

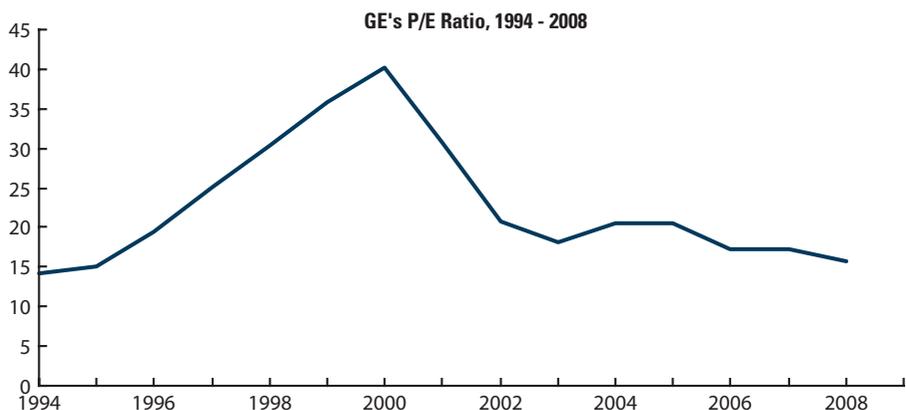
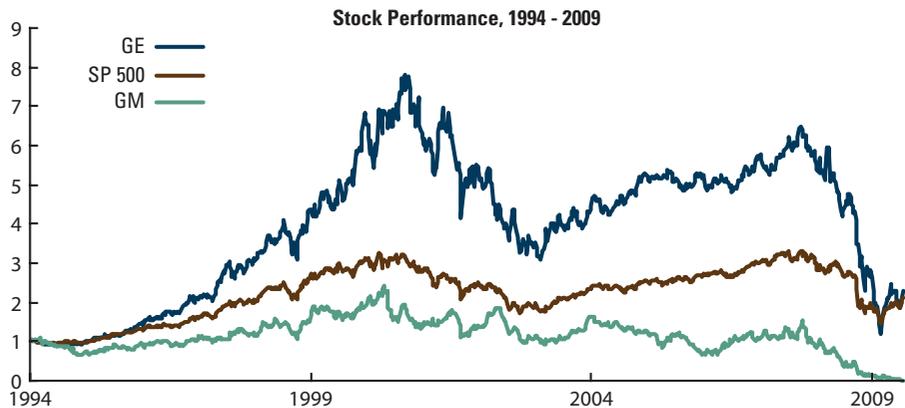
CHAPTER 6

Risk, Return, and the Capital Asset Pricing Model

As sung by the Grateful Dead, "What a long, strange trip it's been!" The chart below provides some insights into the stock market's risks and returns. The top portion shows the relative changes in price since 1994 for General Electric (GE), General Motors (GM), and the S&P 500 Index. The bottom portion shows the price/earnings ratio for GE.

WWW

Updates on stock prices can be found at many Web sites, including <http://finance.yahoo.com>. For updates on P/E ratios, see Yahoo!, Value Line Investment Survey, or GE's annual reports.



Let's take a look at several sub-periods.

1996–2000. These years were wonderful for GE, great for the S&P stocks, and pretty good even for GM. The dramatic increase in P/E ratios indicated that stock prices were going up more as a result of increasing expectations than actual earnings, which was a dangerous sign. Alan Greenspan, Chairman of the Federal Reserve Board at that time, stated that the market was suffering from “irrational exuberance,” but investors paid no attention and kept roaring ahead.

2001–2003. Greenspan was right. The bubble started to leak in 2001, the 9/11 terrorist attacks on the World Trade Center knocked stocks down further, and in 2002 fears of another attack plus a recession drove the market down even more. Those three years cost the average investor almost 50% of his or her beginning-of-2000 market value. P/E ratios plunged, reflecting investors' declining expectations.

2004–2007. Investors had overreacted, so in 2004 the market as measured by the S&P 500 began a rebound, remaining strong through 2007. The economy was robust, profits were rising rapidly, and the Federal Reserve encouraged a bull market by cutting interest rates eleven times. In 2007 the S&P hit an all-time high.

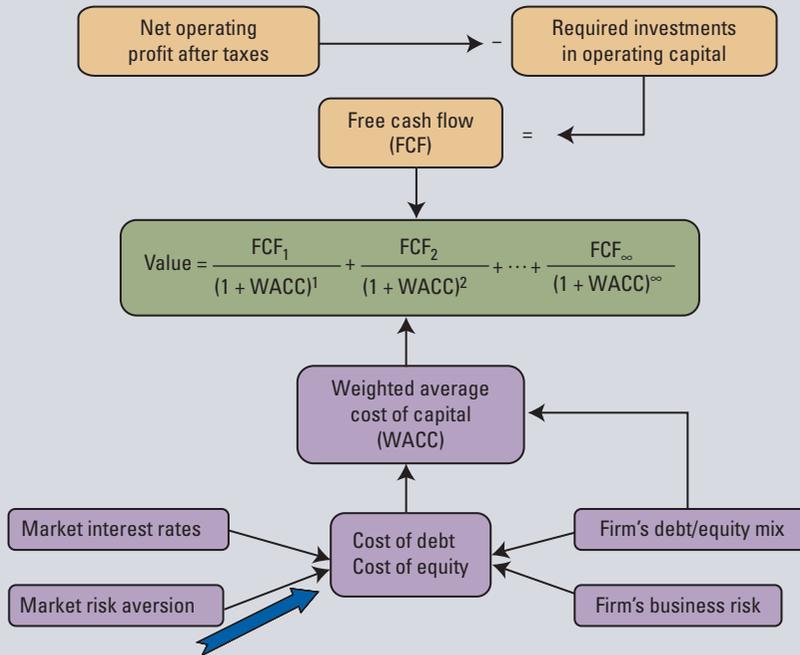
2007–2009. The financial crisis caused by mortgage-backed securities spilled over into the stock markets and the economy, causing a recession.

After 2009: Bull or Bear? We wish we knew! Investing in stocks can be quite profitable, but it means bearing risks. The key to smart investing is to estimate the amount of risk different strategies entail, the returns those strategies are likely to produce, and your own tolerance for risk. We address these topics in this chapter.

Intrinsic Value, Risk, and Return

The intrinsic value of a company is the present value of its expected future free cash flows (FCF) discounted at the weighted average cost of capital (WACC). This chapter shows you how to measure a firm's risk and the rate

of return expected by shareholders, which affects the weighted average cost of capital (WACC). All else held equal, higher risk increases the WACC, which reduces the firm's value.



 **resource**
The textbook's Web site contains an Excel file that will guide you through the chapter's calculations. The file for this chapter is **Ch06 Tool Kit.xls**, and we encourage you to open the file and follow along as you read the chapter.

In this chapter, we start from the basic premise that investors like returns and dislike risk. Therefore, people will invest in relatively risky assets only if they expect to receive relatively high returns—the higher the perceived risk, the higher the expected rate of return an investor will demand. We define exactly what the term *risk* means as it relates to investments, we examine procedures used to measure risk, and we discuss more precisely the relationship between risk and required returns. In later chapters we extend these relationships to show how risk and return interact to determine security prices. Managers must understand and apply these concepts as they plan the actions that will shape their firms' futures, and investors must understand them in order to make appropriate investment decisions.

6.1 RETURNS ON INVESTMENTS

With most investments, an individual or business spends money today with the expectation of earning even more money in the future. The concept of *return* provides investors with a convenient way to express the financial performance of an investment. To illustrate, suppose you buy 10 shares of a stock for \$1,000. The stock pays no dividends, but at the end of 1 year you sell the stock for \$1,100. What is the return on your \$1,000 investment?

One way to express an investment's return is in *dollar terms*:

$$\begin{aligned}\text{Dollar return} &= \text{Amount to be received} - \text{Amount invested} \\ &= \$1,100 - \$1,000 \\ &= \$100\end{aligned}$$

If at the end of the year you sell the stock for only \$900, your dollar return will be $-\$100$.

Although expressing returns in dollars is easy, two problems arise: (1) to make a meaningful judgment about the return, you need to know the scale (size) of the investment; a \$100 return on a \$100 investment is a great return (assuming the investment is held for 1 year), but a \$100 return on a \$10,000 investment would be a poor return. (2) You also need to know the timing of the return; a \$100 return on a \$100 investment is a great return if it occurs after 1 year, but the same dollar return after 20 years is not very good.

The solution to these scale and timing problems is to express investment results as *rates of return*, or *percentage returns*. For example, the rate of return on the 1-year stock investment, when \$1,100 is received after 1 year, is 10%:

$$\begin{aligned}\text{Rate of return} &= \frac{\text{Amount received} - \text{Amount invested}}{\text{Amount invested}} \\ &= \frac{\text{Dollar return}}{\text{Amount invested}} = \frac{\$100}{\$1,000} \\ &= 0.10 = 10\%\end{aligned}$$

The rate of return calculation “standardizes” the dollar return by considering the annual return per unit of investment. Although this example has only one outflow and one inflow, the annualized rate of return can easily be calculated in situations where multiple cash flows occur over time by using time value of money concepts as discussed in Chapter 4.

Self-Test

Differentiate between dollar returns and rates of return.

Why are rates of return superior to dollar returns when comparing different potential investments? (*Hint*: Think about size and timing.)

If you pay \$500 for an investment that returns \$600 in one year, what is your annual rate of return? (**20%**)

6.2 STAND-ALONE RISK

Risk is defined in *Webster's* as “a hazard; a peril; exposure to loss or injury.” Thus, risk refers to the chance that some unfavorable event will occur. If you go skydiving, you are taking a chance with your life—skydiving is risky. If you bet on horse races, you are risking your money. If you invest in speculative stocks (or, really, *any* stock), then you are taking a risk in the hope of earning an appreciable return.

An asset's risk can be analyzed in two ways: (1) on a stand-alone basis, where the asset is considered in isolation, and (2) on a portfolio basis, where the asset is held as one of a number of assets in a portfolio. Thus, an asset's **stand-alone risk** is the risk an investor would face if she held only this one asset. Obviously, most assets are held in portfolios, but it is necessary to understand stand-alone risk in order to understand risk in a portfolio context.

To begin, suppose an investor buys \$100,000 of short-term Treasury bills with an expected return of 5%. In this case, the rate of return on the investment, 5%, can be estimated quite precisely, and the investment is defined as being essentially *risk free*. However, if the \$100,000 were invested in the stock of a company just being organized to prospect for oil in the mid-Atlantic, then the investment's return could not be estimated

precisely. One might analyze the situation and conclude that the *expected* rate of return, in a statistical sense, is 20%, but the investor should recognize that the *actual* rate of return could range from, say, +1,000% to -100%. Because there is a significant danger of actually earning much less than the expected return, this stock would be relatively risky.

No investment should be undertaken unless the expected rate of return is high enough to compensate for the perceived risk. In our example, it is clear that few if any investors would be willing to buy the oil company's stock if its expected return were 5%, the same as that of the T-bill.

Risky assets rarely produce their exact expected rates of return; in general, risky assets earn either more or less than was originally expected. Indeed, if assets always produced their expected returns, they would not be risky. Investment risk, then, is related to the probability of actually earning a low or negative return: The greater the chance of a low or negative return, and the larger the potential loss, the riskier the investment. However, risk can be defined more precisely, and we do so in the next section.

Distributions

An event's *probability* is defined as the chance that the event will occur. For example, a weather forecaster might state: "There is a 40% chance of rain today and a 60% chance that it will not rain." If all possible events, or outcomes, are listed, and if a probability is assigned to each event, then the listing is called a **probability distribution**. Keep in mind that the probabilities must sum to 1.0, or 100%.

With this in mind, consider the possible rates of return—due to dividends or stock price changes—that you might earn next year on a \$10,000 investment in the stock of either Sale.com or Basic Foods Inc. Sale.com is an Internet company that offers deep discounts on factory seconds and overstocked merchandise. Because it faces intense competition, its new services may or may not be competitive in the marketplace, so its future earnings cannot be predicted very well. Indeed, some new company could develop better services and literally bankrupt Sale.com. Basic Foods, on the other hand, distributes essential food staples to grocery stores, and its sales and profits are relatively stable and predictable.

The rate-of-return probability distributions for the two companies are shown in Figure 6-1. There is a 30% chance of strong demand, in which case both companies will have high earnings, pay high dividends, and enjoy capital gains. There is a 40% probability of normal demand and moderate returns and a 30% probability of weak demand, which will mean low earnings and dividends as well as capital losses. Notice, however, that Sale.com's rate of return could vary far more widely than that of Basic Foods. There is a fairly high probability that the value of Sale.com's stock will drop substantially, resulting in a 70% loss, while there is a much smaller possible loss for Basic Foods.¹

Expected Rate of Return

If we multiply each possible outcome by its probability of occurrence and then sum these products, as in Figure 6-2, the result is a *weighted average* of outcomes. The weights are the probabilities, and the weighted average is the **expected rate of return**, \hat{r} , called "r-hat."² The expected rates of return for both Sale.com and Basic Foods are shown in Figure 6-2 to be 15%. This type of table is known as a *payoff matrix*.

¹Note that the following discussion of risk applies to all random variables, not just stock returns.

²In other chapters, we will use \hat{r}_d and \hat{r}_s to signify expected returns on bonds and stocks, respectively. However, this distinction is unnecessary in this chapter, so we just use the general term, \hat{r} , to signify the expected return on an investment.

FIGURE 6-1 Probability Distributions for Sale.com and Basic Foods

	A	B	C	D
26	Demand for the	Probability of this	Rate of Return on Stock	
27	Company's Products	Demand Occurring	if this Demand Occurs	
28			Sale.com	Basic Foods
29	Strong	0.30	90%	45%
30	Normal	0.40	15%	15%
31	Weak	0.30	-60%	-15%
32		1.00		
33				

FIGURE 6-2 Calculation of Expected Rates of Return: Payoff Matrix

	A	B	Sale.com		Basic Foods	
41	Demand for the	Probability of this	Rate of Return		Rate of Return	
42	Company's Products	Demand Occurring	Product	Product	Product	Product
43	(1)	(2)	(3)	(2) x (3) = (4)	(5)	(2) x (5) = (6)
44	Strong	0.3	90%	27.0%	45%	13.0%
45	Normal	0.4	15%	6.0%	15%	6.0%
46	Weak	0.3	-60%	-18.0%	-15%	-4.5%
47		1.0				
48	Expected Rate of Return =		\hat{r} =	15.0%	\hat{r} =	15.0%
49	Sum of Products =					



resource

See Ch06 Tool Kit.xls on
the textbook's Web site.

The calculation for expected rate of return can also be expressed as an equation that does the same thing as the payoff matrix table:

$$\begin{aligned}\text{Expected rate of return} &= \hat{r} = P_1 r_1 + P_2 r_2 + \cdots + P_n r_n \\ &= \sum_{i=1}^n P_i r_i\end{aligned}$$

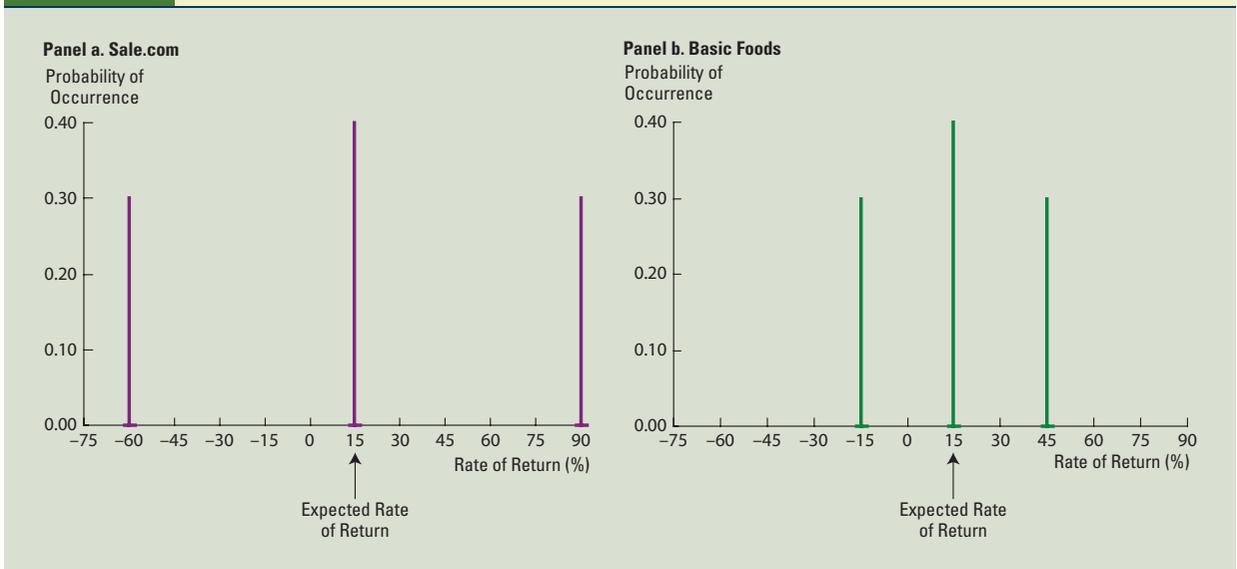
(6-1)

Here r_i is the return if outcome i occurs, P_i is the probability that outcome i occurs, and n is the number of possible outcomes. Thus, \hat{r} is a weighted average of the possible outcomes (the r_i values), with each outcome's weight being its probability of occurrence. Using the data for Sale.com, we obtain its expected rate of return as follows:

$$\begin{aligned}\hat{r} &= P_1(r_1) + P_2(r_2) + P_3(r_3) \\ &= 0.3(90\%) + 0.4(15\%) + 0.3(-60\%) \\ &= 15\%\end{aligned}$$

Basic Foods's expected rate of return is also 15%:

$$\begin{aligned}\hat{r} &= 0.3(45\%) + 0.4(15\%) + 0.3(-15\%) \\ &= 15\%\end{aligned}$$

FIGURE 6-3 Probability Distributions of Sale.com's and Basic Foods's Rates of Return

We can graph the rates of return to obtain a picture of the variability of possible outcomes; this is shown in the bar charts of Figure 6-3. The height of each bar signifies the probability that a given outcome will occur. The range of probable returns for Sale.com is from -60% to $+90\%$, with an expected return of 15% . The expected return for Basic Foods is also 15% , but its range is much narrower.

