

chapter 6

Risk, Return, and the Capital Asset Pricing Model

Skill or luck? That's the question *The Wall Street Journal's* Investment Dartboard Contest sought to answer by comparing the actual investment results of professional analysts against amateurs and dart throwers. Here's how the contest worked. First, *The Wall Street Journal (WSJ)* picked four professional analysts, and each of those pros formed a portfolio by picking four stocks. The stocks had to trade on the NYSE, AMEX, or Nasdaq; have a market capitalization of at least \$50 million and a stock price of at least \$2; and have average daily trades of at least \$100,000. Second, amateurs could enter the contest by e-mailing their pick of a single stock to the *WSJ*, which then picked four amateurs at random and combined their choices to make a four-stock portfolio. Third, a group of *WSJ* editors formed a portfolio by throwing four darts at the stock tables. At the beginning of each contest, the *WSJ* announced the six resulting portfolios, and at the end of six months, the paper announced the results. The top two pros were invited back for the next contest.

Since 1990 there have been 142 completed contests. The pros beat the darts 87 times and lost 55 times. The pros also beat the Dow Jones Industrial Average in 54% of the contests. The pros had an average six-month portfolio return of 10.2%, much higher than either the

DJIA six-month average of 5.6% or the darts' return of only 3.5%. The readers, meantime, lost an average of 4% versus a same-period (30 contests) gain of 7.2% for the pros.

Do these results mean that skill is more important than luck when it comes to investing in stocks? Not necessarily, according to Burton Malkiel, an economics professor at Princeton and the author of the widely read book, *A Random Walk Down Wall Street*. Since the dart-selected portfolios consist of randomly chosen stocks, they should have average risk. However, the pros have consistently picked high-risk stocks. Because there was a bull market during most of the contest, one would expect high-risk stocks to outperform the average stock. According to Malkiel, the pros' performance could be due to a rising market rather than superior analytical skills. The *WSJ* stopped that contest in 2002, so we won't know for sure whether Malkiel was right or wrong.

The *WSJ* now runs a new contest, pitting six amateurs against six darts. In the recently completed Contest No. 21, the darts trounced the readers by gaining 20% versus the readers' loss of -4.1% (the Dow Jones Industrial Average was up 5.1%). Overall, readers have won 8 contests, while the darts have won 13. If you would like to enter the contest, e-mail your stock pick to sundaydartboard@wsj.com.

In this chapter, we start from the basic premise that investors like returns and dislike risk. Therefore, people will invest in riskier assets only if they expect to receive higher returns. We define precisely what the term *risk* means as it relates to investments. We examine procedures managers use to measure risk, and we discuss the relationship between risk and return. In later chapters we extend these relationships to show how risk and return interact to determine security prices. Managers must understand and apply these concepts as they plan the actions that will shape their firms' futures.



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The textbook's Web site contains an *Excel* file that will guide you through the chapter's calculations. The file for this chapter is **FM12 Ch 06 Tool Kit.xls**, and we encourage you to open the file and follow along as you read the chapter.

6.1 Investment Returns

With most investments, an individual or business spends money today with the expectation of earning even more money in the future. The concept of *return* provides investors with a convenient way to express the financial performance of an investment. To illustrate, suppose you buy 10 shares of a stock for \$1,000. The stock pays no dividends, but at the end of one year, you sell the stock for \$1,100. What is the return on your \$1,000 investment?

One way to express an investment return is in *dollar terms*. The dollar return is simply the total dollars received from the investment less the amount invested:

$$\begin{aligned}\text{Dollar return} &= \text{Amount received} - \text{Amount invested} \\ &= \$1,100 - \$1,000 \\ &= 100.\end{aligned}$$

If, at the end of the year, you sell the stock for only \$900, your dollar return will be $-\$100$.

Although expressing returns in dollars is easy, two problems arise: (1) To make a meaningful judgment about the return, you need to know the scale (size) of the investment; a \$100 return on a \$100 investment is a good return (assuming the investment is held for 1 year), but a \$100 return on a \$10,000 investment would be a poor return. (2) You also need to know the timing of the return; a \$100 return on a \$100 investment is a very good return if it occurs after one year, but the same dollar return after 20 years is not very good.

The solution to the scale and timing problems is to express investment results as *rates of return*, or *percentage returns*. For example, the rate of return on the 1-year stock investment, when \$1,100 is received after 1 year, is 10%:

$$\begin{aligned}\text{Rate of return} &= \frac{\text{Amount received} - \text{Amount invested}}{\text{Amount invested}} \\ &= \frac{\text{Dollar return}}{\text{Amount invested}} = \frac{\$100}{\$1,000} \\ &= 0.10 = 10\%.\end{aligned}$$

The rate of return calculation "standardizes" the return by considering the annual return per unit of investment. Although this example has only one outflow and one inflow, the annualized rate of return can easily be calculated in situations where multiple cash flows occur over time by using time value of money concepts.

Corporate Valuation and Risk

In Chapter 1, we told you that managers should strive to make their firms more valuable, and that the value of a firm is determined by the size, timing, and risk of its free cash flows (FCF). This chapter shows you how to measure a firm's risk and the rate of return

expected by shareholders, which affects the weighted average cost of capital (WACC). All else held equal, higher risk increases the WACC, which reduces the firm's value.

$$\text{Value} = \frac{\text{FCF}_1}{(1 + \text{WACC})^1} + \frac{\text{FCF}_2}{(1 + \text{WACC})^2} + \frac{\text{FCF}_3}{(1 + \text{WACC})^3} + \cdots + \frac{\text{FCF}_\infty}{(1 + \text{WACC})^\infty}$$

SELF-TEST

Differentiate between dollar returns and rates of return.

Why are rates of return superior to dollar returns in terms of accounting for the size of investment and the timing of cash flows?

Suppose you pay \$500 for an investment that returns \$600 in 1 year. What is the annual rate of return? (20%)

6.2 Stand-Alone Risk

Risk is defined in *Webster's* as "a hazard; a peril; exposure to loss or injury." Thus, risk refers to the chance that some unfavorable event will occur. If you go skydiving, you are taking a chance with your life—skydiving is risky. If you bet on the horses, you are risking your money. If you invest in speculative stocks (or, really, *any* stock), you are taking a risk in the hope of earning an appreciable return.

An asset's risk can be analyzed in two ways: (1) on a stand-alone basis, where the asset is considered in isolation, and (2) on a portfolio basis, where the asset is held as one of a number of assets in a portfolio. Thus, an asset's **stand-alone risk** is the risk an investor would face if he or she held only this one asset. Obviously, most assets are held in portfolios, but it is necessary to understand stand-alone risk in order to understand risk in a portfolio context.

To illustrate the risk of financial assets, suppose an investor buys \$100,000 of short-term Treasury bills with an expected return of 5%. In this case, the rate of return on the investment, 5%, can be estimated quite precisely, and the investment is defined as being essentially *risk free*. However, if the \$100,000 were invested in the stock of a company just being organized to prospect for oil in the mid-Atlantic, then the investment's return could not be estimated precisely. One might analyze the situation and conclude that the *expected* rate of return, in a statistical sense, is 20%, but the investor should recognize that the *actual* rate of return could range from, say, +1,000% to -100%. Because there is a significant danger of actually earning much less than the expected return, the stock would be relatively risky.

No investment should be undertaken unless the expected rate of return is high enough to compensate the investor for the perceived risk of the investment. In our example, it is clear that few if any investors would be willing to buy the oil company's stock if its expected return were the same as that of the T-bill.

Table 6-1

Probability Distributions for Sale.com and Basic Foods

Demand for the Company's Products	Probability of This Demand Occurring	Rate of Return on Stock if This Demand Occurs	
		Sale.com	Basic Foods
Strong	0.3	100%	40%
Normal	0.4	15	15
Weak	0.3	(70)	(10)
	<u>1.0</u>		

Risky assets rarely actually produce their expected rates of return; generally, risky assets earn either more or less than was originally expected. Indeed, if assets always produced their expected returns, they would not be risky. Investment risk, then, is related to the probability of actually earning a low or negative return: The greater the chance of a low or negative return, the riskier the investment. However, risk can be defined more precisely, and we do so in the next section.

Probability Distributions

An event's *probability* is defined as the chance that the event will occur. For example, a weather forecaster might state, "There is a 40% chance of rain today and a 60% chance that it will not rain." If all possible events, or outcomes, are listed, and if a probability is assigned to each event, the listing is called a **probability distribution**. Keep in mind that the probabilities must sum to 1.0, or 100%.

With this in mind, consider the possible rates of return due to dividends and stock price changes that you might earn next year on a \$10,000 investment in the stock of either Sale.com or Basic Foods Inc. Sale.com is an Internet company offering deep discounts on factory seconds and overstocked merchandise. Because it faces intense competition, its new services may or may not be competitive in the marketplace, so its future earnings cannot be predicted very well. Indeed, some new company could develop better services and literally bankrupt Sale.com. Basic Foods, on the other hand, distributes essential food staples to grocery stores, and its sales and profits are relatively stable and predictable.

The rate-of-return probability distributions for the two companies are shown in Table 6-1. There is a 30% chance of strong demand, in which case both companies will have high earnings, pay high dividends, and enjoy capital gains. There is a 40% probability of normal demand and moderate returns, and there is a 30% probability of weak demand, which will mean low earnings and dividends as well as capital losses. Notice, however, that Sale.com's rate of return could vary far more widely than that of Basic Foods. There is a fairly high probability that the value of Sale.com's stock will drop substantially, resulting in a 70% loss, while there is a much smaller possible loss for Basic Foods.

Note that the following discussion of risk applies to all random variables, not just stock returns.



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Table 6-2

Calculation of Expected Rates of Return: Payoff Matrix

Demand for the Company's Products (1)	Probability of This Demand Occurring (2)	Sale.com		Basic Foods	
		Rate of Return if This Demand Occurs (3)	Product: (2) × (3) = (4)	Rate of Return if This Demand Occurs (5)	Product: (2) × (5) = (6)
Strong	0.3	100%	30%	40%	12%
Normal	0.4	15	6	15	6
Weak	0.3	(70)	(21)	(10)	(3)
	<u>1.0</u>		$\hat{r} = \underline{15\%}$		$\hat{r} = \underline{15\%}$

Expected Rate of Return

If we multiply each possible outcome by its probability of occurrence and then sum these products, as in Table 6-2, we have a *weighted average* of outcomes. The weights are the probabilities, and the weighted average is the **expected rate of return**, \hat{r} , called “r-hat.”¹ The expected rates of return for both Sale.com and Basic Foods are shown in Table 6-2 to be 15%. This type of table is known as a *payoff matrix*.

The expected rate of return calculation can also be expressed as an equation that does the same thing as the payoff matrix table:²

$$\text{Expected rate of return} = \hat{r} = P_1r_1 + P_2r_2 + \cdots + P_nr_n = \sum_{i=1}^n P_i r_i. \quad (6-1)$$

Here r_i is the i th possible outcome, P_i is the probability of the i th outcome, and n is the number of possible outcomes. Thus, \hat{r} is a weighted average of the possible outcomes (the r_i values), with each outcome's weight being its probability of occurrence. Using the data for Sale.com, we obtain its expected rate of return as follows:

$$\begin{aligned} \hat{r} &= P_1(r_1) + P_2(r_2) + P_3(r_3) \\ &= 0.3(100\%) + 0.4(15\%) + 0.3(-70\%) \\ &= 15\%. \end{aligned}$$

Basic Foods' expected rate of return is also 15%:

$$\begin{aligned} \hat{r} &= 0.3(40\%) + 0.4(15\%) + 0.3(-10\%) \\ &= 15\%. \end{aligned}$$

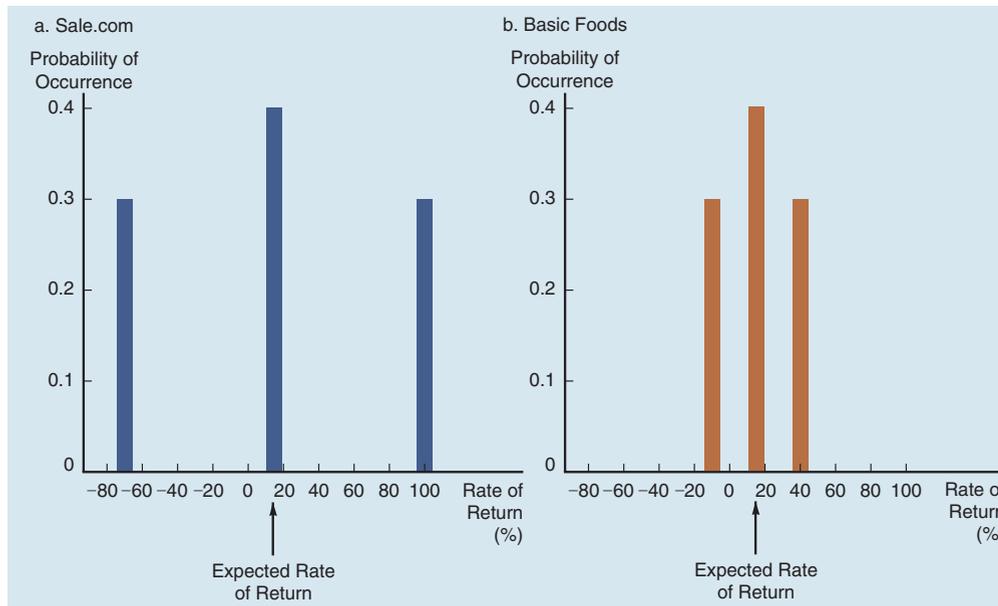
We can graph the rates of return to obtain a picture of the variability of possible outcomes; this is shown in the Figure 6-1 bar charts. The height of each bar signifies the probability that a given outcome will occur. The range of probable

¹In later chapters, we will use \hat{r}_d and \hat{r}_s to signify the returns on bonds and stocks, respectively. However, this distinction is unnecessary in this chapter, so we just use the general term, \hat{r} , to signify the expected return on an investment.

²This equation is valid for any random variable with a discrete probability distribution, not just for stock returns.

Figure 6-1

Probability Distributions of Sale.com's and Basic Foods' Rates of Return



returns for Sale.com is from -70 to $+100\%$, with an expected return of 15% . The expected return for Basic Foods is also 15% , but its range is much narrower.

Thus far, we have assumed that only three situations can exist: strong, normal, and weak demand. Actually, of course, demand could range from a deep depression to a fantastic boom, and there are unlimited possibilities in between. Suppose we had the time and patience to assign a probability to each possible level of demand (with the sum of the probabilities still equaling 1.0) and to assign a rate of return to each stock for each level of demand. We would have a table similar to Table 6-1, except it would have many more entries in each column. This table could be used to calculate expected rates of return as shown previously, and the probabilities and outcomes could be approximated by continuous curves such as those presented in Figure 6-2. Here we have changed the assumptions so that there is essentially a zero probability that Sale.com's return will be less than -70% or more than 100% , or that Basic Foods' return will be less than -10% or more than 40% , but virtually any return within these limits is possible.

The tighter, or more peaked, the probability distribution, the more likely it is that the actual outcome will be close to the expected value, and, consequently, the less likely it is that the actual return will end up far below the expected return. Thus, the tighter the probability distribution, the lower the risk assigned to a stock. Since Basic Foods has a relatively tight probability distribution, its actual return is likely to be closer to its 15% expected return than that of Sale.com.

