

CHAPTER 26

Analysis of Capital Structure Theory

At a meeting of the Financial Management Association, a panel session focused on how firms actually set their target capital structures. The participants included financial managers from Hershey Foods, Verizon, EG&G (a high-tech firm), and a number of other firms in various industries. Although there were minor differences in philosophy and procedures among the companies, several themes emerged.

First, in practice it is difficult to specify an optimal capital structure—indeed, managers feel uncomfortable even about specifying an optimal capital structure range. Thus, financial managers worry primarily about whether their firms are using too little or too much debt, not about the precise optimal amount of debt. Second, even if a firm’s actual capital structure varies widely from the theoretical optimum, this might not have much effect on its stock price. Overall, financial managers believe that capital structure decisions are secondary in importance to operating decisions, especially those relating to capital budgeting and the strategic direction of the firm.

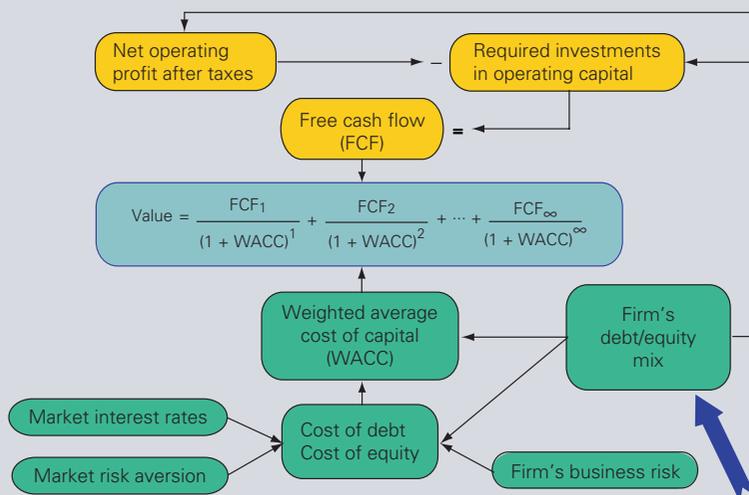
In general, financial managers focus on identifying a “prudent” level of debt rather than on setting a precise optimal level. A prudent level is defined as one that captures most of the benefits of debt yet (1) keeps financial risk at a manageable level, (2) ensures future financing flexibility, and (3) allows the firm to maintain a desirable credit rating. Thus, a prudent level of debt will protect the company against financial distress under all but the worst economic scenarios, and it will ensure access to money and capital markets under most conditions.

As you read this chapter, think about how you would make capital structure decisions if you had that responsibility. At the same time, don’t forget the very important message from the FMA panel session: Establishing the right capital structure is an imprecise process at best, and it should be based on both informed judgment and quantitative analyses.

Corporate Valuation and Capital Structure Decisions

A firm's financing choices obviously have a direct effect on its weighted average cost of capital (WACC). Financing choices also have an indirect effect because they

change the risk and required return of debt and equity. This chapter focuses on the debt–equity choice and its effect on value.



Chapter 15 presented basic material on capital structure, including an introduction to capital structure theory. We saw that debt concentrates a firm's business risk on its stockholders, thus raising stockholders' risk, but it also increases the expected return on equity. We also saw there is some optimal level of debt that maximizes a company's stock price, and we illustrated this concept with a simple model. Now we go into more detail on capital structure theory. This will give you a deeper understanding of the benefits and costs associated with debt financing.



resource

The textbook's Web site contains an Excel file that will guide you through the chapter's calculations. The file for this chapter is **Ch26 Tool Kit.xls**, and we encourage you to open the file and follow along as you read the chapter.

26.1 CAPITAL STRUCTURE THEORY: ARBITRAGE PROOFS OF THE MODIGLIANI-MILLER MODELS

Until 1958, capital structure theory consisted of loose assertions about investor behavior rather than carefully constructed models that could be tested by formal statistical analysis. In what has been called the most influential set of financial papers ever published, Franco Modigliani and Merton Miller (MM) addressed capital structure in a rigorous, scientific fashion, and they set off a chain of research that continues to this day.¹

¹See Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review*, June 1958, pp. 261–297; "The Cost of Capital, Corporation Finance and the Theory of Investment: Reply," *American Economic Review*, September 1958, pp. 655–669; "Taxes and the Cost of Capital: A Correction," *American Economic Review*, June 1963, pp. 433–443; and "Reply," *American Economic Review*, June 1965, pp. 524–527. In a survey of Financial Management Association members, the original MM article was judged to have had the greatest impact on the field of finance of any work ever published. See Philip L. Cooley and J. Louis Heck, "Significant Contributions to Finance Literature," *Financial Management*, Tenth Anniversary Issue, 1981, pp. 23–33. Note that both Modigliani and Miller won Nobel Prizes—Modigliani in 1985 and Miller in 1990.

Assumptions

As we explain in this chapter, MM employed the concept of **arbitrage** to develop their theory. Arbitrage occurs if two similar assets—in this case, levered and unlevered stocks—sell at different prices. Arbitrageurs will buy the undervalued stock and simultaneously sell the overvalued stock, earning a profit in the process, and will continue doing so until market forces of supply and demand cause the prices of the two assets to be equal. For arbitrage to work, the assets must be equivalent, or nearly so. MM show that, under their assumptions, levered and unlevered stocks are sufficiently similar for the arbitrage process to operate.

No one, not even MM, believes their assumptions are sufficiently correct that their models will hold exactly in the real world. However, their models do show how money can be made through arbitrage if one can find ways around problems with the assumptions. Though some of them were later relaxed, here are the initial MM assumptions.

1. There are *no taxes*, either personal or corporate.
2. Business risk can be measured by σ_{EBIT} , and firms with the same degree of business risk are said to be in a *homogeneous risk class*.
3. All present and prospective investors have identical estimates of each firm's future EBIT; that is, investors have *homogeneous expectations* about expected future corporate earnings and the riskiness of those earnings.
4. Stocks and bonds are traded in *perfect capital markets*. This assumption implies, among other things, (a) that there are no brokerage costs and (b) that investors (both individuals and institutions) can borrow at the same rate as corporations.
5. *Debt is riskless*. This applies to both firms and investors, so the interest rate on all debt is the risk-free rate. Further, this situation holds regardless of how much debt a firm (or individual) uses.
6. All cash flows are *perpetuities*; that is, all firms expect zero growth and hence have an “expectationally constant” EBIT, and all bonds are perpetuities. “Expectationally constant” means that the best guess is that EBIT will be constant, although after the fact the realized level could be different from the expected level.

MM without Taxes

MM first analyzed leverage under the assumption that there are no corporate or personal income taxes. On the basis of their assumptions, they stated and algebraically proved two propositions.²

Proposition 1. The value of any firm is established by capitalizing its expected net operating income (EBIT) at a constant rate (r_{sU}) that is based on the firm's risk class:

$$V_L = V_U = \frac{\text{EBIT}}{\text{WACC}} = \frac{\text{EBIT}}{r_{sU}} \quad (26-1)$$

Here the subscript L designates a levered firm and U designates an unlevered firm. Both firms are assumed to be in the same business risk class, and r_{sU} is the required rate of return for an unlevered (i.e., all-equity) firm of this risk class when there are no taxes. For our purposes, it is easiest to think in terms of a single firm that has the

²Modigliani and Miller actually stated and proved three propositions, but the third one is not material to our discussion here.

option of financing either with all equity or with some combination of debt and equity. Hence, L designates a firm that uses some amount of debt and U designates a firm that uses no debt.

As established by Equation 26-1, V is a constant; therefore, *under the MM model, if there are no taxes then the value of the firm is independent of its leverage*. As we shall see, this also implies the following statements.

1. The weighted average cost of capital, WACC, is completely independent of a firm's capital structure.
2. Regardless of the amount of debt a firm uses, its WACC is equal to the cost of equity that it would have if it used no debt.

Proposition II. When there are no taxes, the cost of equity to a levered firm, r_{sL} , is equal to (1) the cost of equity to an unlevered firm in the same risk class, r_{sU} , plus (2) a risk premium whose size depends on (a) the difference between an unlevered firm's costs of debt and equity and (b) the amount of debt used:

$$r_{sL} = r_{sU} + \text{Risk premium} = r_{sU} + (r_{sU} - r_d)(D/S) \quad (26-2)$$

Here D is the market value of the firm's debt, S is the market value of its equity, and r_d is the constant cost of debt. Equation 26-2 states that, *as debt increases, the cost of equity rises in a mathematically precise manner* (even though the cost of debt does not rise).

Taken together, the two MM propositions imply that using more debt in the capital structure will not increase the value of the firm, because the benefits of cheaper debt will be exactly offset by an increase in the riskiness of the equity and hence in its cost. Thus MM argue that, *in a world without taxes, both the value of a firm and its WACC would be unaffected by its capital structure*.

MM's Arbitrage Proof

Propositions I and II are important because they showed for the first time that any valuation effects due to the use of debt must arise from taxes or other market frictions. The technique that MM used to prove these propositions is equally important, however, so we discuss it in detail here. They used an *arbitrage proof* to support their propositions, and this proof technique was later used in the development of option pricing models that revolutionized the securities industry.³ Modigliani and Miller showed that, under their assumptions, if two companies differed only (1) in the way they were financed and (2) in their total market values, then investors would sell shares of the higher-valued firm, buy those of the lower-valued firm, and continue this process until the companies had exactly the same market value. To illustrate, assume that two firms, L and U, are identical in all important respects except that Firm L has \$4,000,000 of 7.5% debt while Firm U uses only equity. Both firms have $EBIT = \$900,000$, and σ_{EBIT} is the same for both firms, so they are in the same business risk class.

Modigliani and Miller assumed that all firms are in a zero-growth situation. In other words, $EBIT$ is expected to remain constant; this will occur if ROE is constant,

³By *arbitrage* we mean the simultaneous buying and selling of essentially identical assets that sell at different prices. The buying increases the price of the undervalued asset, and the selling decreases the price of the overvalued asset. Arbitrage operations will continue until prices have adjusted to the point where the arbitrageur can no longer earn a profit, at which point the market is in equilibrium. In the absence of transaction costs, equilibrium requires that the prices of the two assets be equal.

all earnings are paid out as dividends, and there are no taxes. Under the constant EBIT assumption, the total market value of the common stock, S , is the present value of a perpetuity, which is found as follows:

$$S = \frac{\text{Dividends}}{r_{sL}} = \frac{\text{Net income}}{r_{sL}} = \frac{\text{EBIT} - r_d D}{r_{sL}} \quad (26-3)$$

Equation 26-3 is merely the value of a perpetuity, where the numerator is the net income available to common stockholders (all of which is paid out as dividends) and the denominator is the cost of common equity. Since there are no taxes, the numerator is not multiplied by $(1 - T)$, as it was when we calculated NOPAT in Chapters 2 and 13.

Assume that initially, *before any arbitrage occurs*, both firms have the same equity capitalization rate: $r_{sU} = r_{sL} = 10\%$. Under this condition, according to Equation 26-3, the following situation would exist.

Firm U:

$$\begin{aligned} \text{Value of Firm U's stock} = S_U &= \frac{\text{EBIT} - r_d D}{r_{sU}} \\ &= \frac{\$900,000 - \$0}{0.10} = \$9,000,000 \end{aligned}$$

$$\begin{aligned} \text{Total market value of Firm U} = V_U = D_U + S_U &= \$0 + \$9,000,000 \\ &= \$9,000,000 \end{aligned}$$

Firm L:

$$\begin{aligned} \text{Value of Firm L's stock} = S_L &= \frac{\text{EBIT} - r_d D}{r_{sL}} \\ &= \frac{\$900,000 - 0.075(\$4,000,000)}{0.10} = \frac{\$600,000}{0.10} \\ &= \$6,000,000 \end{aligned}$$

$$\begin{aligned} \text{Total market value of Firm L} = V_L = D_L + S_L &= \$4,000,000 + \$6,000,000 \\ &= \$10,000,000 \end{aligned}$$

Thus, before arbitrage (and assuming that $r_{sU} = r_{sL}$, which implies that capital structure has no effect on the cost of equity), the value of the levered Firm L exceeds that of the unlevered Firm U.