Thus far, we have assumed that only three situations can exist: strong, normal, and weak demand. Actually, of course, demand could range from a deep depression to a fantastic boom, and there are unlimited possibilities in between. Suppose we had the time and patience to assign a probability to each possible level of demand (with the sum of the probabilities still equaling 1.0) and to assign a rate of return to each stock for each level of demand. We would have a table similar to Figure 6-2, except it would have many more entries in each column. This table could be used to calculate expected rates of return using the same approach as shown previously. In fact, the probabilities and outcomes could be approximated by continuous curves such as those presented in Figure 6-4.

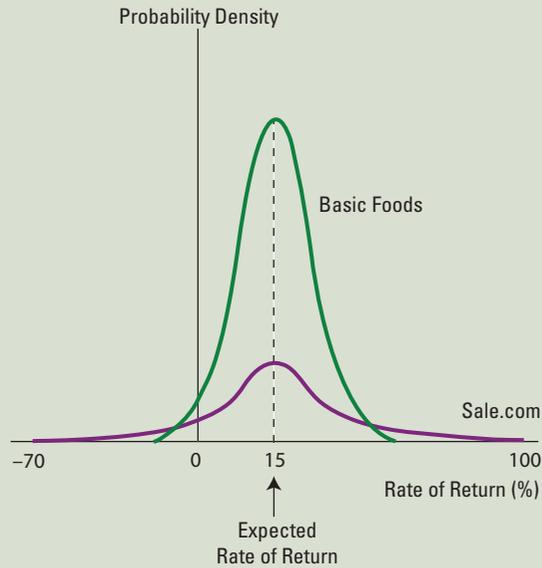
The tighter (or more peaked) the probability distribution, the more likely it is that the actual outcome will be close to the expected value, and hence the less likely it is that the actual return will end up far below the expected return. Thus, the tighter the probability distribution, the lower the risk assigned to a stock. Since Basic Foods has a relatively tight probability distribution, its actual return is likely to be closer to its 15% expected return than that of Sale.com.

Measuring Stand-Alone Risk: The Standard Deviation

Risk is a difficult concept to grasp, and a great deal of controversy has surrounded attempts to define and measure it. However, a common definition that is satisfactory for many purposes is stated in terms of probability distributions such as those presented in Figure 6-4: *The tighter the probability distribution of expected future returns, the smaller the risk of a given investment.* According to this definition, Basic Foods is

FIGURE 6-4

Approximate Continuous Probability Distributions of Sale.com's and Basic Foods's Rates of Return



Note: The assumptions regarding the probabilities of various outcomes have been changed from those in Figure 6-3. There the probability of obtaining exactly 15% was 40%; here it is *much smaller* because there are many possible outcomes instead of just three. With continuous distributions, it is more appropriate to ask what the probability is of obtaining at least some specified rate of return than to ask what the probability is of achieving exactly that rate. This topic is covered in detail in statistics courses.

less risky than Sale.com because there is a smaller chance that its actual return will end up far below its expected return.

To be most useful, any measure of risk should have a definite value—we need a measure of the tightness of the probability distribution. One such measure is the **standard deviation**, the symbol for which is σ , pronounced “sigma.” The smaller the standard deviation, the tighter the probability distribution and, accordingly, the less risky the stock. To calculate the standard deviation, we proceed as shown in Figure 6-5, taking the following steps.³

1. Calculate the expected rate of return:

$$\text{Expected rate of return} = \hat{r} = \sum_{i=1}^n P_i r_i$$

For Sale.com, we previously found $\hat{r} = 15\%$.

2. Subtract the expected rate of return (\hat{r}) from each possible outcome (r_i) to obtain a set of deviations about \hat{r} as shown in Column 4 of Figure 6-5:

$$\text{Deviation}_i = r_i - \hat{r}$$

³These equations are valid for any random variable from a discrete probability distribution, not just for returns.

FIGURE 6-5 Calculating Sale.com's and Basic Foods's Standard Deviations

	A	B	C	D	E	F
89						
90	Panel a.			Sale.com		
	Probability of Occurring	Rate of Return on Stock	Expected Return	Deviation from Expected Return	Squared Deviation	Sq. Dev. x Prob.
91	(1)	(2)	(3)	(2) - (3) = (4)	(4) ² = (5)	(5) x (1) = (6)
92	0.3	90%	15%	75.0%	56.25%	16.88%
93	0.4	15%	15%	0.0%	0.00%	0.00%
94	0.3	-60%	15%	-75.0%	56.25%	16.88%
95	1.0				Sum = Variance =	33.75%
96					Std. Dev. = Square root of variance =	58.09%
97						
98	Panel b.			Basic Foods		
	Probability of Occurring	Rate of Return on Stock	Expected Return	Deviation from Expected Return	Squared Deviation	Sq. Dev. x Prob.
99	(1)	(2)	(3)	(2) - (3) = (4)	(4) ² = (5)	(5) x (1) = (6)
100	0.3	45%	15%	30.0%	9.00%	2.70%
101	0.4	15%	15%	0.0%	0.00%	0.00%
102	0.3	-15%	15%	-30.0%	9.00%	2.70%
103	1.0				Sum = Variance =	5.40%
104					Std. Dev. = Square root of variance =	23.24%
105						



- Square each deviation as shown in Column 5. Then multiply the squared deviations in Column 5 by the probability of occurrence for its related outcome; these products are shown in Column 6. Sum these products to obtain the **variance** of the probability distribution:

$$\text{Variance} = \sigma^2 = \sum_{i=1}^n (r_i - \hat{r})^2 P_i \quad (6-2)$$

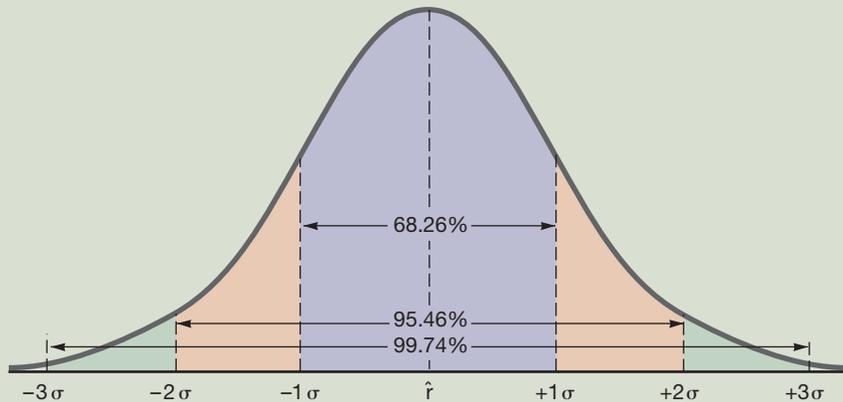
- Finally, find the square root of the variance to obtain the standard deviation:

$$\text{Standard deviation} = \sigma = \sqrt{\sum_{i=1}^n (r_i - \hat{r})^2 P_i} \quad (6-3)$$



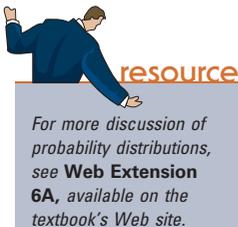
Thus, the standard deviation is essentially a weighted average of the deviations from the expected value, and it provides an idea of how far above or below the expected value the actual value is likely to be. If we use this procedure, Sale.com's standard deviation is seen in Figure 6-5 to be $\sigma = 58.09\%$; we likewise find Basic Foods's standard deviation to be 23.24% . Sale.com has the larger standard deviation,

FIGURE 6-6 Probability Ranges for a Normal Distribution



Notes:

- The area under the normal curve always equals 1.0, or 100%. Thus, the areas under any pair of normal curves drawn on the same scale, whether they are peaked or flat, must be equal.
- Half of the area under a normal curve is to the left of the mean, indicating that there is a 50% probability that the actual outcome will be less than the mean, and half is to the right of \hat{r} , indicating a 50% probability that it will be greater than the mean.
- Of the area under the curve, 68.26% is within $\pm 1\sigma$ of the mean, indicating that the probability is 68.26% that the actual outcome will be within the range $\hat{r} - \sigma$ to $\hat{r} + \sigma$.



which indicates a greater variation of returns and thus a greater chance that the actual return will turn out to be substantially lower than the expected return. Therefore, Sale.com is a riskier investment than Basic Foods when held alone.⁴

If we have a **normal distribution**, then the *actual* return will be within ± 1 standard deviation of the *expected* return 68.26% of the time. Figure 6-6 illustrates this point, and it also shows the situation for $\pm 2\sigma$ and $\pm 3\sigma$. For Sale.com, $\hat{r} = 15\%$ and $\sigma = 58.09\%$, whereas for Basic Foods $\hat{r} = 15\%$ and $\sigma = 23.24\%$. Thus, if the two distributions were normal, there would be a 68.26% probability that Sale.com's actual return would be in the range of $15\% \pm 58.09\%$, or from -43.09% to 73.09% . For Basic Foods, the 68.26% range is $15\% \pm 23.24\%$, or from -8.24% to 38.24% .

⁴As *Cb06 Tool Ki.xls* shows, it is easy to calculate the standard deviation in *Excel*. Calculating by hand is tedious and error-prone:

$$\sigma = \sqrt{(0.3)(0.90 - 0.15)^2 + (0.4)(0.15 - 0.15)^2 + (0.3)(-0.60 - 0.15)^2} = 0.5809$$

Most financial calculators have no built-in formula for finding the expected value or variance for discrete probability distributions, except for the special case in which the probabilities for all outcomes are equal. Therefore, you must go through the processes outlined in Figure 6-2 and 6-5 (i.e., Equations 6-1 and 6-3). For an example of this process using a financial calculator, see Richard W. Taylor, "Discrete Probability Analysis with the BAII Plus Professional Calculator," *Journal of Financial Education*, Winter 2005, pp. 100–106.

What Does Risk Really Mean?

As explained in the text, the probability of being within 1 standard deviation of the expected return is 68.26%, so the probability of being further than 1 standard deviation from the mean is 31.74%. There is an equal probability of being above or below the range, so there is a 15.87% chance of being more than one standard deviation below the mean, which is roughly equal to a 1 in 6 chance (1 in 6 is 16.67%).

For the average firm listed on the New York Stock Exchange, σ has been in the range of 35% to 40% in recent years, with an expected return of around 8% to 12%. One standard deviation below this expected return is about $10\% - 35\% = -25\%$. This means that, for a typical stock in typical year, there is about a 1 in 6 chance of having a 25% loss. You might be thinking that 1 in 6 is a pretty low probability, but what if your chance of getting hit by a car when you crossed a street were 1 in 6? When put that way, 1 in 6 sounds pretty scary.

You might also correctly be thinking that there would be a 1 in 6 chance of getting a return higher than 1 standard deviation above the mean, which would be about 45% for a typical stock. A 45% return is great, but human nature is such that most investors would dislike a 25% loss a whole lot more than they would enjoy a 45% gain.

You might also be thinking that you'll be OK if you hold stock long enough. But even if you buy and hold a diversified portfolio for 10 years, there is still roughly a 10% chance that you will lose money. If you hold it for 20 years, there is about a 4% chance of losing. Such odds wouldn't be worrisome if you were engaged in a game of chance that could be played multiple times, but you have only one life to live and just a few rolls of the dice.

We aren't suggesting that investors shouldn't buy stocks; indeed, we own stock ourselves. But we do believe investors should understand more clearly exactly how much risk stock investing entails.

Using Historical Data to Measure Risk

In our previous example, we described the procedure for finding the mean and standard deviation when the data are in the form of a known probability distribution. This implies that the distribution includes all data points, not a sample of data points from a broader universe of returns. Suppose, however, that only a sample of returns over some past period is available. These past **realized rates of return** are denoted as \bar{r}_t ("r bar t"), where t designates the time period. The average annual return over the last n years is then denoted as \bar{r}_{Avg} :

$$\bar{r}_{Avg} = \frac{\sum_{t=1}^n \bar{r}_t}{n} \quad (6-4)$$

The standard deviation of the sample of returns can then be estimated using this formula:⁵

$$\text{Estimated } \sigma = S = \sqrt{\frac{\sum_{t=1}^n (\bar{r}_t - \bar{r}_{Avg})^2}{n - 1}} \quad (6-5)$$

⁵Because we are estimating the standard deviation from a sample of observations, the denominator in Equation 6-5 is "n - 1" and not just "n." Equations 6-4 and 6-5 are built into all financial calculators. For example, to find the sample standard deviation, enter the rates of return into the calculator and press the key marked S (or S_x) to get the standard deviation. See our tutorials on the textbook's Web site or your calculator's manual for details.

FIGURE 6-7 Standard Deviation Based on a Sample of Historical Data

	A	B	C	D
120				Realized
121			Year	return
122			2008	15.0%
123			2009	-5.0%
124			2010	20.0%
125				
126			Average = AVERAGE(D122:D124) =	10.0%
127			Standard deviation = STDEV(D122:D124) =	13.2%
128				



When estimated from past data, the standard deviation is often denoted by S .

To illustrate, consider the historical returns in Figure 6-7. Using Equations 6-4 and 6-5, the estimated average and standard deviation are, respectively,

$$\bar{r}_{\text{Avg}} = \frac{15\% - 5\% + 20\%}{3} = 10.0\%$$

$$\text{Estimated } \sigma \text{ (or } S) = \sqrt{\frac{(15\% - 10\%)^2 + (-5\% - 10\%)^2 + (20\% - 10\%)^2}{3-1}}$$

$$= 13.2\%$$

The average and standard deviation can also be calculated using *Excel's* built-in functions, shown below using numerical data rather than cell ranges as inputs:

$$= \text{AVERAGE}(0.15, -0.05, 0.20) = 10.0\%$$

$$= \text{STDEV}(0.15, -0.05, 0.20) = 13.2\%$$

The historical standard deviation is often used as an estimate of the future variability. Because past variability is likely to be repeated, past variability may be a reasonably good estimate of future risk. However, it is usually incorrect to use \bar{r}_{Avg} based on a past period as an estimate of \hat{r} , the expected future return. For example, just because a stock had a 75% return in the past year, there is no reason to expect a 75% return this year.

Measuring Stand-Alone Risk: The Coefficient of Variation

If a choice has to be made between two investments that have the same expected returns but different standard deviations, most people would choose the one with the lower standard deviation and, therefore, the lower risk. Similarly, given a choice between two investments with the same risk (standard deviation) but different expected returns, investors would generally prefer the investment with the higher expected return. To most people, this is common sense—return is “good,” risk is “bad,” and consequently investors want as much return and as little risk as possible. But how do we choose between two investments if one has a higher expected return and the other a lower standard deviation? To help answer this question, we often use another measure of risk, the **coefficient of variation (CV)**, which is the standard deviation divided by the expected return:

$$\text{Coefficient of variation} = CV = \frac{\sigma}{\hat{r}}$$

(6-6)

The Trade-off between Risk and Return

The table accompanying this box summarizes the historical trade-off between risk and return for different classes of investments. The assets that produced the highest average returns also had the highest standard deviations and the widest ranges of returns. For example, small-company stocks had the highest average annual return, but their standard deviation of returns was also the highest. In contrast, U.S. Treasury bills had the lowest standard deviation, but they also had the lowest average return.

Note that a T-bill is riskless *if you hold it until maturity*, but if you invest in a rolling portfolio of T-bills and hold the portfolio for a number of years, then your investment income will vary depending on what happens to the level of interest rates in each year. You can be sure of the return you will earn on an individual T-bill, but you cannot be sure of the return you will earn on a portfolio of T-bills held over a number of years.

Distribution of Realized Returns, 1926–2008

	Small Company Stocks	Large Company Stocks	Long-Term Corporate Bonds	Long-Term Government Bonds	U.S. Treasury Bills	Inflation
Average return	16.4%	11.7%	6.2%	6.1%	3.8%	3.1%
Standard deviation	33.0	20.6	8.4	9.4	3.1	4.2
Excess return over T-bonds ^a	10.3	5.6	0.1			

^aThe excess return over T-bonds is called the “historical risk premium.” This excess return will also be the current risk premium that is reflected in security prices if and only if investors expect returns in the future to be similar to returns earned in the past.

Sources: Based on *Stocks, Bonds, Bills, and Inflation: Valuation Edition 2009 Yearbook* (Chicago: Ibbotson Associates, 2009).

The coefficient of variation shows the risk per unit of return, and it provides a more meaningful basis for comparison than σ when the expected returns on two alternatives are different. Since Basic Foods and Sale.com have the same expected return, 15%, the coefficient of variation is not necessary in this case: The firm with the larger standard deviation, Sale.com, must have the larger coefficient of variation when the means are equal. In fact, the coefficient of variation for Sale.com is $58.09/15 = 3.87$ and that for Basic Foods is $23.24/15 = 1.55$. Thus, Sale.com is more than three times as risky as Basic Foods on the basis of this criterion. Because the coefficient of variation captures the effects of both risk and return, it is a better measure than the standard deviation when evaluating stand-alone risk in situations in which different investments have substantially different expected returns.

Risk Aversion and Required Returns

Suppose you have worked hard and saved \$1 million, which you now plan to invest for 1 year. You can buy a 5% U.S. Treasury security, and at the end of the year you will have a sure \$1.05 million, which is your original investment plus \$50,000 in interest. Alternatively, you can buy stock in Genetic Advances Inc. If Genetic Advances’s research programs are successful, your stock will increase in value to \$2.1 million. However, if the research is a failure, the value of your stock will go to zero, and you will be penniless. You regard Genetic Advances’s chances of

success or failure as being 50-50, so the expected value of the stock investment is $0.5(\$0) + 0.5(\$2,100,000) = \$1,050,000$. Subtracting the \$1 million cost of the stock leaves an expected profit of \$50,000, or an expected (but risky) 5% rate of return: $\$50,000/\$1,000,000 = 0.05 = 5\%$.

