization.

Securitizations undertaken primarily to capture the spread between the underlying loan interest and the coupon rates of the liabilities are sometimes called *arbitrage CDOs*, while securitizations motivated largely for balance sheet relief are termed *balance-sheet CDOs*. However, while the motivations are conceptually distinct, it is hard to distinguish securitizations this way.

Among the factors that tend to lower the spreads on securitization liabilities are loan pool diversification and an originator’s reputation for high underwriting standards. Originators that have issued securitization deals with less-than-stellar performance may be obliged by the market to pay higher spreads on future deals. Issuer spread differentials are quite persistent. These factors also enable the issuer to lower the credit enhancement levels of the senior bonds that have the narrowest spreads, increasing the proceeds the issuer can borrow through securitization and decreasing the weighted-average financing cost.

Idiosyncratic credit risk can be hard to expunge entirely from credit portfolios, limiting the funding advantage securitization can achieve for some lending sectors. This limitation is important for sectors such as credit card and auto loans, where a high degree of granularity in loan pools can be achieved. As noted, commercial mortgage pools are particularly hard to diversify. Residential mortgage pools can be quite granular, but both commercial and residential mortgage loans have a degree of systematic risk

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<sup>1</sup>The discussion in Chapter 12 provides a fuller appreciation of these issues.

that many market participants and regulators vastly underestimated prior to the subprime crisis.

The interest rates on the underlying loans, the default rate, the potential credit subordination level, and the spreads on the bonds interact to determine if securitization is economically superior to the alternatives. If, as is often the case, the issuer retains the servicing rights for the loans, and enjoys significant economies of scale in servicing, securitization permits him to increase servicing profits, raising the threshold interest rates on the liabilities at which securitization becomes attractive.

Looking now at the originator's alternative of selling the loans after origination, secondary trading markets exist for large corporate and commercial real estate loans, in which purchasers take an ongoing monitoring role. It is more difficult to sell most consumer loans to another financial intermediary. One of the impediments to secondary-market loan sales is the twin problem of monitoring and asymmetric information. The credit quality of loans is hard to assess and monitor over the life of the loan. The mere fact that the originator is selling a loan may indicate he possesses information suggesting the loan is of poorer quality than indicated by the securitization disclosures—the “lemons” problem (see Chapter 6). The originator's superior information on the loan and the borrower often puts him in the best position to monitor the loan and take mitigating action if the borrower has trouble making payments.

These problems can be mitigated if equity or other subordinated tranches, or parts of the underlying loans themselves, are either retained by the loan originator or by a firm with the capability to monitor the underlying collateral. Their first-loss position then provides an incentive to exercise care in asset selection, monitoring and pool management that protects the interests of senior tranches as well. As discussed in Chapter 15, *risk retention* has been viewed as a panacea for the conflicts of interest inherent in securitization and has been enshrined in the Dodd-Frank regulatory changes. Rules embodying the legislation have not yet been promulgated but will likely bar issuers of most securitizations from selling all tranches in their entirety. Other mitigants include legal representations by the loan seller regarding the underwriting standards and quality of the loans.

The loan purchaser has legal rights against the seller if these representations are violated, for example, by applying lower underwriting standards than represented. In the wake of the subprime crisis, a number of legal actions have been brought by purchasers of loans as well as structured credit investors on these grounds. These mitigants suggest the difficulty of economically separating originators from loans, that is, of achieving genuine *credit risk transfer*, regardless of how legally robust is the sale of the loans into the securitization trust. The ambiguities of credit risk transfer also arise in credit

derivatives transactions and in the creation of off-balance sheet vehicles by intermediaries, and contribute to financial instability by making it harder for market participants to discern issuers' asset volume and leverage.

### 9.5.2 Incentives of Investors

To understand why securitizations take place, we also need to understand the incentives of investors. Securitization enables capital markets investors to participate in diversified loan pools in sectors that would otherwise be the province of banks alone, such as mortgages, credit card, and auto loans.

Tranching technology provides additional means of risk sharing over and above diversification. Investors, not issuers, motivate credit tranching beyond the issuers' retained interests. Issuers' needs are met by pooling and securitization—they don't require the tranching. Tranching enables investors to obtain return distributions better-tailored to their desired risk profile. A pass-through security provides only the benefit of diversification.