Modigliani and Miller argued that this result is a disequilibrium that cannot persist. To see why, suppose you owned 10% of L's stock and so the market value of your investment was $0.10(\$6,000,000) = \$600,000$. According to MM, you could increase your income without increasing your exposure to risk. For example, you could (1) sell your stock in L for \$600,000, (2) borrow an amount equal to 10% of L's debt (\$400,000), and then (3) buy 10% of U's stock for \$900,000. Note that you would receive \$1,000,000 from the sale of your 10% of L's stock plus your borrowing, and you would be spending only \$900,000 on U's stock. Hence you would have an extra \$100,000, which you could invest in riskless debt to yield 7.5%, or \$7,500 annually.

Now consider your income positions:

| <u>Old Portfolio</u> | | <u>New Portfolio</u> | |
|---------------------------------------|-----------------|---------------------------------------|-----------------|
| 10% of L's \$600,000 equity income | \$60,000 | 10% of U's \$900,000 equity income | \$90,000 |
| | | Less 7.5% interest on \$400,000 loan | (30,000) |
| | | Plus 7.5% interest on extra \$100,000 | <u>7,500</u> |
| Total income | <u>\$60,000</u> | Total income | <u>\$67,500</u> |

Thus, your net income from common stock would be exactly the same as before, \$60,000, but you would have \$100,000 left over for investment in riskless debt and this would increase your income by \$7,500. Therefore, the total return on your \$600,000 net worth would rise to \$67,500. And your risk, according to MM, would be the same as before, because you would have simply substituted \$400,000 of “home-made” leverage for your 10% share of Firm L’s \$4 million of corporate leverage. Thus, neither your “effective” debt nor your risk would have changed. Therefore, you would have increased your income without raising your risk, which is obviously desirable.

Modigliani and Miller argued that this arbitrage process would actually occur, with sales of L’s stock driving its price down and purchases of U’s stock driving its price up, until the market values of the two firms were equal. Until this equality was established, gains could be obtained by switching from one stock to the other; hence the profit motive would force equality to be reached. When equilibrium is established, the values of Firms L and U must be equal, which is what Proposition I states. If their values are equal, then Equation 26-1 implies that $WACC = r_{SU}$. Because there are no taxes, we have

$$WACC = [D/(D + S)]r_d + [S/(D + S)]r_{SL}$$

and a little algebra then yields

$$r_{SL} = r_{SU} + (r_{SU} - r_d)(D/S)$$

which is what Proposition II states. Thus, according to MM, both a firm’s value and its WACC must be independent of capital structure.

Note that each of the assumptions listed at the beginning of this section is necessary for the arbitrage proof to work exactly. For example, if the companies did not have identical business risk or if transaction costs were significant, then the arbitrage process could not be invoked. We discuss other implications of the assumptions later in the chapter.

Arbitrage with Short Sales

Even if you did not own any stock in L, you still could reap benefits if U and L did not have the same total market value. Your first step would be to sell short \$600,000 of stock in L. To do this, your broker would let you borrow stock in L from another client. Your broker would then sell the stock for you and give you the proceeds, or \$600,000 in cash. You would supplement this \$600,000 by borrowing \$400,000. With the \$1 million total, you would buy 10% of the stock in U for \$900,000 and have \$100,000 remaining.

Your position would then consist of \$100,000 in cash and two portfolios. The first portfolio would contain \$900,000 of stock in U, which would generate \$90,000 of income. Because you would own the stock, we’ll call it the “long” portfolio. The

other portfolio would consist of \$600,000 of stock in L and \$400,000 of debt. The value of this portfolio is \$1 million, and it would generate \$60,000 of dividends and \$30,000 of interest. However, you would not own this second portfolio—you would “owe” it. Since you borrowed the \$400,000, you would owe the \$30,000 in interest. And since you borrowed the stock in L, you would “owe the stock” to the client from whom it was borrowed. Therefore, you would have to pay your broker the \$60,000 of dividends paid by L, which the broker would then pass on to the client from whom the stock was borrowed. Thus your net cash flow from the second portfolio would be a negative \$90,000. Because you would “owe” this portfolio, we’ll call it the “short” portfolio.

Where would you get the \$90,000 that you must pay on the short portfolio? The good news is that this is exactly the amount of cash flow generated by your long portfolio. Because the cash flows generated by each portfolio are the same, the short portfolio “replicates” the long portfolio.

Here is the bottom line. You started out with no money of your own. By selling L short, borrowing \$400,000, and purchasing stock in U, you ended up with \$100,000 in cash plus the two portfolios. The portfolios mirror one another, so their net cash flow is zero. This is perfect arbitrage: You invest none of your own money, you have no risk, you have no future negative cash flows, but you end up with cash in your pocket.

Not surprisingly, many traders would want to do this. The selling pressure on L would cause its price to fall, and the buying pressure on U would cause its price to rise, until the two companies’ values were equal. To put it another way, *if the long and short replicating portfolios have the same cash flows, then arbitrage will force them to have the same value.*

This is one of the most important ideas in modern finance. Not only does it give us insights into capital structure, but it is the fundamental building block underlying the valuation of real and financial options and derivatives as discussed in Chapter 8 and 23. Without the concept of arbitrage, the options and derivatives markets we have today simply would not exist.

MM with Corporate Taxes

Modigliani and Miller’s original work, published in 1958, assumed zero taxes. In 1963, they published a second article that incorporated corporate taxes. With corporate income taxes, they concluded that leverage will increase a firm’s value. This occurs because interest is a tax-deductible expense; hence more of a levered firm’s operating income flows through to investors.

Later in this chapter we present a proof of the MM propositions when personal taxes as well as corporate taxes are allowed. The situation when there are corporate taxes but no personal taxes is a special instance of the situation with both personal and corporate taxes, so we only present results in this case.

Proposition I. The value of a levered firm is equal to the value of an unlevered firm in the same risk class (V_U) *plus* the value of the tax shield ($V_{\text{Tax shield}}$) due to the tax deductibility of interest expenses. The value of the tax shield, which is often called *the gain from leverage*, is the present value of the annual tax savings. The annual tax saving is equal to the interest payment multiplied by the tax rate, T :

$$\text{Annual tax saving} = r_d D(T)$$

Modigliani and Miller assume a no-growth firm, so the present value of the annual tax saving is the present value of a perpetuity. They assume that the appropriate discount rate for the tax shield is the interest rate on debt, so the value of the tax shield is

$$V_{\text{Tax shield}} = \frac{r_d D(T)}{r_d} = TD$$

Therefore, the value of a levered firm is

$$\begin{aligned} V_L &= V_U + V_{\text{Tax shield}} \\ &= V_U + TD \end{aligned} \quad (26-4)$$

The important point here is that, when corporate taxes are introduced, the value of the levered firm exceeds that of the unlevered firm by the amount TD. Since the gain from leverage increases as debt increases, this implies that a firm's value is maximized at 100% debt financing.

Because all cash flows are assumed to be perpetuities, the value of the unlevered firm can be found by using Equation 26-3 and incorporating taxes. With zero debt ($D = \$0$), the value of the firm is its equity value:

$$V_U = S = \frac{\text{EBIT}(1 - T)}{r_{sU}} \quad (26-5)$$

Note that the discount rate, r_{sU} , is not necessarily equal to the discount rate in Equation 26-1. The r_{sU} from Equation 26-1 is the required discount rate in a world with no taxes, whereas the r_{sU} in Equation 26-5 is the required discount rate in a world with taxes.

Proposition II. The cost of equity to a levered firm is equal to (1) the cost of equity to an unlevered firm in the same risk class plus (2) a risk premium whose size depends on (a) the difference between the costs of equity and debt to an unlevered firm, (b) the amount of financial leverage used, and (c) the corporate tax rate:

$$r_{sL} = r_{sU} + (r_{sU} - r_d)(1 - T)(D/S) \quad (26-6)$$

Observe that Equation 26-6 is identical to the corresponding without-tax equation (26-2) except for the term $(1 - T)$, which appears only in Equation 26-6. Because $(1 - T)$ is less than 1, corporate taxes cause the cost of equity to rise less rapidly with leverage than it would in the absence of taxes. Proposition II, coupled with the reduction (due to taxes) in the effective cost of debt, is what produces the Proposition I result—namely, that the firm's value increases as its leverage increases.

As shown in Chapter 15, Professor Robert Hamada extended the MM analysis to define the relationship between a firm's beta, b , and the amount of leverage it has. The beta of an unlevered firm is denoted by b_U , and Hamada's equation is

$$b = b_U[1 + (1 - T)(D/S)] \quad (26-7)$$

Note that beta, like the cost of stock shown in Equation 26-6, increases with leverage.



Illustration of the MM Models

To illustrate the MM models, assume that the following data and conditions hold for Fredrickson Water Company, an established firm that supplies water to residential customers in several no-growth upstate New York communities.

1. Fredrickson currently has no debt; it is an all-equity company.
2. Expected EBIT = \$2,400,000. This value is not expected to increase over time, so Fredrickson is in a no-growth situation.
3. Because it does not need new capital, Fredrickson pays out all of its income as dividends.
4. If Fredrickson begins to use debt, it can borrow at a rate $r_d = 8\%$. This borrowing rate is constant—it does not increase regardless of the amount of debt used. Any money raised by selling debt would be used to repurchase common stock, so *Fredrickson's assets would remain constant.*
5. The business risk inherent in Fredrickson's assets, and thus in its EBIT, is such that its beta is 0.80; this is called the unlevered beta, b_U , because Fredrickson has no debt. The risk-free rate is 8%, and the market risk premium (RP_M) is 5%. Using the Capital Asset Pricing Model (CAPM), Fredrickson's required rate of return on stock, r_{sU} , is 12% if no debt is used:

$$r_{sU} = r_{RF} + b_U(RP_M) = 8\% + 0.80(5\%) = 12\%$$

With Zero Taxes. To begin, assume that there are no taxes and so $T = 0\%$. At any level of debt, Proposition I (Equation 26-1) can be used to find Fredrickson's value in an MM world, \$20 million:

$$V_L = V_U = \frac{\text{EBIT}}{r_{sU}} = \frac{\$2.4 \text{ million}}{0.12} = \$20.0 \text{ million}$$

If Fredrickson uses \$10 million of debt, then the value of its stock must be \$10 million:

$$S = V - D = \$20 \text{ million} - \$10 \text{ million} = \$10 \text{ million}$$

We can also find Fredrickson's cost of equity, r_{sL} , and its WACC at a debt level of \$10 million. First, we use Proposition II (Equation 26-2) to find r_{sL} , Fredrickson's levered cost of equity:

$$\begin{aligned} r_{sL} &= r_{sU} + (r_{sU} - r_d)(D/S) \\ &= 12\% + (12\% - 8\%)(\$10 \text{ million}/\$10 \text{ million}) \\ &= 12\% + 4.0\% = 16.0\% \end{aligned}$$