Thus, you have a choice between a sure \$50,000 profit (representing a 5% rate of return) on the Treasury security and a risky expected \$50,000 profit (also representing a 5% expected rate of return) on the Genetic Advances stock. Which one would you choose? *If you choose the less risky investment, you are risk averse. Most investors are indeed risk averse, and certainly the average investor is risk averse with regard to his “serious money.” Because this is a well-documented fact, we shall assume **risk aversion** throughout the remainder of the book.*

What are the implications of risk aversion for security prices and rates of return? The answer is that, other things held constant, the higher a security's risk, the lower its price and the higher its required return. To see how risk aversion affects security prices, consider again Basic Foods and Sale.com. Suppose each stock is expected to pay an annual dividend of \$15 forever. We know that the dividend could be higher or lower, but \$15 is our best guess. Under these conditions, the price of each stock can be found as the present value of a perpetuity. If each stock had an expected return of 15%, then each stock's price must be $P = \$15/0.15 = \100 . However, investors are averse to risk, so under these conditions there would be a general preference for Basic Foods—it has the same expected return as Sale.com but less risk. People with money to invest would bid for Basic Foods rather than Sale.com stock, and Sale.com stockholders would start selling their stock and using the money to buy Basic Foods. Buying pressure would drive up Basic Foods's stock price, and selling pressure would simultaneously cause Sale.com's price to decline.

These price changes, in turn, would cause changes in the expected rates of return on the two securities. Suppose, for example, that Basic Foods's stock price was bid up from \$100 to \$150, whereas Sale.com's stock price declined from \$100 to \$75. This would cause Basic Foods's expected return to fall to 10%, while Sale.com's expected return would rise to 20%.⁶ The difference in returns, $20\% - 10\% = 10\%$, is a **risk premium, RP**, which represents the additional compensation investors require for assuming the additional risk of Sale.com stock.

This example demonstrates a fundamentally important principle: *In a market dominated by risk-averse investors, riskier securities must have higher expected returns, as estimated by the marginal investor, than less risky securities. If this situation does not already exist, then buying and selling in the marketplace will force it to occur.* We will consider the question of how much higher the returns on risky securities must be later in the chapter, after we see how diversification affects risk and the way it should be measured. Then, in later chapters, we will see how risk-adjusted rates of return affect the prices that investors are willing to pay for bonds and stocks.

Self-Test

What does “investment risk” mean?

Set up an illustrative probability distribution for an investment.

What is a payoff matrix?

⁶Recall that the present value of a perpetuity is $P = \text{PMT}/I$, where PMT is the constant annual cash flow of the perpetuity and I is the rate of return. For stocks, we use r for the expected rate of return. Solving for r, the expected return for Basic Foods is $\$15/\$150 = 0.10 = 10\%$ and that for Sale.com is $\$15/\$75 = 0.20 = 20\%$.

Which of the two stocks graphed in Figure 6-4 is less risky? Why?

How does one calculate the standard deviation?

Which is a better measure of risk when assets have different expected returns:

(1) the standard deviation or (2) the coefficient of variation? Why?

Discuss the following statement: “Most investors are risk averse.”

How does risk aversion affect rates of return on securities?

An investment has a 20% chance of producing a 25% return, a 60% chance of producing a 10% return, and a 20% chance of producing a –15% return. What is its expected return? **(8%)** What is its standard deviation? **(12.9%)**

A stock’s returns for the past 3 years were 10%, –15%, and 35%. What is the historical average return? **(10%)** What is the historical sample standard deviation? **(25%)**

An investment has an expected return of 15% and a standard deviation of 30%. What is its coefficient of variation? **(2.0)**

6.3 RISK IN A PORTFOLIO CONTEXT

In the preceding section we considered the risk of assets held in isolation. Now we analyze the risk of assets held in portfolios. As we shall see, an asset held as part of a portfolio is less risky than the same asset held in isolation. Therefore, most financial assets are actually held as parts of portfolios. Banks, pension funds, insurance companies, mutual funds, and other financial institutions are required by law to hold diversified portfolios. Even individual investors—at least those whose security holdings constitute a significant part of their total wealth—generally hold portfolios, not the stock of only one firm. This being the case, from an investor’s standpoint the fact that a particular stock goes up or down is not the key issue: What’s important are the portfolio’s return and its risk. Logically, then, the risk and return of an individual security should be analyzed in terms of how that security affects the risk and return of the portfolio in which it is held.

To illustrate, Pay Up Inc. collects debts for other firms and operates nationwide through 37 offices. The company is not well known, its stock is not very liquid, its earnings have fluctuated quite a bit in the past, and it doesn’t pay a dividend. All this suggests that Pay Up is risky and that the required rate of return on its stock should be relatively high. However, Pay Up’s required rate of return in 2008, and all other years, was quite low relative to those of most other companies. Thus, investors regard Pay Up as being a low-risk company in spite of its uncertain profits. This is counterintuitive, but it is caused by diversification and its effect on risk. Pay Up’s earnings rise during recessions, whereas most other companies’ earnings tend to decline when the economy slumps. The stock is like a fire insurance policy—it pays off when other things go badly. Therefore, adding Pay Up to a portfolio of “normal” stocks tends to stabilize returns on the entire portfolio, thus making the portfolio less risky.

Portfolio Returns

The **expected return on a portfolio**, \hat{r}_p , is simply the weighted average of the expected returns on the individual assets in the portfolio. Suppose there are n stocks. The expected return on Stock i is \hat{r}_i . The fraction of the portfolio’s dollar value invested in Stock i (that is, the value of the investment in Stock i divided by the total value of the portfolio) is w_i , and all the w_i must sum to 1.0. The expected return on the portfolio is

FIGURE 6-8 Expected Returns on a Portfolio of Stocks

	A	B	C	D	E
150	Stock	Amount of Investment	Portfolio Weight	Expected Return	Weighted Expected Return
151	Southwest Airlines	\$300,000	0.3	15.0%	4.5%
152	Starbucks	\$100,000	0.1	12.0%	1.2%
153	FedEx	\$200,000	0.2	10.0%	2.0%
154	Dell	\$400,000	0.4	9.0%	3.6%
155	Total investment =	\$1,000,000	1.0		
156					
157			Portfolio's Expected Return =		<u>11.3%</u>



$$\begin{aligned}\hat{r}_p &= w_1\hat{r}_1 + w_2\hat{r}_2 + \cdots + w_n\hat{r}_n \\ &= \sum_{i=1}^n w_i\hat{r}_i\end{aligned}$$

(6-7)

To illustrate, assume that a security analyst estimated the upcoming year's returns on the stocks of four large companies, as shown in Figure 6-8. A client wishes to invest \$1 million, divided among the stocks as shown in the figure. Notice that the \$300,000 investment in Southwest Airlines means that its weight in the portfolio is $0.3 = \$300,000/\$1,000,000$. The expected portfolio return is:

$$\begin{aligned}\hat{r}_p &= w_1\hat{r}_1 + w_2\hat{r}_2 + w_3\hat{r}_3 + w_4\hat{r}_4 \\ &= 0.3(15\%) + 0.1(12\%) + 0.2(10\%) + 0.4(9\%) \\ &= 11.3\%\end{aligned}$$

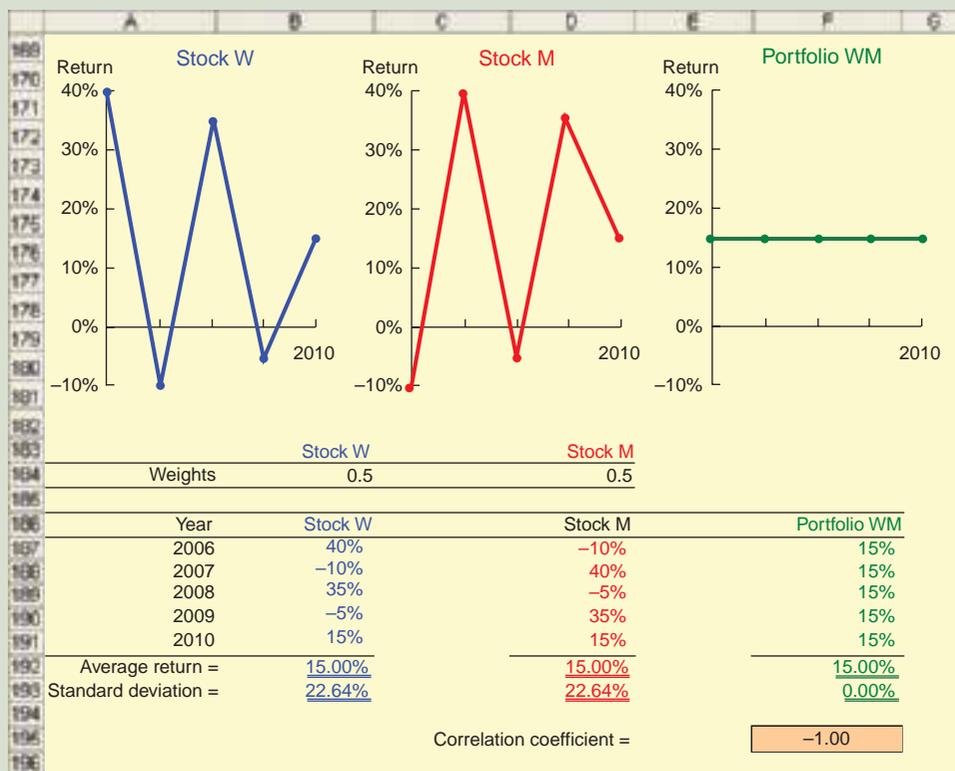
Of course, the actual realized rates of return almost certainly will be different from their expected values, so the realized portfolio return, \bar{r}_p , will be different from the expected return. For example, Starbucks might double and provide a return of +100%, whereas Dell might have a terrible year, fall sharply, and have a return of -75%. Note, though, that those two events would be somewhat offsetting, so the portfolio's return might still be close to its expected return.

Portfolio Risk

As we just saw, the expected return on a portfolio is simply the weighted average of the expected returns on the individual assets in the portfolio. However, unlike returns, the risk of a portfolio, σ_p , is generally not the weighted average of the standard deviations of the individual assets in the portfolio. Indeed, the portfolio's standard deviation will (almost always) be smaller than the assets' weighted standard deviations, and it is theoretically possible to combine stocks that are individually quite risky as measured by their standard deviations and form a portfolio that is completely riskless, with $\sigma_p = 0$.

To illustrate the effect of combining assets, consider first the situation in Figure 6-9. The bottom section gives data on rates of return for Stocks W and M as well as for a

FIGURE 6-9

Rates of Return for Two Perfectly Negatively Correlated Stocks ($\rho = -1.0$) and for Portfolio WM

portfolio invested 50% in each stock. (Note: These stocks are called W and M because the graphs of their returns in Figure 6-9 resemble a W and an M.) The three graphs plot the data in a time-series format. Note that the portfolio's return is 15% in every year. Therefore, although the two stocks would be quite risky if they were held in isolation, when combined to form Portfolio WM they are not risky at all.

The reason Stocks W and M can be combined to form a riskless portfolio is that their returns move countercyclically to each other—when W's returns fall, those of M rise, and vice versa. The tendency of two variables to move together is called **correlation**, and the **correlation coefficient** measures this tendency.⁷ The symbol for the correlation coefficient is the Greek letter rho, ρ (pronounced roe). In statistical terms, we say that the returns on Stocks W and M are *perfectly negatively correlated*, with $\rho = -1.0$.

The estimate of correlation from a sample of historical data is often called "R." Here is the formula to estimate the correlation between stocks i and j ($\bar{r}_{i,t}$ is the actual return for Stock i in period t , and $\bar{r}_{i,Avg}$ is the average return during the n -period sample; similar notation is used for stock j):

⁷The correlation coefficient, ρ , can range from +1.0, denoting that the two variables move up and down in perfect synchronization, to -1.0, denoting that the variables always move in exactly opposite directions. A correlation coefficient of zero indicates that the two variables are not related to each other—that is, changes in one variable are independent of changes in the other.

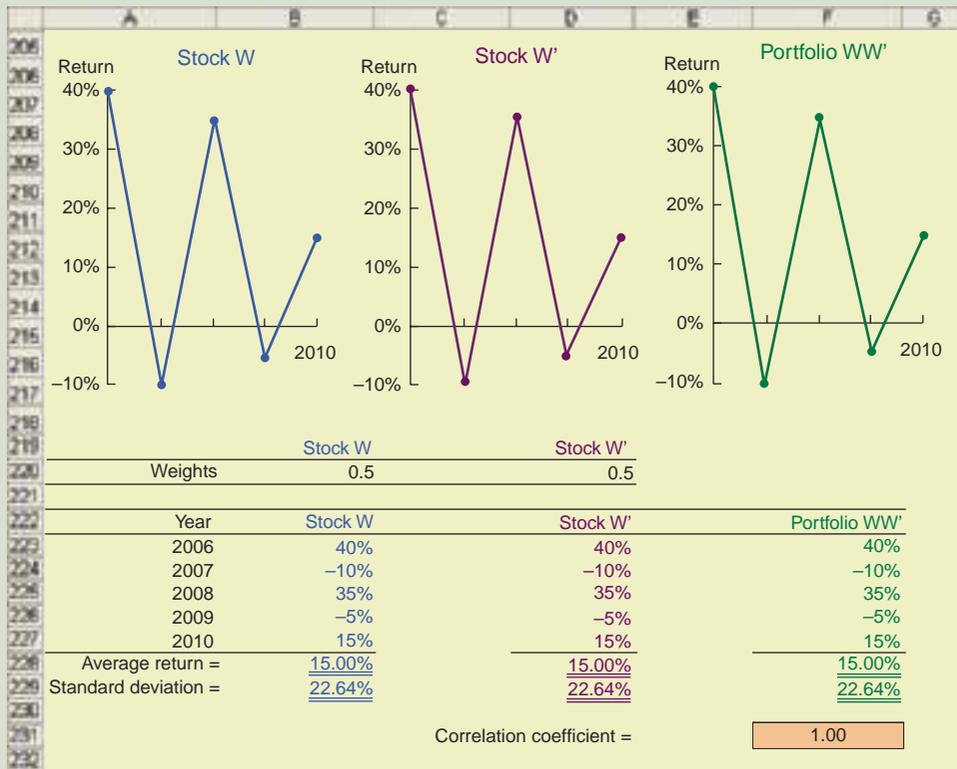
$$\text{Estimated } \rho = R = \frac{\sum_{t=1}^n (\bar{r}_{i,t} - \bar{r}_{i,Avg})(\bar{r}_{j,t} - \bar{r}_{j,Avg})}{\sqrt{\left[\sum_{t=1}^n (\bar{r}_{i,t} - \bar{r}_{i,Avg})^2 \right] \left[\sum_{t=1}^n (\bar{r}_{j,t} - \bar{r}_{j,Avg})^2 \right]}} \quad (6-8)$$



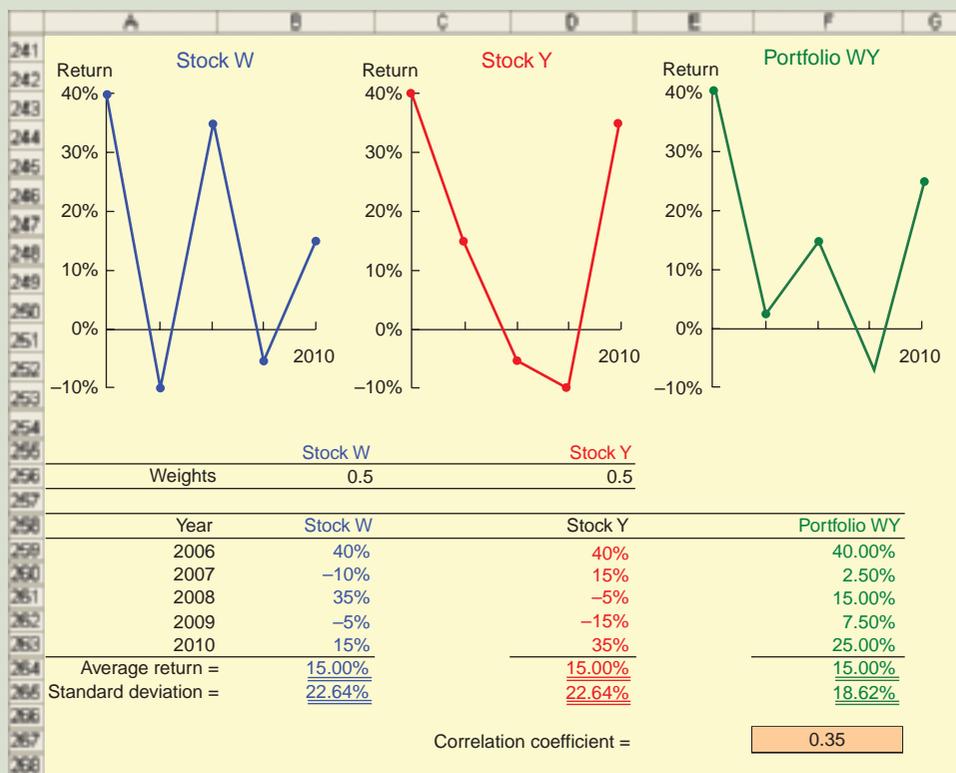
Fortunately, it is easy to estimate the correlation coefficients with a financial calculator or *Excel*. With a calculator, simply enter the returns of the two stocks and then press a key labeled “r.”⁸ In *Excel*, use the CORREL function. See *Ch06 Tool Kit.xls*, where we calculate the correlation between Stocks W and M.

The opposite of perfect negative correlation, with $\rho = -1.0$, is *perfect positive correlation*, with $\rho = +1.0$. Returns on two perfectly positively correlated stocks move up and down together, and a portfolio consisting of two such stocks would be exactly as risky as each individual stock. This point is illustrated in Figure 6-10, where we see that the portfolio’s standard deviation is equal to that of the individual stocks.