Measuring Stand-Alone Risk: The Standard Deviation

Risk is a difficult concept to grasp, and a great deal of controversy has surrounded attempts to define and measure it. However, a common definition, and one that is satisfactory for many purposes, is stated in terms of probability distributions such

as those presented in Figure 6-2: *The tighter the probability distribution of expected future returns, the smaller the risk of a given investment.* According to this definition, Basic Foods is less risky than Sale.com because there is a smaller chance that its actual return will end up far below its expected return.

To be most useful, any measure of risk should have a definite value—we need a measure of the tightness of the probability distribution. One such measure is the **standard deviation**, the symbol for which is σ , pronounced “sigma.” The smaller the standard deviation, the tighter the probability distribution, and, accordingly, the less risky the stock. To calculate the standard deviation, we proceed as shown in Table 6-3, taking the following steps:³

1. Calculate the expected rate of return:

$$\text{Expected rate of return} = \hat{r} = \sum_{i=1}^n P_i r_i.$$

For Sale.com, we previously found $\hat{r} = 15\%$.

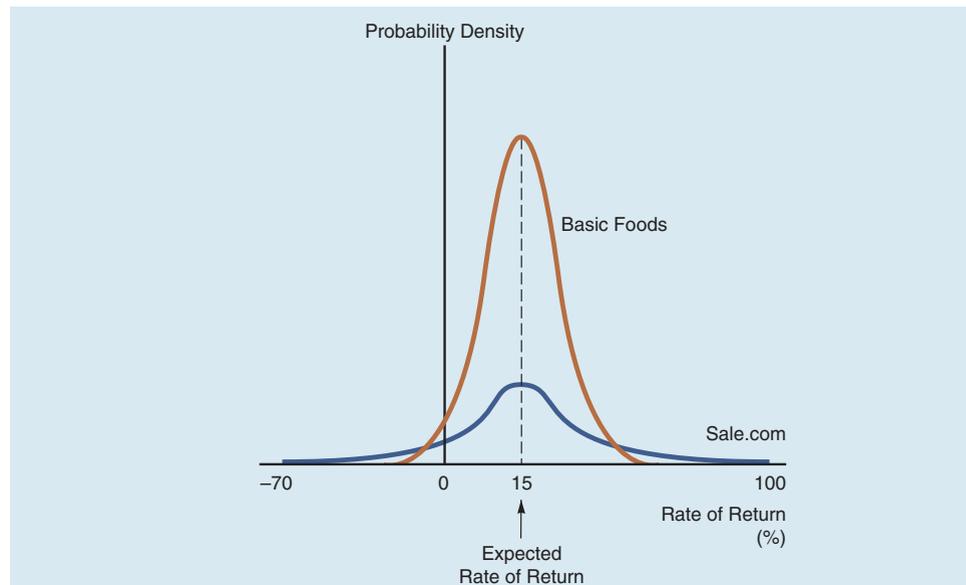
2. Subtract the expected rate of return (\hat{r}) from each possible outcome (r_i) to obtain a set of deviations about \hat{r} as shown in Column 1 of Table 6-3:

$$\text{Deviation}_i = r_i - \hat{r}.$$

3. Square each deviation, then multiply the result by the probability of occurrence for its related outcome, and then sum these products to obtain the

Figure 6-2

Continuous Probability Distributions of Sale.com’s and Basic Foods’ Rates of Return



Note: The assumptions regarding the probabilities of various outcomes have been changed from those in Figure 6-1. There the probability of obtaining exactly 15% was 40%; here it is *much smaller* because there are many possible outcomes instead of just three. With continuous distributions, it is more appropriate to ask what the probability is of obtaining at least some specified rate of return than to ask what the probability is of obtaining exactly that rate. This topic is covered in detail in statistics courses.

³These equations are valid for any random variable from a discrete probability distribution, not just for returns.

Table 6-3

Calculating Sale.com's Standard Deviation

(Values for r_i and \hat{r} are percentages.)

$r_i - \hat{r}$ (1)	$(r_i - \hat{r})^2$ (2)	$(r_i - \hat{r})^2 P_i$ (3)
$100 - 15 = 85\%$	7,225%	$(7,225)(0.3) = 2,167.5\%$
$15 - 15 = 0$	0	$(0)(0.04) = 0.0$
$-70 - 15 = -85$	7,225	$(7,225)(0.3) = 2,167.5$
		Variance = $\sigma^2 = 4,335.0\%$
		Standard deviation = $\sigma = \sqrt{\sigma^2} = \sqrt{4,335\%} = 65.84\%$



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variance of the probability distribution as shown in Columns 2 and 3 of the table:

$$\text{Variance} = \sigma^2 = \sum_{i=1}^n (r_i - \hat{r})^2 P_i. \quad (6-2)$$

4. Finally, find the square root of the variance to obtain the standard deviation:

$$\text{Standard deviation} = \sigma = \sqrt{\sum_{i=1}^n (r_i - \hat{r})^2 P_i}. \quad (6-3)$$

Thus, the standard deviation is essentially a weighted average of the deviations from the expected value, and it provides an idea of how far above or below the expected value the actual value is likely to be. Sale.com's standard deviation is seen in Table 6-3 to be $\sigma = 65.84\%$. Using these same procedures, we find Basic Foods' standard deviation to be 19.36% . Sale.com has the larger standard deviation, which indicates a greater variation of returns and thus a greater chance that the actual return may be substantially lower than the expected return. Therefore, Sale.com is a riskier investment than Basic Foods when held alone.⁴

If a probability distribution is normal, the *actual* return will be within ± 1 standard deviation of the *expected* return 68.26 percent of the time. Figure 6-3 illustrates this point, and it also shows the situation for $\pm 2 \sigma$ and $\pm 3 \sigma$. For Sale.com, $\hat{r} = 15\%$ and $\sigma = 65.84\%$, whereas $\hat{r} = 15\%$ and $\sigma = 19.36\%$ for Basic Foods. Thus, if the two distributions were normal, there would be a 68.26% probability that Sale.com's actual return would be in the range of $15 \pm 65.84\%$, or from -50.84 to 80.84% . For Basic Foods, the 68.26% range is $15 \pm 19.36\%$, or from -4.36 to 34.36% . For the average firm listed on the New York Stock Exchange, σ has generally been in the range of 35 to 40% in recent years.

⁴Most financial calculators have no built-in formula for finding the expected value or variance for discrete probability distributions, except for the special case in which the probabilities for all outcomes are equal. Therefore, you must go through the processes outlined in Tables 6-2 and 6-3 [i.e., Equations 6-1 and 6-3]. For an example of this process using a financial calculator, see Richard W. Taylor, "Discrete Probability Analysis with the BALL Plus Professional Calculator," *Journal of Financial Education*, Winter 2005, pp. 100–106. Excel also has no built-in formula for discrete distributions, although it is possible to find free add-ins on the Web that do the calculations for discrete distributions.

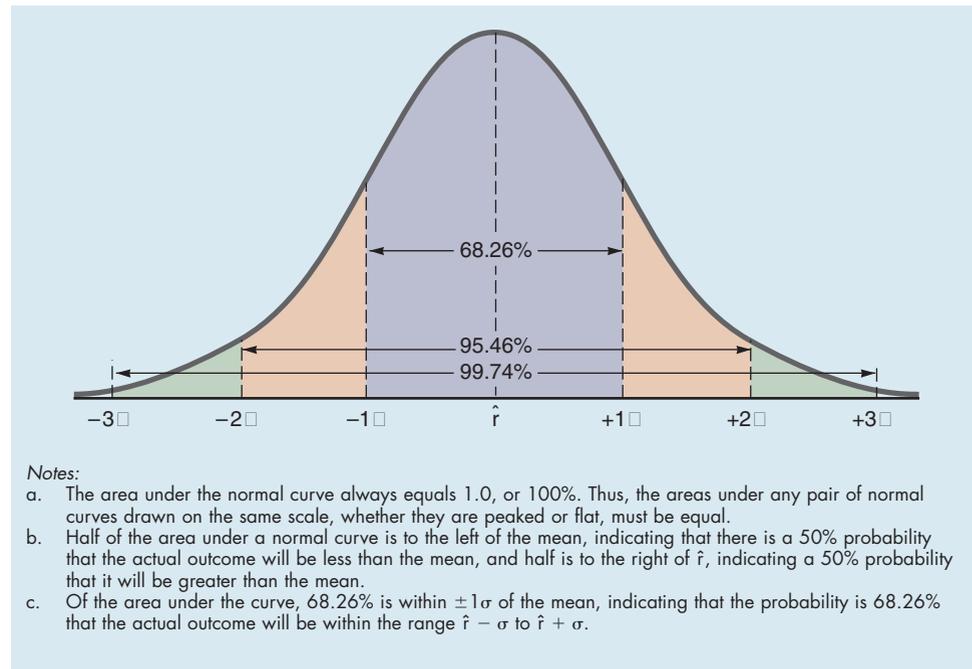
Figure 6-3

Probability Ranges for a Normal Distribution



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For more discussion of probability distributions, see **Web Extension 6A** at the textbook's Web site.



Using Historical Data to Measure Risk

In the previous example, we described the procedure for finding the mean and standard deviation when the data are in the form of a known probability distribution. Suppose only sample returns data over some past period are available. The past **realized rate of return** in period t is denoted by \bar{r}_t ("r bar t"). The average annual return over the last n years is \bar{r}_{Avg} :

$$\bar{r}_{\text{Avg}} = \frac{\sum_{t=1}^n \bar{r}_t}{n} \quad (6-4)$$

The standard deviation of a sample of returns can be estimated using this formula:

$$\text{Estimated } \sigma = S = \sqrt{\frac{\sum_{t=1}^n (\bar{r}_t - \bar{r}_{\text{Avg}})^2}{n - 1}} \quad (6-5)$$

When estimated from past data, the standard deviation is often denoted by S . Here is an example:⁵

⁵Because we are estimating the standard deviation from a sample of observations, the denominator in Equation 6-5 is " $n - 1$ " and not just " n ." Equations 6-4 and 6-5 are built into all financial calculators. For example, to find the sample standard deviation, enter the rates of return into the calculator and press the key marked S (or S_x) to get the standard deviation. See our tutorials or your calculator manual for details.

Year	\bar{r}_t
2005	15%
2006	-5
2007	20

$$\bar{r}_{\text{Avg}} = \frac{(15 - 5 + 20)}{3} = 10.0\%,$$

$$\begin{aligned} \text{Estimated } \sigma \text{ (or } S) &= \sqrt{\frac{(15 - 10)^2 + (-5 - 10)^2 + (20 - 10)^2}{3 - 1}} \\ &= \sqrt{\frac{350}{2}} = 13.2\%. \end{aligned}$$

In *Excel*, the average can be found using a built-in function: **=AVERAGE(0.15,-0.05,0.20)** = 10.0%. For the sample standard deviation, the function is **=STDEV(0.15,-0.05,0.20)** = 13.2%.

The historical σ is often used as an estimate of the future σ . Because past variability is likely to be repeated, S may be a reasonably good estimate of future risk. However, it is usually incorrect to use \bar{r}_{Avg} for some past period as an estimate of \hat{r} , the expected future return. For example, just because a stock had a 75% return in the past year, there is no reason to expect a 75% return this year.



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Measuring Stand-Alone Risk: The Coefficient of Variation

If a choice has to be made between two investments that have the same expected returns but different standard deviations, most people would choose the one with the lower standard deviation and, therefore, the lower risk. Similarly, given a choice between two investments with the same risk (standard deviation) but different expected returns, investors would generally prefer the investment with the higher expected return. To most people, this is common sense—return is “good,” risk is “bad,” and consequently investors want as much return and as little risk as possible. But how do we choose between two investments if one has a higher expected return but the other a lower standard deviation? To help answer this question, we often use another measure of risk, the **coefficient of variation (CV)**, which is the standard deviation divided by the expected return:

$$\text{Coefficient of variation} = CV = \frac{\sigma}{\hat{r}}. \quad (6-6)$$

The coefficient of variation shows the risk per unit of return, and it provides a more meaningful basis for comparison when the expected returns on two alternatives are not the same. Since Basic Foods and Sale.com have the same expected return, the coefficient of variation is not necessary in this case. The firm with the larger standard deviation, Sale.com, must have the larger coefficient of variation when the means are equal. In fact, the coefficient of variation for Sale.com is $65.84/15 = 4.39$ and that for Basic Foods is $19.36/15 = 1.29$. Thus, Sale.com is more than three times as risky as Basic Foods on the basis of this criterion. Because the coefficient of variation captures the effects of both risk and return, it is a better measure than just standard deviation for evaluating stand-alone risk in situations where two or more investments have substantially different expected returns.

The Trade-Off between Risk and Return

The table accompanying this box summarizes the historical trade-off between risk and return for different classes of investments. Those assets that produced the highest average returns also had the highest standard deviations and the widest ranges of returns. For example, small-company stocks had the highest average annual return, but their standard deviation of returns was also the highest. By contrast, U.S. Treasury bills had the lowest standard deviation, but they also had the lowest average return.

Note that a T-bill is riskless *if you hold it until maturity*, but if you invest in a rolling portfolio of T-bills and hold the portfolio for a number of years, your investment income will vary depending on what happens to the level of interest rates in each year. While you can be sure of the return you will earn on a T-bill, you cannot be sure of the return you will earn on a portfolio of T-bills over a number of years.

Distribution of Realized Returns, 1926–2005

	Small Company Stocks	Large Company Stocks	Long-Term Corporate Bonds	Long-Term Government Bonds	U.S. Treasury Bills	Inflation
Average return	17.4%	12.3%	6.2%	5.8%	3.8%	3.1%
Standard deviation	32.9	20.2	8.5	9.2	3.1	4.3
Excess return over T-bonds ^a	11.6	6.5	0.4			

^aThe excess return over T-bonds is called the “historical risk premium.” If and only if investors expect returns in the future to be similar to returns earned in the past, the excess return will also be the current risk premium that is reflected in security prices.

Source: Based on *Stocks, Bonds, Bills, and Inflation: Valuation Edition 2006 Yearbook* (Chicago: Ibbotson Associates, 2006).

Risk Aversion and Required Returns

Suppose you have worked hard and saved \$1 million, which you now plan to invest. You can buy a 5% U.S. Treasury security, and at the end of 1 year you will have a sure \$1.05 million, which is your original investment plus \$50,000 in interest. Alternatively, you can buy stock in Genetic Advances. If Genetic Advances’ research programs are successful, your stock will increase in value to \$2.1 million. However, if the research is a failure, the value of your stock will go to zero, and you will be penniless. You regard Genetic Advances’ chance of success or failure as being 50–50, so the expected value of the stock investment is $0.5(\$0) + 0.5(\$2,100,000) = \$1,050,000$. Subtracting the \$1 million cost of the stock leaves an expected profit of \$50,000, or an expected (but risky) 5% rate of return: $\$50,000/\$1,000,000 = 0.05 = 5\%$.