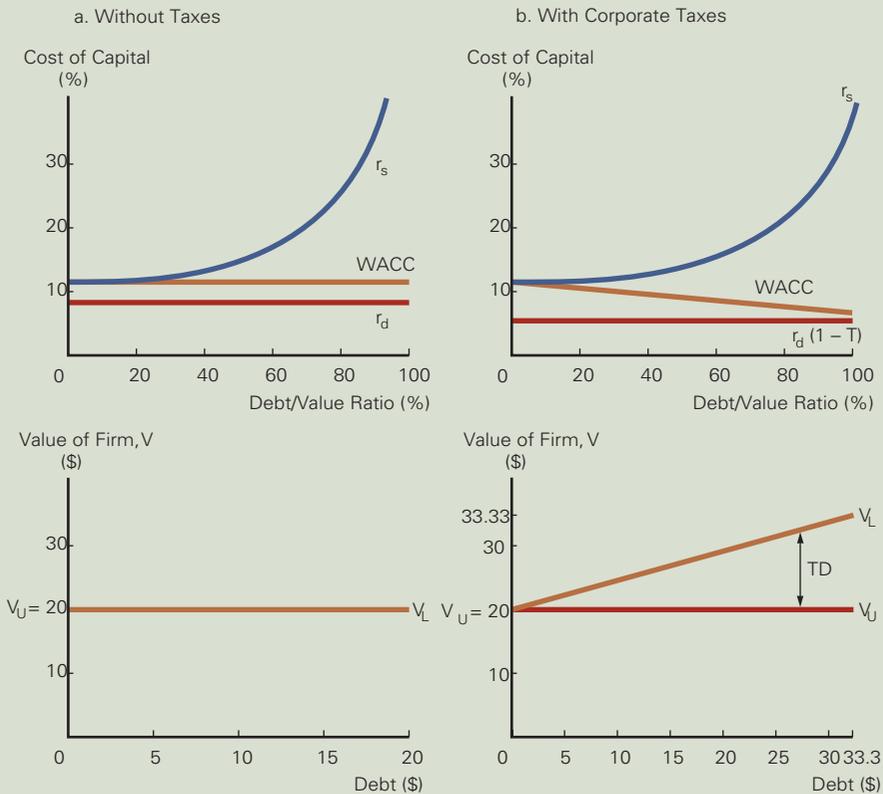
Now we can find the company's weighted average cost of capital:

$$\begin{aligned} \text{WACC} &= (D/V)(r_d)(1 - T) + (S/V)r_{sL} \\ &= (\$10/\$20)(8\%)(1.0) + (\$10/\$20)(16.0\%) = 12.0\% \end{aligned}$$

Fredrickson's value and cost of capital based on the MM model without taxes at various debt levels are shown in Panel a on the left side of Figure 26-1. Here we see that, in an MM world without taxes, financial leverage simply does not matter: *The value of the firm, and its overall cost of capital, are both independent of the amount of debt.*

With Corporate Taxes. To illustrate the MM model with corporate taxes, assume that all of the previous conditions hold except for the following changes:

FIGURE 26-1 Effects of Leverage: MM Models (Millions of Dollars)



MM without Taxes

| D | V | S | D/V | r_d | r_s | WACC |
|------|---------|---------|--------|-------|--------|--------|
| \$ 0 | \$20.00 | \$20.00 | 0.00% | 8.0% | 12.00% | 12.00% |
| 5 | 20.00 | 15.00 | 25.00 | 8.0 | 13.33 | 12.00 |
| 10 | 20.00 | 10.00 | 50.00 | 8.0 | 16.00 | 12.00 |
| 15 | 20.00 | 5.00 | 75.00 | 8.0 | 24.00 | 12.00 |
| 20 | 20.00 | 0.00 | 100.00 | 12.0 | — | 12.00 |

MM with Corporate Taxes

| D | V | S | D/V | r_d | r_s | WACC |
|-------|---------|---------|--------|-------|--------|--------|
| \$ 0 | \$20.00 | \$20.00 | 0.00% | 8.0% | 12.00% | 12.00% |
| 5 | 22.00 | 17.00 | 22.73 | 8.0 | 12.71 | 10.91 |
| 10 | 24.00 | 14.00 | 41.67 | 8.0 | 13.71 | 10.00 |
| 15 | 26.00 | 11.00 | 57.69 | 8.0 | 15.27 | 9.23 |
| 20 | 28.00 | 8.00 | 71.43 | 8.0 | 18.00 | 8.57 |
| 25 | 30.00 | 5.00 | 83.33 | 8.0 | 24.00 | 8.00 |
| 30 | 32.00 | 2.00 | 93.75 | 8.0 | 48.00 | 7.50 |
| 33.33 | 33.33 | 0.00 | 100.00 | 12.0 | — | 12.00 |

1. Expected EBIT = \$4,000,000.⁴
2. Fredrickson has a 40% federal-plus-state tax rate, so $T = 40\%$.

⁴If we had left Fredrickson's EBIT at \$2.4 million, then introducing corporate taxes would have reduced the firm's value from \$20 million to \$12 million:

$$V_U = \frac{\text{EBIT}(1 - T)}{r_{sU}} = \frac{\$2.4 \text{ million}(0.6)}{0.12} = \$12.0 \text{ million}$$

Corporate taxes reduce the amount of operating income available to investors in an unlevered firm by the factor $(1 - T)$, so the value of the firm would be reduced by the same amount, holding r_{sU} constant.

Other things held constant, the introduction of corporate taxes would lower Fredrickson's net income and hence its value, so we increased EBIT from \$2.4 million to \$4 million to facilitate comparisons between the two models.

When Fredrickson has zero debt but pays taxes, Equation 26-5 can be used to find its value:

$$V_U = \frac{\text{EBIT}(1 - T)}{r_{sU}} = \frac{\$4 \text{ million}(0.6)}{0.12} = \$20 \text{ million}$$

If Fredrickson now uses \$10 million of debt in a world with taxes, we see by Proposition I (Equation 26-4) that its total market value rises from \$20 to \$24 million:

$$V_L = V_U + TD = \$20 \text{ million} + 0.4(\$10 \text{ million}) = \$24 \text{ million}$$

Therefore, the implied value of Fredrickson's equity is \$14 million:

$$S = V - D = \$24 \text{ million} - \$10 \text{ million} = \$14 \text{ million}$$

We can also find Fredrickson's cost of equity, r_{sL} , and its WACC at a debt level of \$10 million. First, we use Proposition II (Equation 26-6) to find r_{sL} , the levered cost of equity:

$$\begin{aligned} r_{sL} &= r_{sU} + (r_{sU} - r_d)(1 - T)(D/S) \\ &= 12\% + (12\% - 8\%)(0.6)(\$10 \text{ million}/\$14 \text{ million}) \\ &= 12\% + 1.71\% = 13.71\% \end{aligned}$$

The company's weighted average cost of capital is then

$$\begin{aligned} \text{WACC} &= (D/V)(r_d)(1 - T) + (S/V)r_{sL} \\ &= (\$10/\$24)(8\%)(0.6) + (\$14/\$24)(13.71\%) = 10.0\% \end{aligned}$$

Note that we can also find the levered beta and then the levered cost of equity. First, we apply Hamada's equation to find the levered beta:

$$\begin{aligned} b &= b_U[1 + (1 - T)(D/S)] \\ &= 0.80[1 + (1 - 0.4)(\$10 \text{ million}/\$14 \text{ million})] \\ &= 1.1429 \end{aligned}$$

Applying the CAPM then yields the levered cost of equity as

$$r_{sL} = r_{RF} + b(\text{RP}_M) = 8\% + 1.1429(5\%) = 0.1371 = 13.71\%$$

Observe that this is the same levered cost of equity that we obtained directly using Equation 26-6.

Fredrickson's value and cost of capital at various debt levels with corporate taxes are shown in Panel b on the right side of Figure 26-1. In an MM world with corporate taxes, financial leverage does matter: The value of the firm is maximized—and its overall cost of capital is minimized—if it uses almost 100% debt financing. The

increase in value is due solely to the tax deductibility of interest payments, which lowers both the cost of debt and the equity risk premium by $(1 - T)$.⁵

To conclude this section, compare the “Without Taxes” and “With Corporate Taxes” sections of Figure 26-1. Without taxes, both WACC and the firm’s value (V) are constant. With corporate taxes, WACC declines and V rises as more and more debt is used; thus, under MM with corporate taxes, the optimal capital structure is 100% debt.

Self-Test

Is there an optimal capital structure under the MM zero-tax model?

What is the optimal capital structure under the MM model with corporate taxes?

How does the Proposition I equation differ between the two models?

How does the Proposition II equation differ between the two models?

Why do taxes result in a “gain from leverage” in the MM model with corporate taxes?

An unlevered firm has a value of \$100 million. An otherwise identical but levered firm has \$30 million in debt. Under the MM zero-tax model, what is the value of the levered firm? (**\$100 million**) Under the MM corporate tax model, what is the value of a levered firm if the corporate tax rate is 40%? (**\$112 million**)

26.2 INTRODUCING PERSONAL TAXES: THE MILLER MODEL

Although MM included *corporate taxes* in the second version of their model, they did not extend the model to include *personal taxes*. However, in his presidential address to the American Finance Association, Merton Miller presented a model to show how leverage affects firms’ values when both personal and corporate taxes are taken into account.⁶ To explain Miller’s model, we begin by defining T_c as the corporate tax rate, T_s as the personal tax rate on income from stocks, and T_d as the personal tax rate on income from debt. Note that stock returns are expected to come partly as dividends and partly as capital gains, so T_s is a weighted average of the effective tax rates on dividends and capital gains. However, essentially all debt income comes from interest, which is effectively taxed at investors’ top rates; thus T_d is higher than T_s .

⁵In the limit case where the firm used 100% debt financing, the bondholders would own the entire company and so would bear all the business risk. (Up until this point, MM assume that stockholders bear all the risk.) If the bondholders bear all the risk, then the capitalization rate on the debt should be equal to the equity capitalization rate at zero debt, $r_d = r_{sU} = 12\%$.

The income stream to the stockholders in the all-equity case was $\$4,000,000(1 - T) = \$2,400,000$, and the value of the firm was

$$V_U = \frac{\$2,400,000}{0.12} = \$20,000,000$$

With all debt, the entire \$4,000,000 of EBIT would be used to pay interest charges: r_d would be 12%, so $I = 0.12(\text{Debt}) = \$4,000,000$. Taxes would be zero, so the investors (bondholders) would get the entire \$4,000,000 of operating income (they would not have to share it with the government). Thus, the value of the firm at 100% debt would be

$$V_L = \frac{\$4,000,000}{0.12} = \$33,333,333 = D$$

There is, of course, a transition problem in all this. Modigliani and Miller assume that $r_d = 8\%$ regardless of how much debt the firm has until debt reaches 100%, at which point r_d jumps to 12%, the cost of equity. As we shall see later in the chapter, r_d actually rises as the risk of financial distress increases.

⁶See Merton H. Miller, “Debt and Taxes,” *Journal of Finance*, May 1977, pp. 261–275.

With personal taxes included and under the same set of assumptions used in the earlier MM models, the value of an unlevered firm is found as follows:

$$V_U = \frac{\text{EBIT}(1 - T_c)}{r_{sU}} = \frac{\text{EBIT}(1 - T_c)(1 - T_s)}{r_{sU}(1 - T_s)} \quad (26-8)$$

The $(1 - T_s)$ term takes account of personal taxes. Note that, in order to find the value of the unlevered firm, we can either discount pre-personal-tax cash flows at the pre-personal-tax rate of r_{sU} or discount after-personal-tax cash flows at the after-personal-tax rate of $r_{sU}(1 - T_s)$. Therefore, the numerator in the second line of Equation 26-8 shows how much of the firm's operating income is left after the unlevered firm pays corporate income taxes and its stockholders subsequently pay personal taxes on their equity income. Note also that the discount rate, r_{sU} , in Equation 26-8 is not necessarily equal to the discount rate in Equation 26-5. The r_{sU} from Equation 26-5 is the required discount rate in a world with corporate taxes but no personal taxes; the r_{sU} in Equation 26-8 is the required discount rate in a world with both corporate and personal taxes.