FIGURE 6-10 Rates of Return for Two Perfectly Positively Correlated Stocks ($\rho = +1.0$) and for Portfolio WW'



⁸See our tutorial or your calculator manual for the exact steps. Also, note that the correlation coefficient is often denoted by the term “r.” We use ρ here to avoid confusion with r, which is used to denote the rate of return.

FIGURE 6-11 Rates of Return for Two Partially Correlated Stocks ($\rho = +0.35$) and for Portfolio WY


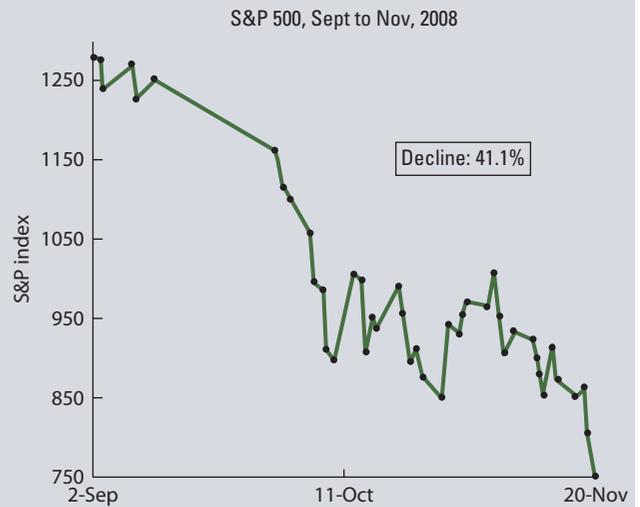
Thus, *diversification does nothing to reduce risk if the portfolio consists of stocks that are perfectly positively correlated.*

Figures 6-9 and 6-10 show that when stocks are perfectly negatively correlated ($\rho = -1.0$), all risk can be diversified away, but when stocks are perfectly positively correlated ($\rho = +1.0$), diversification does no good whatsoever. In reality, virtually all stocks are positively correlated, but not perfectly so. Past studies have estimated that, on average, the correlation coefficient for the monthly returns on two randomly selected stocks is in the range of 0.28 to 0.35.⁹ *Under this condition, combining stocks into portfolios reduces but does not completely eliminate risk.* Figure 6-11 illustrates this point with two stocks whose correlation coefficient is $\rho = +0.35$. The portfolio's average return is 15%, which is exactly the same as the average return for our other two illustrative portfolios, but its standard deviation is 18.6%, which is between the other two portfolios' standard deviations.

⁹During the period 1968–1998, the average correlation coefficient between two randomly selected stocks was 0.28, while the average correlation coefficient between two large-company stocks was 0.33; see Louis K. C. Chan, Jason Karceski, and Josef Lakonishok, "On Portfolio Optimization: Forecasting Covariance and Choosing the Risk Model," *The Review of Financial Studies*, Vol. 12, No. 5, Winter 1999, pp. 937–974. The average correlation fell from around 0.35 in the late 1970s to less than 0.10 by the late 1990s; see John Y. Campbell, Martin Lettau, Burton G. Malkiel, and Yexiao Xu, "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *Journal of Finance*, February 2001, pp. 1–43.

How Risky Is a Large Portfolio of Stocks?

Many investors, including the authors of this text, buy “index” mutual funds that hold the S&P 500. Such funds are obviously well diversified. However, as you can see from the accompanying graph, diversification didn’t help much when the market crashed in 2008. In just 2½ months, the market lost over 40% of its value. Someone with a \$1 million nest egg invested in this “safe” portfolio suddenly experienced a \$411,000 loss. Diversification helps, but it doesn’t eliminate stock market risk.



These examples demonstrate that in one extreme case ($\rho = -1.0$), risk can be completely eliminated, while in the other extreme case ($\rho = +1.0$), diversification does not affect risk at all. The real world lies between these extremes, so combining stocks into portfolios reduces—but does not eliminate—the risk inherent in the individual stocks. Also, we should note that in the real world it is *impossible* to find stocks like *W* and *M*, whose returns are expected to be perfectly negatively correlated. Therefore, *it is impossible to form completely riskless stock portfolios*. Diversification can reduce risk but not eliminate it, so the real world is similar to the situation depicted in Figure 6-11.

What would happen if we included more than two stocks in the portfolio? As a rule, *the risk of a portfolio declines as the number of stocks in the portfolio increases*. If we added enough partially correlated stocks, could we completely eliminate risk? The answer is “no,” but adding stocks to a portfolio reduces its risk to an extent that depends on the *degree of correlation* among the stocks: The smaller the stocks’ correlation coefficients, the lower the portfolio’s risk. If we could find stocks with correlations of -1.0 , all risk could be eliminated. However, *in the real world the correlations among the individual stocks are generally positive but less than $+1.0$, so some (but not all) risk can be eliminated*.

In general, there are higher correlations between the returns on two companies in the same industry than for two companies in different industries. There are also higher correlations among similar “style” companies, such as large versus small and growth versus value. Thus, to minimize risk, portfolios should be diversified across industries and styles.

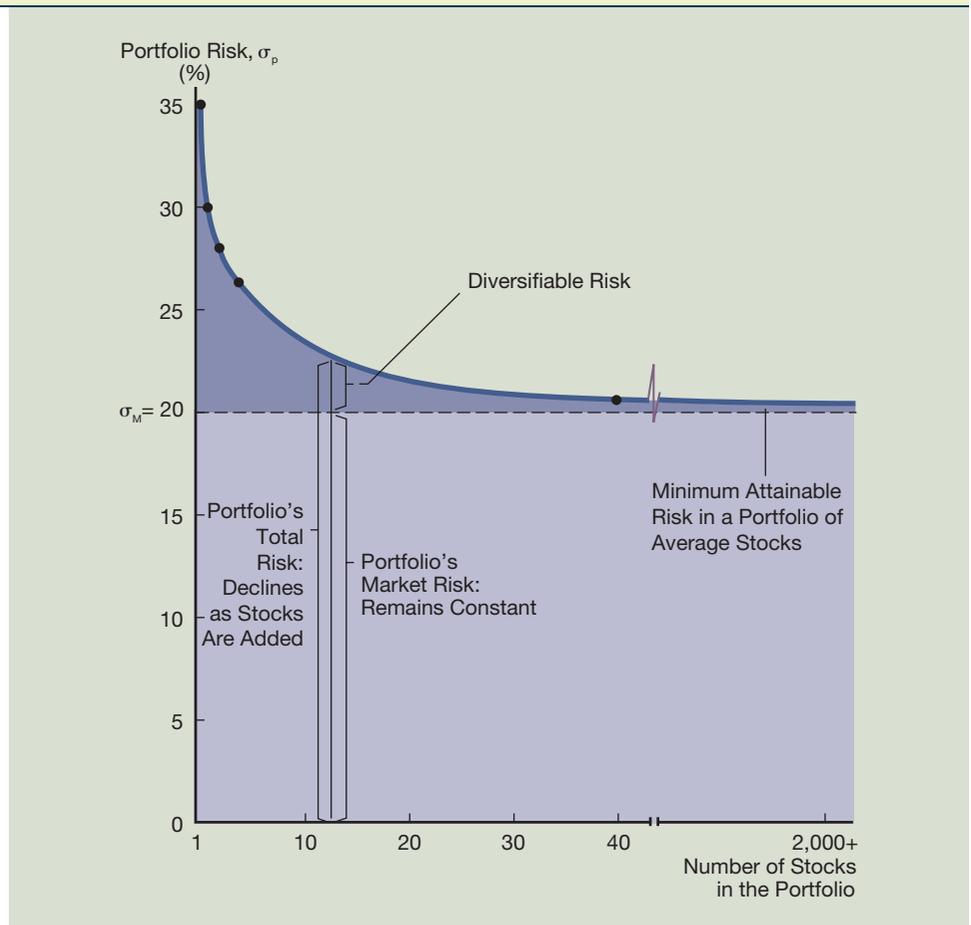
Diversifiable Risk versus Market Risk

As already mentioned, it’s difficult if not impossible to find stocks whose expected returns are negatively correlated—most stocks tend to do well when the national economy is strong and badly when it is weak. Thus, even very large portfolios end up with a substantial amount of risk, but not as much risk as if all the money were invested in only one stock.

To see more precisely how portfolio size affects portfolio risk, consider Figure 6-12, which shows how portfolio risk is affected by forming larger and larger portfolios of randomly selected New York Stock Exchange (NYSE) stocks. Standard deviations are plotted for an average one-stock portfolio, an average two-stock portfolio, and so on, up to a portfolio consisting of all 2,000-plus common stocks that were listed on the NYSE at the time the data were plotted. The graph illustrates that, in general, the risk of a portfolio consisting of large-company stocks tends to decline and to approach some limit as the size of the portfolio increases. According to data accumulated in recent years, σ_1 , the standard deviation of a one-stock portfolio (or an average stock), is approximately 35%. However, a portfolio consisting of all stocks, which is called the **market portfolio**, would have a standard deviation, σ_M , of only about 20%, which is shown as the horizontal dashed line in Figure 6-12.

Thus, *almost half of the risk inherent in an average individual stock can be eliminated if the stock is held in a reasonably well-diversified portfolio, which is one containing forty or more stocks in a number of different industries.* Some risk always remains—terrorists can attack, recessions can get out of hand, meteors can strike, and so forth—so it is impossible to diversify away the effects of broad stock market movements that affect virtually all stocks.

FIGURE 6-12 Effects of Portfolio Size on Portfolio Risk for Average Stocks



The part of a stock's risk that *can* be eliminated is called *diversifiable risk*, while the part that *cannot* be eliminated is called *market risk*.¹⁰ The fact that a large part of the risk of any individual stock can be eliminated is vitally important, because rational investors *will* eliminate it and thus render it irrelevant.

Diversifiable risk is caused by such random events as lawsuits, strikes, successful and unsuccessful marketing programs, winning or losing a major contract, and other events that are unique to a particular firm. Because these events are random, their effects on a portfolio can be eliminated by diversification—bad events in one firm will be offset by good events in another. **Market risk**, on the other hand, stems from factors that systematically affect most firms: war, inflation, recessions, and high interest rates. Because most stocks are negatively affected by these factors, market risk cannot be eliminated by diversification.

We know that investors demand a premium for bearing risk; that is, the higher the risk of a security, the higher its expected return must be to induce investors to buy (or to hold) it. However, if investors are primarily concerned with the risk of their *portfolios* rather than the risk of the individual securities in the portfolio, then how should the risk of an individual stock be measured? One answer is provided by the **Capital Asset Pricing Model (CAPM)**, an important tool used to analyze the relationship between risk and rates of return.¹¹ The primary conclusion of the CAPM is this: *The relevant risk of an individual stock is its contribution to the risk of a well-diversified portfolio.* A stock might be quite risky if held by itself, but—since about half of its risk can be eliminated by diversification—the stock's **relevant risk** is its *contribution to the portfolio's risk*, which is much smaller than its stand-alone risk.

A simple example will help make this point clear. Suppose you are offered the chance to flip a coin. If it comes up heads, you win \$20,000, but if it's tails, you lose \$16,000. This is a good bet—the expected return is $0.5(\$20,000) + 0.5(-\$16,000) = \$2,000$. However, it's a highly risky proposition because you have a 50% chance of losing \$16,000. Thus, you might well refuse to make the bet. Alternatively, suppose that you were to flip 100 coins and that you would win \$200 for each head but lose \$160 for each tail. It is theoretically possible that you would flip all heads and win \$20,000, and it is also theoretically possible that you would flip all tails and lose \$16,000, but the chances are very high that you would actually flip about 50 heads and about 50 tails, winning a net of about \$2,000. Although each individual flip is a risky bet, collectively you have a low-risk proposition because most of the risk has been diversified away. This is the idea behind holding portfolios of stocks rather than just one stock. The difference is that, with stocks, not all of the risk can be eliminated by diversification—those risks related to broad, systematic changes in the stock market will remain.

Are all stocks equally risky in the sense that adding them to a well-diversified portfolio will have the same effect on the portfolio's risk? The answer is “no.” Different stocks will affect the portfolio differently, so different securities have different degrees of relevant risk. How can the relevant risk of an individual stock be measured? As we have seen, all risk except that related to broad market movements can, and presumably will, be diversified away. After all, why accept risk that can be eliminated easily? *The risk that remains after diversifying is called market risk, the risk that is inherent in the market.* In the

¹⁰Diversifiable risk is also known as *company-specific*, or *unsystematic*, risk. Market risk is also known as *nondiversifiable*, *systematic*, or *beta*, risk; it is the risk that remains after diversification.

¹¹Indeed, Nobel Prizes were awarded to the developers of the CAPM, Professors Harry Markowitz and William F. Sharpe. The CAPM is a relatively complex theory, and only its basic elements are presented in this chapter.

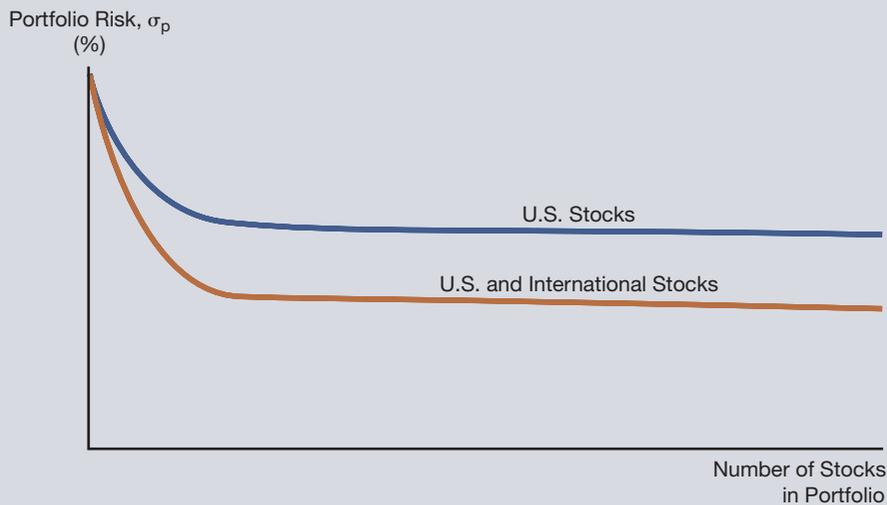
The Benefits of Diversifying Overseas

Figure 6-12 shows that an investor can significantly reduce portfolio risk by holding a large number of stocks. The figure accompanying this box suggests that investors may be able to reduce risk even further by holding stocks from all around the world, because the returns on domestic and international stocks are not perfectly correlated.

Although U.S. investors have traditionally been relatively reluctant to hold international assets, it is a

safe bet that in the years ahead U.S. investors will shift more and more of their assets to overseas investments.

Source: For further reading, see Kenneth Kasa, "Measuring the Gains from International Portfolio Diversification," *Federal Reserve Bank of San Francisco Weekly Letter*, no. 94-14 (April 8, 1994).



next section, we develop a measure of a stock's market risk and then, in a later section, we introduce an equation for determining the required rate of return on a stock, given its market risk.

Contribution to Market Risk: Beta

The primary conclusion reached in the preceding section is that the relevant risk of an individual stock is the amount of risk the stock contributes to a well-diversified portfolio. The benchmark for a well-diversified stock portfolio is the market portfolio, which is a portfolio containing all stocks. Therefore, the relevant risk of an individual stock, which is measured by its **beta coefficient**, is defined under the CAPM as the amount of risk that the stock contributes to the market portfolio. In CAPM terminology, ρ_{iM} is the correlation between Stock i 's return and the market return, σ_i is the standard deviation of Stock i 's return, and σ_M is the standard deviation of the market's return. The beta coefficient of Stock i , denoted by b_i , is found as follows:

$$b_i = \left(\frac{\sigma_i}{\sigma_M} \right) \rho_{iM}$$

(6-9)

This tells us that a stock with a high standard deviation, σ_i , will tend to have a high beta, which means that, other things held constant, the stock contributes a lot of risk to a well-diversified portfolio. This makes sense, because a stock with high stand-alone risk will tend to destabilize the portfolio. Note too that a stock with a high correlation with the market, ρ_{iM} , will also tend to have a large beta and hence be risky. This also makes sense, because a high correlation means that diversification is not helping much, with most of the stock's risk affecting the portfolio's risk.

It is also useful to transform the variables in Equation 6-9 to form the **covariance between Stock i and the market, COV_{iM}** , defined as¹²

$$COV_{iM} = \rho_{iM}\sigma_i\sigma_M \quad (6-10)$$

Substituting Equation 6-10 into 6-9 provides another frequently used expression for calculating beta:

$$b_i = \frac{COV_{iM}}{\sigma_M^2} \quad (6-11)$$

Calculators and spreadsheets can calculate the components of Equation 6-9 (ρ_{iM} , σ_i , and σ_M), which can then be used to calculate beta, but there is another way. Suppose you plotted the stock's returns on the y-axis of a graph and the market portfolio's returns on the x-axis. The formula for the slope of a regression line is exactly equal to the formula for beta in Equation 6-11. Therefore, to estimate beta for a security, you can just estimate a regression with the stock's returns on the y-axis and the market's returns on the x-axis, which we do in the next section.

Individual Stocks' Betas

The tendency of a stock to move up and down with the market is reflected in its beta coefficient. An *average-risk stock* is defined as one with a beta equal to 1 ($b = 1.0$). Such a stock's returns tend to move up and down, on average, with the market, which is measured by some index such as the S&P 500 Index. A portfolio of such $b = 1.0$ stocks will move up and down with the broad market indexes, and it will be just as risky as the market. A portfolio of $b = 0.5$ stocks tends to move in the same direction as the market, but to a lesser degree. On the other hand, a portfolio of $b = 2.0$ stocks also tends to move with the market, but it will have even bigger swings than the market.