Thus, you have a choice between a sure \$50,000 profit (representing a 5% rate of return) on the Treasury security and a risky expected \$50,000 profit (also representing a 5% expected rate of return) on the Genetic Advances stock. Which one would you choose? *If you choose the less risky investment, you are risk averse. Most investors are indeed risk averse, and certainly the average investor is risk averse with regard to his or her “serious money.” Because this is a well-documented fact, we shall assume risk aversion throughout the remainder of the book.*

What are the implications of risk aversion for security prices and rates of return? The answer is that, other things held constant, the higher a security’s risk, the lower its price and the higher its required return. To see how risk aversion affects security prices, consider again Basic Foods and Sale.com. Suppose each stock is expected to pay an annual dividend of \$15 forever. Under these conditions,

the price of each stock is just the present value of a perpetuity. If each stock had an expected return of 15%, then each stock's price would be $P = \$15/0.15 = \100 . Investors are averse to risk, so under these conditions there would be a general preference for Basic Foods—it has the same expected return as Sale.com but less risk. People with money to invest would bid for Basic Foods rather than Sale.com stock, and Sale.com stockholders would start selling their stock and using the money to buy Basic Foods. Buying pressure would drive up Basic Foods' stock, and selling pressure would simultaneously cause Sale.com's price to decline.

These price changes, in turn, would cause changes in the expected rates of return on the two securities. Suppose, for example, that Basic Foods' stock price was bid up from \$100 to \$150, whereas Sale.com's stock price declined from \$100 to \$75. This would cause Basic Foods' expected return to fall to 10%, while Sale.com's expected return would rise to 20%.⁶ The difference in returns, $20\% - 10\% = 10\%$, is a **risk premium, RP**, which represents the additional compensation investors require for assuming the additional risk of Sale.com stock.

This example demonstrates a very important principle: *In a market dominated by risk-averse investors, riskier securities must have higher expected returns, as estimated by the marginal investor, than less risky securities. If this situation does not exist, buying and selling in the market will force it to occur.* We will consider the question of how much higher the returns on risky securities must be later in the chapter, after we see how diversification affects the way risk should be measured. Then, in later chapters, we will see how risk-adjusted rates of return affect the prices investors are willing to pay for bonds and stocks.

SELF-TEST

What does “investment risk” mean?

Set up an illustrative probability distribution for an investment.

What is a payoff matrix?

Which of the two stocks graphed in Figure 6-2 is less risky? Why?

How does one calculate the standard deviation?

Which is a better measure of risk if assets have different expected returns: (1) the standard deviation or (2) the coefficient of variation? Why?

Explain the following statement: “Most investors are risk averse.”

How does risk aversion affect rates of return?

An investment has a 30% chance of producing a 25% return, a 40% chance of producing a 10% return, and a 30% chance of producing a –15% return. What is its expected return? (7%) What is its standard deviation? (15.7%)

A stock's returns for the past 3 years are 10%, –15%, and 35%. What is the historical average return? (10%) What is the historical sample standard deviation? (25%)

An investment has an expected return of 15% and a standard deviation of 25%. What is its coefficient of variation? (2.0)

6.3 Risk in a Portfolio Context

In the preceding section, we considered the risk of assets held in isolation. Now we analyze the risk of assets held in portfolios. As we shall see, an asset held as part of a portfolio is less risky than the same asset held in isolation. Accordingly, most

⁶Recall that the present value of a perpetuity is $P = CF/r$, where CF is the constant annual cash flow of the perpetuity. Solving for r , the expected return for Basic Foods is $\$15/\$150 = 0.10 = 10\%$. The expected return for Sale.com is $\$15/\$75 = 0.20 = 20\%$.

financial assets are actually held as parts of portfolios. Banks, pension funds, insurance companies, mutual funds, and other financial institutions are required by law to hold diversified portfolios. Even individual investors—at least those whose security holdings constitute a significant part of their total wealth—generally hold portfolios, not the stock of only one firm. This being the case, from an investor’s standpoint the fact that a particular stock goes up or down is not very important; *what is important is the return on his or her portfolio, and the portfolio’s risk. Logically, then, the risk and return of an individual security should be analyzed in terms of how that security affects the risk and return of the portfolio in which it is held.*

To illustrate, Pay Up Inc. is a collection agency that operates nationwide through 37 offices. The company is not well known, its stock is not very liquid, its earnings have fluctuated quite a bit in the past, and it doesn’t pay a dividend. All this suggests that Pay Up is risky and that the required rate of return on its stock, r , should be relatively high. However, Pay Up’s required rate of return in 2006, and all other years, was quite low in relation to those of most other companies. This indicates that investors regard Pay Up as being a low-risk company in spite of its uncertain profits. The reason for this counterintuitive fact has to do with diversification and its effect on risk. Pay Up’s earnings rise during recessions, whereas most other companies’ earnings tend to decline when the economy slumps. It’s like fire insurance—it pays off when other things go badly. Therefore, adding Pay Up to a portfolio of “normal” stocks tends to stabilize returns on the entire portfolio, thus making the portfolio less risky.

Portfolio Returns

The **expected return on a portfolio**, \hat{r}_p , is simply the weighted average of the expected returns on the individual assets in the portfolio, with the weights being the fraction of the total portfolio invested in each asset:

$$\begin{aligned}\hat{r}_p &= w_1\hat{r}_1 + w_2\hat{r}_2 + \cdots + w_n\hat{r}_n \\ &= \sum_{i=1}^n w_i\hat{r}_i.\end{aligned}\tag{6-7}$$

Here the \hat{r}_i ’s are the expected returns on the individual stocks, the w_i ’s are the weights, and there are n stocks in the portfolio. Note that (1) w_i is the fraction of the portfolio’s dollar value invested in Stock i (that is, the value of the investment in Stock i divided by the total value of the portfolio) and (2) the w_i ’s must sum to 1.0.

Assume that in August 2007, a security analyst estimated that the following returns could be expected on the stocks of four large companies:

	Expected Return, \hat{r}
Southwest Airlines	15.0%
Starbucks	12.0
FedEx	10.0
Dell	9.0

If we formed a \$100,000 portfolio, investing \$25,000 in each stock, the expected portfolio return would be 11.5%:

$$\begin{aligned}\hat{r}_p &= w_1\hat{r}_1 + w_2\hat{r}_2 + w_3\hat{r}_3 + w_4\hat{r}_4 \\ &= 0.25(15\%) + 0.25(12\%) + 0.25(10\%) + 0.25(9\%) \\ &= 11.5\%.\end{aligned}$$

Of course, the actual realized rates of return will almost certainly be different from their expected values, so the realized portfolio return, \bar{r}_p , will be different from the expected return. For example, Starbucks might double and provide a return of +100%, whereas Dell might have a terrible year, fall sharply, and have a return of -75%. Note, though, that those two events would be somewhat offsetting, so the portfolio's return might still be close to its expected return, even though the individual stocks' actual returns were far from their expected returns.

Portfolio Risk

As we just saw, the expected return on a portfolio is simply the weighted average of the expected returns on the individual assets in the portfolio. However, unlike returns, the risk of a portfolio, σ_p , is generally not the weighted average of the standard deviations of the individual assets in the portfolio; the portfolio's risk will almost always be smaller than the weighted average of the assets' σ 's. In fact, it is theoretically possible to combine stocks that are individually quite risky as measured by their standard deviations to form a portfolio that is completely riskless, with $\sigma_p = 0$.

To illustrate the effect of combining assets, consider the situation in Figure 6-4. The bottom section gives data on rates of return for Stocks W and M individually, as well as for a portfolio invested 50% in each stock. The three graphs plot the data in a time series format. The two stocks would be quite risky if they were held in isolation, but when they are combined to form Portfolio WM, they are not risky at all. (Note: These stocks are called W and M because the graphs of their returns in Figure 6-4 resemble a W and an M.)

The reason Stocks W and M can be combined to form a riskless portfolio is that their returns move countercyclically to each other—when W's returns fall, those of M rise, and vice versa. The tendency of two variables to move together is called **correlation**, and the **correlation coefficient** measures this tendency.⁷ The symbol for the correlation coefficient is the Greek letter rho, ρ (pronounced roe). In statistical terms, we say that the returns on Stocks W and M are *perfectly negatively correlated*, with $\rho = -1.0$.

The estimate of correlation from a sample of historical data is often called "R." Here is the formula to estimate the correlation between stocks i and j ($\bar{r}_{i,t}$ is the actual return for Stock i in period t, and $\bar{r}_{i,Avg}$ is the average return during the n-period sample; similar notation is used for Stock j):

$$\text{Estimated } \rho = R = \frac{\sum_{t=1}^n (\bar{r}_{i,t} - \bar{r}_{i,Avg})(\bar{r}_{j,t} - \bar{r}_{j,Avg})}{\sqrt{\sum_{t=1}^n (\bar{r}_{i,t} - \bar{r}_{i,Avg})^2} \sqrt{\sum_{t=1}^n (\bar{r}_{j,t} - \bar{r}_{j,Avg})^2}} \quad (6-8)$$

Fortunately, it is easy to estimate the correlation coefficients with a financial calculator. Simply enter the returns on the two stocks and then press a key labeled "r."⁸ In *Excel*, use the CORREL function. See *FM12 Ch 06 Tool Kit.xls* for the calculation of correlation between Stocks W and M.

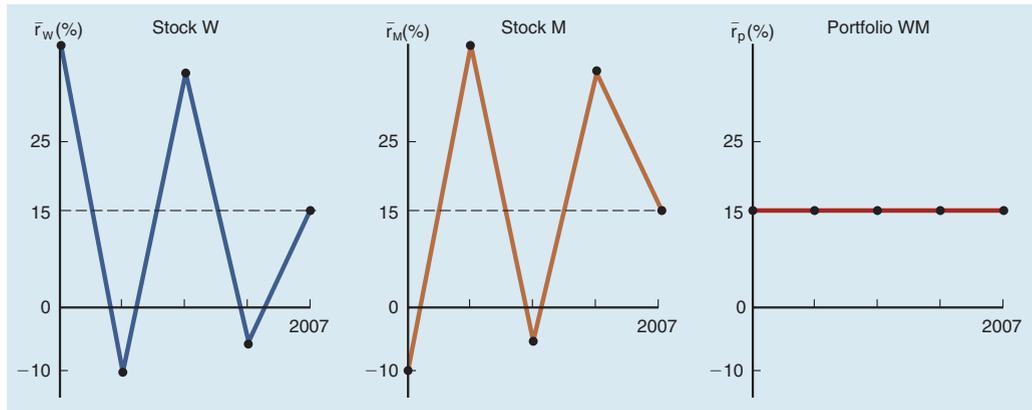
The opposite of perfect negative correlation, with $\rho = +1.0$, is *perfect positive correlation*, with $\rho = +1.0$. Returns on two perfectly positively correlated stocks

⁷The correlation coefficient, ρ , can range from +1.0, denoting that the two variables move up and down in perfect synchronization, to -1.0, denoting that the variables always move in exactly opposite directions. A correlation coefficient of zero indicates that the two variables are not related to each other—that is, changes in one variable are independent of changes in the other.

⁸See our tutorial or your calculator manual for the exact steps. Also, note that the correlation coefficient is often denoted by the term "r." We use ρ here to avoid confusion with r as used to denote the rate of return.

Figure 6-4

Rates of Return for Two Perfectly Negatively Correlated Stocks ($\rho = -1.0$) and for Portfolio WM



Year	Stock W (\bar{r}_W)	Stock M (\bar{r}_M)	Portfolio WM (\bar{r}_P)
2003	40.0%	(10.0%)	15.0%
2004	(10.0)	40.0	15.0
2005	35.0	(5.0)	15.0
2006	(5.0)	35.0	15.0
2007	<u>15.0</u>	<u>15.0</u>	<u>15.0</u>
Average return	<u>15.0%</u>	<u>15.0%</u>	<u>15.0%</u>
Standard deviation	<u>22.6%</u>	<u>22.6%</u>	<u>0.0%</u>

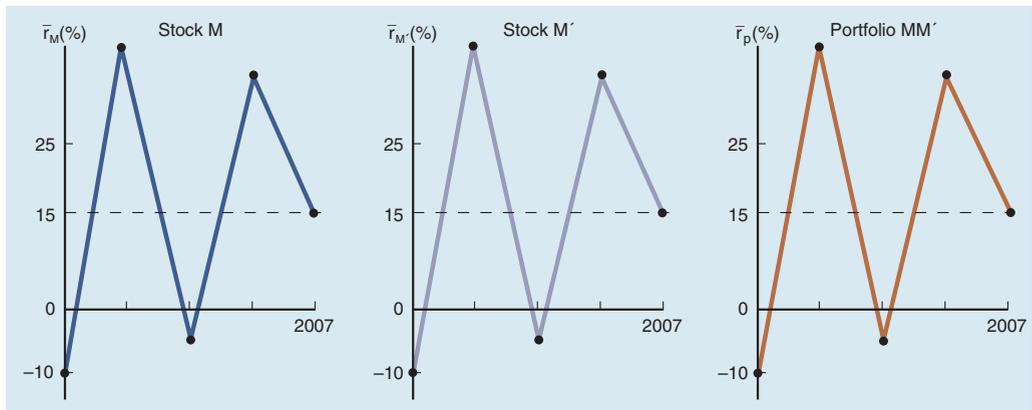


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See **FM12 Ch 06 Tool Kit.xls** at the textbook's Web site for all calculations.

Figure 6-5

Rates of Return for Two Perfectly Positively Correlated Stocks ($\rho = +1.0$) and for Portfolio MM'



Year	Stock M (\bar{r}_M)	Stock M' ($\bar{r}_{M'}$)	Portfolio MM' (\bar{r}_P)
2003	(10.0%)	(10.0%)	(10.0%)
2004	40.0	40.0	40.0
2005	(5.0)	(5.0)	(5.0)
2006	35.0	35.0	35.0
2007	<u>15.0</u>	<u>15.0</u>	<u>15.0</u>
Average return	<u>15.0%</u>	<u>15.0%</u>	<u>15.0%</u>
Standard deviation	<u>22.6%</u>	<u>22.6%</u>	<u>22.6%</u>



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See **FM12 Ch 06 Tool Kit.xls** at the textbook's Web site for all calculations.

(M and M') would move up and down together, and a portfolio consisting of two such stocks would be exactly as risky as each individual stock. This point is illustrated in Figure 6-5, where we see that the portfolio's standard deviation is equal to that of the individual stocks. *Thus, diversification does nothing to reduce risk if the portfolio consists of perfectly positively correlated stocks.*

Figures 6-4 and 6-5 demonstrate that when stocks are perfectly negatively correlated ($\rho = -1.0$), all risk can be diversified away, but when stocks are perfectly positively correlated ($\rho = +1.0$), diversification does no good whatsoever. In reality, virtually all stocks are positively correlated, but not perfectly so. Past studies have estimated that on average the correlation coefficient for the monthly returns on two randomly selected stocks is about 0.3.⁹ *Under this condition, combining stocks into portfolios reduces risk but does not completely eliminate it.* Figure 6-6 illustrates this point with two stocks whose correlation coefficient is $\rho = +0.35$. The portfolio's average return is 15%, which is exactly the same as the average return for our other two illustrative portfolios, but its standard deviation is 18.6%, which is between the other two portfolios' standard deviations.

These examples demonstrate that in one extreme case ($\rho = -1.0$), risk can be completely eliminated, while in the other extreme case ($\rho = +1.0$), diversification does no good whatsoever. The real world lies between these extremes, so combining stocks into portfolios reduces, but does not eliminate, the risk inherent in the individual stocks. Also, we should note that in the real world, it is *impossible* to find stocks like W and M, whose returns are expected to be perfectly negatively correlated. *Therefore, it is impossible to form completely riskless stock portfolios.* Diversification can reduce risk but not eliminate it, so the real world is similar to the situation depicted in Figure 6-6.