Miller's formula can be proved by an arbitrage proof similar to the one we presented earlier. However, the alternative proof shown below is easier to follow. To begin, we partition the levered firm's annual cash flows, CF_L , into those going to stockholders and those going to bondholders *after* corporate and personal taxes:

$$CF_L = \text{Net CF to stockholders} + \text{Net CF to bondholders} \\ = (\text{EBIT} - I)(1 - T_c)(1 - T_s) + I(1 - T_d) \quad (26-9)$$

where I is the annual interest payment. Equation 26-9 can be rearranged as follows:

$$CF_L = [\text{EBIT}(1 - T_c)(1 - T_s)] - [I(1 - T_c)(1 - T_s)] + [I(1 - T_d)] \quad (26-9a)$$

The first term in Equation 26-9a is identical to the after-personal-tax cash flow of an unlevered firm as shown in the numerator of Equation 26-8, and its present value is found by discounting the perpetual cash flow by $r_{sU}(1 - T_s)$.

The second and third terms reflect leverage and result from the cash flows associated with debt financing, which under the MM assumptions are riskless (because the firm's debt is riskless under those assumptions). We can either discount pre-personal-tax interest payments at the pre-personal-tax rate of r_d or discount after-personal-tax interest payments at the after-personal-tax rate of $r_d(1 - T_d)$. Because they are after-personal-tax cash flows to debtholders, the present value of the last two right-hand terms in Equation 26-9a can be obtained by discounting at the after-personal-tax cost of debt, $r_d(1 - T_d)$. Combining the present values of the three terms, we obtain this value for the levered firm:

$$V_L = \frac{\text{EBIT}(1 - T_c)(1 - T_s)}{r_{sU}(1 - T_s)} - \frac{I(1 - T_c)(1 - T_s)}{r_d(1 - T_d)} + \frac{I(1 - T_d)}{r_d(1 - T_d)} \quad (26-10)$$

The first right-hand term in Equation 26-10 is identical to V_U in Equation 26-8. Recognizing this and consolidating the second two terms, we obtain

$$V_L = V_U + \left[1 - \frac{(1 - T_c)(1 - T_s)}{(1 - T_d)} \right] \left[\frac{I(1 - T_d)}{r_d(1 - T_d)} \right] \quad (26-10a)$$

Now recognize that the after-tax perpetual interest payment divided by the after-tax required rate of return on debt, $I(1 - T_d)/r_d(1 - T_d)$, is equal to the market value of the perpetual debt, D :

$$D = \frac{I}{r_d} = \frac{I(1 - T_d)}{r_d(1 - T_d)} \quad (26-11)$$

Substituting D into Equation 26-10a and rearranging, we obtain the following expression, which is called the **Miller model**:

$$\text{Miller model: } V_L = V_U + \left[1 - \frac{(1 - T_c)(1 - T_s)}{(1 - T_d)} \right] D \quad (26-12)$$

The Miller model provides an estimate of the value of a levered firm in a world with both corporate and personal taxes.

The Miller model has several important implications, as follows.

1. The term in brackets,

$$\left[1 - \frac{(1 - T_c)(1 - T_s)}{(1 - T_d)} \right]$$

when multiplied by D , represents the gain from leverage. The bracketed term thus replaces the corporate tax rate, T , in the earlier MM model with corporate taxes ($V_L = V_U + TD$).

2. If we ignore all taxes (i.e., if $T_c = T_s = T_d = 0$) then the bracketed term is zero, so in this case Equation 26-12 is the same as the original MM model without taxes.
3. If we ignore personal taxes (i.e., if $T_s = T_d = 0$) then the bracketed term reduces to $[1 - (1 - T_c)] = T_c$, so in this case Equation 26-12 is the same as the MM model with corporate taxes.
4. If the effective personal tax rates on stock and bond incomes were equal (i.e., if $T_s = T_d$), then $(1 - T_s)$ and $(1 - T_d)$ would cancel and so the bracketed term would again reduce to T_c .
5. If $(1 - T_c)(1 - T_s) = (1 - T_d)$, then the bracketed term would be zero and so the value of using leverage would also be zero. This implies that the tax advantage of debt to the firm would be exactly offset by the personal tax advantage of equity. Under this condition, capital structure would have no effect on a firm's value or its cost of capital, so we would be back to MM's original zero-tax proposition.
6. Because taxes on capital gains are lower than on ordinary income and can be deferred, the effective tax rate on stock income is normally less than that on bond income. This being the case, what would the Miller model predict as the gain from leverage? To answer this question, assume the tax rate on corporate income is $T_c = 34\%$, the effective rate on bond income is $T_d = 28\%$, and the effective rate on stock income is $T_s = 15\%$. Using these values in the Miller model, we

find that a levered firm's value exceeds that of an unlevered firm by 22% of the market value of corporate debt:

$$\begin{aligned}\text{Gain from leverage} &= \left[1 - \frac{(1 - T_c)(1 - T_s)}{(1 - T_d)} \right] D \\ &= \left[1 - \frac{(1 - 0.34)(1 - 0.15)}{(1 - 0.28)} \right] D \\ &= (1 - 0.78)D \\ &= 0.22D\end{aligned}$$

Note that the MM model with corporate taxes would indicate a gain from leverage of $T_c(D) = 0.34D$, or 34% of the amount of corporate debt. Thus, with these assumed tax rates, adding personal taxes to the model lowers but does not eliminate the benefit from corporate debt. In general, whenever the effective tax rate on income from stock is less than the effective rate on income from bonds, the Miller model produces a lower gain from leverage than is produced by the MM model with taxes.

In his paper, Miller argued that firms in the aggregate would issue a mix of debt and equity securities such that the before-tax yields on corporate securities and the personal tax rates of the investors who bought these securities would adjust until an equilibrium was reached. At equilibrium, $(1 - T_d)$ would equal $(1 - T_c)(1 - T_s)$ and so, as we noted in item 5 above, the tax advantage of debt to the firm would be exactly offset by personal taxation and thus capital structure would have no effect on a firm's value or its cost of capital. Hence, according to Miller, the conclusions derived from the original MM zero-tax model are correct!

Others have extended and tested Miller's analysis. Generally, these extensions question Miller's conclusion that there is no advantage to the use of corporate debt. In fact, Equation 26-12 shows that both T_c and T_s must be less than T_d if there is to be zero gain from leverage. For most U.S. corporations and investors, the effective tax rate on income from stock is less than the rate on income from bonds; that is, $T_s < T_d$. However, many corporate bonds are held by tax-exempt institutions, and in those cases T_c is generally greater than T_d . Also, for those high-tax-bracket individuals with $T_d > T_c$, T_s may be large enough that $(1 - T_c)(1 - T_s)$ is less than $(1 - T_d)$; in this case there would be an advantage to using corporate debt. Still, Miller's work does show that personal taxes offset some of the benefits of corporate debt. This means that the tax advantages of corporate debt are less than were implied by the earlier MM model, where only corporate taxes were considered.

As we discuss in the next section, both the MM and the Miller models are based on strong and unrealistic assumptions, so we should regard our examples as indicating the general effects of leverage on a firm's value and not a precise relationship.

Self-Test

How does the Miller model differ from the MM model with corporate taxes?

What are the implications of the Miller model if $T_c = T_s = T_d = 0$? If $T_s = T_d = 0$?

Considering the current tax structure in the United States, what is the primary implication of the Miller model?

An unlevered firm has a value of \$100 million. An otherwise identical but levered firm has \$30 million in debt. Use the Miller model to calculate the value of a levered firm if the corporate tax rate is 40%, the personal tax rate on equity is 15%, and the personal tax rate on debt is 35%. (**\$106.46 million**)

26.3 CRITICISMS OF THE MM AND MILLER MODELS

The conclusions of the MM and Miller models follow logically from their initial assumptions. However, both academicians and executives have voiced concerns over the validity of the MM and Miller models, and virtually no one believes they hold precisely. The MM zero-tax model leads to the conclusion that capital structure doesn't matter, yet we observe systematic capital structure patterns within industries. Further, when used with "reasonable" tax rates, both the MM model with corporate taxes and the Miller model lead to the conclusion that firms should use 100% debt financing, but real-life firms do not (deliberately) go to that extreme.

People who disagree with the MM and Miller theories generally attack them on the grounds that their assumptions are invalid. Here are the main objections.

1. Both MM and Miller assume that personal and corporate leverage are perfect substitutes. However, an individual investing in a levered firm has less loss exposure as a result of corporate *limited liability* than if she used "homemade" leverage. For example, in our earlier illustration of the MM arbitrage argument, it should be noted that only the \$600,000 our investor had in Firm L would be lost if that firm went bankrupt. However, if the investor engaged in arbitrage transactions and employed "homemade" leverage to invest in Firm U, then she could lose \$900,000—the original \$600,000 investment plus the \$400,000 loan less the \$100,000 investment in riskless bonds. This increased personal risk exposure would tend to restrain investors from engaging in arbitrage, and that could cause the equilibrium values of V_L , V_U , r_{sL} , and r_{sU} to be different from those specified by MM. Restrictions on institutional investors, who dominate capital markets today, may also hinder the arbitrage process, because many institutional investors cannot legally borrow to buy stocks and hence are prohibited from engaging in homemade leverage.

However, even though limited liability may present a problem to individuals, it does *not* present a problem to corporations that are set up to undertake **leveraged buyouts (LBOs)**. Thus, after MM's work became widely known, literally hundreds of LBO firms were established whose founders made billions by recapitalizing underleveraged firms. "Junk bonds" were created to aid in the process, and the managers of underleveraged firms who did not want their firms to be taken over increased debt usage on their own. Thus, MM's work raised the level of debt in corporate America, which probably raised the level of economic efficiency.

2. If a levered firm's operating income declined, then it would sell assets and take other measures to raise the cash necessary to meet its interest obligations and thus avoid bankruptcy. If our illustrative unlevered firm experienced the same decline in operating income, it would probably take the less drastic measure of cutting dividends rather than selling assets. But if dividends were cut then investors who employed homemade leverage would not receive cash to pay the interest on their debt. Thus, homemade leverage puts stockholders in greater danger of bankruptcy than does corporate leverage.
3. Brokerage costs were assumed away by MM and Miller, which makes the switch from L to U costless. However, brokerage and other transaction costs do exist, and they also impede the arbitrage process.
4. Modigliani and Miller initially assumed that corporations and investors could borrow at the risk-free rate. Although risky debt has been introduced into the analysis by others, to reach the MM and Miller conclusions it is still necessary to assume that both corporations and investors can borrow at the same rate. Although major institutional investors probably can borrow at the corporate rate,

many institutions are not allowed to borrow to buy securities. Furthermore, most individual investors must borrow at higher rates than those paid by large corporations.

5. In his article, Miller concluded that an equilibrium would be reached, but to reach his equilibrium the tax benefit from corporate debt must be the same for all firms and must also be constant for an individual firm regardless of the amount of leverage used. However, we know that tax benefits vary from firm to firm: Highly profitable companies gain the maximum tax benefit from leverage, whereas the benefits to firms that are struggling are much smaller. Moreover, some firms have other tax shields (e.g., high depreciation, pension plan contributions, operating loss carryforwards), and these shields reduce the tax savings from interest payments.⁷ It also is simplistic to assume that the expected tax shield is unaffected by the amount of debt used. Higher leverage increases the probability that the firm will not be able to use the full tax shield in the future, because higher leverage increases the probability of future unprofitability and consequently lower tax rates. Note also that large, diversified corporations can use losses in one division to offset profits in another. Thus, the tax shelter benefit is more certain in such firms than in smaller, single-product companies. All things considered, it appears likely that the interest tax shield from corporate debt is more valuable to some firms than to others.
6. MM and Miller assume that there are no costs associated with financial distress, and they also ignore agency costs. Further, they assume that all market participants have identical information about firms' prospects, which is clearly an oversimplification.