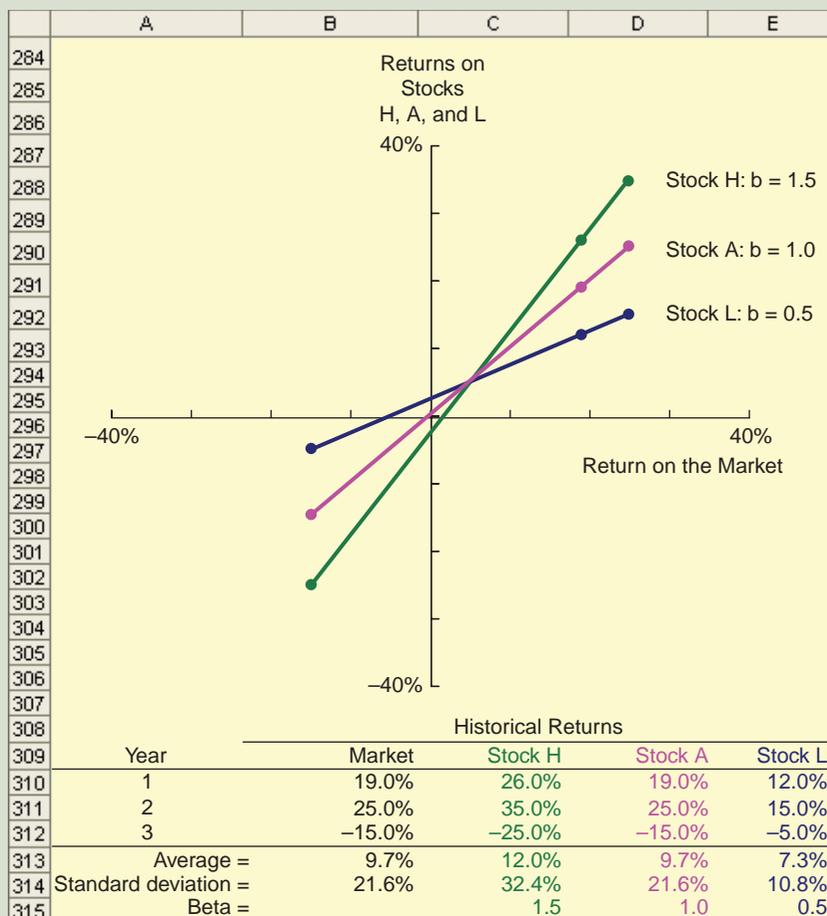
Figure 6-13 shows a graph of the historical returns of three stocks versus the market. The data below the graph show that in Year 1 the "market," defined as a portfolio consisting of all stocks, had a total return (dividend yield plus capital gains yield) of $\bar{r}_M = 19\%$ and that Stocks H, A, and L (for High, Average, and Low risk) had returns of 26%, 19%, and 12%, respectively. In Year 2, the market went up sharply,

¹²Using historical data, the sample covariance can be calculated as

$$\text{Sample covariance from historical data} = COV_{iM} = \frac{\sum_{t=1}^n (\bar{r}_{i,t} - \bar{r}_{i,Avg})(\bar{r}_{M,t} - \bar{r}_{M,Avg})}{n - 1}$$

Calculating the covariance is somewhat easier than calculating the correlation. So if you have already calculated the standard deviations, it is easier to calculate the covariance and then calculate the correlation as $\rho_{iM} = COV_{iM}/(\sigma_i\sigma_M)$.

FIGURE 6-13 Relative Returns of Stocks H, A, and L



Note: These three stocks plot exactly on their regression lines. This indicates that they are exposed only to market risk. Portfolios that concentrate on stocks with betas of 1.5, 1.0, and 0.5 have patterns similar to those shown in the graph.



and the return on the market portfolio was $\bar{r}_M = 25\%$. Returns on the three stocks also went up: H soared to 35%; A went up to 25%, the same as the market; and L went up only to 15%. The market dropped in Year 3, when the market return was $\bar{r}_M = -15\%$. The three stocks' returns also fell: H plunging to -25% , A falling to -15% , and L going down to $\bar{r}_L = -5\%$. Thus, the three stocks all moved in the same direction as the market, but H was by far the most volatile; A was just as volatile as the market; and L was less volatile than the market.

Beta measures a stock's tendency to move up and down with the market. By definition, then, the market has $b = 1.0$. As noted previously, the slope of a regression line shows how a stock moves in response to a movement in the general market. Most stocks have betas in the range of 0.50 to 1.50, and the average beta for all stocks is 1.0 by definition.

Theoretically, it is possible for a stock to have a negative beta. In this case, the stock's returns would tend to rise whenever the returns on other stocks fall. In practice, few if

any stocks have a negative beta. Keep in mind that a stock in a given period may move counter to the overall market even though the stock's "true" beta is positive. If a stock has a positive beta, we would *expect* its return to increase whenever the overall stock market rises. However, company-specific factors may cause the stock's realized return in a given period to decline, even though the market's return is positive.

Portfolio Betas

An important aspect of the CAPM is that the beta of a portfolio is a weighted average of its individual securities' betas:

$$b_p = w_1b_1 + w_2b_2 + \dots + w_nb_n$$

$$= \sum_{i=1}^n w_ib_i$$
(6-12)

Here b_p is the beta of the portfolio, which shows its tendency to move with the market; w_i is the fraction of the portfolio invested in Stock i ; and b_i is the beta coefficient of Stock i . For example, if an investor holds a \$100,000 portfolio consisting of \$33,333.333 invested in each of three stocks, and if each of the stocks has a beta of 0.70, then the portfolio's beta will be $b_p = 0.70$:

$$b_p = 0.3333(0.70) + 0.3333(0.70) + 0.3333(0.70) = 0.70$$

Such a portfolio will be less risky than the market, so it should experience relatively narrow price swings and have relatively small fluctuations in its rates of return. In terms of Figure 6-13, the slope of its regression line would be 0.70, which is less than that for a portfolio of average stocks.

Now suppose that one of the existing stocks is sold and replaced by a stock with $b_i = 2.00$. This action will increase the beta of the portfolio from $b_{p1} = 0.70$ to $b_{p2} = 1.13$:

$$b_{p2} = 0.3333(0.70) + 0.3333(0.70) + 0.3333(2.00)$$

$$= 1.13$$

Had a stock with $b_i = 0.20$ been added, the portfolio beta would have declined from 0.70 to 0.53. Adding a low-beta stock, therefore, would reduce the risk of the portfolio. Consequently, adding new stocks to a portfolio can change the risk of that portfolio. *Since a stock's beta measures its contribution to the risk of a portfolio, beta is the theoretically correct measure of the stock's risk.*

Some Other Points Related to Beta

The preceding analysis of risk in a portfolio context is part of the CAPM, and we highlight the key points below.

1. A stock's risk consists of two components, market risk and diversifiable risk.
2. Diversifiable risk can be eliminated by diversification, and most investors do indeed diversify, either by holding large portfolios or by purchasing shares in a mutual fund. We are left, then, with market risk, which is caused by general movements in the stock market and which reflects the fact that most stocks are systematically affected by events like war, recessions, and inflation. Market risk is the only risk relevant to a rational, diversified investor because such an investor can eliminate diversifiable risk.

3. Investors must be compensated for bearing risk: The greater the risk of a stock, the higher its required return. However, compensation is required only for risk that cannot be eliminated by diversification. If stocks had risk premiums due to diversifiable risk, then well-diversified investors would start buying those securities (which the investors would not consider especially risky) and bidding up their prices. The stocks' final (equilibrium) expected returns would reflect only nondiversifiable market risk.
4. The market risk of a stock is measured by its beta coefficient, and beta is the proper measure of the stock's relevant risk. If b equals 1.0, then the stock is about as risky as the market, assuming it is held in a diversified portfolio. If b is less than 1.0 then the stock is less risky than the market; if beta is greater than 1.0, the stock is more risky.
5. The beta of a portfolio is a weighted average of the individual securities' betas.
6. *Since a stock's beta coefficient determines how the stock affects the risk of a diversified portfolio, beta is the most relevant measure of any stock's risk.*

Self-Test

Explain the following statement: "An asset held as part of a portfolio is generally less risky than the same asset held in isolation."

What is meant by *perfect positive correlation*, *perfect negative correlation*, and *zero correlation*?

In general, can the risk of a portfolio be reduced to zero by increasing the number of stocks in the portfolio? Explain.

What is the average beta? If a stock has the average beta, what does that imply about its risk relative to the market?

Why is beta the theoretically correct measure of a stock's risk?

If you plotted the returns on a particular stock versus those on the Dow Jones Index over the past 5 years, what would the slope of the regression line tell you about the stock's market risk?

An investor has a three-stock portfolio with \$25,000 invested in Dell, \$50,000 invested in Ford, and \$25,000 invested in Wal-Mart. Dell's beta is estimated to be 1.20, Ford's beta is estimated to be 0.80, and Wal-Mart's beta is estimated to be 1.0. What is the estimated beta of the investor's portfolio? **(0.95)**

6.4 CALCULATING BETA COEFFICIENTS

The CAPM is an *ex ante* model, which means that all of the variables represent before-the-fact, *expected* values. In particular, the beta coefficient used by investors should reflect the relationship between a stock's expected return and the market's return during some *future* period. However, people generally calculate betas using data from some *past* period and then assume that the stock's risk will be the same in the future as it was in the past.

Table 6-1 shows the betas for some well-known companies as provided by two different financial organizations, Zacks and Yahoo! Finance. Notice that their estimates of beta usually differ because they calculate it in slightly different ways. Given these differences, many analysts choose to calculate their own betas or else average the published betas.

Recall from Figure 6-13 how betas can be calculated. The actual historical returns for a company are plotted on the y-axis and the market portfolio's returns are plotted on the x-axis. A regression line is then fitted through the points, and the slope of that line provides an estimate of the stock's beta. It is possible to compute beta coefficients with a calculator, but in the real world a computer is typically used, either with a statistical



To see updated estimates, go to <http://www.zacks.com> and enter the ticker symbol; select Detailed Quotes for beta. Or go to <http://finance.yahoo.com> and enter the ticker symbol. When the results page comes up, select Key Statistics from the left panel to find beta.

TABLE 6-1 Beta Coefficients for Some Actual Companies

STOCK (TICKER SYMBOL)	ZACKS	YAHOO! FINANCE
Amazon.com (AMZN)	1.88	1.95
Cisco Systems (CSCO)	1.28	1.32
Dell Computer (DELL)	1.36	1.37
Microsoft Corp. (MSFT)	1.03	1.08
General Electric (GE)	1.11	1.01
Empire District Electric (EDE)	0.65	0.56
Coca-Cola (KO)	0.59	0.63
Heinz (HNZ)	0.64	0.66
Energen Corp. (EGN)	1.21	1.19
Procter & Gamble (PG)	0.56	0.54

Sources: <http://www.zacks.com> and <http://finance.yahoo.com>, February 2009.

software program or a spreadsheet program. The chapter's *Excel Tool Kit* model shows how GE's beta can be calculated using *Excel's* regression function.¹³

The first step in a regression analysis is getting the data. Most analysts use 4 to 5 years of monthly data, although some use 52 weeks of weekly data. We decided to use 4 years (48 months) of monthly data, so we began by downloading 49 months of stock prices for GE from the Yahoo! Finance Web site (we needed 49 months of data to get 48 rates of return). We used the S&P 500 Index as the market portfolio because it is representative of the market and because many analysts use this index. Figure 6-14 shows a portion of these data; the full data set is in the chapter's *Tool Kit*.

The second step is to convert the stock prices into rates of return. For example, to find the March 2009 return for GE, we find the percentage change from the previous month: $(\$10.11 - \$8.51)/\$8.51 = 0.188 = 18.8\%$.¹⁴ We also find the percent change of the S&P Index level and use this as the market return.

As the lower portion of Figure 6-14 shows, GE had an average annual return of -22.9% during this 4-year period, while the market had an average annual return of -8.5%. As we noted before, it is usually unreasonable to think that the future expected return for a stock will equal its average historical return over a relatively short period, such as 4 years. If this were not true, then why would anyone buy either the S&P or GE if they expected the same negative returns as were earned in the past? However, we might well expect past volatility to be a reasonable estimate of fu-

¹³For an explanation of computing beta with a financial calculator, see *Web Extension 6B* on the textbook's Web site.

¹⁴The prices reported in Yahoo! Finance are adjusted for dividends and stock splits, so we can calculate the return as the percentage change in the adjusted price. If you use a source that reports actual market prices, then you must make the adjustment yourself when calculating returns. For example, suppose the stock price is \$100 in January, the company has a 2-for-1 split, and the actual price is then \$60 in February. The reported adjusted price for February would be \$60, but the reported adjusted price for January would be lowered to \$50 to reflect the stock split. This gives an accurate stock return of 20%: $(\$60 - \$50)/\$50 = 20\%$, the same as if there had not been a split, in which case the return would have been $(\$120 - \$100)/\$100 = 20\%$. Or suppose the actual price in March was \$50, the company paid a \$10 dividend, and the actual price in April was \$60. Shareholders have earned a return of $(\$60 + \$10 - \$50)/\$50 = 40\%$. Yahoo! Finance reports an adjusted price of \$60 for April and an adjusted price of \$42.857 for March, which gives a return of $(\$60 - \$42.857)/\$42.857 = 40\%$. Again, the percentage change in the adjusted price accurately reflects the actual return.



FIGURE 6-14 Stock Return Data for General Electric and the S&P 500 Index

	A	B	C	D	E
	Month	Market Level (S&P 500 Index) at Month End	Market's Return	GE Adjusted Stock Price at Month End	GE's Return
396	March 2009	797.87	8.5%	\$10.11	18.8%
397	February 2009	735.09	-11.0%	\$8.51	-27.8%
398	January 2009	825.88	8.6%	\$11.78	-25.2%
399	December 2008	903.25	0.8%	\$15.74	-3.8%
441	June 2005	1,191.33	0.0%	\$29.68	-4.4%
442	May 2005	1,191.50	3.0%	\$31.05	0.07%
443	April 2005	1,156.85	-20%	\$30.82	0.4%
444	March 2005	1,180.59	NA	\$30.70	NA
445	Description of Data				
447	Average return (annual):		-8.5%	-22.9%	
448	Standard deviation (annual):		15.9%	28.9%	
449	Minimum monthly return:		-16.8%	-27.8%	
450	Maximum monthly return:		8.5%	18.8%	
451	Correlation between GE and the market:			0.76	
452	Beta: $b_{GE} = \rho_{GE,M}(\sigma_{GE}/\sigma_M)$			1.37	

Note: The data for July 2005 through November 2008 are not shown but are included in all calculations.

WWW

You can get historical stock returns for GE and the S&P 500 index (its symbol is ^SPX) from <http://finance.yahoo.com>.

ture volatility, at least during the next couple of years. Note that the annualized standard deviation for GE's return during this period was 28.9% versus 15.9% for the market. The range between GE's minimum and maximum returns is also greater than the corresponding range for the market. Thus, GE's volatility is greater than the market's volatility. This is what we would expect, since the market is a well-diversified portfolio and so much of its risk has been diversified away. The correlation between GE's stock returns and the market returns is 0.76, which is somewhat higher than the correlation between a typical stock and the market.

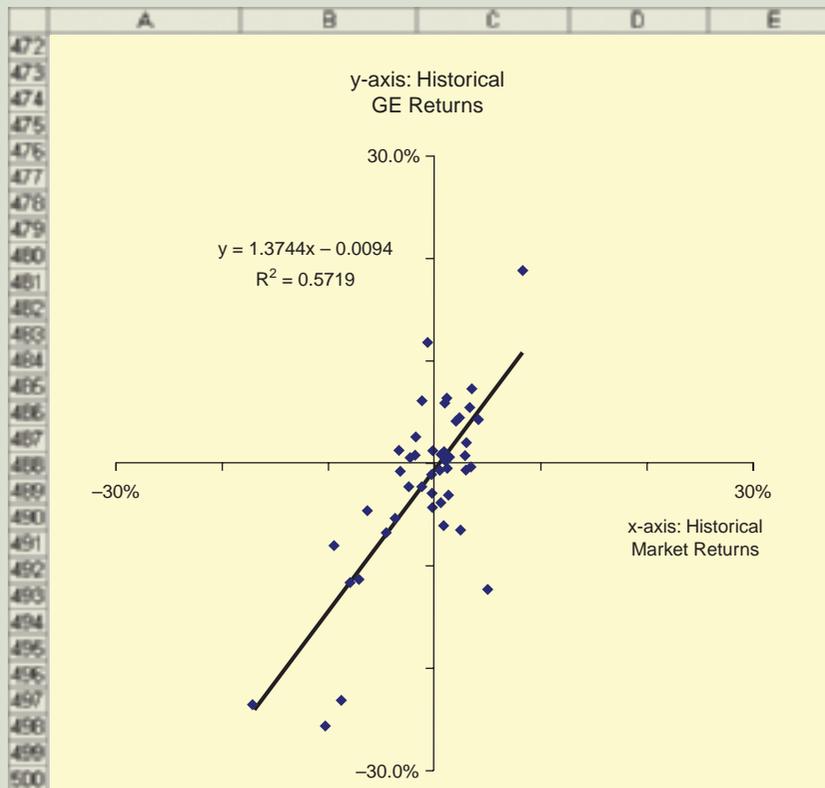
We obtained inputs from Figure 6-14 and used Equation 6-9 to approximate GE's beta:

$$b_i = \left(\frac{\sigma_i}{\sigma_M} \right) \rho_{iM} = \left(\frac{0.289}{0.159} \right) (0.76) = 1.38 \approx 1.37$$

Other than a small difference due to rounding in intermediate steps, this is the same result reported in Figure 6-14.

A picture is worth a thousand words, so Figure 6-15 shows a plot of GE's returns against the market returns. As you will notice if you look in the *Excel Tool Kit* file, we used the *Excel* chart feature to add a trend line and to display the equation and R^2 value on the chart itself. We also could have used the *Excel* regression analysis feature, which would have provided more detailed data.