What would happen if we included more than two stocks in the portfolio? *As a rule, the risk of a portfolio will decline as the number of stocks in the portfolio increases.* If we added enough partially correlated stocks, could we completely eliminate risk? In general, the answer is no, but the extent to which adding stocks to a portfolio reduces its risk depends on the *degree of correlation* among the stocks: The smaller the positive correlation coefficients, the lower the risk in a large portfolio. If some stocks had correlations of -1.0 , all risk could be eliminated. *In the real world, where the correlations among the individual stocks are generally positive but less than $+1.0$, some, but not all, risk can be eliminated.*

In general, there are higher correlations between the returns on two companies in the same industry than for two companies in different industries. Thus, to minimize risk, portfolios should be diversified across industries.

Diversifiable Risk versus Market Risk

As noted above, it is difficult if not impossible to find stocks whose expected returns are negatively correlated—most stocks tend to do well when the national economy is strong and badly when it is weak. Thus, even very large portfolios end up with a substantial amount of risk, but not as much risk as if all the money were invested in only one stock.

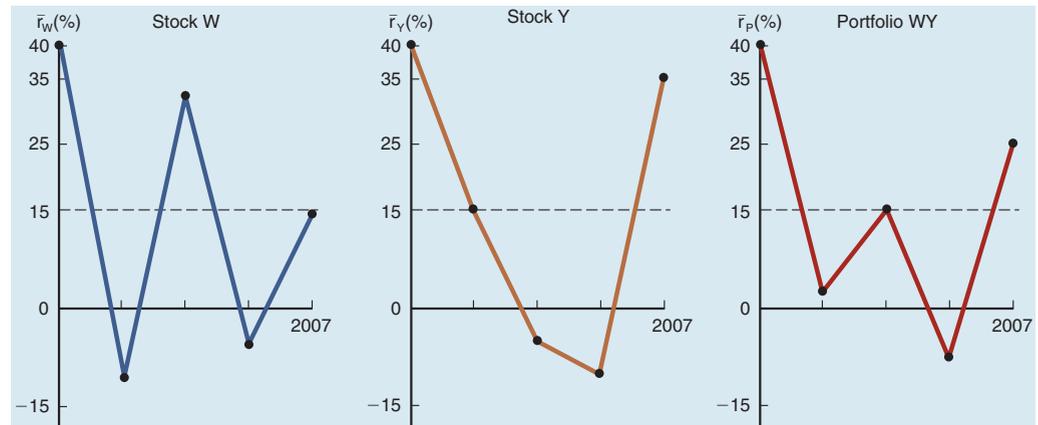
⁹A recent study by Chan, Karceski, and Lakonishok (1999) estimated that the average correlation coefficient between two randomly selected stocks was 0.28, while the average correlation coefficient between two large-company stocks was 0.33. The time period of their sample was 1968 to 1998. See Louis K. C. Chan, Jason Karceski, and Josef Lakonishok, "On Portfolio Optimization: Forecasting Covariance and Choosing the Risk Model," *The Review of Financial Studies*, Vol. 12, No. 5, Winter 1999, pp. 937–974. A study by Campbell, Lettau, Malkiel, and Xu found that the average correlation fell from around 0.35 in the late 1970s to less than 0.10 by the late 1990s; see John Y. Campbell, Martin Lettau, Burton G. Malkiel, and Yexiao Xu, "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *Journal of Finance*, February 2001, pp. 1–43.

Figure 6-6

Rates of Return for Two Partially Correlated Stocks ($\rho = +0.35$) and for Portfolio WY

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See **FM12 Ch 06 Tool Kit.xls**, at the text-book's Web site for all calculations.



Year	Stock W (\bar{r}_W)	Stock Y (\bar{r}_Y)	Portfolio WY (\bar{r}_P)
2003	40.0%	40.0%	40.0%
2004	(10.0)	15.0	2.5
2005	35.0	(5.0)	15.0
2006	(5.0)	(10.0)	(7.5)
2007	15.0	35.0	25.0
Average return	<u>15.0%</u>	<u>15.0%</u>	<u>15.0%</u>
Standard deviation	<u>22.6%</u>	<u>22.6%</u>	<u>18.6%</u>

To see more precisely how portfolio size affects portfolio risk, consider Figure 6-7, which shows how portfolio risk is affected by forming larger and larger portfolios of randomly selected New York Stock Exchange (NYSE) stocks. Standard deviations are plotted for an average one-stock portfolio, a two-stock portfolio, and so on, up to a portfolio consisting of all 2,000-plus common stocks that were listed on the NYSE at the time the data were graphed. The graph illustrates that, in general, the risk of a portfolio consisting of large-company stocks tends to decline and to approach some limit as the size of the portfolio increases. According to data accumulated in recent years, σ_1 , the standard deviation of a one-stock portfolio (or an average stock), is approximately 35%. A portfolio consisting of all stocks, which is called the **market portfolio**, would have a standard deviation, σ_M , of about 20%, which is shown as the horizontal dashed line in Figure 6-7.

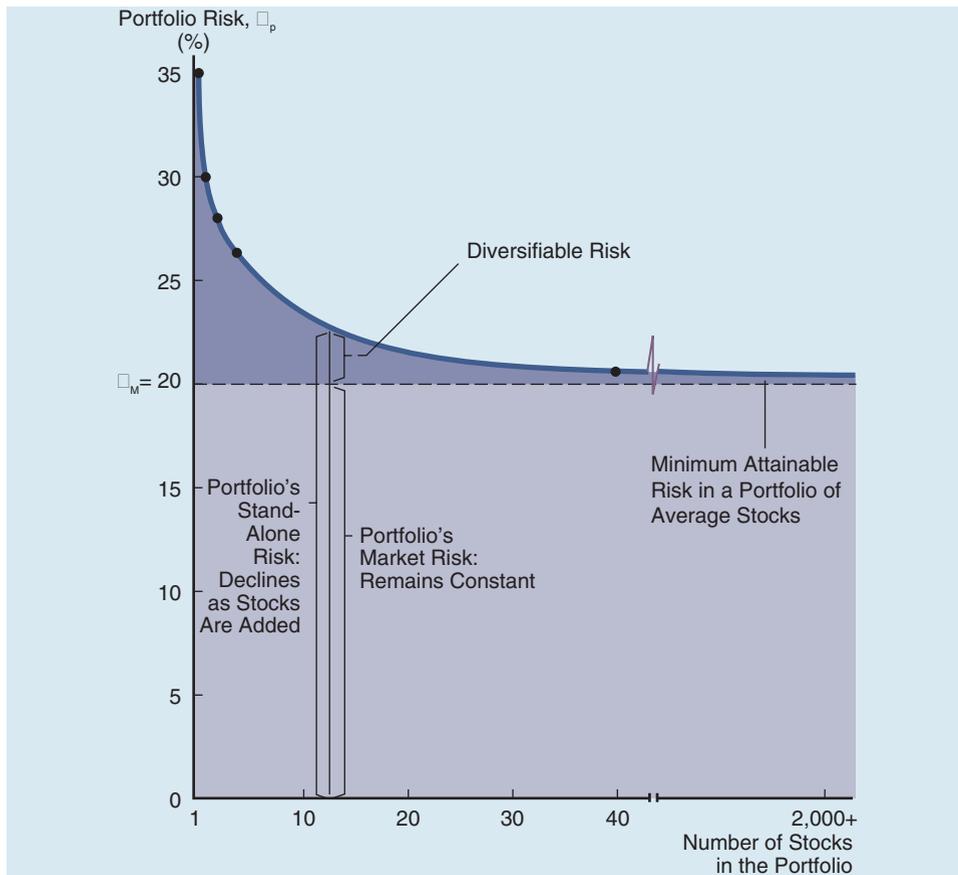
Thus, almost half of the risk inherent in an average individual stock can be eliminated if the stock is held in a reasonably well-diversified portfolio, which is one containing 40 or more stocks in a number of different industries. Some risk always remains, however, so it is virtually impossible to diversify away the effects of broad stock market movements that affect almost all stocks.

The part of a stock's risk that *can* be eliminated is called *diversifiable risk*, while the part that *cannot* be eliminated is called *market risk*.¹⁰ The fact that a large part of the risk of any individual stock can be eliminated is vitally important, because rational investors *will* eliminate it and thus render it irrelevant.

¹⁰Diversifiable risk is also known as *company-specific*, or *unsystematic*, risk. Market risk is also known as *nondiversifiable*, *systematic*, or *beta*, risk; it is the risk that remains after diversification.

Figure 6-7

Effects of Portfolio Size on Portfolio Risk for Average Stocks



Diversifiable risk is caused by such random events as lawsuits, strikes, successful and unsuccessful marketing programs, the winning or losing of a major contract, and other events that are unique to a particular firm. Because these events are random, their effects on a portfolio can be eliminated by diversification—bad events in one firm will be offset by good events in another. **Market risk**, on the other hand, stems from factors that systematically affect most firms: war, inflation, recessions, and high interest rates. Since most stocks are negatively affected by these factors, market risk cannot be eliminated by diversification.

We know that investors demand a premium for bearing risk; that is, the higher the risk of a security, the higher its expected return must be to induce investors to buy (or to hold) it. However, if investors are primarily concerned with the risk of their *portfolios* rather than the risk of the individual securities in the portfolio, how should the risk of an individual stock be measured? One answer is provided by the **Capital Asset Pricing Model (CAPM)**, an important tool used to analyze the relationship between risk and rates of return.¹¹ The primary conclusion of the CAPM is this: *The relevant risk of an individual stock is its contribution to the risk of a well-diversified portfolio.* A stock might be quite risky if held by itself, but if half of its risk can be eliminated by diversification, then its **relevant risk**, which is its *contribution to the portfolio's risk*, is much smaller than its stand-alone risk.

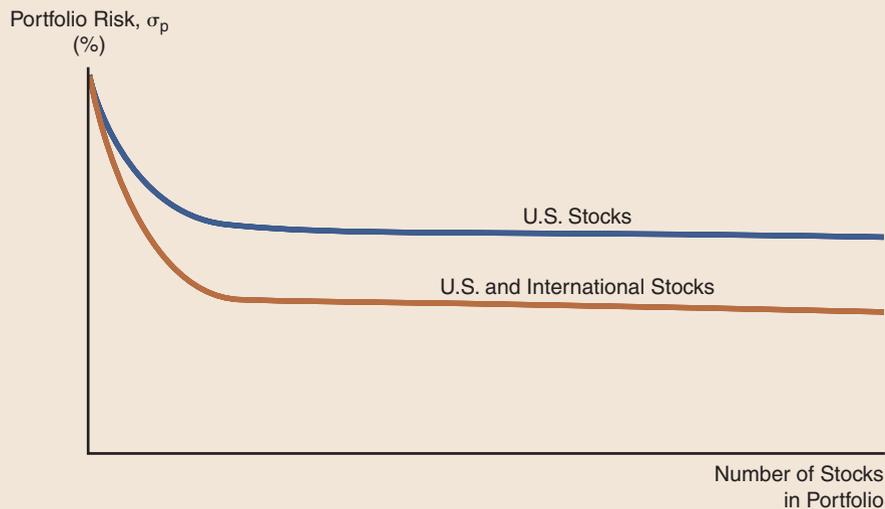
¹¹Indeed, the 1990 Nobel Prize was awarded to the developers of the CAPM, Professors Harry Markowitz and William F. Sharpe. The CAPM is a relatively complex theory, and only its basic elements are presented in this chapter. A more in-depth presentation appears in Chapter 7.

The Benefits of Diversifying Overseas

Figure 6-7 shows that an investor can significantly reduce portfolio risk by holding a large number of stocks. The figure accompanying this box suggests that investors may be able to reduce risk even further by holding a large portfolio of stocks from all around the world, because the returns of domestic and international stocks are not perfectly correlated.

Although U.S. investors have traditionally been relatively reluctant to hold international assets, it is a safe bet that in the years ahead U.S. investors will shift more and more of their assets to overseas investments.

Source: For further reading, see Kenneth Kasa, "Measuring the Gains from International Portfolio Diversification," *Federal Reserve Bank of San Francisco Weekly Letter*, no. 94-14 (April 8, 1994).



A simple example will help make this point clear. Suppose you are offered the chance to flip a coin once. If it's heads, you win \$20,000, but if it's tails, you lose \$16,000. This is a good bet—the expected return is $0.5(\$20,000) + 0.5(-\$16,000) = \$2,000$. However, it is a highly risky proposition, because you have a 50% chance of losing \$16,000. Thus, you might well refuse to make the bet. Alternatively, suppose you were offered the chance to flip a coin 100 times, and you would win \$200 for each head but lose \$160 for each tail. It is theoretically possible that you would flip all heads and win \$20,000, and it is also theoretically possible that you would flip all tails and lose \$16,000, but the chances are very high that you would actually flip about 50 heads and about 50 tails, winning a net of about \$2,000. Although each individual flip is a risky bet, collectively you have a low-risk proposition because most of the risk has been diversified away. This is the idea behind holding portfolios of stocks rather than just one stock, except that with stocks all of the risk cannot be eliminated by diversification—those risks related to broad, systematic changes in the stock market will remain.

Are all stocks equally risky in the sense that adding them to a well-diversified portfolio will have the same effect on the portfolio's riskiness? The answer is no. Different stocks will affect the portfolio differently, so different securities have different degrees of relevant risk. How can the relevant risk of an individual stock be measured? As we have seen, all risk except that related to broad market movements can, and presumably will, be diversified away. After all, why accept risk that can be easily eliminated? *The risk that remains after diversifying is market risk, or the risk that is inherent in the market, and it can be measured by the degree to which a given*

stock tends to move up or down with the market. In the next section, we develop a measure of a stock's market risk, and then, in a later section, we introduce an equation for determining the required rate of return on a stock, given its market risk.

Contribution to Market Risk: Beta

As we noted above, the primary conclusion of the CAPM is that the relevant risk of an individual stock is the amount of risk the stock contributes to a well-diversified portfolio. The benchmark for a well-diversified stock portfolio is the market portfolio, which is a portfolio containing all stocks. Therefore, the relevant risk of an individual stock, which is called its **beta coefficient**, is defined under the CAPM as the amount of risk that the stock contributes to the market portfolio. In CAPM terminology, ρ_{iM} is the correlation between the i th stock's return and the return on the market, σ_i is the standard deviation of the i th stock's return, and σ_M is the standard deviation of the market's return. The beta coefficient of the i th stock, denoted by b_i , is defined as follows:

$$b_i = \left(\frac{\sigma_i}{\sigma_M} \right) \rho_{iM}. \quad (6-9)$$

This tells us that a stock with a high standard deviation, σ_i , will tend to have a high beta, which means that it contributes a relatively large amount of risk to a well-diversified portfolio. This makes sense, because if all other things are equal, a stock with high stand-alone risk will contribute a lot of risk to the portfolio. Note too that a stock with a high correlation with the market, ρ_{iM} , will also have a large beta, and hence be risky. This also makes sense, because a high correlation means that diversification is not helping much; hence the stock contributes a lot of risk to the portfolio.