These six points all suggest that the MM and Miller models lead to questionable conclusions and that the models would be better if certain of their assumptions could be relaxed. We discuss an extension of the models in the next section.

Self-Test

Should we accept that one of the models presented thus far (MM with zero taxes, MM with corporate taxes, or Miller) is correct? Why or why not?

Are any of the assumptions used in the models worrisome to you, and what does "worrisome" mean in this context?

26.4 AN EXTENSION OF THE MM MODEL: NONZERO GROWTH AND A RISKY TAX SHIELD

In this section, we discuss an extension of the MM model that incorporates growth and different discount rates for the debt tax shield.⁸

Modigliani and Miller assumed that firms pay out all of their earnings as dividends and therefore do not grow. However, most firms do grow, and growth affects the MM and Hamada results (as found in the first part of this chapter). Recall that, for

⁷For a discussion of the impact of tax shields, see Harry DeAngelo and Ronald W. Masulis, "Optimal Capital Structure under Corporate and Personal Taxation," *Journal of Financial Economics*, March 1980, pp. 3–30; Thomas W. Downs, "Corporate Leverage and Nondebt Tax Shields: Evidence on Crowding-Out," *The Financial Review*, November 1993, pp. 549–583; John R. Graham, "Taxes and Corporate Finance: A Review," *Review of Financial Studies*, Winter 2003, pp. 1075–1129; and Jeffrey K. Mackie-Mason, "Do Taxes Affect Corporate Financing Decisions?" *Journal of Finance*, December 1990, pp. 1471–1493.

⁸See Michael C. Ehrhardt and Phillip R. Daves, "Corporate Valuation: The Combined Impact of Growth and the Tax Shield of Debt on the Cost of Capital and Systematic Risk," *Journal of Applied Finance*, Fall/Winter 2002, pp. 31–38.

an unlevered firm, the WACC is just the unlevered cost of equity: $WACC = r_{sU}$. If g is the constant growth rate and FCF is the expected free cash flow, then the corporate value model from Chapter 13 shows that

$$V_U = \frac{FCF}{r_{sU} - g} \quad (26-13)$$

As shown by Equation 26-4, the value of the levered firm is equal to the value of the unlevered firm plus gain from leverage, which is the value of the tax shield:

$$V_L = V_U + V_{\text{Tax shield}} \quad (26-4a)$$

However, when there is growth, the value of the tax shield is not equal to TD as it is in the MM model with corporate taxes. If the firm uses debt and if g is positive then, as the firm grows, the amount of debt will increase over time; hence the size of the annual tax shield will also increase at the rate g , provided the debt ratio remains constant. Moreover, the value of this growing tax shield is greater than the value of the constant tax shield in the MM analysis.

Modigliani and Miller assumed that corporate debt was riskless and that the firm would always be able to use its tax savings. Therefore, they discounted the tax savings at the cost of debt, r_d , which is the risk-free rate. However, corporate debt is not risk free—firms do occasionally default on their loans. Also, a firm may not be able to use tax savings from debt in the current year if it already has a pre-tax loss from operations. Therefore, the flow of tax savings to the firm is not risk-free and hence it should be discounted at a higher rate than the risk-free rate. In addition, since debt is safer than equity to an investor because it has a higher priority claim on the firm's cash flows, its discount rate should be no greater than the unlevered cost of equity. For now, assume that the appropriate discount rate for the tax savings is r_{TS} , which is greater than or equal to the cost of debt, r_d , and less than or equal to the unlevered cost of equity, r_{sU} .

If r_{TS} is the appropriate discount rate for the tax shield, r_d is the interest rate on the debt, T is the corporate tax rate, and D is the current amount of debt, then the present value of this growing tax shield is

$$V_{\text{Tax shield}} = \frac{r_d TD}{r_{TS} - g} \quad (26-14)$$

This formula is similar to the dividend growth formula from Chapter 7, except it has $r_d TD$ as the growing cash flow generated by the tax savings and r_{TS} as the discount rate. Substituting Equation 26-14 into Equation 26-4a yields a valuation expression that incorporates constant growth:

$$V_L = V_U + \left(\frac{r_d}{r_{TS} - g} \right) TD \quad (26-15)$$

The difference between Equation 26-15 for the value of the levered firm and the expression given in Equation 26-4 is the $r_d/(r_{TS} - g)$ term in large parentheses, which reflects the added value of the tax shield due to growth. In the MM model, $r_{TS} = r_d = r_{RF}$ and $g = 0$, so the term in parentheses is equal to 1.0.

If $r_{TS} < r_{sU}$, then growth can actually cause the levered cost of equity to be *less* than the unlevered cost of equity.⁹ This happens because the combination of rapid growth and a low discount rate for the tax shield causes the value of the tax shield to dominate the unlevered value of the firm. If this were true, then high-growth firms would tend to have larger amounts of debt than low-growth firms. However, this is not consistent with either intuition or what we observe in the market: High-growth firms actually tend to have lower levels of debt. Regardless of the growth rate, firms with more debt should have a higher cost of equity than firms with no debt. These inconsistencies can be resolved if $r_{TS} = r_{sU}$. Given this equality, the value of the levered firm becomes¹⁰

$$V_L = V_U + \left(\frac{r_d TD}{r_{sU} - g} \right) \quad (26-16)$$

In view of this valuation equation, expressions for the levered cost of equity and the levered beta (corresponding to Equations 26-6 and 26-7) become

$$r_{sL} = r_{sU} + (r_{sU} - r_d) \frac{D}{S} \quad (26-17)$$

and

$$b = b_U + (b_U - b_D) \frac{D}{S} \quad (26-18)$$

As in Chapter 15, b_U is the beta of an unlevered firm and b is the beta of a levered firm. Because debt is not riskless, it has a beta (b_D).

Although the derivations of Equations 26-17 and 26-18 reflect corporate taxes and growth, neither of these expressions includes the corporate tax rate or the growth rate. This means that the expression for the levered required rate of return, Equation 26-17, is exactly the same as MM's expression for the levered required rate of return *without taxes*, Equation 26-2. And the expression for the levered beta, Equation 26-18, is exactly the same as Hamada's equation (with risky debt) but *without taxes*. The reason the tax rate and the growth rate drop out of these two expressions is that the growing tax shield is discounted at the unlevered cost of equity, r_{sU} , not at the cost of debt as in the MM model. The tax rate drops out because, no matter how high the level of T , the total risk of the firm will not be changed: the unlevered cash flows and the tax shield are discounted at the same rate. The growth rate drops out for the same reason: An increasing debt level will not change the riskiness of the entire firm no matter what rate of growth prevails.¹¹

⁹See the paper by Ehrhardt and Daves cited in footnote 8.

¹⁰For a discussion of the *compressed APV* valuation method, which assumes that $r_{TS} = r_{sU}$, see Steven N. Kaplan and Richard S. Ruback, "The Valuation of Cash Flow Forecasts: An Empirical Analysis," *Journal of Finance*, September 1995, pp. 1059–1093.

¹¹Of course, Equations 26-14, 26-15, and 26-16 also apply to firms that don't happen to be growing. In this special case, the difference between the Ehrhardt and Daves extension and the MM with taxes treatment is that MM assume that the tax shield should be discounted at the risk-free rate, whereas this extension of their model shows it is more reasonable for the tax shield to be discounted at the unlevered cost of equity, r_{sU} . Because r_{sU} is greater than the risk-free rate, the value of a nongrowing tax shield will be lower when discounted at this higher rate, giving a lower value of the levered firm than what MM would predict.

Observe that Equation 26-18 includes the term b_D . Since MM and Hamada assumed that corporate debt is riskless, its beta should be zero. However, if corporate debt is not riskless then its beta, b_D , may not be zero. If we assume that bonds lie on the Security Market Line, then a bond's required return, r_d , can be expressed as $r_d = r_{RF} + b_D RP_M$. Solving for b_D then gives $b_D = (r_d - r_{RF})/RP_M$.

Illustration of the MM Extension with Growth

Earlier in this chapter we examined Fredrickson Water Company, a zero-growth firm with unlevered value of \$20 million. To see how growth affects the levered value of the firm and the levered cost of equity, let's look at Peterson Power Inc., which is similar to Fredrickson except that it is growing. Peterson's expected free cash flow is \$1 million, which is expected to grow at a rate of 7%. Like Fredrickson, Peterson has an unlevered cost of equity of 12% and faces a 40% tax rate. Peterson's unlevered value is $V_U = \$1 \text{ million}/(0.12 - 0.07) = \20 million , the same as Fredrickson's.

Suppose now that Peterson, like Fredrickson, uses \$10 million of debt with a cost of 8%. We see from Equation 26-16 that

$$V_L = \$20 \text{ million} + \left(\frac{0.08 \times 0.40 \times \$10 \text{ million}}{0.12 - 0.07} \right) = \$26.4 \text{ million}$$

and that the implied value of equity is

$$S = V_L - D = \$26.4 \text{ million} - \$10 \text{ million} = \$16.4 \text{ million}$$

The increase in value due to leverage when there is 7% growth is \$6.4 million, compared with the increase in value of only \$4 million for Fredrickson. The reason for this difference is that, even though the debt tax shield is currently $(0.08)(0.40)(10 \text{ million}) = \0.32 million for each company, this tax shield will grow at an annual rate of 7% for Peterson but will remain fixed over time for Fredrickson. And even though Peterson and Fredrickson have the same initial dollar value of debt, their debt weights, w_d , are not the same. Peterson's w_d is $D/V_L = \$10/\$26.4 = 37.88\%$, whereas Fredrickson's w_d is $\$10/\$24 = 41.67\%$.

With \$10 million in debt, Peterson's new cost of equity is given by Equation 26-17:

$$r_{sL} = 12\% + (12\% - 8\%) \frac{0.3788}{0.6212} = 14.44\%$$

This is higher than Fredrickson's levered cost of equity of 13.71%. Finally, Peterson's new WACC is $(1.0 - 0.3788)14.44\% + 0.3788(1 - 0.40)8\% = 10.78\%$ versus Fredrickson's WACC of 10.0%.

In sum, using the MM and Hamada models to calculate the value of a levered firm and its cost of capital when there is growth will: (1) underestimate the value of the levered firm, because these models underestimate the value of the growing tax shield; and (2) underestimate the levered WACC and levered cost of capital because, for a given initial amount of debt, these models overestimate the firm's w_d .

Self-Test

Why is the value of the tax shield different when a firm grows?

Why would it be inappropriate to discount tax shield cash flows at the risk-free rate as MM do?

How will your estimates of the levered cost of equity be biased if you use the MM or Hamada models when growth is present? Why does this matter?

An unlevered firm has a value of \$100 million. An otherwise identical but levered firm has \$30 million in debt. Suppose both firms are growing at a constant rate of 5%, the corporate tax rate is 40%, the cost of debt is 6%, and the unlevered cost of equity is 8% (assume r_{sU} is the appropriate discount rate for the tax shield). What is the value of the levered firm? (**\$124 million**) What is the value of the stock? (**\$94 million**) What is the levered cost of equity? (**8.64%**)

26.5 RISKY DEBT AND EQUITY AS AN OPTION

In the previous sections, we evaluated equity and debt using the standard discounted cash flow techniques. However, we learned in Chapter 11 that if there is an opportunity for management to make a change as a result of new information after a project or investment has been started, then there might be an option component to the project or investment being evaluated. This is the case with equity. To see why, consider Kunkel Inc., a small manufacturer of electronic wiring harnesses and instrumentation located in Minot, North Dakota. Kunkel's current value (debt plus equity) is \$20 million, and its debt consists of \$10 million face value of 5-year zero coupon bonds. What decision does management make when the debt comes due? In most cases, it would pay the \$10 million that is due. But what if the company has done poorly and the firm is worth only \$9 million? In that case, the firm is technically bankrupt, since its value is less than the amount of debt due. Management will choose to default on the loan; in this case, the firm will be liquidated or sold for \$9 million, the debtholders will get all \$9 million, and the stockholders will get nothing. Of course, if the firm is worth \$10 million or more then management will choose to repay the loan. The ability to make this decision—to pay or not to pay—looks very much like an option, and the techniques we developed in Chapter 8 can be used to value it.