Figure 6-15 indicates that GE's estimated beta is about 1.37, as shown by the slope coefficient in the regression equation displayed on the chart. This means that GE's beta is greater than the 1.0 average beta. Therefore, GE's returns tend to move up and down (on average) by more than the market's returns. Note, however, that the points are only loosely clustered around the regression line. Sometimes GE does much better than the

FIGURE 6-15 Stock Return Data for General Electric and the S&P 500 Index

market, while at other times it does much worse. The R^2 value shown in the chart measures the degree of dispersion about the regression line. Statistically speaking, it measures the percentage of the variance that is explained by the regression equation. An R^2 of 1.0 indicates that all points lie exactly on the line and hence that all of the variations in the y-variable are explained by the x-variable. GE's R^2 is about 0.57, which is somewhat higher than the typical stock R^2 of 0.32. This indicates that about 57% of the variance in GE's returns is explained by the market returns versus only 32% of the explained variance of a typical stock. If we had done a similar analysis for a portfolio of forty randomly selected stocks, then the points would probably have been clustered tightly around the regression line and the R^2 probably would have exceeded 0.90.

Finally, observe that the intercept shown in the regression equation on the chart is -0.0094 . This indicates that GE's average monthly return was -0.94% less than that of a typical company during these 4 years, or $12(-0.94\%) = -11.28\%$ less per year as a result of factors other than the general decline in stock prices.

Self-Test

What types of data are needed to calculate a beta coefficient for an actual company? What does the R^2 measure? What is the R^2 for a typical company?

6.5 THE RELATIONSHIP BETWEEN RISK AND RETURN

In the preceding section we saw that, under the CAPM theory, beta is the proper measure of a stock's relevant risk. However, we need to quantify how risk affects required

returns: For a given level of risk as measured by beta, what rate of return do investors require to compensate them for bearing that risk? To begin, let us define the following terms.

\hat{r}_i = *Expected* rate of return on Stock *i*.

r_i = *Required* rate of return on Stock *i*. This is the minimum expected return that is required to induce an average investor to purchase the stock.

\bar{r} = Realized, after-the-fact return.

r_{RF} = Risk-free rate of return. In this context, r_{RF} is generally measured by the expected return on long-term U.S. Treasury bonds.

b_i = Beta coefficient of Stock *i*.

r_M = Required rate of return on a portfolio consisting of all stocks, which is called the *market portfolio*.

RP_M = Risk premium on “the market.” $RP_M = (r_M - r_{RF})$ is the additional return over the risk-free rate required to induce an average investor to invest in the market portfolio.

RP_i = Risk premium on Stock *i*: $RP_i = (RP_M)b_i$.

The **market risk premium, RP_M** , is the premium that investors require for bearing the risk of an average stock, and it depends on the degree of risk aversion that investors on average have. Assume that Treasury bonds yield $r_{RF} = 6\%$ and that the stock market has a required return of $r_M = 11\%$. Under these conditions, the market risk premium, RP_M , is 5%:

$$RP_M = r_M - r_{RF} = 11\% - 6\% = 5\%$$

We can measure a stock’s relative risk by its beta coefficient and then calculate its individual risk premium as follows:

$$\text{Risk premium for Stock } i = RP_i = (RP_M)b_i$$

(6-13)

For example, if $b_i = 0.5$ and $RP_M = 5\%$, then RP_i is 2.5%:

$$\begin{aligned} RP_i &= (5\%)(0.5) \\ &= 2.5\% \end{aligned}$$

The required return for any investment can be expressed in general terms as

$$\text{Required return} = \text{Risk-free return} + \text{Premium for risk}$$

Here the risk-free return includes a premium for expected inflation, and we assume that the assets under consideration have similar maturities and liquidity. Under these conditions, the relationship between risk and required returns can be found as specified in the **Security Market Line (SML)**:

$$\begin{aligned} \text{SML equation: } \text{Required return on Stock } i &= \text{Risk-free rate} + \left(\text{Market risk premium} \right) \left(\text{Beta of Stock } i \right) \\ r_i &= r_{RF} + (r_M - r_{RF})b_i \\ &= r_{RF} + (RP_M)b_i \end{aligned}$$

(6-14)

The required return for our illustrative Stock i is then found as follows:

$$\begin{aligned} r_i &= 6\% + 5\%(0.5) \\ &= 8.5\% \end{aligned}$$

If some other Stock j were riskier than Stock i and had $b_j = 2.0$, then its required rate of return would be 16%:

$$r_j = 6\% + (5\%)2.0 = 16\%$$

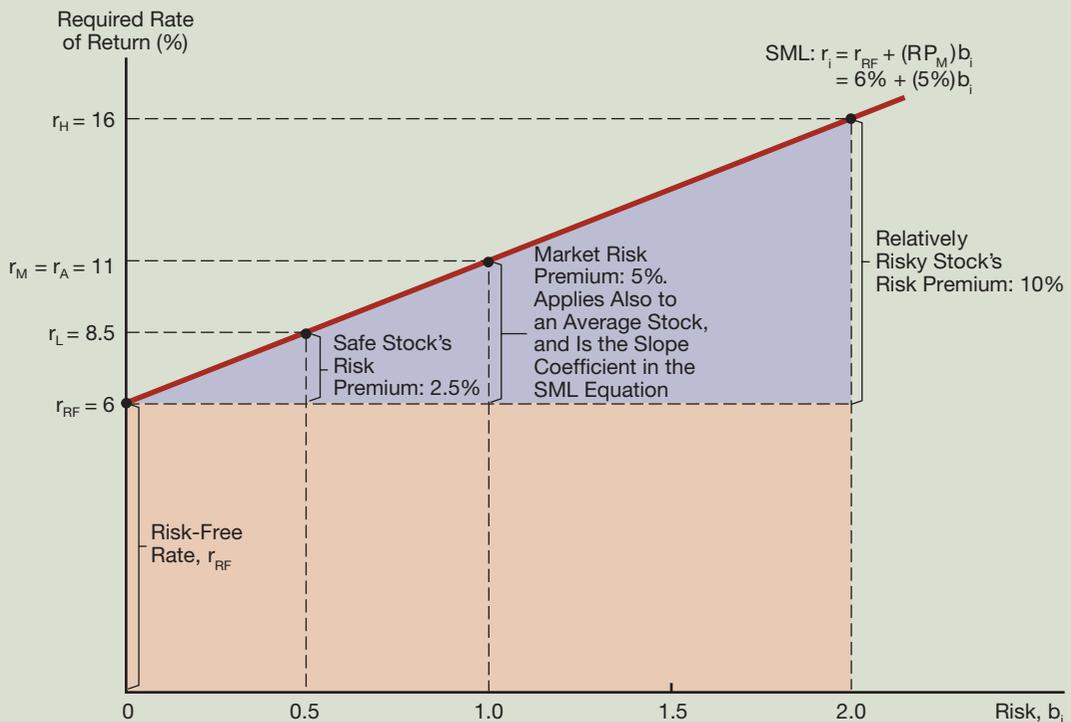
An average stock, with $b = 1.0$, would have a required return of 11%, the same as the market return:

$$r_A = 6\% + (5\%)1.0 = 11\% = r_M$$

Equation 6-14 is called the Security Market Line (SML) equation, and it is often expressed in graph form; see Figure 6-16, which shows the SML when $r_{RF} = 6\%$ and $RP_M = 5\%$. Note the following points.

1. Required rates of return are shown on the vertical axis, while risk as measured by beta is shown on the horizontal axis. This graph is quite different from the one shown in Figure 6-13, where the returns on individual stocks were plotted on the vertical axis and returns on the market index were shown on the horizontal axis. The slopes of the three lines in Figure 6-13 were used to calculate the three stocks' betas, and those betas were then plotted as points on the horizontal axis of Figure 6-16.

FIGURE 6-16 The Security Market Line (SML)



2. Riskless securities have $b_i = 0$; therefore, r_{RF} appears as the vertical axis intercept in Figure 6-16. If we could construct a portfolio that had a beta of zero, then it would have a required return equal to the risk-free rate.
3. The slope of the SML (5% in Figure 6-16) reflects the degree of risk aversion in the economy: The greater the average investor's aversion to risk, then (a) the steeper the slope of the line, (b) the greater the risk premium for all stocks, and (c) the higher the required rate of return on all stocks.¹⁵ These points are discussed further in a later section.
4. The values we worked out for stocks with $b_i = 0.5$, $b_i = 1.0$, and $b_i = 2.0$ agree with the values shown on the graph for r_L , r_A , and r_H .
5. Negative betas are rare, but they can occur. For example, some stocks associated with gold, such as a mining operation, occasionally have a negative beta. Based on the SML, a stock with a negative beta should have a required return less than the risk-free rate. In fact, a stock with a very large but negative beta might have negative required return! This means that when the market is doing well, this stock will do poorly. But it also implies the opposite: When the market is doing poorly, a negative-beta stock should have a positive return. In other words, the negative-beta stock acts like an insurance policy. Therefore, an investor might be willing to accept a negative return on the stock during good times if it is likely to provide a positive return in bad times.

What would happen if a stock's expected return, \hat{r}_i , were greater than its required return, r_i ? In other words, suppose investors thought they could get a 14% return even though the stock's risk only justified an 11% return? If all investors felt this way, then demand for the stock would soar as investors tried to purchase it. But if everyone tried to buy the stock, its price would go up. As the price went up, the extra expected returns would evaporate until the expected return equaled the required return. The reverse would happen if the expected return were less than the required return. Therefore, it seems reasonable to expect that investors' actions would tend to drive the expected return toward the required return.

Unexpected news about a stock's cash flow prospects would certainly change the stock's expected return. A stock's required return can also change because the Security Market Line and a company's position on it can change over time as a result of changes in interest rates, investors' aversion to risk, and individual companies' betas. Such changes are discussed in the following sections.

The Impact of Changes in Inflation and Interest Rates

Interest is the same as "rent" on borrowed money, or the price of money. Thus, r_{RF} is the price of money to a riskless borrower. The risk-free rate as measured by the rate on U.S. Treasury securities is called the *nominal*, or *quoted*, *rate*, and it consists of two elements: (1) a *real inflation-free rate of return*, r^* ; and (2) an *inflation premium*, IP , equal to the anticipated rate of inflation.¹⁶ Thus, $r_{RF} = r^* + IP$. The real rate on

¹⁵Students sometimes confuse beta with the slope of the SML. This is a mistake. The slope of any straight line is equal to the "rise" divided by the "run," or $(Y_1 - Y_0)/(X_1 - X_0)$. Consider Figure 6-16. If we let $Y = r$ and $X = \text{beta}$ and if we go from the origin to $b = 1.0$, then we see that the slope is $(r_M - r_{RF})/(b_M - b_{RF}) = (11\% - 6\%)/(1 - 0) = 5\%$. Thus, the slope of the SML is equal to $(r_M - r_{RF})$, the market risk premium. In Figure 6-16, $r_i = 6\% + 5\%(b_i)$, so an increase of beta from 1.0 to 2.0 would produce a 5-percentage-point increase in r_i .

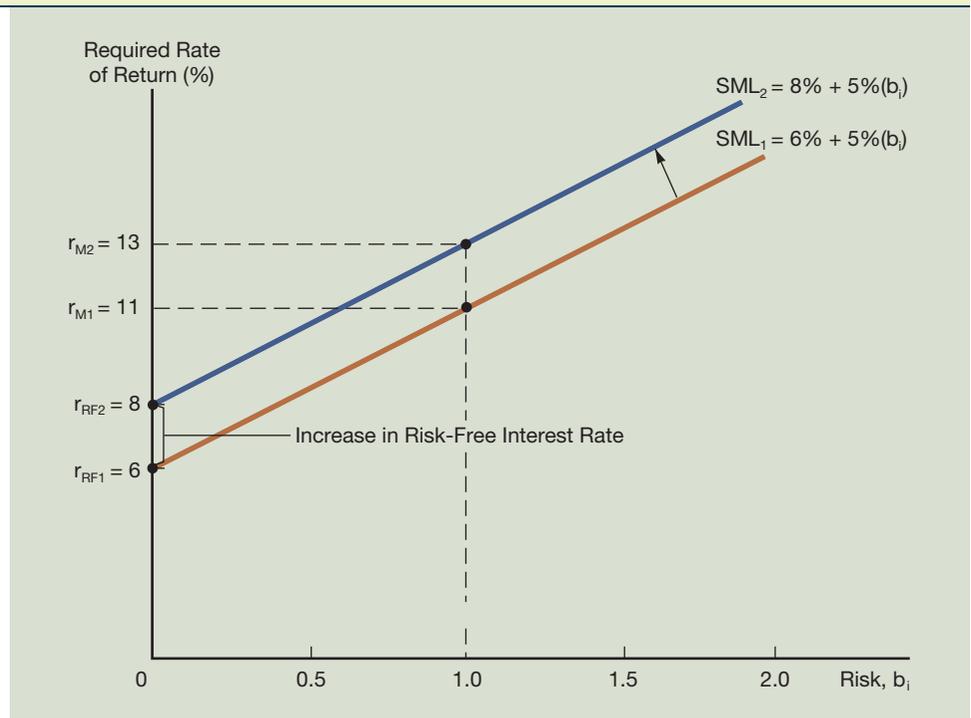
¹⁶In addition to anticipated inflation, the inflation premium may also include a premium for bearing inflation risk. Long-term Treasury bonds also contain a maturity risk premium, MRP. Here we include the MRP in r^* to simplify the discussion. See Chapter 5 for more on bond pricing and bond risk premiums.

long-term Treasury bonds has historically ranged from 2% to 4% with a mean of about 3%. Therefore, the 6% r_{RF} shown in Figure 6-16 might be thought of as consisting of a 3% real risk-free rate of return plus a 3% inflation premium: $r_{RF} = r^* + IP = 3\% + 3\% = 6\%$.

The nominal risk-free rate could change as a result of changes in anticipated inflation or changes in the real interest rate. Consider a recession, such as the one that began in 2007. If consumers and businesses decide to cut back on spending, this will reduce the demand for funds, and that will, other things held constant, lower the risk-free rate and thus the required return on other investments.¹⁷ A key point to note is that a change in r_{RF} will not necessarily cause a change in the market risk premium. Thus, as r_{RF} changes, so will the required return on the market, and this will, other things held constant, keep the market risk premium stable.

Suppose the risk-free interest rate increases to 8% from some combination of an increase in real rates and in anticipated inflation. Such a change is shown in Figure 6-17. Notice that, under the CAPM, the increase in r_{RF} leads to an *identical* increase in the rate of return on all assets, because the same risk-free rate is built into the required rate of return on all assets. For example, the rate of return on an average stock, r_M , increases from 11% to 13%. Other risky securities' returns also rise by 2 percentage points.

FIGURE 6-17 Shift in the SML Caused by an Increase in Interest Rates



¹⁷Think of a sailboat floating in a harbor. The distance from the ocean floor to the ocean surface is like the risk-free rate, and it moves up and down with the tides. The distance from the top of the ship's mast to the ocean floor is like the required market return: It too moves up and down with the tides. The distance from the mast-top to the ocean surface is like the market risk premium—it also stays the same, even though tides move the ship up and down. Thus, other things held constant, a change in the risk-free rate also causes an identical change in the required market return, r_M , resulting in a relatively stable market risk premium, $r_M - r_{RF}$.

Changes in Risk Aversion

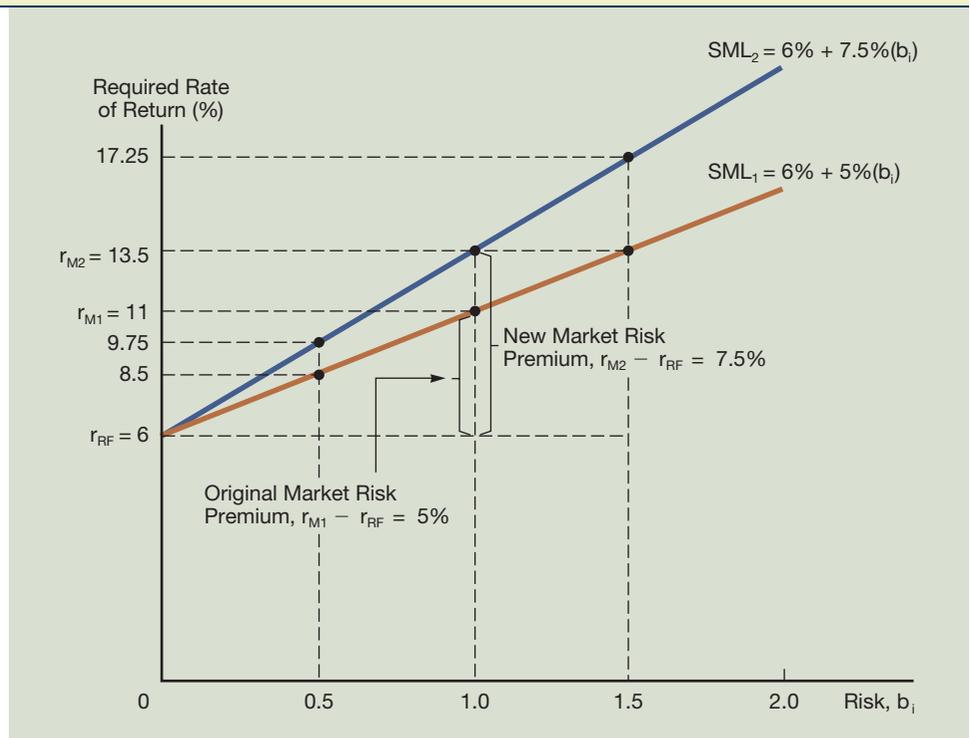
The slope of the Security Market Line reflects the extent to which investors are averse to risk: The steeper the slope of the line, the greater the average investor's aversion to risk. Suppose all investors were indifferent to risk—that is, suppose they were *not* risk averse. If r_{RF} were 6%, then risky assets would also provide an expected return of 6%, because if there were no risk aversion then there would be no risk premium, and the SML would be plotted as a horizontal line. As risk aversion increases, so does the risk premium, and this causes the slope of the SML to become steeper.