The **covariance between stock i and the market**, COV_{iM} , is defined as¹²

$$\text{COV}_{iM} = \rho_{iM} \sigma_i \sigma_M. \quad (6-10)$$

Substituting Equation 6-10 into 6-9 provides another frequently used expression for beta:

$$b_i = \frac{\text{COV}_{iM}}{\sigma_M^2}. \quad (6-11)$$

Calculators and spreadsheets can calculate the components of Equation 6-9 (ρ_{iM} , σ_i , and σ_M), which can then be used to calculate beta, but there is another way. Suppose you plotted the stock's returns on the y -axis of a graph and the market portfolio's returns on the x -axis, as shown in Figure 6-8. The formula for the slope

¹²Using historical data, the sample covariance can be calculated as

$$\text{Sample covariance from historical data} = \text{COV}_{iM} = \frac{\sum_{t=1}^n (\bar{r}_{i,t} - \bar{r}_{i, \text{Avg}}) (\bar{r}_{M,t} - \bar{r}_{M, \text{Avg}})}{n - 1}.$$

Calculating the covariance is somewhat easier than calculating the correlation. So if you have already calculated the standard deviations, then it is easier to calculate the covariance and then calculate the correlation as:

$$\rho_{iM} = \text{COV}_{iM} / (\sigma_i \sigma_M).$$

of a regression line is exactly equal to the formula for beta in Equation 6-11. Therefore, to estimate beta for a security, you can just estimate a regression with the stock's returns on the y-axis and the market's returns on the x-axis.

Individual Stock Betas

The tendency of a stock to move up and down with the market is reflected in its beta coefficient. An *average-risk stock* is defined as one with a beta equal to 1.0. Such a stock's returns tend to move up and down, on average, with the market, which is measured by some index such as the Dow Jones Industrials, the S&P 500, or the New York Stock Exchange Composite Index. A portfolio of such $b = 1.0$ stocks will move up and down with the broad market indexes, and it will be just as risky as the indexes. A portfolio of $b = 0.5$ stocks will be half as risky as the market. On the other hand, a portfolio of $b = 2.0$ stocks will be twice as risky as the market.

Figure 6-8 shows a graph of the historical returns of three stocks and the market. The data below the graph assume that in Year 1 the "market," defined as a portfolio consisting of all stocks, had a total return (dividend yield plus capital gains yield) of $\bar{r}_M = 10\%$ and Stocks H, A, and L (for High, Average, and Low risk) also all had returns of 10%. In Year 2, the market went up sharply, and the return on the market portfolio was $\bar{r} = 20\%$. Returns on the three stocks also went up: H soared to 30%; A went up to 20%, the same as the market; and L only went up to 15%. The market dropped in Year 3, and the market return was $\bar{r}_M = -10\%$. The three stocks' returns also fell, H plunging to -30% , A falling to -10% , and L going down to $\bar{r}_L = 0\%$. Thus, the three stocks all moved in the same direction as the market, but H was by far the most volatile; A was just as volatile as the market; and L was less volatile.

Beta measures a stock's volatility relative to the market, which by definition has $b = 1.0$. As we noted above, the slope of a regression line shows how a stock moves in response to a movement in the general market. Most stocks have betas in the range of 0.50 to 1.50, and the average beta for all stocks is 1.0 by definition.

Theoretically, it is possible for a stock to have a negative beta. In this case, the stock's returns would tend to rise whenever the returns on other stocks fall. In practice, very few stocks have a negative beta. Keep in mind that a stock in a given period may move counter to the overall market, even though the stock's beta is positive. If a stock has a positive beta, we would *expect* its return to increase whenever the overall stock market rises. However, company-specific factors may cause the stock's realized return to decline, even though the market's return is positive.

Portfolio Betas

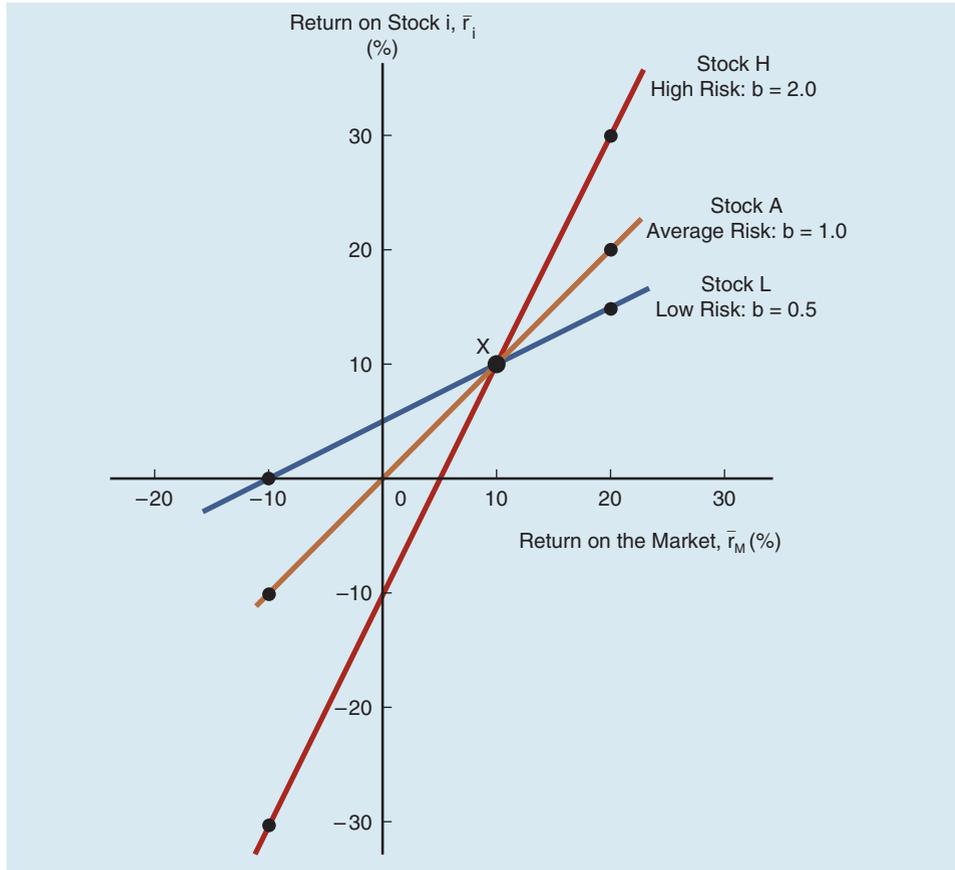
A very important feature of beta is that the beta of a portfolio is a weighted average of its individual securities' betas:

$$\begin{aligned} b_p &= w_1b_1 + w_2b_2 + \cdots + w_nb_n \\ &= \sum_{i=1}^n w_ib_i. \end{aligned} \tag{6-12}$$

Here b_p is the beta of the portfolio, and it shows how volatile the portfolio is in relation to the market; w_i is the fraction of the portfolio invested in the

Figure 6-8

Relative Volatility of Stocks H, A, and L



Year	\bar{r}_H	\bar{r}_A	\bar{r}_L	\bar{r}_M
1	10%	10%	10%	10%
2	30	20	15	20
3	(30)	(10)	0	(10)
Beta	2.0	1.0	0.5	

Note: These three stocks plot exactly on their regression lines. This indicates that they are exposed only to market risk. Mutual funds that concentrate on stocks with betas of 2.0, 1.0, and 0.5 will have patterns similar to those shown in the graph.

i th stock; and b_i is the beta coefficient of the i th stock. For example, if an investor holds a \$100,000 portfolio consisting of \$33,333.33 invested in each of three stocks, and if each of the stocks has a beta of 0.7, then the portfolio's beta will be $b_p = 0.7$:

$$b_p = 0.3333(0.7) + 0.3333(0.7) + 0.3333(0.7) = 0.7.$$

Such a portfolio will be less risky than the market, so it should experience relatively narrow price swings and have relatively small rate-of-return fluctuations. In terms of Figure 6-8, the slope of its regression line would be 0.7, which is less than that for a portfolio of average stocks.



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Now suppose one of the existing stocks is sold and replaced by a stock with $b_i = 2.0$. This action will increase the beta of the portfolio from $b_{p1} = 0.7$ to $b_{p2} = 1.13$:

$$\begin{aligned} b_{p2} &= 0.3333(0.7) + 0.3333(0.7) + 0.3333(2.0) \\ &= 1.13. \end{aligned}$$

Had a stock with $b_i = 0.2$ been added, the portfolio beta would have declined from 0.7 to 0.53. Adding a low-beta stock, therefore, would reduce the risk of the portfolio. Consequently, adding new stocks to a portfolio can change the riskiness of that portfolio. *Thus, since a stock's beta measures its contribution to the risk of a portfolio, beta is the theoretically correct measure of the stock's risk.*

Key Points Related to Beta

The preceding analysis of risk in a portfolio context is part of the Capital Asset Pricing Model (CAPM), and we can highlight the key points as follows:

1. A stock's risk consists of two components, market risk and diversifiable risk.
2. Diversifiable risk can be eliminated by diversification, and most investors do indeed diversify, either by holding large portfolios or by purchasing shares in a mutual fund. We are left, then, with market risk, which is caused by general movements in the stock market and which reflects the fact that most stocks are systematically affected by events like war, recessions, and inflation. Market risk is the only risk relevant to a rational, diversified investor because such an investor would eliminate diversifiable risk.
3. Investors must be compensated for bearing risk—the greater the risk of a stock, the higher its required return. However, compensation is required only for risk that cannot be eliminated by diversification. If risk premiums existed on stocks due to diversifiable risk, well-diversified investors would start buying those securities (which would not be especially risky to such investors) and bidding up their prices. The stocks' final (equilibrium) expected returns would reflect only nondiversifiable market risk.
4. The market risk of a stock is measured by its beta coefficient, which is an index of the stock's relative volatility. If b equals 1.0, then the stock is about as risky as the market, if held in a diversified portfolio. If b is less than 1.0, the stock is less risky than the market. If beta is greater than 1.0, the stock is more risky.
5. The beta of a portfolio is a weighted average of the individual securities' betas.
6. *Since a stock's beta coefficient determines how the stock affects the risk of a diversified portfolio, beta is the most relevant measure of any stock's risk.*

SELF-TEST

Explain the following statement: "An asset held as part of a portfolio is generally less risky than the same asset held in isolation."

What is meant by *perfect positive correlation*, *perfect negative correlation*, and *zero correlation*?

In general, can the risk of a portfolio be reduced to zero by increasing the number of stocks in the portfolio? Explain.

What is the beta of a stock that is as risky as the market?

Why is beta the theoretically correct measure of a stock's risk?

If you plotted the returns on a particular stock versus those on the Dow Jones Index over the past 5 years, what would the slope of the regression line you obtained indicate about the stock's market risk?

An investor has a three-stock portfolio with \$25,000 invested in Dell, \$50,000 invested in Ford, and \$25,000 invested in Wal-Mart. Dell's beta is estimated to be 1.20, Ford's beta is estimated to be 0.80, and Wal-Mart's beta is estimated to be 1.0. What is the estimated beta of the investor's portfolio? (0.95)

6.4 Calculating Beta Coefficients

The CAPM is an *ex ante* model, which means that all of the variables represent before-the-fact, *expected* values. In particular, the beta coefficient used by investors should reflect the expected volatility of a given stock's return versus the return on the market during some *future* period. However, people generally calculate betas using data from some *past* period, and then assume that the stock's relative volatility will be the same in the future as it was in the past.

Table 6-4 shows the betas for some well-known companies, as provided by two different financial organizations, Zacks and Yahoo!Finance. Notice that their estimates of beta usually differ, because they calculate beta in slightly different ways. Given these differences, many analysts choose to calculate their own betas.

Recall from Figure 6-8 how betas are calculated. The actual historical returns for a company are plotted on the y-axis and the market portfolio's returns are plotted on the x-axis. A regression line is then fitted through the points, and the slope of the regression line provides an estimate of the stock's beta. Although it is possible to calculate beta coefficients with a calculator, they are usually calculated with a computer, either with a statistical software program or a spreadsheet program. The file *FM12 Ch 06 Tool Kit.xls* shows how GE's beta coefficient is calculated using *Excel's* regression function.¹³

The first step in a regression analysis is compiling the data. Most analysts use 4 to 5 years of monthly data, although some use 52 weeks of weekly data. We decided to use 4 years of monthly data, so we began by downloading 49 months of stock prices for GE from the Yahoo!Finance Web site. We used the S&P 500 Index as the market portfolio because it is representative of the market and because many analysts use this index. Table 6-5 shows a portion of this data; the full data set is in the file *FM12 Ch 06 Tool Kit.xls*.



e-resource

See **FM12 Ch 06 Tool Kit.xls** at the textbook's Web site for all calculations.

Table 6-4

Beta Coefficients for Some Actual Companies

Stock (Ticker Symbol)	Zacks	Yahoo!Finance
Amazon.com (AMZN)	2.53	2.93
Cisco Systems (CSCO)	1.99	1.56
Coca-Cola (KO)	0.38	0.82
Dell Computer (DELL)	1.16	1.05
Empire District Electric (EDE)	0.45	0.75
Energen Corp. (EGN)	0.57	0.93
General Electric (GE)	0.90	0.44
Heinz (HNZ)	0.32	0.42
Merrill Lynch (MER)	1.68	1.40
Microsoft Corp. (MSFT)	1.23	0.35
Procter & Gamble (PG)	0.10	0.76

Sources: <http://www.zacks.com> and <http://finance.yahoo.com>.



To see updated estimates, go to <http://www.zacks.com> and enter the ticker symbol; select Detailed Quotes for beta. Or go to <http://finance.yahoo.com> and enter the ticker symbol. When the results page comes up, select Key Statistics from the left panel to find beta.

¹³For an explanation of calculating beta with a financial calculator, see **Web Extension 6B** at the textbook's Web site.

Table 6-5

Stock Return Data for General Electric

Date	Market Level (S&P 500 Index)	Market Return	GE Adjusted Stock Price	GE Return
May 2006	1,280.16	-2.3%	34.33	-0.8%
April 2006	1,310.61	1.2	34.59	-0.5
March 2006	1,294.87	1.1	34.78	5.8
February 2006	1,280.66	0.0	32.87	1.1
.
.
.
August 2002	916.07	0.5	27.30	-6.4
July 2002	911.62	-7.9	29.16	10.8
June 2002	989.82	-7.2	26.31	-6.1
May 2002	1,067.14	NA	28.02	NA
Average return (annual)		<u>5.4%</u>		<u>6.9%</u>
Standard deviation (annual)		<u>13.0%</u>		<u>19.1%</u>
Correlation between GE and the market		<u>0.49</u>		

The second step is to convert the stock prices into rates of return. For example, to find the May 2006 return for GE, we find the percentage change from the previous month: $(\$34.59 - \$34.33)/\$34.59 = -0.0008 = -0.83\%$.¹⁴ We also find the percent change of the S&P Index level, and use this as the market return.

As Table 6-5 shows, GE had an average annual return of 6.9% during this 4-year period, while the market had an average annual return of 5.4%. As we noted before, it is usually unreasonable to think that the future expected return for a stock will equal its average historical return over a relatively short period, such as 4 years. However, we might well expect past volatility to be a reasonable estimate of future volatility, at least during the next couple of years. Note that the standard deviation for GE's return during this period was 19.1% versus 13.0% for the market. Thus, the market's volatility is less than that of GE. This is what we would expect, since the market is a well-diversified portfolio and thus much of its risk has been diversified away. The correlation between GE's stock returns and the market returns is about 0.49, which is a little higher than the correlation for a typical stock.