Using the Black-Scholes Option Pricing Model to Value Equity

To put this decision into an option context, suppose P is Kunkel's total value when the debt matures. Then, if the debt is paid off, Kunkel's stockholders will receive the equivalent of $P - \$10$ million if $P > \$10$ million.¹² They will receive nothing if $P \leq \$10$ million because management will default on the bond. These facts can be summarized as follows:

$$\text{Payoff to stockholders} = \text{MAX}(P - \$10 \text{ million}, 0)$$

This is exactly the same payoff as a European call option on the total value (P) of the firm with a strike price equal to the face value of the debt, \$10 million. We can use the Black-Scholes option pricing model from Chapter 8 to determine the value of this asset.

Recall from Chapter 8 that the value of a call option depends on five things: the price of the underlying asset, the strike price, the risk-free rate, the time to expiration, and the volatility of the market value of the underlying asset. Here the underlying asset is the total value of the firm. If we assume that volatility is 40% and that the risk-free rate is 6%, then the inputs for the Black-Scholes model are as follows:



¹²Actually, rather than receive cash of $P - \$10$ million, the stockholders will keep the company (which is worth $P - \$10$ million) rather than turn it over to the bondholders.

$$\begin{aligned}
 P &= \$20 \text{ million} \\
 X &= \$10 \text{ million} \\
 t &= 5 \text{ years} \\
 r_{RF} &= 6\% \\
 \sigma &= 40\%
 \end{aligned}$$

The value of a European call option, as shown in Chapter 8, is

$$V = P[N(d_1)] - Xe^{-r_{RF}t}[N(d_2)] \quad (26-19)$$

where

$$d_1 = \frac{\ln(P/X) + (r_{RF} + \sigma^2/2)t}{\sigma\sqrt{t}} \quad (26-20)$$

and

$$d_2 = d_1 - \sigma\sqrt{t} \quad (26-21)$$

For Kunkel Inc.,

$$d_1 = \frac{\ln(20/10) + (0.06 + 0.40^2/2)5}{0.40\sqrt{5}} = 1.5576$$

$$d_2 = 1.5576 - 0.40\sqrt{5} = 0.6632$$

Using the *Excel* NORMSDIST function gives $N(d_1) = N(1.5576) = 0.9403$, $N(d_2) = N(0.6632) = 0.7464$, and $V = \$20(0.9403) - \$10e^{-0.06(5)}(0.7464) = \13.28 million. So Kunkel's equity is worth \$13.28 million, and its debt must be worth what is left over: $\$20 - \$13.28 = \$6.72$ million. Since this is 5-year, zero coupon debt, its yield must be

$$\text{Yield on debt} = \left(\frac{10}{6.72}\right)^{1/5} - 1 = 0.0827 = 8.27\%$$

Thus, when Kunkel issued the debt, it received \$6.72 million and the yield on the debt was 8.27%. Notice that the yield on the debt, 8.27%, is greater than the 6% risk-free rate. This is because the firm might default if its value falls enough, so the bonds are risky. Note also that the yield on the debt depends on the value of the option and hence on the riskiness of the firm. The debt will have a lower value—and a higher yield—the more the option is worth.

Managerial Incentives

The only decision an investor in a stock option can make, once the option is purchased, is whether and when to exercise it. However, this restriction does not apply to equity when it is viewed as an option on the total value of the firm. Management has some leeway to affect the riskiness of the firm through its capital budgeting and investment decisions, and it can affect the amount of capital invested in the firm through its dividend policy.

Capital Budgeting Decisions

When Kunkel issued the \$10 million face value debt discussed previously, the yield was determined in part by Kunkel's riskiness, which in turn was determined in part by what management intended to do with the \$6.72 million it raised. We know from our analysis in Chapter 8 that options are worth more when volatility is higher. This means that if Kunkel's management can find a way to increase its riskiness without decreasing the total value of the firm, then doing so will increase the equity's value while decreasing the debt's value. Management can accomplish this by selecting risky rather than safe investment projects. Table 26-1 shows the value of equity, the value of debt, and the yield on debt for a range of possible volatilities. See *Ch26 Tool Kit.xls* for the calculations.

Kunkel's current volatility is 40%, so its equity is worth \$13.28 million and its debt is worth \$6.72 million. But if, after incurring the debt, management undertakes projects that increase its riskiness from a volatility of 40% to a volatility of 80%, then the value of Kunkel's equity will increase by \$2.53 million to \$15.81 million and the value of its debt will decrease by the same amount. This 19% increase in the value of the equity represents a transfer of wealth from bondholders to stockholders. A corresponding transfer of wealth from stockholders to bondholders would occur if Kunkel undertook projects that were safer than originally planned. Table 26-1 shows that if management undertakes safe projects and drives the volatility down to 30%, then stockholders will lose (and bondholders will gain) \$0.45 million.

Such a strategy of investing borrowed funds in risky assets is called **bait and switch** because the firm obtains the money by promising one investment policy and then switching to another policy. The bait-and-switch problem is more severe when a firm's value is low relative to its level of debt. If Kunkel's total value is \$20 million, then doubling its volatility from 40% to 80% increases its equity value by 19%. But if Kunkel had done poorly in recent years and its total value were only \$10 million, then the impact of increasing volatility would be much greater. Table 26-2 shows that if Kunkel's total value were only \$10 million and it issued \$10 million face value of 5-year, zero coupon debt, then its equity would be worth \$4.46 million at a volatility of 40%. Doubling the volatility to 80% would increase the value of the equity to \$6.83 million, or by 53%. The incentive for management to "roll the dice" with



TABLE 26-1 The Value of Kunkel's Debt and Equity for Various Levels of Volatility (Millions of Dollars)

| STANDARD DEVIATION | EQUITY | PROCEEDS FROM DEBT | DEBT YIELD |
|--------------------|--------------|--------------------|-------------|
| 20% | \$12.62 | \$7.38 | 6.25% |
| 30 | 12.83 | 7.17 | 6.89 |
| 40 | 13.28 | 6.72 | 8.27 |
| 50 | 13.86 | 6.14 | 10.25 |
| 60 | 14.51 | 5.49 | 12.74 |
| 70 | 15.17 | 4.83 | 15.66 |
| 80 | 15.81 | 4.19 | 18.99 |
| 90 | 16.41 | 3.59 | 22.74 |
| 100 | 16.96 | 3.04 | 26.92 |
| 110 | 17.46 | 2.54 | 31.56 |
| 120 | 17.90 | 2.10 | 36.68 |

TABLE 26-2

Debt and Equity Values for Various Levels of Volatility When the Firm's Total Value is \$10 Million (Millions of Dollars)

| STANDARD DEVIATION | EQUITY | VALUE OF DEBT | DEBT YIELD |
|--------------------|-------------|---------------|--------------|
| 20% | \$3.16 | \$6.84 | 7.90% |
| 30 | 3.80 | 6.20 | 10.02 |
| 40 | 4.46 | 5.54 | 12.52 |
| 50 | 5.10 | 4.90 | 15.35 |
| 60 | 5.72 | 4.28 | 18.49 |
| 70 | 6.30 | 3.70 | 21.98 |
| 80 | 6.83 | 3.17 | 25.81 |
| 90 | 7.31 | 2.69 | 30.04 |
| 100 | 7.74 | 2.26 | 34.68 |
| 110 | 8.13 | 1.87 | 39.77 |
| 120 | 8.46 | 1.54 | 45.36 |

borrowed funds can be enormous, and if management owns many stock options then their payoff from rolling the dice is even greater than the payoff to stockholders!

Bondholders are aware of these incentives and write covenants into debt issues that restrict management's ability to invest in riskier projects than originally promised. However, their attempts to protect themselves are not always successful, as the failures of Enron and Global Crossing demonstrate. The combination of a risky industry, high levels of debt, and option-based compensation has proven to be very dangerous.

Equity with Risky Coupon Debt

We have analyzed the simple case when a firm has zero coupon debt outstanding. The analysis becomes much more complicated when a firm has debt that requires periodic interest payments, because then management can decide whether or not to default on each interest payment date. For example, suppose Kunkel's \$10 million of debt is a 1-year, 8% loan with semiannual payments. The scheduled payments are \$400,000 in 6 months, and then \$10.4 million at the end of the year. If management makes the scheduled \$400,000 interest payment, then the stockholders will acquire the right to make the next payment of \$10.4 million. If it does not make the \$400,000 payment, then by defaulting the stockholders lose the right to make that next payment and hence lose the firm.¹³ In other words, at the beginning of the year the stockholders have an option to purchase an option. The option they own has an exercise price of \$400,000 and it expires in 6 months, and if they exercise it, they will acquire an option to purchase the entire firm for \$10.4 million in another 6 months.

If the debt were 2-year debt, then there would be four decision points for management and the stockholders' position would be like an option on an option on an option on an option! These types of options are called **compound options**, and techniques for valuing them are beyond the scope of this book. However, the

¹³Actually, bankruptcy is far more complicated than our example suggests. As a firm approaches default it can take a number of actions, and even after filing for bankruptcy the stockholders can substantially delay a takeover by bondholders, during which time the value of the firm can deteriorate further. As a result, stockholders can often extract concessions from bondholders in situations where it would seem that the bondholders should get all of the firm's value. Bankruptcy is discussed in more detail in Chapter 22.

incentives discussed previously for the case when a firm has risky zero coupon debt still apply when the firm has periodic interest payments to make.¹⁴

Self-Test

Discuss how equity can be viewed as an option. Who has the option and what decision can they make?

Why would management want to increase the riskiness of the firm? Why would this make bondholders unhappy?

What can bondholders do to limit management's ability to bait and switch?

26.6 CAPITAL STRUCTURE THEORY: OUR VIEW

The great contribution of the capital structure models developed by MM, Miller, and their followers is that these models identified the specific benefits and costs of using debt: the tax benefits, financial distress costs, and so on. Prior to MM, no capital structure theory existed and so we had no systematic way of analyzing the effects of debt financing.