Introducing tranching and structure can reduce default risk for higher tranches, though at the price of potentially greater exposure to systematic risk. Thinner subordinate tranches draw investors desiring higher risk and returns. Some securitization tranches provide *embedded leverage*, which we discuss further in Chapter 12. Thicker senior tranches draw investors seeking lower-risk bonds in most states of the economy, but potentially severe losses in extremely bad states, and willing to take that type of risk in exchange for additional yield.

However, these features are useful to investors only if they carry out the due diligence needed to understand the return distribution accurately. Some institutional investors, particularly pension funds, have high demand for high-quality fixed-income securities that pay even a modest premium over risk-free or high-grade corporate bonds. This phenomenon, often called "searching" or "reaching for yield," arises because institutional investors deploy large sums of capital, while being required to reach particular return targets. Securitization is founded to a large extent on institutional demand for senior bonds. In the presence of regulatory safe harbors and imperfect governance mechanisms, this can lead to inadequate due diligence of the systematic risks of securitized credit products.

Mezzanine tranches, as we have seen, are an odd duck. Depending on the default probability, correlation, and tranche size, they may behave much like a senior tranche. That is, they have a low probability of loss, but high systematic risk; expected loss in the event of impairment is high, and impairment is likeliest in an adverse scenario for the economy as a whole. They may, in a different structure and environment, behave more like an equity tranche, with a high probability of impairment, but a respectable

probability of a low loss. A mezzanine tranche may also switch from one behavior to another. In consequence, mezzanine tranches have less of a natural investor base than other securitized credit products. One result was that many mezzanine tranches were sold into CDOs, the senior tranches of which could be sold to yield-seeking investors uncritically buying structured products on the basis of yield and rating.

Fees also provide incentives to loan originators and issuers to create securitizations. A financial intermediary may earn a higher return from originating and, possibly, servicing loans than from retaining them and earning the loan interest. Securitization then provides a way for the intermediary to remove the loans from its balance sheet after origination. This motivation is related to *regulatory arbitrage*, discussed further in Chapter 15. For example, an intermediary may be able to retain some of the risk exposure and return from a loan pool while drastically reducing the regulatory capital required through securitization.

Off-balance sheet vehicles, and thus, ultimately, money market mutual funds, were also important investors in securitizations. We return to these in Chapter 12 on liquidity risk and Chapter 15 on financial regulation.

## **FURTHER READING**

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Rutledge and Raynes (2010) is a quirky, but comprehensive overview of structured finance, with particularly useful material on legal and structure issues. Textbook introductions to structured credit products include the somewhat untimely-titled Kothari (2006) and (2009), and Mounfield (2009). Meissner (2008) is a useful collection of articles on structured credit. Many of the references following Chapters 1, 7 and 8, 14, and 15 are also useful here.

Gibson (2004) provides a similar analysis to that of this chapter of a structured credit product, but focusing on synthetic CDOs and carrying out the analysis using the single-factor model. Duffie and Gârleanu (2001) is an introduction to structured product valuation. Schwarcz (1994) is an introduction from a legal standpoint, which is important in all matters pertaining to credit and particularly so where securitization is concerned.

Gibson (2004) and Coval, Jurek, and Stafford (2009) discuss the embedded leverage in structured products and the motivations of investors. Zimmerman (2007), Ashcraft and Schuermann (2008) and Gorton (2008) provide thorough discussions of the securitization markets for subprime residential mortgages. Ashcraft and Schuermann (2008) also provide a detailed accounting of the information cost, monitoring, and other conflict-of-interest issues arising at different stages of the securitization. The discussion

is useful both for the risk analysis of securitizations and as an illustration of the role of information cost issues in credit transactions generally. Benmelech and Dlugosz (2009) describes the ratings process for a class of structured products,

Li (2000) was an early application of copula theory to structured credit modeling. Tranche sensitivities are explained in Schloegl and Greenberg (2003).

Belsham, Vause, and Wells (2005); and Amato and Gyntelberg (2005) are introductions to credit correlation concepts. O’Kane and Livesey (2004), Kakodkar, Martin, and Galiani (2003), and Kakodkar, Galiani, and Shchetkovskiy (2004) are introductions by trading desk strategists to structured credit correlations. Amato and Remolona (2005) discusses the difficulty, compared with equities, of reducing idiosyncratic risk in credit portfolios and applies this finding to the risk of structured credit.