Figure 6-9 shows a plot of GE's returns against the market returns. As you will notice if you look in the file *FM12 Ch 06 Tool Kit.xls*, we used the *Excel Chart*

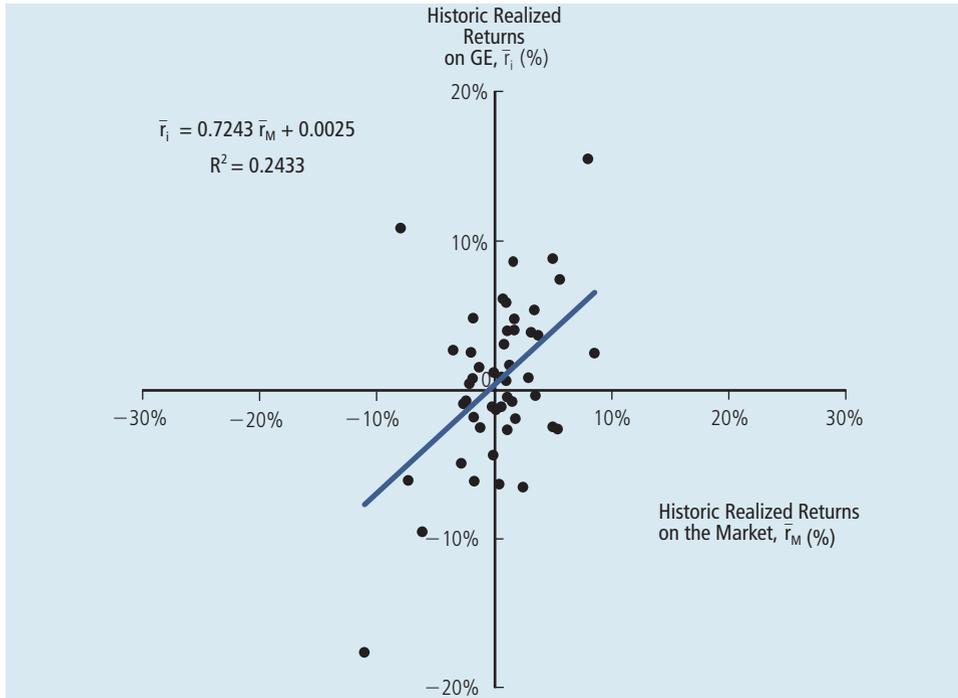


Check out <http://finance.yahoo.com> for General Electric using its ticker symbol of GE. You can also download data for the S&P 500 Index using its symbol of ^SPX.

¹⁴The prices reported in Yahoo!Finance are adjusted for dividends and stock splits so we can calculate the return as the percentage change in the adjusted price. If you use a source that reports actual market prices, then you have to make the adjustment yourself when calculating returns. For example, suppose the stock price is \$100 in July, the company has a 2-for-1 split, and the actual price is then \$60 in August. The reported adjusted price for August would be \$60, but the reported price for July would be lowered to \$50 to reflect the stock split. This gives an accurate stock return of 20%: $(\$60 - \$50)/\$50 = 20\%$, the same as if there had not been a split, in which case the return would have been $(\$120 - \$100)/\$100 = 20\%$. Or suppose the actual price in September was \$50, the company paid a \$10 dividend, and the actual price in October was \$60. Shareholders have earned a return of $(\$60 + \$10 - \$50)/\$50 = 40\%$. Yahoo!Finance reports an adjusted price of \$60 for October, and an adjusted price of \$42.857 for September, which gives a return of $(\$60 - \$42.857)/\$42.857 = 40\%$. Again, the percentage change in the adjusted price accurately reflects the actual return.

Figure 6-9

Calculating a Beta Coefficient for General Electric



feature to add a trend line and to display the equation and R^2 value on the chart itself. Alternatively, we could have used the *Excel* regression analysis feature, which would have provided more detailed data.

Figure 6-9 shows that GE's beta is about 0.72, as shown by the slope coefficient in the regression equation displayed on the chart. This means that GE's beta is less than the 1.0 average beta. Thus, GE moves up and down less than the market. Note, however, that the points are not clustered very tightly around the regression line. Sometimes GE does much better than the market, while at other times it does much worse. The R^2 value shown in the chart measures the degree of dispersion about the regression line. Statistically speaking, it measures the percentage of the variance that is explained by the regression equation. An R^2 of 1.0 indicates that all points lie exactly on the line, hence that all of the variance of the y-variable is explained by the x-variable. GE's R^2 is about 0.24, which is fairly typical for an individual stock. This indicates that about 24% of the variance in GE's returns is explained by the market returns. If we had done a similar analysis for a portfolio of 40 randomly selected stocks, then the points would probably have been clustered tightly around the regression line, and the R^2 would have probably been over 0.9.

Finally, note that the intercept shown in the regression equation on the chart is about 0.0025. Since the regression equation is based on monthly data, this means that over this period GE's stock earned about 0.25% more per month than an average stock as a result of factors other than a general increase in stock prices.

SELF-TEST

What types of data are needed to calculate a beta coefficient for an actual company?
 What does the R^2 measure? What is the R^2 for a typical company?

6.5 The Relationship between Risk and Rates of Return

In the preceding section, we saw that under the CAPM theory, beta is the appropriate measure of a stock's relevant risk. Now we must specify the relationship between risk and return: For a given level of risk as measured by beta, what rate of return should investors require to compensate them for bearing that risk? To begin, let us define the following terms:

\hat{r}_i	=	expected rate of return on the i th stock.
r_i	=	required rate of return on the i th stock. This is the minimum expected return that is required to induce an average investor to purchase the stock.
\bar{r}	=	realized, after-the-fact return.
r_{RF}	=	risk-free rate of return. In this context, r_{RF} is generally measured by the expected return on long-term U.S. Treasury bonds.
b_i	=	beta coefficient of the i th stock.
r_M	=	required rate of return on a portfolio consisting of all stocks, which is called the <i>market portfolio</i> .
RP_M	=	risk premium on "the market." $RP_M = (r_M - r_{RF})$ is the additional return over the risk-free rate required to induce an average investor to invest in the market portfolio.
RP_i	=	risk premium on the i th stock: $RP_i = (RP_M)b_i$.

The **market risk premium**, RP_M , shows the premium investors require for bearing the risk of an average stock, and it depends on the degree of risk aversion that investors on average have. Let us assume that Treasury bonds yield $r_{RF} = 6\%$, and the market has a required return of $r_M = 11\%$. The market risk premium is 5%:

$$RP_M = r_M - r_{RF} = 11\% - 6\% = 5\%.$$

We can measure a stock's relative riskiness by its beta coefficient. The risk premium for the i th stock is

$$\text{Risk premium for Stock } i = RP_i = (RP_M)b_i. \quad (6-13)$$

If we know the market risk premium, RP_M , and the stock's risk as measured by its beta coefficient, b_i , we can find the stock's risk premium as the product $(RP_M)b_i$. For example, if $b_i = 0.5$ and $RP_M = 5\%$, then RP_i is 2.5%:

$$\begin{aligned} RP_i &= (5\%)(0.5) \\ &= 2.5\%. \end{aligned}$$

The required return for any investment can be expressed in general terms as

$$\text{Required return} = \text{Risk-free return} + \text{Premium for risk.}$$

Here the risk-free return includes a premium for expected inflation, and we assume that the assets under consideration have similar maturities and liquidity. Under these conditions, the relationship between the required return and risk is called the **Security Market Line (SML)**:

SML Equation: Required return on Stock i = Risk-free rate + (Market risk premium) (Stock i's beta)

$$r_i = r_{RF} + (r_M - r_{RF})b_i \quad (6-14)$$

$$= r_{RF} + (RP_M)b_i.$$

The required return for Stock i can be written as follows:

$$r_i = 6\% + 5\%(0.5)$$

$$= 8.5\%.$$

If some other Stock j were riskier than Stock i and had $b_j = 2.0$, then its required rate of return would be 16%:

$$r_j = 6\% + (5\%)2.0 = 16\%.$$

An average stock, with $b = 1.0$, would have a required return of 11%, the same as the market return:

$$r_A = 6\% + (5\%)1.0 = 11\% = r_M.$$

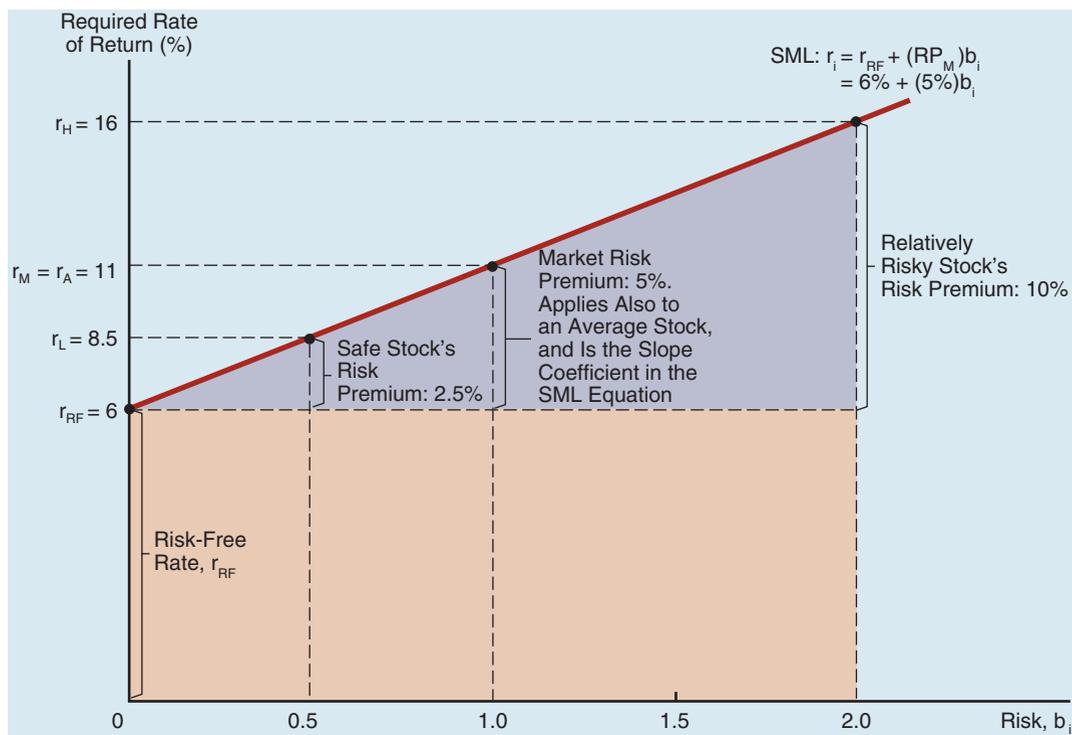
As noted above, Equation 6-14 is called the Security Market Line (SML) equation, and it is often expressed in graph form, as in Figure 6-10, which shows the SML when $r_{RF} = 6\%$ and $RP_M = 5\%$. Note the following points:

1. Required rates of return are shown on the vertical axis, while risk as measured by beta is shown on the horizontal axis. This graph is quite different from the one shown in Figure 6-8, where the returns on individual stocks were plotted on the vertical axis and returns on the market index were shown on the horizontal axis. The slopes of the three lines in Figure 6-8 were used to calculate the three stocks' betas, and those betas were then plotted as points on the horizontal axis of Figure 6-10.
2. Riskless securities have $b_i = 0$; therefore, r_{RF} appears as the vertical axis intercept in Figure 6-10. If we could construct a portfolio that had a beta of zero, it would have a required return equal to the risk-free rate.
3. The slope of the SML (5% in Figure 6-10) reflects the degree of risk aversion in the economy—the greater the average investor's aversion to risk, then (a) the steeper the slope of the line, (b) the greater the risk premium for all stocks, and (c) the higher the required rate of return on all stocks.¹⁵ These points are discussed further in a later section.
4. The values we worked out for stocks with $b_i = 0.5$, $b_i = 1.0$, and $b_i = 2.0$ agree with the values shown on the graph for r_L , r_A , and r_H .
5. Negative betas are rare but can occur. For example, some stocks associated with gold, such as a mining operation, occasionally have a negative beta. Based on the SML, a stock with a negative beta should have a required return less than the risk-free rate. In fact, a stock with a very large but negative beta might have negative required return! This means that when the market is doing well, this stock will do poorly. But it also implies the opposite: When the market is doing poorly, a negative beta stock should have a positive return. In other words, the negative beta stock acts as insurance. Therefore, an

¹⁵Students sometimes confuse beta with the slope of the SML. This is a mistake. The slope of any straight line is equal to the "rise" divided by the "run," or $(Y_1 - Y_0)/(X_1 - X_0)$. Consider Figure 6-10. If we let $Y = r$ and $X = \text{beta}$, and we go from the origin to $b = 1.0$, we see that the slope is $(r_M - r_{RF})/(b_M - b_{RF}) = (11\% - 6\%)/(1 - 0) = 5\%$. Thus, the slope of the SML is equal to $(r_M - r_{RF})$, the market risk premium. In Figure 6-10, $r_i = 6\% + 5\%b_i$, so an increase of beta from 1.0 to 2.0 would produce a 5 percentage point increase in r_i .

Figure 6-10

The Security Market Line (SML)



investor might be willing to accept a negative return on the stock during the good times if it is likely to provide a positive return in bad times.

Both the Security Market Line and a company's position on it change over time due to changes in interest rates, investors' aversion to risk, and individual companies' betas. Such changes are discussed in the following sections.

The Impact of Inflation

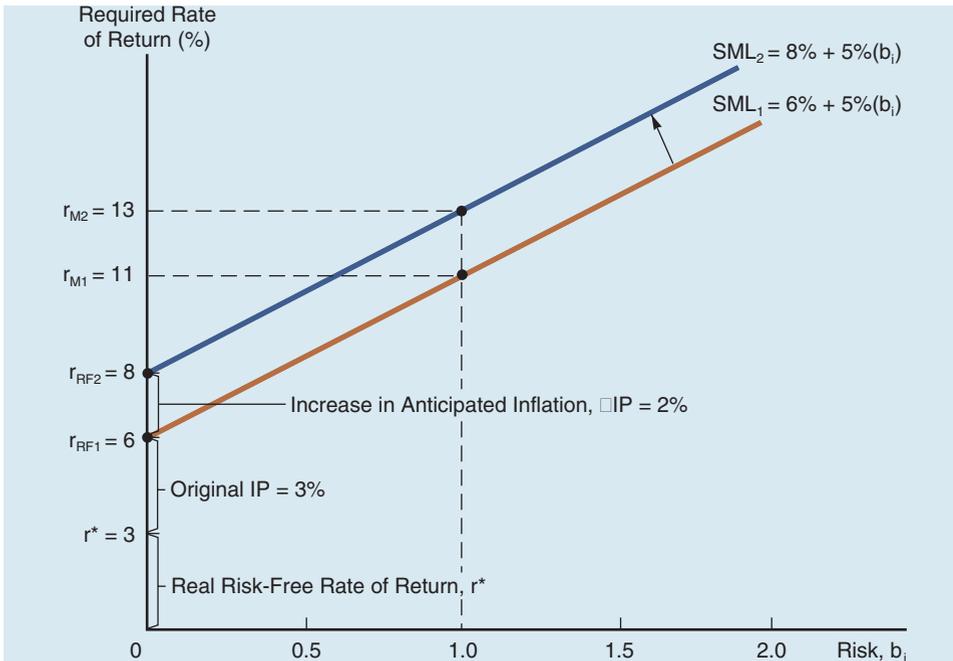
Interest is the same as "rent" on borrowed money, or the price of money. Thus, r_{RF} is the price of money to a riskless borrower. The risk-free rate as measured by the rate on U.S. Treasury securities is called the *nominal*, or *quoted*, rate, and it consists of two elements: (1) a *real inflation-free rate of return*, r^* , and (2) an *inflation premium*, IP , equal to the anticipated rate of inflation.¹⁶ Thus, $r_{RF} = r^* + IP$. The real rate on long-term Treasury bonds has historically ranged from 2% to 4%, with a mean of about 3%. Therefore, if no inflation were expected, long-term Treasury bonds would yield about 3%. However, as the expected rate of inflation increases, a premium must be added to the real risk-free rate of return to compensate investors for the loss of purchasing power that results from inflation. Therefore, the 6% r_{RF} shown in Figure 6-10 might be thought of as consisting of a 3% real risk-free rate of return plus a 3% inflation premium: $r_{RF} = r^* + IP = 3\% + 3\% = 6\%$.