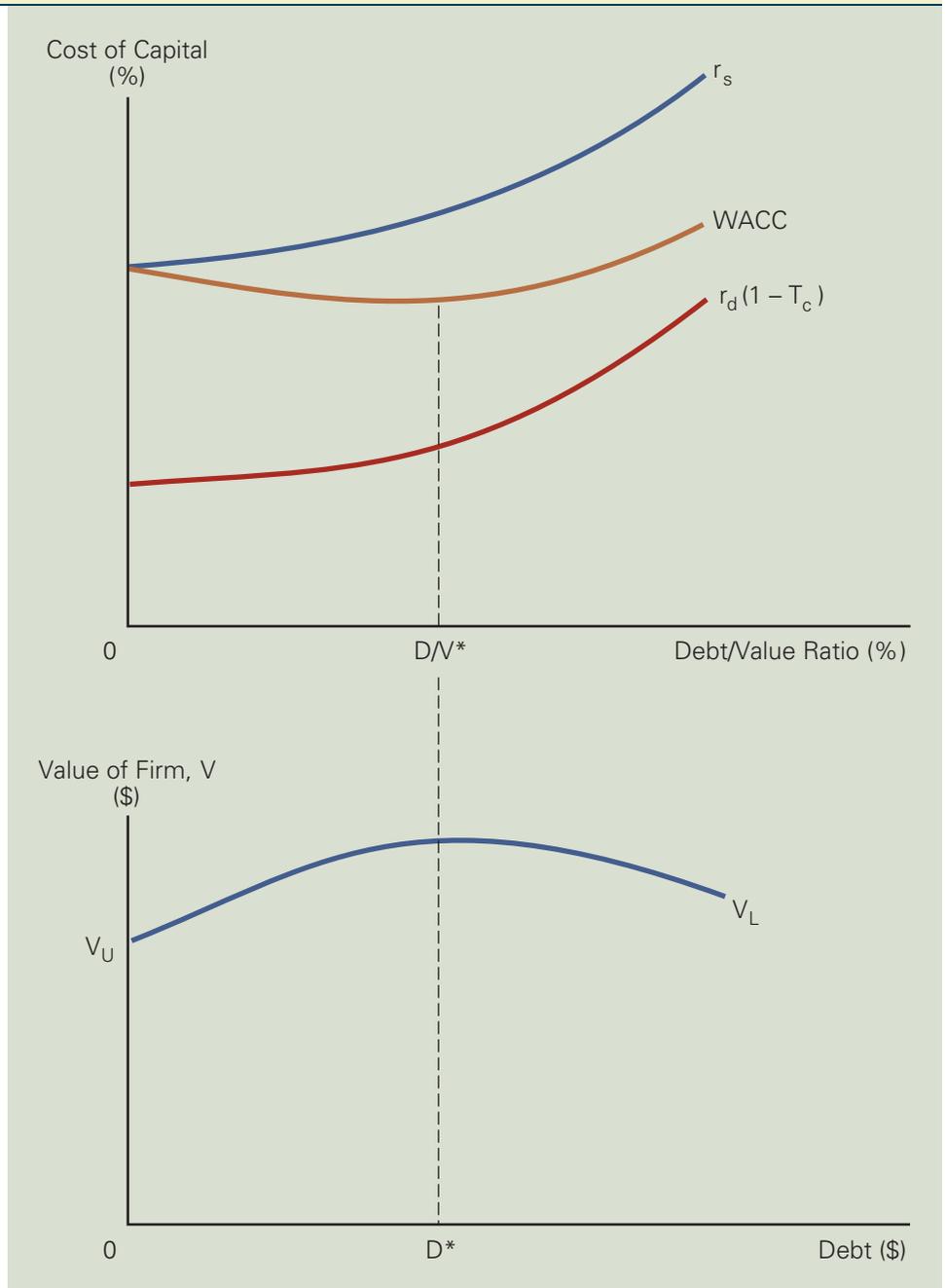
The trade-off model discussed in Chapter 15 is summarized graphically in Figure 26-2. The top graph shows the relationships between the debt ratio and the cost of debt, the cost of equity, and the WACC. Both r_s and $r_d(1 - T_c)$ rise steadily with increases in leverage, but the rate of increase accelerates at higher debt levels; this reflects agency costs and the increased probability of financial distress. Under increasing leverage the WACC first declines, then hits a minimum at D/V^* , and then begins to rise. Note that the value of D in D/V^* in the upper graph is D^* , the level of debt in the lower graph that maximizes the firm's value. Thus, a firm's WACC is minimized and its value is maximized at the same capital structure. Note also that the general shapes of the curves apply regardless of whether we are using the modified MM with corporate taxes model, the Miller model, or a variant of these models.

Unfortunately, it is impossible to quantify accurately the costs and benefits of debt financing, so it is impossible to pinpoint D/V^* , the capital structure that maximizes a firm's value. Most experts believe that such a structure exists for every firm but that it changes over time as a firm's operations and investor preferences change. Most experts also believe that, as shown in Figure 26-2, the relationship between value and leverage is relatively flat over a fairly broad range, so large deviations from the optimal capital structure can occur without materially affecting the stock price.

Now consider signaling theory, which we discussed in Chapter 15. Because of asymmetric information, investors know less about a firm's prospects than its managers know. Furthermore, managers try to maximize value for *current* stockholders, not new ones. Hence, if the firm has excellent prospects then management will not want to issue new shares, but if things look bleak then a new stock offering would benefit current stockholders. Investors therefore view a stock offering as a signal of bad news, so stock prices tend to decline when new issues are announced. As a result, new equity financings are relatively expensive. The net effect of signaling is to motivate firms to maintain a reserve borrowing capacity so that future investment opportunities can be financed by debt if internal funds are not available.

By combining the trade-off and asymmetric information theories, we obtain the following explanation for firms' behavior.

¹⁴For more on viewing equity as an option, see D. Galai and R. Masulis, "The Option Pricing Model and the Risk Factor of Stock," *Journal of Financial Economics*, Vol. 3, 1976, pp. 53–81. For a discussion on compound options, see Robert Geske, "The Valuation of Corporate Liabilities as Compound Options," *Journal of Financial and Quantitative Analysis*, June 1984, pp. 541–552.

FIGURE 26-2 Effects of Leverage: The Trade-off Models

1. Debt financing provides benefits because of the tax deductibility of interest, so firms should have some debt in their capital structures.
2. However, financial distress and agency costs place limits on debt usage—beyond some point, these costs offset the tax advantage of debt. The costs of financial distress are especially harmful to firms whose values consist primarily of intangible

- growth options, such as research and development. Such firms should have lower levels of debt than firms whose asset bases consist mostly of tangible assets.
3. Because of problems resulting from asymmetric information and flotation costs, low-growth firms should follow a pecking order by raising capital first from internal sources, then by borrowing, and finally by issuing new stock. In fact, such low-growth firms rarely need to issue external equity. High-growth firms whose growth occurs primarily through increases in tangible assets should follow the same pecking order, but usually they will need to issue new stock as well as debt. High-growth firms whose values consist primarily of intangible growth options may run out of internally generated cash, but they should emphasize stock rather than debt because of the severe problems that financial distress imposes on such firms.
 4. Managers have better information than investors about a firm's prospects. This informational asymmetry causes investors to view a stock issue as a negative signal, which leads to a decline in stock price. To prevent this, firms should maintain a reserve of borrowing capacity so they can take advantage of investment opportunities without having to issue stock at low prices. This reserve will cause the actual debt ratio to be lower than that suggested by the trade-off models.

There is some evidence that managers do attempt to behave in ways that are consistent with this view of capital structure. In a survey of CFOs, about two-thirds said they follow a “hierarchy in which the most advantageous sources of funds are exhausted before other sources are used.” The hierarchy usually followed the pecking order of first internally generated cash flow, then debt, and finally external equity, which is consistent with the predicted behavior of most low-growth firms. But there were occasions in which external equity was the first source of financing, which would be consistent with the theory for either high-growth firms or firms whose agency costs and level of financial distress have exceeded the benefit of tax savings.¹⁵

Self-Test

Summarize the trade-off and signaling theories of capital structure. Are the trade-off and signaling theories mutually exclusive or might *both* be correct? Does capital structure theory provide managers with a model that can be used to set a precise optimal capital structure?

Summary

In this chapter we discussed a variety of topics related to capital structure decisions. The key concepts covered are listed below.

- In 1958, **Franco Modigliani and Merton Miller (MM)** proved, under a restrictive set of assumptions including zero taxes, that capital structure is irrelevant; thus, according to the original MM article, a firm's value is not affected by its financing mix.
- Modigliani and Miller later added **corporate taxes** to their model and reached the conclusion that capital structure does matter. Indeed, their model led to the conclusion that firms should use 100% debt financing.
- MM's model with corporate taxes demonstrated that the primary benefit of debt stems from the **tax deductibility of interest payments**.

¹⁵For more on capital budgeting issues, see Ravindra R. Kamath, “Long-Term Financing Decisions: Views and Practices of Financial Managers of NYSE Firms,” *The Financial Review*, May 1997, pp. 350–356; Michael T. Dugan and Keith A. Shriver, “An Empirical Comparison of Alternative Methods for Estimating the Degree of Operating Leverage,” *The Financial Review*, May 1992, pp. 309–321; and Dilip K. Ghosh, “Optimum Capital Structure Redefined,” *The Financial Review*, August 1992, pp. 411–429.

- Later, Miller extended the theory to include **personal taxes**. The introduction of personal taxes reduces, but does not eliminate, the benefits of debt financing. Thus, the **Miller model** also leads to 100% debt financing.
- The introduction of growth changes the MM and Hamada results for the levered cost of equity and the levered beta.
- If the firm is growing at a constant rate, the debt tax shield is discounted at r_{sU} , and debt remains a constant proportion of the capital structure, then

$$r_{sL} = r_{sU} + (r_{sU} - r_d) \frac{D}{S}$$

and

$$b = b_U + (b_U - b_D) \frac{D}{S}$$

- When debt is risky, management may choose to default. If the debt is zero coupon debt, then this makes equity like an option on the value of the firm with a strike price equal to the face value of the debt. If the debt has periodic interest payments then the equity is like an option on an option, or a **compound option**.
- When a firm has risky debt and equity is like an option, management has an incentive to increase the firm's risk in order to increase the equity value at the expense of the debt value. This is called **bait and switch**.

Questions

- (26-1) Define each of the following terms:
- MM Proposition I without taxes and with corporate taxes
 - MM Proposition II without taxes and with corporate taxes
 - Miller model
 - Financial distress costs
 - Agency costs
 - Trade-off model
 - Value of debt tax shield
 - Equity as an option
- (26-2) Explain, in words, how MM use the arbitrage process to prove the validity of Proposition I. Also, list the major MM assumptions and explain why each of these assumptions is necessary in the arbitrage proof.
- (26-3) A utility company is allowed to charge prices high enough to cover all costs, including its cost of capital. Public service commissions are supposed to take actions that stimulate companies to operate as efficiently as possible in order to keep costs, and hence prices, as low as possible. Some time ago, AT&T's debt ratio was about 33%. Some individuals (Myron J. Gordon, in particular) argued that a higher debt ratio would lower AT&T's cost of capital and permit it to charge lower rates for telephone service. Gordon thought an optimal debt ratio for AT&T was about 50%. Do the theories presented in the chapter support or refute Gordon's position?
- (26-4) Modigliani and Miller assumed that firms do not grow. How does positive growth change their conclusions about the value of the levered firm and its cost of capital?

- (26–5) Your firm’s CEO has just learned about options and how your firm’s equity can be viewed as an option. Why might he want to increase the riskiness of the firm, and why might the bondholders be unhappy about this?

Self-Test Problem

Solution Appears in Appendix A

(ST–1)
MM with Financial
Distress Costs

B. Gibbs Inc. is an unlevered firm, and it has constant expected operating earnings (EBIT) of \$2 million per year. The firm’s tax rate is 40%, and its market value is $V = S = \$12$ million. Management is considering the use of some debt financing. (Debt would be issued and used to buy back stock, so the size of the firm would remain constant.) Because interest expense is tax deductible, the value of the firm would tend to increase as debt is added to the capital structure, but there would be an offset in the form of a rising risk of financial distress. The firm’s analysts have estimated, as an approximation, that the present value of any future financial distress costs is \$8 million and that the probability of distress would increase with leverage according to the following schedule:

| Value of Debt | Probability of Financial Distress |
|---------------|-----------------------------------|
| \$ 2,500,000 | 0.00% |
| 5,000,000 | 1.25 |
| 7,500,000 | 2.50 |
| 10,000,000 | 6.25 |
| 12,500,000 | 12.50 |
| 15,000,000 | 31.25 |
| 20,000,000 | 75.00 |

- What is the firm’s cost of equity and WACC at this time?
- According to the MM model with corporate taxes, what is the optimal level of debt?
- What is the optimal capital structure when the costs of financial distress are included?
- Plot the value of the firm, with and without distress costs, as a function of the level of debt.