Figure 6-18 illustrates an increase in risk aversion. The market risk premium rises from 5% to 7.5%, causing r_M to rise from $r_{M1} = 11\%$ to $r_{M2} = 13.5\%$. The returns on other risky assets also rise, and the effect of this shift in risk aversion is greater for riskier securities. For example, the required return on a stock with $b_i = 0.5$ increases by only 1.25 percentage points, from 8.5% to 9.75%; that on a stock with $b_i = 1.0$ increases by 2.5 percentage points, from 11.0% to 13.5%; and that on a stock with $b_i = 1.5$ increases by 3.75 percentage points, from 13.5% to 17.25%.

Changes in a Stock's Beta Coefficient

Given risk aversion and a positively sloped SML as in Figure 6-18, the higher a stock's beta, the higher its required rate of return. As we shall see later in the book, a firm can influence its beta through changes in the composition of its assets and also through its use of debt: Acquiring riskier assets will increase beta, as will a change in capital structure that calls for a higher debt ratio. A company's beta can also change as a result of external factors such as increased competition in its industry, the expiration of basic patents, and the like. When such changes lead to a higher or lower beta, the required rate of return will also change.

FIGURE 6-18 Shift in the SML Caused by Increased Risk Aversion



Another Kind of Risk: The Bernie Madoff Story

In the fall of 2008, Bernard Madoff's massive Ponzi scheme was exposed, revealing an important type of risk that's not dealt with in this chapter. Madoff was a money manager in the 1960s, and apparently through good luck he produced above-average results for several years. His clients then told their friends about Madoff's success, and those friends sent in money for him to invest. Madoff's actual returns then dropped, but he didn't tell his clients that they were losing money. Rather, he told them that returns were holding up well, and he used new incoming money to pay dividends and meet withdrawal requests. The idea of using new money to pay off old investors is called a Ponzi scheme, named after Charles Ponzi, a Bostonian who set up the first widely publicized such scheme in the early 1900s.

Madoff perfected the system, ran his scheme for about 40 years, and attracted about \$50 billion of investors' funds. His investors ranged from well-known billionaires to retirees who invested their entire life savings. His advertising was strictly word-of-mouth, and clients telling potential clients about the many wealthy and highly regarded people who invested with him certainly helped. All of his investors assumed that someone else had done the "due diligence" and found the operation to be clean. A few investors who actually did some due diligence were suspicious and didn't invest with him, but for the most part people just blindly followed the others.

The hedge fund boom that began in the 1990s was a big help. "Funds of funds," which are hedge funds set up to investigate and then invest in other hedge funds, gained momentum, and some of those funds were Madoff's biggest investors. They were a scam in their own right, taking fees to investigate other hedge funds and money managers to find the best of them, but really doing virtually no investigating on their own and simply sending the money entrusted to them to Bernie Madoff. Madoff also paid some "finders" to frequent country club bars, brag about how well they were doing in the market, and induce other fish to swim into Madoff's net.

All Ponzi schemes crash when something occurs that causes some investors to seek to withdraw funds in

amounts greater than the incoming funds from new investors. Someone tries to get out, can't do it, tells others who worry and try to get out too, and almost overnight the scam unravels. That happened to Madoff in 2008, when the stock market crash caused some of his investors to seek withdrawals and few new dollars were coming in. In the end, his investors lost billions; some lost their entire life savings, and several have committed suicide. Moreover, the Madoff revelations have led to the exposure of other, similar (but smaller), frauds. Indeed, people—especially the rich and famous—are reluctant to admit they've been taken, so it's likely that lots of frauds are never reported.

How can you guard against a fraud like Madoff's? For a scheme as sophisticated as that of Madoff, it's not easy, but there are some steps that can and should be taken.

1. *Use your basic financial knowledge and some common sense.* Earlier in this chapter we gave you an idea about the kinds of returns that have been earned in the past on different classes of investments with different risk profiles. If someone offers you an investment that seems too good to be true, it probably is, so you should drill deep to see if it is really what its sponsor says it is before you invest. If you don't have the time and expertise for such investigation, you should probably avoid the investment.
2. *Deal with reputable institutions.* We hate to recommend avoiding small and new organizations, but unless you have the ability to check out an investment manager carefully, that's probably a good idea. There are institutions with the expertise to conduct such investigations, so you might be wise to leave the too-good-to-be-true investments to them.
3. *Diversify.* Many people were burned by Bernie Madoff, but the ones who were hurt worst were those who trusted most or all of their funds to him. Those who diversified their holdings were hurt, but they weren't wiped out. So diversify!

Self-Test

Differentiate among the expected rate of return (\hat{r}), the required rate of return (r), and the realized, after-the-fact return (\hat{r}) on a stock. Which must be larger to get you to buy the stock, \hat{r} or r ? Would \hat{r} , r , and \bar{r} typically be the same or different for a given company, say on January 1, 2010?

What are the differences between the relative returns graph (Figure 6-13), where “betas are made,” and the SML graph (Figure 6-16), where “betas are used”? Discuss both how the graphs are constructed and the information they convey. What happens to the SML graph in Figure 6-16 when inflation increases or decreases?

What happens to the SML graph when risk aversion increases or decreases? What would the SML look like if investors were completely indifferent to risk—that is, had zero risk aversion?

How can a firm influence its market risk as reflected in its beta?

A stock has a beta of 1.4. Assume that the risk-free rate is 5.5% and that the market risk premium is 5%. What is the stock’s required rate of return? (12.5%)



Kenneth French’s Web site, <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html> is an excellent resource for data and information regarding factors related to stock returns.

6.6 SOME CONCERNS ABOUT BETA AND THE CAPM

The Capital Asset Pricing Model is more than just an abstract theory described in textbooks. It has great intuitive appeal, and it is widely used by analysts, investors, and corporations. However, a number of recent studies have raised concerns about its validity. For example, a study by Eugene Fama of the University of Chicago and Kenneth French of Dartmouth found no historical relationship between stocks’ returns and their market betas, confirming a position long held by some professors and stock market analysts.¹⁸

As an alternative to the traditional CAPM, researchers and practitioners are developing models with more explanatory variables than just beta. These multi-factor models represent an attractive generalization of the traditional CAPM model’s insight that market risk—risk that cannot be diversified away—underlies the pricing of assets. In the multi-variable models, risk is assumed to be caused by a number of different factors, including size of firm, market/book ratios, measures of liquidity, and the like, whereas the CAPM gauges risk only relative to returns on the market portfolio. The multi-variable models represent a potentially important step forward in finance theory, but they also have some deficiencies when applied in practice. As a result, the basic CAPM is still the most widely used method for thinking about required rates of return on stocks.

Self-Test

Have there been any studies that question the validity of the CAPM? Explain.

6.7 SOME CONCLUDING THOUGHTS: IMPLICATIONS FOR CORPORATE MANAGERS AND INVESTORS

The connection between risk and return is an important concept, and it has numerous implications for both corporate managers and investors. As we will see in later chapters, corporate managers spend a great deal of time assessing the risk and returns of individual projects. Indeed, given their concerns about the risk of individual projects, it might be fair to ask why we spend so much time discussing the riskiness of stocks. Why not begin by looking at the riskiness of such business assets as plant and equipment? The reason is that, *for a management whose primary goal is to maximize*

¹⁸See Eugene F. Fama and Kenneth R. French, “The Cross-Section of Expected Stock Returns,” *Journal of Finance*, Vol. 47 (1992), pp. 427–465; and Eugene F. Fama and Kenneth R. French, “Common Risk Factors in the Returns on Stocks and Bonds,” *Journal of Financial Economics*, Vol. 33 (1993), pp. 3–56. They found that stock returns are related to firm size and market/book ratios. Small firms and those firms with low market/book ratios had higher returns; however, they found no relationship between returns and beta.

intrinsic value, the overriding consideration is the riskiness of the firm's stock, and the relevant risk of any physical asset must be measured in terms of its effect on the stock's risk as seen by investors. For example, suppose Goodyear is considering a major investment in a new product, recapped tires. Sales of recaps and hence earnings on the new operation are highly uncertain, so on a stand-alone basis the new venture appears to be quite risky. However, suppose returns in the recap business are negatively correlated with Goodyear's other operations: When times are good and people have plenty of money, they buy new cars with new tires, but when times are bad, they tend to keep their old cars and buy recaps for them. Therefore, returns would be high on regular operations and low on the recap division during good times, but the opposite would be true during recessions. The result might be a pattern like that shown earlier in Figure 6-9 for Stocks W and M. Thus, what appears to be a risky investment when viewed on a stand-alone basis might not be so risky when viewed within the context of the company as a whole.

This analysis can be extended to the corporation's stockholders. Because Goodyear's stock is owned by diversified stockholders, the real issue each time management makes an investment decision is this: How will this investment affect the risk of our stockholders? Again, the stand-alone risk of an individual project may look quite high; however, when viewed in the context of the project's effect on stockholder risk, it may not be as large. We will address this issue again in Chapter 11, where we examine the effects of capital budgeting on companies' beta coefficients and thus on stockholders' risks.

These concepts are obviously important for individual investors, but they are also important for corporate managers. Here we summarize some key ideas that all investors should consider.

1. *There is a trade-off between risk and return.* The average investor likes higher returns but dislikes risk. It follows that higher-risk investments need to offer investors higher expected returns. Put another way: If you are seeking higher returns, you must be willing to assume higher risks.
2. *Diversification is crucial.* By diversifying wisely, investors can dramatically reduce risk without reducing their expected returns. Don't put all of your money in one or two stocks, or in one or two industries. A huge mistake many people make is to invest a high percentage of their funds in their employer's stock. Then, if the company goes bankrupt, they lose not only their job but also their invested capital. Although no stock is completely riskless, you can smooth out the bumps somewhat by holding a well-diversified portfolio.
3. *Real returns are what matters.* All investors should understand the difference between nominal and real returns. When assessing performance, the real return (what you have left after inflation) is what really matters. It follows that, as expected inflation increases, investors need to earn higher nominal returns.
4. *The risk of an investment often depends on how long you plan to hold the investment.* Common stocks, for example, can be extremely risky for short-term investors. However, over the long haul the bumps tend to even out, so stocks are less risky when held as part of a long-term portfolio. Indeed, in his best-selling book *Stocks for the Long Run*, Jeremy Siegel of the University of Pennsylvania concludes: "The safest long-term investment for the preservation of purchasing power has clearly been stocks, not bonds."
5. The past gives us insights into the risk and returns on various investments, but *there is no guarantee that the future will repeat the past.* Stocks that have performed well in recent years might tumble, while stocks that have struggled may rebound.

The same thing can hold true for the stock market as a whole. Even Jeremy Siegel, who has preached that stocks have historically been good long-term investments, has also argued that there is no assurance that returns in the future will be as strong as they have been in the past. More importantly, when purchasing a stock you always need to ask: “Is this stock fairly valued, or is it currently priced too high?” We discuss this issue more completely in the next chapter.

Self-Test

Explain the following statement: “The stand-alone risk of an individual corporate project may be quite high, but viewed in the context of its effect on stockholders’ risk, the project’s true risk may be much lower.”

How does the correlation between returns on a project and returns on the firm’s other assets affect the project’s risk?

What are some important concepts for individual investors to consider when evaluating the risk and returns of various investments?

Summary

This chapter focuses on the trade-off between risk and return. We began by discussing how to estimate risk and return for both individual assets and portfolios. In particular, we differentiated between stand-alone risk and risk in a portfolio context, and we explained the benefits of diversification. Finally, we introduced the CAPM, which describes how risk affects rates of return. In the chapters that follow, we will give you the tools to estimate the required rates of return for bonds, preferred stock, and common stock, and we will explain how firms use these rates of return to estimate their costs of capital. As you will see, the cost of capital is a basic element in the capital budgeting process. The key concepts covered in this chapter are listed below.

- **Risk** can be defined as the chance that some unfavorable event will occur.
- The risk of an asset’s cash flows can be considered on a **stand-alone basis** (each asset all by itself) or in a **portfolio context**, in which the investment is combined with other assets and its risk is reduced through **diversification**.
- Most rational investors hold **portfolios of assets**, and they are more concerned with the risk of their portfolios than with the risk of individual assets.
- The **expected return** on an investment is the mean value of its probability distribution of returns.
- The **greater the probability** that the actual return will be far below the expected return, the **greater the asset’s stand-alone risk**.
- The average investor is **risk averse**, which means that he or she must be compensated for holding risky assets. Therefore, riskier assets have higher required returns than less risky assets.
- An asset’s risk has two components: (1) **diversifiable risk**, which can be eliminated by diversification, and (2) **market risk**, which cannot be eliminated by diversification.
- Market risk is measured by the standard deviation of returns on a well-diversified portfolio, one that consists of all stocks traded in the market. Such a portfolio is called the **market portfolio**.
- The **relevant risk** of an individual asset is its contribution to the risk of a well-diversified portfolio. Since market risk cannot be eliminated by diversification, investors must be compensated for bearing it.
- A stock’s **beta coefficient, b** , is a measure of its market risk. Beta measures the extent to which the stock’s returns move relative to the market.

- A **high-beta stock** has stock returns that tend to move up and down by more than the returns on an average stock, while the opposite is true for a **low-beta stock**. An average stock has $b = 1.0$, as does the market portfolio.
- The **beta of a portfolio** is a **weighted average** of the betas of the individual securities in the portfolio.
- The **Security Market Line (SML)** equation shows the relationship between a security's market risk and its required rate of return. The return required for any security i is equal to the **risk-free rate** plus the **market risk premium** multiplied by the security's **beta**: $r_i = r_{RF} + (RP_M)b_i$.
- In equilibrium, the expected rate of return on a stock must equal its required return. However, a number of things can happen to cause the required rate of return to change: (1) **the risk-free rate can change** because of changes in either real rates or expected inflation, (2) **a stock's beta can change**, and (3) **investors' aversion to risk can change**.
- Because returns on assets in different countries are not perfectly correlated, **global diversification** may result in lower risk for multinational companies and globally diversified portfolios.
- The CAPM is conceptually based on **expected returns**. However, only historical returns are available to test it. Various tests have been conducted, and none has "proved" that the CAPM actually describes how investors behave. Indeed, evidence exists to suggest that investors regard factors other than just beta when analyzing risk. The 2008–2009 market crash suggests that, in addition to risk as measured by beta, liquidity is important as well.
- Two web extensions accompany this chapter: *Web Extension 6A* provides a discussion of **continuous probability distributions**, and *Web Extension 6B* shows how to calculate beta with a financial calculator.

Questions

- (6–1) Define the following terms, using graphs or equations to illustrate your answers where feasible.
- Risk in general; stand-alone risk; probability distribution and its relation to risk
 - Expected rate of return, \hat{r}
 - Continuous probability distribution
 - Standard deviation, σ ; variance, σ^2 ; coefficient of variation, CV
 - Risk aversion; realized rate of return, \bar{r}
 - Risk premium for Stock i , RP_i ; market risk premium, RP_M
 - Capital Asset Pricing Model (CAPM)
 - Expected return on a portfolio, \hat{r}_p ; market portfolio
 - Correlation as a concept; correlation coefficient, ρ
 - Market risk; diversifiable risk; relevant risk
 - Beta coefficient, b ; average stock's beta
 - Security Market Line (SML); SML equation
 - Slope of SML and its relationship to risk aversion
- (6–2) The probability distribution of a less risky return is more peaked than that of a riskier return. What shape would the probability distribution have for (a) completely certain returns and (b) completely uncertain returns?

- (6-3) Security A has an expected return of 7%, a standard deviation of returns of 35%, a correlation coefficient with the market of -0.3 , and a beta coefficient of -1.5 . Security B has an expected return of 12%, a standard deviation of returns of 10%, a correlation with the market of 0.7 , and a beta coefficient of 1.0 . Which security is riskier? Why?
- (6-4) Suppose you owned a portfolio consisting of \$250,000 of U.S. government bonds with a maturity of 30 years.
- Would your portfolio be riskless?
 - Now suppose you hold a portfolio consisting of \$250,000 of 30-day Treasury bills. Every 30 days your bills mature, and you reinvest the principal (\$250,000) in a new batch of bills. Assume that you live on the investment income from your portfolio and that you want to maintain a constant standard of living. Is your portfolio truly riskless?
 - Can you think of any asset that would be completely riskless? What security comes closest to being riskless? Explain.
- (6-5) If investors' aversion to risk *increased*, would the risk premium on a high-beta stock increase by more or less than that on a low-beta stock? Explain.
- (6-6) If a company's beta were to double, would its expected return double?
- (6-7) In the real world, is it possible to construct a portfolio of stocks that has an expected return equal to the risk-free rate?

Self-Test Problems

Solutions Appear in Appendix A

- (ST-1) Stocks A and B have the following historical returns:
- Realized Rates of Return

Year	\bar{r}_A	\bar{r}_B
2006	-18%	-24%
2007	44	24
2008	-22	-4
2009	22	8
2010	34	56

- Calculate the average rate of return for each stock during the 5-year period. Assume that someone held a portfolio consisting of 50% of Stock A and 50% of Stock B. What would have been the realized rate of return on the portfolio in each year? What would have been the average return on the portfolio for the 5-year period?
- Now calculate the standard deviation of returns for each stock and for the portfolio. Use Equation 6-5.
- Looking at the annual returns data on the two stocks, would you guess that the correlation coefficient between returns on the two stocks is closer to 0.8 or to -0.8 ?
- If you added more stocks at random to the portfolio, which of the following is the most accurate statement of what would happen to σ_p ?
 - σ_p would remain constant.
 - σ_p would decline to somewhere in the vicinity of 20%.
 - σ_p would decline to zero if enough stocks were included.