If the expected inflation rate rose by 2%, to $3\% + 2\% = 5\%$, this would cause r_{RF} to rise to 8%. Such a change is shown in Figure 6-11. Notice that under the CAPM, the

¹⁶Long-term Treasury bonds also contain a maturity risk premium, MRP. Here we include the MRP in r^* to simplify the discussion. See Chapter 5 for more on bond pricing and bond risk premiums.

Figure 6-11

Shift in the SML Caused by an Increase in Inflation



increase in r_{RF} leads to an *equal* increase in the rate of return on all risky assets, because the same inflation premium is built into the required rate of return of both riskless and risky assets. For example, the rate of return on an average stock, r_M , increases from 11 to 13%. Other risky securities' returns also rise by 2 percentage points.

The discussion above also applies to any change in the nominal risk-free interest rate, whether it is caused by a change in expected inflation or in the real interest rate. The key point to remember is that a change in r_{RF} will not necessarily cause a change in the market risk premium, which is the required return on the market, r_M , minus the risk-free rate, r_{RF} . In other words, as r_{RF} changes, so may the required return on the market, keeping the market risk premium stable. Think of a sailboat floating in a harbor. The distance from the ocean floor to the ocean surface is like the risk-free rate, and it moves up and down with the tides. The distance from the top of the ship's mast to the ocean floor is like the required market return: It, too, moves up and down with the tides. But the distance from the mast-top to the ocean surface is like the market risk premium—it generally stays the same, even though tides move the ship up and down. In other words, a change in the risk-free rate also causes a change in the required market return, r_M , resulting in a relatively stable market risk premium, $r_M - r_{RF}$.

Changes in Risk Aversion

The slope of the Security Market Line reflects the extent to which investors are averse to risk—the steeper the slope of the line, the greater the average investor's risk aversion. Suppose investors were indifferent to risk; that is, they were not risk averse. If r_{RF} were 6%, then risky assets would also provide an expected return of 6%, because if there were no risk aversion, there would be no risk premium, and the SML would be plotted as a horizontal line. As risk aversion increases, so does the risk premium, and this causes the slope of the SML to become steeper.

Figure 6-12

Shift in the SML Caused by Increased Risk Aversion

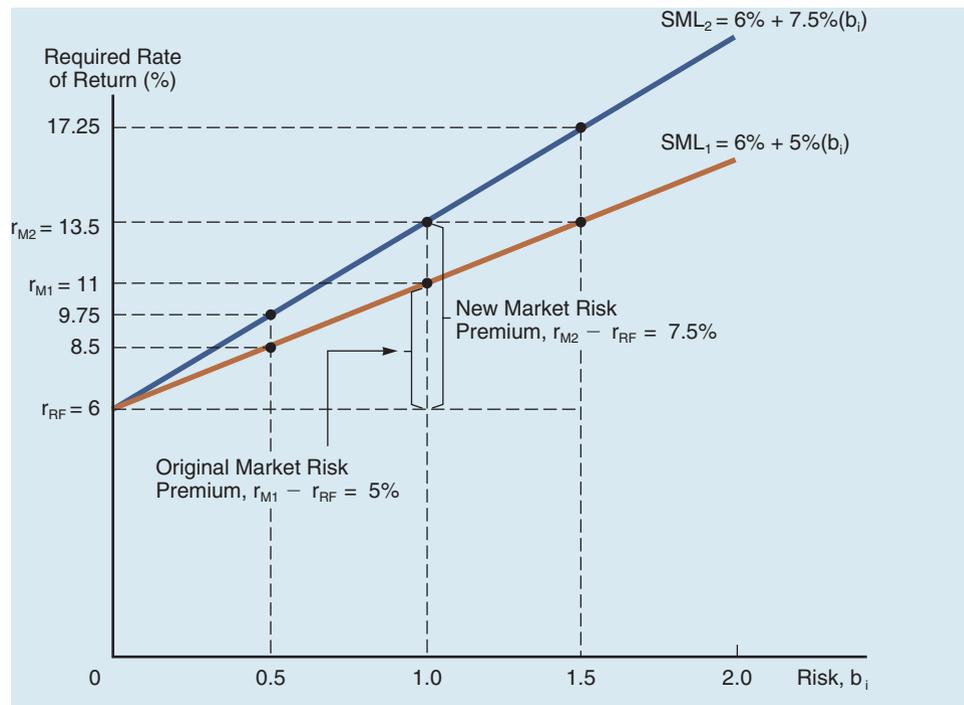


Figure 6-12 illustrates an increase in risk aversion. The market risk premium rises from 5 to 7.5%, causing r_M to rise from $r_{M1} = 11\%$ to $r_{M2} = 13.5\%$. The returns on other risky assets also rise, and the effect of this shift in risk aversion is more pronounced on riskier securities. For example, the required return on a stock with $b_i = 0.5$ increases by only 1.25 percentage points, from 8.5 to 9.75%, whereas that on a stock with $b_i = 1.5$ increases by 3.75 percentage points, from 13.5 to 17.25%.

Changes in a Stock's Beta Coefficient

As we shall see later in the book, a firm can influence its market risk, hence its beta, through changes in the composition of its assets and also through its use of debt. A company's beta can also change as a result of external factors such as increased competition in its industry, the expiration of basic patents, and the like. When such changes occur, the required rate of return also changes.

SELF-TEST

Differentiate among the expected rate of return (\hat{r}), the required rate of return (r), and the realized, after-the-fact return (\bar{r}) on a stock. Which would have to be larger to get you to buy the stock, \hat{r} or r ? Would \hat{r} , r , and \bar{r} typically be the same or different for a given company?

What are the differences between the relative volatility graph (Figure 6-8), where "betas are made," and the SML graph (Figure 6-10), where "betas are used"? Discuss both how the graphs are constructed and the information they convey.

What happens to the SML graph in Figure 6-10 when inflation increases or decreases?

What happens to the SML graph when risk aversion increases or decreases? What would the SML look like if investors were indifferent to risk, that is, had zero risk aversion?

How can a firm influence its market risk as reflected in its beta?

A stock has a beta of 1.4. Assume that the risk-free rate is 5.5% and the market risk premium is 5%. What is the stock's required rate of return? (12.5%)

6.6 The CAPM, Risk, and Return: Is Something Missing?

The Holy Grail of finance is the search for the relationship between risk and required rates of return. This relationship affects the securities purchased and sold by investors, the strategies chosen by portfolio managers, and the projects selected by corporate managers. In fact, most decisions in finance boil down to the trade-off between risk and return: Does the security or project in question have enough return to justify its risk? To answer this question, you must be able to specify the relationship between required return and risk. If the security or project provides at least the required return, then it is acceptable.

The Capital Asset Pricing Model (CAPM) was the first theory of risk and return to become widely used by analysts, investors, and corporations. One of its key contributions is the insight that required returns should not be affected by diversifiable risk and that only nondiversifiable risk matters. Indeed, investors have become more diversified as the CAPM has become more widely known.¹⁷ However, despite the CAPM's intuitive appeal, a number of studies have raised concerns about its validity. In particular, a study by Eugene Fama of the University of Chicago and Kenneth French of Yale casts doubt on the CAPM.¹⁸ Fama and French found two variables that are consistently related to stock returns: (1) the firm's size and (2) its market/book ratio. After adjusting for other factors, they found that smaller firms have provided relatively high returns and that returns are relatively high on stocks with low market/book ratios. At the same time, and contrary to the CAPM, they found no relationship between a stock's beta and its return.

As an alternative to the traditional CAPM, researchers and practitioners have begun to look to more general multifactor models that expand on the CAPM and address its shortcomings. The multifactor model is an attractive generalization of the traditional CAPM model's insight that market risk, or the risk that cannot be diversified away, underlies the pricing of assets. In a multifactor model, market risk is measured relative to a set of risk factors that determine the behavior of asset returns, whereas the CAPM gauges risk only relative to the market return. It is important to note that the risk factors in the multifactor models are all nondiversifiable sources of risk. Empirical research investigating the relationship between economic risk factors and security returns is ongoing, but it has discovered several risk factors—including the bond default premium, the bond term structure premium, and inflation—that affect most securities.

An underlying assumption of the CAPM (and most other risk-return models) is that investors are rational, or at least the large investors whose buying and selling actions determine security prices are rational. However, psychologists have long known that humans aren't always rational, and this has led to a new field in finance called behavioral finance. Behavioral finance seeks to explain why investors and managers make certain decisions, even if those decisions seem to contradict rational pricing models such as the CAPM.

We discuss the Fama-French models, the multifactor models, and behavioral finance in more detail in Chapter 7. And as we will discuss in Chapter 10, it is not

¹⁷There is evidence suggesting that investors still do not diversify enough. See Meir Statman, "The Diversification Puzzle," *Financial Analysts Journal*, 2004, pp. 44–53.

¹⁸See Eugene F. Fama and Kenneth R. French, "The Cross-Section of Expected Stock Returns," *Journal of Finance*, 1992, pp. 427–465; and Eugene F. Fama and Kenneth R. French, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 1993, pp. 3–56.

always easy to estimate beta or the market risk premium for the CAPM. Despite these issues, however, the CAPM is still the most widely used risk-return model for corporate finance applications.

SELF-TEST

Are there any reasons to question the validity of the CAPM? Explain.

Summary

In this chapter, we described the trade-off between risk and return. We began by discussing how to calculate risk and return for both individual assets and portfolios. In particular, we differentiated between stand-alone risk and risk in a portfolio context, and we explained the benefits of diversification. Finally, we developed the CAPM, which explains how risk affects rates of return. In the chapters that follow, we will give you the tools to estimate the required rates of return for bonds, preferred stock, and common stock, and we will explain how firms use these returns to develop their costs of capital. As you will see, the cost of capital is an important element in the firm's capital budgeting process. The key concepts covered in this chapter are listed below.

- **Risk** can be defined as the chance that some unfavorable event will occur.
- The risk of an asset's cash flows can be considered on a **stand-alone basis** (each asset by itself) or in a **portfolio context**, where the investment is combined with other assets and its risk is reduced through **diversification**.
- Most rational investors hold **portfolios of assets**, and they are more concerned with the riskiness of their portfolios than with the risk of individual assets.
- The **expected return** on an investment is the mean value of its probability distribution of returns.
- The **greater the probability** that the actual return will be far below the expected return, the **greater the stand-alone risk** associated with an asset.
- The average investor is **risk averse**, which means that he or she must be compensated for holding risky assets. Therefore, riskier assets have higher required returns than less risky assets.
- An asset's risk consists of (1) **diversifiable risk**, which can be eliminated by diversification, plus (2) **market risk**, which cannot be eliminated by diversification.
- The **relevant risk** of an individual asset is its contribution to the riskiness of a well-diversified **portfolio**, which is the asset's **market risk**. Since market risk cannot be eliminated by diversification, investors must be compensated for bearing it.
- A stock's **beta coefficient, b** , is a measure of its market risk. Beta measures the extent to which the stock's returns move relative to the market.
- A **high-beta stock** is more volatile than an average stock, while a **low-beta stock** is less volatile than an average stock. An average stock has $b = 1.0$.
- The **beta of a portfolio** is a **weighted average** of the betas of the individual securities in the portfolio.
- The **Security Market Line (SML)** equation shows the relationship between a security's market risk and its required rate of return. The return required for any security i is equal to the **risk-free rate** plus the **market risk premium** times the security's beta: $r_i = r_{RF} + (RP_M)b_i$.
- Even though the expected rate of return on a stock is generally equal to its required return, a number of things can happen to cause the required rate of

return to change: (1) **the risk-free rate can change** because of changes in either real rates or anticipated inflation, (2) **a stock's beta can change**, and (3) **investors' aversion to risk can change**.

- Because returns on assets in different countries are not perfectly correlated, **global diversification** may result in lower risk for multinational companies and globally diversified portfolios.

Questions

- (6-1) Define the following terms, using graphs or equations to illustrate your answers wherever feasible:
- Stand-alone risk; risk; probability distribution
 - Expected rate of return, \hat{r}
 - Continuous probability distribution
 - Standard deviation, σ ; variance, σ^2 ; coefficient of variation, CV
 - Risk aversion; realized rate of return, \bar{r}
 - Risk premium for Stock i , RP_i ; market risk premium, RP_M
 - Capital Asset Pricing Model (CAPM)
 - Expected return on a portfolio, \hat{r}_p ; market portfolio
 - Correlation coefficient, ρ ; correlation
 - Market risk; diversifiable risk; relevant risk
 - Beta coefficient, b ; average stock's beta, b_A
 - Security Market Line (SML); SML equation
 - Slope of SML as a measure of risk aversion
- (6-2) The probability distribution of a less risky return is more peaked than that of a riskier return. What shape would the probability distribution have for (a) completely certain returns and (b) completely uncertain returns?
- (6-3) Security A has an expected return of 7%, a standard deviation of returns of 35%, a correlation coefficient with the market of -0.3 , and a beta coefficient of -1.5 . Security B has an expected return of 12%, a standard deviation of returns of 10%, a correlation with the market of 0.7, and a beta coefficient of 1.0. Which security is riskier? Why?
- (6-4) Suppose you owned a portfolio consisting of \$250,000 worth of long-term U.S. government bonds.
- Would your portfolio be riskless?
 - Now suppose you hold a portfolio consisting of \$250,000 worth of 30-day Treasury bills. Every 30 days your bills mature, and you reinvest the principal (\$250,000) in a new batch of bills. Assume that you live on the investment income from your portfolio and that you want to maintain a constant standard of living. Is your portfolio truly riskless?
 - Can you think of any asset that would be completely riskless? Could someone develop such an asset? Explain.
- (6-5) If investors' aversion to risk increased, would the risk premium on a high-beta stock increase more or less than that on a low-beta stock? Explain.
- (6-6) If a company's beta were to double, would its expected return double?

- (6-7) Is it possible to construct a portfolio of stocks which has an expected return equal to the risk-free rate?

Self-Test Problems Solutions Appear in Appendix A

- (ST-1) Stocks A and B have the following historical returns:

Realized Rates of Return

Year	Stock A's Returns, \bar{r}_A	Stock B's Returns, \bar{r}_B
2003	(18%)	(24%)
2004	44	24
2005	(22)	(4)
2006	22	8
2007	34	56

- Calculate the average rate of return for each stock during the 5-year period. Assume that someone held a portfolio consisting of 50% of Stock A and 50% of Stock B. What would have been the realized rate of return on the portfolio in each year? What would have been the average return on the portfolio during this period?
- Now calculate the standard deviation of returns for each stock and for the portfolio. Use Equation 6-5.
- Looking at the annual returns data on the two stocks, would you guess that the correlation coefficient between returns on the two stocks is closer to 0.8 or to -0.8 ?
- If you added more stocks at random to the portfolio, which of the following is the most accurate statement of what would happen to σ_p ?
 - σ_p would remain constant.
 - σ_p would decline to somewhere in the vicinity of 20%.
 - σ_p would decline to zero if enough stocks were included.