Problems

Answers Appear in Appendix B

EASY PROBLEMS 1–3

- (26–1) An unlevered firm has a value of \$500 million. An otherwise identical but levered firm has \$50 million in debt. Under the MM zero-tax model, what is the value of the levered firm?
- MM Model with Zero Taxes
- (26–2) An unlevered firm has a value of \$800 million. An otherwise identical but levered firm has \$60 million in debt. Assuming the corporate tax rate is 35%, use the MM model with corporate taxes to determine the value of the levered firm.
- MM Model with Corporate Taxes

(26-3)

Miller Model with
Corporate and
Personal Taxes

An unlevered firm has a value of \$600 million. An otherwise identical but levered firm has \$240 million in debt. Under the Miller model, what is the value of the levered firm if the corporate tax rate is 34%, the personal tax rate on equity is 10%, and the personal tax rate on debt is 35%?

INTERMEDIATE PROBLEMS 4-7

(26-4)

Business and Financial
Risk—MM Model

Air Tampa has just been incorporated, and its board of directors is currently grappling with the question of optimal capital structure. The company plans to offer commuter air services between Tampa and smaller surrounding cities. Jaxair has been around for a few years, and it has about the same basic business risk as Air Tampa would have. Jaxair's market-determined beta is 1.8, and it has a current market value debt ratio (total debt to total assets) of 50% and a federal-plus-state tax rate of 40%. Air Tampa expects to be only marginally profitable at start-up; hence its tax rate would only be 25%. Air Tampa's owners expect that the total book and market value of the firm's stock, if it uses zero debt, would be \$10 million. Air Tampa's CFO believes that the MM and Hamada formulas for the value of a levered firm and the levered firm's cost of capital should be used. (These are given in Equations 26-4, 26-6, and 26-7.)

- Estimate the beta of an unlevered firm in the commuter airline business based on Jaxair's market-determined beta. (*Hint:* This is a levered beta; use Equation 26-7 and solve for b_U .)
- Now assume that $r_d = r_{RF} = 10\%$ and that the market risk premium $RP_M = 5\%$. Find the required rate of return on equity for an unlevered commuter airline.
- Air Tampa is considering three capital structures: (1) \$2 million debt, (2) \$4 million debt, and (3) \$6 million debt. Estimate Air Tampa's r_s for these debt levels.
- Calculate Air Tampa's r_s at \$6 million debt while assuming its federal-plus-state tax rate is now 40%. Compare this with your corresponding answer to part c. (*Hint:* The increase in the tax rate causes V_U to drop to \$8 million.)

(26-5)

MM without Taxes

Companies U and L are identical in every respect except that U is unlevered while L has \$10 million of 5% bonds outstanding. Assume that (1) there are no corporate or personal taxes, (2) all of the other MM assumptions are met, (3) EBIT is \$2 million, and (4) the cost of equity to Company U is 10%.

- What value would MM estimate for each firm?
- What is r_s for Firm U? For Firm L?
- Find S_L , and then show that $S_L + D = V_L = \$20$ million.
- What is the WACC for Firm U? For Firm L?
- Suppose $V_U = \$20$ million and $V_L = \$22$ million. According to MM, are these values consistent with equilibrium? If not, explain the process by which equilibrium would be restored.

(26-6)

MM with Corporate
Taxes

Companies U and L are identical in every respect except that U is unlevered while L has \$10 million of 5% bonds outstanding. Assume that (1) all of the MM assumptions are met, (2) both firms are subject to a 40% federal-plus-state corporate tax rate, (3) EBIT is \$2 million, and (4) the unlevered cost of equity is 10%.

- What value would MM now estimate for each firm? (*Hint:* Use Proposition I.)
- What is r_s for Firm U? For Firm L?
- Find S_L , and then show that $S_L + D = V_L$ results in the same value as obtained in part a.
- What is the WACC for Firm U? For Firm L?

(26-7)

Miller Model

Companies U and L are identical in every respect except that U is unlevered while L has \$10 million of 5% bonds outstanding. Assume that (1) all of the MM assumptions are met, (2) both firms are subject to a 40% federal-plus-state corporate tax rate, (3) EBIT is \$2 million, (4) investors in both firms face a tax rate of $T_d = 28\%$ on debt income and $T_s = 20\%$ (on average) on stock income, and (5) the appropriate required pre-personal-tax rate r_{sU} is 10%.

- What is the value V_U of the unlevered firm? (Note that V_U is now reduced by the personal tax on stock income, so $V_U = \$12$ million as in Problem 26-6.)
- What is the value of V_L ?
- What is the gain from leverage in this situation? Compare this with the gain from leverage in Problem 26-6.
- Set $T_c = T_s = T_d = 0$. What is the value of the levered firm? The gain from leverage?
- Now suppose $T_s = T_d = 0$ and $T_c = 40\%$. What are the value of the levered firm and the gain from leverage?
- Assume that $T_d = 28\%$, $T_s = 28\%$, and $T_c = 40\%$. Now what are the value of the levered firm and the gain from leverage?

CHALLENGING

PROBLEMS 8-10

(26-8)

MM Extension with Growth

Schwarztraub Industries' expected free cash flow for the year is \$500,000; in the future, free cash flow is expected to grow at a rate of 9%. The company currently has no debt, and its cost of equity is 13%. Its tax rate is 40%. (*Hint:* Use Equations 26-16 and 26-17.)

- Find V_U .
- Find V_L and r_{sL} if Schwarztraub uses \$5 million in debt with a cost of 7%. Use the extension of the MM model that allows for growth.
- Based on V_U from part a, find V_L and r_{sL} using the MM model (with taxes) if Schwarztraub uses \$5 million in 7% debt.
- Explain the difference between your answers to parts b and c.

(26-9)

MM with and without Taxes

International Associates (IA) is about to commence operations as an international trading company. The firm will have book assets of \$10 million, and it expects to earn a 16% return on these assets before taxes. However, because of certain tax arrangements with foreign governments, IA will not pay any taxes; that is, its tax rate will be zero. Management is trying to decide how to raise the required \$10 million. It is known that the capitalization rate r_U for an all-equity firm in this business is 11%, and IA can borrow at a rate $r_d = 6\%$. Assume that the MM assumptions apply.

- According to MM, what will be the value of IA if it uses no debt? If it uses \$6 million of 6% debt?
- What are the values of the WACC and r_s at debt levels of $D = \$0$, $D = \$6$ million, and $D = \$10$ million? What effect does leverage have on firm value? Why?
- Assume the initial facts of the problem ($r_d = 6\%$, EBIT = \$1.6 million, $r_{sU} = 11\%$), but now assume that a 40% federal-plus-state corporate tax rate exists. Use the MM formulas to find the new market values for IA with zero debt and with \$6 million of debt.
- What are the values of the WACC and r_s at debt levels of $D = \$0$, $D = \$6$ million, and $D = \$10$ million if we assume a 40% corporate tax rate? Plot the relationship between the value of the firm and the debt ratio as well as that between capital costs and the debt ratio.

- e. What is the maximum dollar amount of debt financing that can be used? What is the value of the firm at this debt level? What is the cost of this debt?
- f. How would each of the following factors tend to change the values you plotted in your graph?
 - (1) The interest rate on debt increases as the debt ratio rises.
 - (2) At higher levels of debt, the probability of financial distress rises.

(26-10)

Equity Viewed as an Option

- A. Fethe Inc. is a custom manufacturer of guitars, mandolins, and other stringed instruments that is located near Knoxville, Tennessee. Fethe's current value of operations, which is also its value of debt plus equity, is estimated to be \$5 million. Fethe has \$2 million face value, zero coupon debt that is due in 2 years. The risk-free rate is 6%, and the standard deviation of returns for companies similar to Fethe is 50%. Fethe's owners view their equity investment as an option and would like to know the value of their investment.
- a. Using the Black-Scholes option pricing model, how much is Fethe's equity worth?
 - b. How much is the debt worth today? What is its yield?
 - c. How would the equity value and the yield on the debt change if Fethe's managers could use risk management techniques to reduce its volatility to 30%? Can you explain this?

SPREADSHEET PROBLEM**(26-11)**

Build a Model: Equity Viewed as an Option

Start with the partial model in the file *Ch26 P11 Build a Model.xls* on the textbook's Web site. Rework Problem 26-10 using a spreadsheet model. After completing the problem as it appears, answer the following related questions.

- a. Graph the cost of debt versus the face value of debt for values of the face value from \$0.5 to \$8 million.
- b. Graph the values of debt and equity for volatilities from 0.10 to 0.90 when the face value of the debt is \$2 million.
- c. Repeat part b, but instead using a face value of debt of \$5 million. What can you say about the difference between the graphs in part b and part c?

**Mini Case**

David Lyons, CEO of Lyons Solar Technologies, is concerned about his firm's level of debt financing. The company uses short-term debt to finance its temporary working capital needs, but it does not use any permanent (long-term) debt. Other solar technology companies average about 30% debt, and Mr. Lyons wonders why they use so much more debt and how it affects stock prices. To gain some insights into the matter, he poses the following questions to you, his recently hired assistant.

- a. *BusinessWeek* recently ran an article on companies' debt policies, and the names Modigliani and Miller (MM) were mentioned several times as leading researchers on the theory of capital structure. Briefly, who are MM, and what assumptions are embedded in the MM and Miller models?
- b. Assume that Firms U and L are in the same risk class and that both have EBIT = \$500,000. Firm U uses no debt financing, and its cost of equity is $r_{sU} = 14\%$. Firm L has \$1 million of debt outstanding at a cost of $r_d = 8\%$. There are no taxes. Assume that the MM assumptions hold.

- (1) Find V , S , r_s , and WACC for Firms U and L.
 - (2) Graph (a) the relationships between capital costs and leverage as measured by D/V and (b) the relationship between V and D .
- c. Now assume that Firms L and U are both subject to a 40% corporate tax rate. Using the data given in part b, repeat the analysis called for in b(1) and b(2) under the MM model with taxes.
- d. Suppose investors are subject to the following tax rates: $T_d = 30\%$ and $T_s = 12\%$.
- (1) According to the Miller model, what is the gain from leverage?
 - (2) How does this gain compare with the gain in the MM model with corporate taxes?
 - (3) What does the Miller model imply about the effect of corporate debt on the value of the firm; that is, how do personal taxes affect the situation?
- e. What capital structure policy recommendations do the three theories (MM without taxes, MM with corporate taxes, and Miller) suggest to financial managers? Empirically, do firms appear to follow any one of these guidelines?
- f. How is the analysis in part c different if Firms U and L are growing? Assume both firms are growing at a rate of 7% and that the investment in net operating assets required to support this growth is 10% of EBIT.
- g. What if L's debt is risky? For the purpose of this example, assume that the value of L's operations is \$4 million (the value of its debt plus equity). Assume also that its debt consists of 1-year, zero coupon bonds with a face value of \$2 million. Finally, assume that L's volatility σ is 0.60 and that the risk-free rate r_{RF} is 6%.
- h. What is the value of L's stock for volatilities between 0.20 and 0.95? What incentives might the manager of L have if she understands this relationship? What might debtholders do in response?

SELECTED ADDITIONAL CASES

The following cases from Textchoice, Cengage Learning's online library, cover many of the concepts discussed in this chapter and are available at <http://www.textchoice2.com>.

Klein-Brigham Series:

Case 7, "Seattle Steel Products," Case 9, "Kleen Kar, Inc.," Case 10, "Aspeon Sparkling Water," Case 43, "Mountain Springs," Case 57, "Greta Cosmetics," Case 74, "The Western Company," Case 83, "Armstrong Production Company," and Case 99, "Moore Plumbing Supply Company," focus on capital structure theory. Case 8, "Johnson Window Company," and Case 56, "Isle Marine Boat Company," cover operating and financial leverage.

Brigham-Buzzard Series:

Case 8, "Powerline Network Corporation," covers operating leverage, financial leverage, and the optimal capital structure.

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