(ST-2)
Beta and
Required Rate of
Return

ECRI Corporation is a holding company with four main subsidiaries. The percentage of its business coming from each of the subsidiaries, and their respective betas, are as follows:

Subsidiary	Percentage of Business	Beta
Electric utility	60%	0.70
Cable company	25	0.90
Real estate	10	1.30
International/special projects	5	1.50

- What is the holding company's beta?
- Assume that the risk-free rate is 6% and that the market risk premium is 5%. What is the holding company's required rate of return?
- ECRI is considering a change in its strategic focus: It will reduce its reliance on the electric utility subsidiary so that the percentage of its business from this subsidiary will be 50%. At the same time, ECRI will increase its reliance on the international/special projects division, and the percentage of its business from that subsidiary will rise to 15%. What will be the shareholders' required rate of return if management adopts these changes?

Problems

Answers Appear in Appendix B

EASY PROBLEMS 1-3

(6-1)
Portfolio Beta

An individual has \$35,000 invested in a stock with a beta of 0.8 and another \$40,000 invested in a stock with a beta of 1.4. If these are the only two investments in her portfolio, what is her portfolio's beta?

(6-2)
Required Rate of
Return

Assume that the risk-free rate is 6% and that the expected return on the market is 13%. What is the required rate of return on a stock that has a beta of 0.7?

(6-3)
Required Rates of
Return

Assume that the risk-free rate is 5% and that the market risk premium is 6%. What is the required return on the market, on a stock with a beta of 1.0, and on a stock with a beta of 1.2?

INTERMEDIATE PROBLEMS 4-9

(6-4)
Expected Return:
Discrete Distribution

A stock's return has the following distribution:

Demand for the Company's Products	Probability of This Demand Occurring	Rate of Return If This Demand Occurs (%)
Weak	0.1	-50%
Below average	0.2	(5)
Average	0.4	16
Above average	0.2	25
Strong	<u>0.1</u>	60
	<u>1.0</u>	

Calculate the stock's expected return, standard deviation, and coefficient of variation.

(6-5)

Expected Returns:
Discrete
Distribution

The market and Stock J have the following probability distributions:

Probability	r_M	r_J
0.3	15%	20%
0.4	9	5
0.3	18	12

- Calculate the expected rates of return for the market and Stock J.
- Calculate the standard deviations for the market and Stock J.
- Calculate the coefficients of variation for the market and Stock J.

(6-6)

Required Rate of
ReturnSuppose $r_{RF} = 5\%$, $r_M = 10\%$, and $r_A = 12\%$.

- Calculate Stock A's beta.
- If Stock A's beta were 2.0, then what would be A's new required rate of return?

(6-7)

Required Rate of
ReturnSuppose $r_{RF} = 9\%$, $r_M = 14\%$, and $b_i = 1.3$.

- What is r_i , the required rate of return on Stock i?
- Now suppose r_{RF} (1) increases to 10% or (2) decreases to 8%. The slope of the SML remains constant. How would this affect r_M and r_i ?
- Now assume r_{RF} remains at 9% but r_M (1) increases to 16% or (2) falls to 13%. The slope of the SML does not remain constant. How would these changes affect r_i ?

(6-8)

Portfolio Beta

Suppose you hold a diversified portfolio consisting of a \$7,500 investment in each of 20 different common stocks. The portfolio's beta is 1.12. Now, suppose you sell one of the stocks with a beta of 1.0 for \$7,500 and use the proceeds to buy another stock whose beta is 1.75. Calculate your portfolio's new beta.

(6-9)

Portfolio Required
Return

Suppose you manage a \$4 million fund that consists of four stocks with the following investments:

Stock	Investment	Beta
A	\$ 400,000	1.50
B	600,000	-0.50
C	1,000,000	1.25
D	2,000,000	0.75

If the market's required rate of return is 14% and the risk-free rate is 6%, what is the fund's required rate of return?

CHALLENGING PROBLEMS
10-13

(6-10)

Portfolio Beta

You have a \$2 million portfolio consisting of a \$100,000 investment in each of 20 different stocks. The portfolio has a beta of 1.1. You are considering selling \$100,000 worth of one stock with a beta of 0.9 and using the proceeds to purchase another stock with a beta of 1.4. What will the portfolio's new beta be after these transactions?

(6-11) Stock R has a beta of 1.5, Stock S has a beta of 0.75, the expected rate of return on an average stock is 13%, and the risk-free rate is 7%. By how much does the required return on the riskier stock exceed that on the less risky stock?

(6-12) Stocks A and B have the following historical returns:

Year	\bar{r}_A	\bar{r}_B
2006	-18.00%	-14.50%
2007	33.00	21.80
2008	15.00	30.50
2009	-0.50	-7.60
2010	27.00	26.30

- Calculate the average rate of return for each stock during the 5-year period.
- Assume that someone held a portfolio consisting of 50% of Stock A and 50% of Stock B. What would have been the realized rate of return on the portfolio in each year? What would have been the average return on the portfolio during this period?
- Calculate the standard deviation of returns for each stock and for the portfolio.
- Calculate the coefficient of variation for each stock and for the portfolio.
- If you are a risk-averse investor then, assuming these are your only choices, would you prefer to hold Stock A, Stock B, or the portfolio? Why?

(6-13) You have observed the following returns over time:

Historical Returns:
Expected and
Required Rates of
Return

Year	Stock X	Stock Y	Market
2006	14%	13%	12%
2007	19	7	10
2008	-16	-5	-12
2009	3	1	1
2010	20	11	15

Assume that the risk-free rate is 6% and the market risk premium is 5%.

- What are the betas of Stocks X and Y?
- What are the required rates of return on Stocks X and Y?
- What is the required rate of return on a portfolio consisting of 80% of Stock X and 20% of Stock Y?
- If Stock X's expected return is 22%, is Stock X under- or overvalued?

SPREADSHEET PROBLEM

(6-14) Start with the partial model in the file *Cb06 P14 Build a Model.xls* on the textbook's Web site. The file contains hypothetical data for working this problem. Bartman Industries' and Reynolds Incorporated's stock prices and dividends, along with the Market Index, are shown below. Stock prices are reported for December 31 of each year, and dividends reflect those paid during the year. The market data are adjusted to include dividends.



Year	<u>Bartman Industries</u>		<u>Reynolds Incorporated</u>		<u>Market Index</u>
	Stock Price	Dividend	Stock Price	Dividend	Includes Dividends
2010	\$17.25	\$1.15	\$48.75	\$3.00	11,663.98
2009	14.75	1.06	52.30	2.90	8,785.70
2008	16.50	1.00	48.75	2.75	8,679.98
2007	10.75	0.95	57.25	2.50	6,434.03
2006	11.37	0.90	60.00	2.25	5,602.28
2005	7.62	0.85	55.75	2.00	4,705.97

- Use the data given to calculate annual returns for Bartman, Reynolds, and the Market Index, and then calculate average annual returns for the two stocks and the index. (*Hint:* Remember, returns are calculated by subtracting the beginning price from the ending price to get the capital gain or loss, adding the dividend to the capital gain or loss, and then dividing the result by the beginning price. Assume that dividends are already included in the index. Also, you cannot calculate the rate of return for 2005 because you do not have 2004 data.)
- Calculate the standard deviations of the returns for Bartman, Reynolds, and the Market Index. (*Hint:* Use the sample standard deviation formula given in the chapter, which corresponds to the STDEV function in *Excel*.)
- Now calculate the coefficients of variation for Bartman, Reynolds, and the Market Index.
- Construct a scatter diagram graph that shows Bartman's returns on the vertical axis and the Market Index's returns on the horizontal axis. Construct a similar graph showing Reynolds's stock returns on the vertical axis.
- Estimate Bartman's and Reynolds's betas as the slopes of regression lines with stock return on the vertical axis (y-axis) and market return on the horizontal axis (x-axis). (*Hint:* Use *Excel*'s SLOPE function.) Are these betas consistent with your graph?
- The risk-free rate on long-term Treasury bonds is 6.04%. Assume that the market risk premium is 5%. What is the required return on the market? Now use the SML equation to calculate the two companies' required returns.
- If you formed a portfolio that consisted of 50% Bartman stock and 50% Reynolds stock, what would be its beta and its required return?
- Suppose an investor wants to include some Bartman Industries stock in his portfolio. Stocks A, B, and C are currently in the portfolio, and their betas are 0.769, 0.985, and 1.423, respectively. Calculate the new portfolio's required return if it consists of 25% Bartman, 15% Stock A, 40% Stock B, and 20% Stock C.

THOMSON ONE | Business School Edition

Problem

Use the Thomson ONE—Business School Edition online database to work this chapter's questions.

USING PAST INFORMATION TO ESTIMATE REQUIRED RETURNS

In the Capital Asset Pricing Model (CAPM) discussion, beta is identified as the correct measure of risk for diversified shareholders. Recall that beta measures the extent to which the returns of a given stock move with the stock market. When using the

CAPM to estimate required returns, we would ideally like to know how the stock will move with the market in the future, but since we don't have a crystal ball we generally use historical data to estimate this relationship.

As noted in the chapter, beta can be estimated by regressing the individual stock's returns against the returns of the overall market. As an alternative to running our own regressions, we can instead rely on reported betas from a variety of sources. These published sources make it easy to obtain beta estimates for most large publicly traded corporations. However, a word of caution is in order. Beta estimates can often be quite sensitive to the time period in which the data are estimated, the market index used, and the frequency of the data used. Therefore, it is not uncommon to find a wide range of beta estimates among the various published sources. Indeed, Thomson ONE reports multiple beta estimates. These multiple estimates reflect the fact that Thomson ONE puts together data from a variety of different sources.

Thomson ONE—BSE Discussion Questions

1. Begin by taking a look at the historical performance of the overall stock market. If you want to see, for example, the performance of the S&P 500, select INDICES and enter S&PCOMP. Click on PERFORMANCE and you will immediately see a quick summary of the market's performance in recent months and years. How has the market performed over the past year? The past 3 years? The past 5 years? The past 10 years?
2. Now let's take a closer look at the stocks of four companies: Colgate Palmolive (CL), Gillette (G), Heinz (HNZ), and Microsoft (MSFT). Before looking at the data, which of these companies would you expect to have a relatively high beta (greater than 1.0), and which of these companies would you expect to have a relatively low beta (less than 1.0)?
3. Select one of the four stocks listed in question 2 by selecting COMPANIES, entering the company's ticker symbol, and clicking on GO. On the overview page, you should see a chart that summarizes how the stock has done relative to the S&P 500 over the past 6 months. Has the stock outperformed or underperformed the overall market during this time period?
4. Return to the overview page for the stock you selected. If you scroll down the page you should see an estimate of the company's beta. What is the company's beta? What was the source of the estimated beta?
5. Click on the tab labeled PRICES. What is the company's current dividend yield? What has been its total return to investors over the past 6 months? Over the past year? Over the past 3 years? (Remember that total return includes the dividend yield plus any capital gains or losses.)
6. What is the estimated beta on this page? What is the source of the estimated beta? Why might different sources produce different estimates of beta? (*Note:* if you want to see even more beta estimates, click OVERVIEWS on the second line of tabs and then select SEC DATABASE MARKET DATA. Scroll through the STOCK OVERVIEW SECTION and you will see a range of different beta estimates.)
7. Select a beta estimate that you believe is best. (If you are not sure, you may want to consider an average of the given estimates.) Assume that the risk-free rate is 5% and that the market risk premium is 6%. What is the required return on the company's stock?

8. Repeat the same exercise for each of the three remaining companies. Do the reported betas confirm your earlier intuition? In general, do you find that the higher-beta stocks tend to do better in up markets and worse in down markets? Explain.

Mini Case

Assume that you recently graduated with a major in finance and that you just landed a job as a financial planner with Barney Smith Inc., a large financial services corporation. Your first assignment is to invest \$100,000 for a client. Because the funds are to be invested in a new business that the client plans to start at the end of 1 year, you have been instructed to plan for a 1-year holding period. Further, your boss has restricted you to the investment alternatives shown in the table below. (Disregard for now the items at the bottom of the data; you will fill in the blanks later.)

Barney Smith's economic forecasting staff has developed probability estimates for the state of the economy, and its security analysts have developed a sophisticated computer program that was used to estimate the rate of return on each alternative under each state of the economy. Alta Industries is an electronics firm; Repo Men Inc. collects past-due debts; and American Foam manufactures mattresses and various other foam products. Barney Smith also maintains an "index fund" that owns a market-weighted fraction of all publicly traded stocks; you can invest in that fund and thus obtain average stock market results. Given the situation as described, answer the following questions.

Estimated Returns on Alternative Investments

State of the Economy	Probability	T-Bills	Alta Industries	Repo Men	American Foam	Market Portfolio	2-Stock Portfolio
Recession	0.1	8.0%	-22.0%	28.0%	10.0% ^a	-13.0%	3.0%
Below average	0.2	8.0	-2.0	14.7	-10.0	1.0	
Average	0.4	8.0	20.0	0.0	7.0	15.0	10.0
Above average	0.2	8.0	35.0	-10.0	45.0	29.0	
Boom	<u>0.1</u>	<u>8.0</u>	<u>50.0</u>	<u>-20.0</u>	<u>30.0</u>	<u>43.0</u>	<u>15.0</u>
\hat{r}		8.0%	—	1.7%	13.8%	15.0%	—
σ		0.0%	—	13.4%	18.8%	15.3%	—
CV			—	7.9	1.4	1.0	—
b			—	-0.86	0.68		—

^aNote that the estimated returns of American Foam do not always move in the same direction as the overall economy. For example, when the economy is below average, consumers purchase fewer mattresses than they would if the economy were stronger. However, if the economy is in a flat-out recession, a large number of consumers who were planning to purchase a more expensive inner spring mattress may purchase, instead, a cheaper foam mattress. Under these circumstances, we would expect American Foam's stock price to be higher if there is a recession than if the economy was just below average.

- a. What are investment returns? What is the return on an investment that costs \$1,000 and is sold after 1 year for \$1,100?

- b. (1) Why is the T-bill's return independent of the state of the economy? Do T-bills promise a completely risk-free return? (2) Why are Alta Industries's returns expected to move with the economy whereas Repo Men's are expected to move counter to the economy?
- c. Calculate the expected rate of return on each alternative and fill in the blanks in the row for \hat{r} in the table.
- d. You should recognize that basing a decision solely on expected returns is appropriate only for risk-neutral individuals. Because your client, like virtually everyone, is risk averse, the riskiness of each alternative is an important aspect of the decision. One possible measure of risk is the standard deviation of returns. (1) Calculate this value for each alternative, and fill in the blank in the row for σ in the table. (2) What type of risk is measured by the standard deviation? (3) Draw a graph that shows *roughly* the shape of the probability distributions for Alta Industries, American Foam, and T-bills.
- e. Suppose you suddenly remembered that the coefficient of variation (CV) is generally regarded as being a better measure of stand-alone risk than the standard deviation when the alternatives being considered have widely differing expected returns. Calculate the missing CVs, and fill in the blanks in the row for CV in the table. Does the CV produce the same risk rankings as the standard deviation?
- f. Suppose you created a two-stock portfolio by investing \$50,000 in Alta Industries and \$50,000 in Repo Men. (1) Calculate the expected return (\hat{r}_p), the standard deviation (σ_p), and the coefficient of variation (CV_p) for this portfolio and fill in the appropriate blanks in the table. (2) How does the risk of this two-stock portfolio compare with the risk of the individual stocks if they were held in isolation?
- g. Suppose an investor starts with a portfolio consisting of one randomly selected stock. As more and more randomly selected stocks are added to the portfolio, what happens to the portfolio's risk and its expected return? What is the implication for investors? Draw a graph of the two portfolios to illustrate your answer.
- h. (1) Should portfolio effects influence how investors think about the risk of individual stocks? (2) If you decided to hold a one-stock portfolio and consequently were exposed to more risk than diversified investors, could you expect to be compensated for all of your risk; that is, could you earn a risk premium on that part of your risk that you could have eliminated by diversifying?
- i. How is market risk measured for individual securities? How are beta coefficients calculated?
- j. Suppose you have the following historical returns for the stock market and for the company P. Q. Unlimited. Explain how to calculate beta, and use the historical stock returns to calculate the beta for PQU. Interpret your results.

Year	Market	PQU
1	25.7%	40.0%
2	8.0	-15.0
3	-11.0	-15.0
4	15.0	35.0
5	32.5	10.0
6	13.7	30.0
7	40.0	42.0
8	10.0	-10.0
9	-10.8	-25.0
10	-13.1	25.0

- k. The expected rates of return and the beta coefficients of the alternatives, as supplied by Barney Smith's computer program, are as follows:

Security	Return (\hat{r})	Risk (Beta)
Alta Industries	17.4%	1.29
Market	15.0	1.00
American Foam	13.8	0.68
T-bills	8.0	0.00
Repo Men	1.7	-0.86

- (1) Do the expected returns appear to be related to each alternative's market risk? (2) Is it possible to choose among the alternatives on the basis of the information developed thus far?
- l. (1) Write out the Security Market Line (SML) equation, use it to calculate the required rate of return on each alternative, and then graph the relationship between the expected and required rates of return. (2) How do the expected rates of return compare with the required rates of return? (3) Does it make sense that Repo Men has an expected return that is less than the T-bill rate? (4) What would be the market risk and the required return of a 50-50 portfolio of Alta Industries and Repo Men? Of Alta Industries and American Foam?
- m. (1) Suppose investors raised their inflation expectations by 3 percentage points over current estimates as reflected in the 8% T-bill rate. What effect would higher inflation have on the SML and on the returns required on high- and low-risk securities? (2) Suppose instead that investors' risk aversion increased enough to cause the market risk premium to increase by 3 percentage points. (Assume inflation remains constant.) What effect would this have on the SML and on returns of high- and low-risk securities?

SELECTED ADDITIONAL CASES

The following cases from Textchoice, Cengage Learning's online library, cover many of the concepts discussed in this chapter and are available at <http://www.textchoice2.com>.

Klein-Brigham Series:

Case 2, "Peachtree Securities, Inc. (A)."

Brigham-Buzzard Series:

Case 2, "Powerline Network Corporation (Risk and Return)."

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