- (ST-2) ECRI Corporation is a holding company with four main subsidiaries. The percentage of its business coming from each of the subsidiaries, and their respective betas, are as follows:

Beta and Required Rate of Return

Subsidiary	Percentage of Business	Beta
Electric utility	60%	0.70
Cable company	25	0.90
Real estate	10	1.30
International/special projects	5	1.50

- What is the holding company's beta?
- Assume that the risk-free rate is 6% and the market risk premium is 5%. What is the holding company's required rate of return?
- ECRI is considering a change in its strategic focus: it will reduce its reliance on the electric utility subsidiary, so the percentage of its business from this subsidiary will be 50%. At the same time, ECRI will increase its reliance on the international/special projects division, so the percentage of its business from that subsidiary will rise to 15%. What will be the shareholders' required rate of return if they adopt these changes?

Problems Answers Appear in Appendix B

Easy Problems 1–3

- (6-1) **Portfolio Beta** An individual has \$35,000 invested in a stock which has a beta of 0.8 and \$40,000 invested in a stock with a beta of 1.4. If these are the only two investments in her portfolio, what is her portfolio's beta?
- (6-2) **Required Rate of Return** Assume that the risk-free rate is 6% and the expected return on the market is 13%. What is the required rate of return on a stock that has a beta of 0.7?
- (6-3) **Expected and Required Rates of Return** Assume that the risk-free rate is 5% and the market risk premium is 6%. What is the expected return for the overall stock market? What is the required rate of return on a stock that has a beta of 1.2?

Intermediate Problems 4–9

- (6-4) **Expected Return—Discrete Distribution**

A stock's return has the following distribution:

Demand for the Company's Products	Probability of This Demand Occurring	Rate of Return if This Demand Occurs
Weak	0.1	(50%)
Below average	0.2	(5)
Average	0.4	16
Above average	0.2	25
Strong	<u>0.1</u>	60
	<u>1.0</u>	

Calculate the stock's expected return, standard deviation, and coefficient of variation.

- (6-5) **Expected Returns—Discrete Distribution**

The market and Stock J have the following probability distributions:

Probability	r_M	r_J
0.3	15%	20%
0.4	9	5
0.3	18	12

- a. Calculate the expected rates of return for the market and Stock J.
 b. Calculate the standard deviations for the market and Stock J.
 c. Calculate the coefficients of variation for the market and Stock J.
- (6-6) **Required Rate of Return** Suppose $r_{RF} = 5%$, $r_M = 10%$, and $r_A = 12%$.
 a. Calculate Stock A's beta.
 b. If Stock A's beta were 2.0, what would be A's new required rate of return?
- (6-7) **Required Rate of Return** Suppose $r_{RF} = 9%$, $r_M = 14%$, and $b_i = 1.3$.
 a. What is r_i , the required rate of return on Stock i?
 b. Now suppose r_{RF} (1) increases to 10% or (2) decreases to 8%. The slope of the SML remains constant. How would this affect r_M and r_i ?

- c. Now assume r_{RF} remains at 9% but r_M (1) increases to 16% or (2) falls to 13%. The slope of the SML does not remain constant. How would these changes affect r_i ?

(6-8) Portfolio Beta Suppose you hold a diversified portfolio consisting of a \$7,500 investment in each of 20 different common stocks. The portfolio beta is equal to 1.12. Now, suppose you have decided to sell one of the stocks in your portfolio with a beta equal to 1.0 for \$7,500 and to use these proceeds to buy another stock for your portfolio. Assume the new stock's beta is equal to 1.75. Calculate your portfolio's new beta.

(6-9) Portfolio Required Return Suppose you are the money manager of a \$4 million investment fund. The fund consists of four stocks with the following investments and betas:

Stock	Investment	Beta
A	\$ 400,000	1.50
B	600,000	(0.50)
C	1,000,000	1.25
D	2,000,000	0.75

If the market required rate of return is 14% and the risk-free rate is 6%, what is the fund's required rate of return?

Challenging Problems 10–13

(6-10) Portfolio Beta You have a \$2 million portfolio consisting of a \$100,000 investment in each of 20 different stocks. The portfolio has a beta equal to 1.1. You are considering selling \$100,000 worth of one stock which has a beta equal to 0.9 and using the proceeds to purchase another stock which has a beta equal to 1.4. What will be the new beta of your portfolio following this transaction?

(6-11) Required Rate of Return Stock R has a beta of 1.5, Stock S has a beta of 0.75, the expected rate of return on an average stock is 13%, and the risk-free rate of return is 7%. By how much does the required return on the riskier stock exceed the required return on the less risky stock?

(6-12) Historical Realized Rates of Return Stocks A and B have the following historical returns:

Year	Stock A's Returns, \bar{r}_A	Stock B's Returns, \bar{r}_B
2003	(18.00%)	(14.50%)
2004	33.00	21.80
2005	15.00	30.50
2006	(0.50)	(7.60)
2007	27.00	26.30

- Calculate the average rate of return for each stock during the 5-year period.
- Assume that someone held a portfolio consisting of 50% of Stock A and 50% of Stock B. What would have been the realized rate of return on the portfolio in each year? What would have been the average return on the portfolio during this period?
- Calculate the standard deviation of returns for each stock and for the portfolio.
- Calculate the coefficient of variation for each stock and for the portfolio.
- If you are a risk-averse investor, would you prefer to hold Stock A, Stock B, or the portfolio? Why?

(6-13)
Historical Returns:
Expected and Required
Rates of Return

You have observed the following returns over time:

Year	Stock X	Stock Y	Market
2003	14%	13%	12%
2004	19	7	10
2005	216	25	212
2006	3	1	1
2007	20	11	15

Assume that the risk-free rate is 6% and the market risk premium is 5%.

- What are the betas of Stocks X and Y?
- What are the required rates of return for Stocks X and Y?
- What is the required rate of return for a portfolio consisting of 80% of Stock X and 20% of Stock Y?
- If Stock X's expected return is 22%, is Stock X under- or overvalued?

Spreadsheet Problem

(6-14)
Evaluating Risk
and Return

Start with the partial model in the file *FM12 Ch 06 P14 Build a Model.xls* from the textbook's Web site. Bartman Industries' and Reynolds Incorporated's stock prices and dividends, along with the Market Index, are shown below. Stock prices are reported for December 31 of each year, and dividends reflect those paid during the year. The market data are adjusted to include dividends.



Year	Bartman Industries		Reynolds Incorporated		Market Index
	Stock Price	Dividend	Stock Price	Dividend	includes Divs.
2007	\$17.250	\$1.15	\$48.750	\$3.00	11,663.98
2006	14.750	1.06	52.300	2.90	8,785.70
2005	16.500	1.00	48.750	2.75	8,679.98
2004	10.750	0.95	57.250	2.50	6,434.03
2003	11.375	0.90	60.000	2.25	5,602.28
2002	7.625	0.85	55.750	2.00	4,705.97

- Use the data given to calculate annual returns for Bartman, Reynolds, and the Market Index, and then calculate average returns over the 5-year period. (Hint: Remember, returns are calculated by subtracting the beginning price from the ending price to get the capital gain or loss, adding the dividend to the capital gain or loss, and dividing the result by the beginning price. Assume that dividends are already included in the index. Also, you cannot calculate the rate of return for 2002 because you do not have 2001 data.)
- Calculate the standard deviations of the returns for Bartman, Reynolds, and the Market Index. (Hint: Use the sample standard deviation formula given in the chapter, which corresponds to the STDEV function in *Excel*.)
- Now calculate the coefficients of variation for Bartman, Reynolds, and the Market Index.
- Construct a scatter diagram graph that shows Bartman's and Reynolds's returns on the vertical axis and the Market Index's returns on the horizontal axis.

- e. Estimate Bartman's and Reynolds's betas as the slope of a regression with stock return on the vertical axis (y-axis) and market return on the horizontal axis (x-axis). (Hint: use *Excel's* SLOPE function.) Are these betas consistent with your graph?
- f. The risk-free rate on long-term Treasury bonds is 6.04%. Assume that the market risk premium is 5%. What is the expected return on the market? Now use the SML equation to calculate the two companies' required returns.
- g. If you formed a portfolio that consisted of 50% of Bartman stock and 50% of Reynolds stock, what would be its beta and its required return?
- h. Suppose an investor wants to include Bartman Industries' stock in his or her portfolio. Stocks A, B, and C are currently in the portfolio, and their betas are 0.769, 0.985, and 1.423, respectively. Calculate the new portfolio's required return if it consists of 25% of Bartman, 15% of Stock A, 40% of Stock B, and 20% of Stock C.



e-resource

Cyberproblem

Please go to the textbook's Web site to access any Cyberproblems.

Mini Case

Assume that you recently graduated with a major in finance, and you just landed a job as a financial planner with Barney Smith Inc., a large financial services corporation. Your first assignment is to invest \$100,000 for a client. Because the funds are to be invested in a business at the end of 1 year, you have been instructed to plan for a 1-year holding period. Further, your boss has restricted you to the investment alternatives shown in the table with their probabilities and associated outcomes. (Disregard for now the items at the bottom of the data; you will fill in the blanks later.)

		Returns on Alternative Investments					
		Estimated Rate of Return					
State of the Economy	Probability	T-Bills	Alta Industries	Repo Men	American Foam	Market Portfolio	2-Stock Portfolio
Recession	0.1	8.0%	(22.0%)	28.0%	10.0% ^a	(13.0%)	3.0%
Below average	0.2	8.0	(2.0)	14.7	(10.0)	1.0	
Average	0.4	8.0	20.0	0.0	7.0	15.0	10.0
Above average	0.2	8.0	35.0	(10.0)	45.0	29.0	
Boom	<u>0.1</u>	<u>8.0</u>	<u>50.0</u>	<u>(20.0)</u>	<u>30.0</u>	<u>43.0</u>	<u>15.0</u>
\hat{r}			1.7%	13.8%	15.0%		
σ		0.0		13.4	18.8	15.3	
CV			7.9	1.4	1.0		
b			-0.86	0.68			

^aNote that the estimated returns of American Foam do not always move in the same direction as the overall economy. For example, when the economy is below average, consumers purchase fewer mattresses than they would if the economy were stronger. However, if the economy is in a flat-out recession, a large number of consumers who were planning to purchase a more expensive inner spring mattress may purchase, instead, a cheaper foam mattress. Under these circumstances, we would expect American Foam's stock price to be higher if there is a recession than if the economy was just below average.

Barney Smith's economic forecasting staff has developed probability estimates for the state of the economy, and its security analysts have developed a sophisticated computer program that was used to estimate the rate of return on each alternative under each state of the economy. Alta Industries is an electronics firm; Repo Men Inc. collects past-due debts; and American Foam manufactures mattresses and various other foam products. Barney Smith also maintains an "index fund" which owns a market-weighted fraction of all publicly traded stocks; you can invest in that fund, and thus obtain average stock market results. Given the situation as described, answer the following questions.

- a. What are investment returns? What is the return on an investment that costs \$1,000 and is sold after 1 year for \$1,100?
- b. (1) Why is the T-bill's return independent of the state of the economy? Do T-bills promise a completely risk-free return? (2) Why are Alta Industries' returns expected to move with the economy whereas Repo Men's are expected to move counter to the economy?
- c. Calculate the expected rate of return on each alternative and fill in the blanks in the row for \hat{r} in the table.
- d. You should recognize that basing a decision solely on expected returns is appropriate only for risk-neutral individuals. Because your client, like virtually everyone, is risk averse, the riskiness of each alternative is an important aspect of the decision. One possible measure of risk is the standard deviation of returns. (1) Calculate this value for each alternative, and fill in the blank in the row for σ in the table. (2) What type of risk is measured by the standard deviation? (3) Draw a graph that shows *roughly* the shape of the probability distributions for Alta Industries, American Foam, and T-bills.
- e. Suppose you suddenly remembered that the coefficient of variation (CV) is generally regarded as being a better measure of stand-alone risk than the standard deviation when the alternatives being considered have widely differing expected returns. Calculate the missing CVs, and fill in the blanks in the row for CV in the table. Does the CV produce the same risk rankings as the standard deviation?
- f. Suppose you created a 2-stock portfolio by investing \$50,000 in Alta Industries and \$50,000 in Repo Men. (1) Calculate the expected return (\hat{r}_p), the standard deviation (σ_p), and the coefficient of variation (CV_p) for this portfolio and fill in the appropriate blanks in the table. (2) How does the risk of this 2-stock portfolio compare with the risk of the individual stocks if they were held in isolation?
- g. Suppose an investor starts with a portfolio consisting of one randomly selected stock. What would happen (1) to the risk and (2) to the expected return of the portfolio as more and more randomly selected stocks were added to the portfolio? What is the implication for investors? Draw a graph of the two portfolios to illustrate your answer.
- h. (1) Should portfolio effects impact the way investors think about the risk of individual stocks? (2) If you decided to hold a 1-stock portfolio, and consequently were exposed to more risk than diversified investors, could you expect to be compensated for all of your risk; that is, could you earn a risk premium on that part of your risk that you could have eliminated by diversifying?
- i. How is market risk measured for individual securities? How are beta coefficients calculated?
- j. Suppose you have the following historical returns for the stock market and for another company, P. Q. Unlimited. Explain how to calculate beta, and use the historical stock returns to calculate the beta for PQU. Interpret your results.

Year	Market	PQU
1	25.7%	40.0%
2	8.0	215.0
3	211.0	215.0
4	15.0	35.0
5	32.5	10.0
6	13.7	30.0
7	40.0	42.0
8	10.0	210.0
9	210.8	225.0
10	213.1	25.0

- k. The expected rates of return and the beta coefficients of the alternatives as supplied by Barney Smith's computer program are as follows:

Security	Return (\hat{r})	Risk (Beta)
Alta Industries	17.4%	1.29
Market	15.0	1.00
American Foam	13.8	0.68
T-bills	8.0	0.00
Repo Men	1.7	(0.86)

- (1) Do the expected returns appear to be related to each alternative's market risk? (2) Is it possible to choose among the alternatives on the basis of the information developed thus far?
- l. (1) Write out the Security Market Line (SML) equation, use it to calculate the required rate of return on each alternative, and then graph the relationship between the expected and required rates of return. (2) How do the expected rates of return compare with the required rates of return? (3) Does the fact that Repo Men has an expected return that is less than the T-bill rate make any sense? (4) What would be the market risk and the required return of a 50–50 portfolio of Alta Industries and Repo Men? Of Alta Industries and American Foam?
- m. (1) Suppose investors raised their inflation expectations by 3 percentage points over current estimates as reflected in the 8% T-bill rate. What effect would higher inflation have on the SML and on the returns required on high- and low-risk securities? (2) Suppose instead that investors' risk aversion increased enough to cause the market risk premium to increase by 3 percentage points. (Inflation remains constant.) What effect would this have on the SML and on returns of high- and low-risk securities?

Selected Additional Cases

The following cases from Textchoice, Thomson Learning's online library, cover many of the concepts discussed in this chapter and are available at <http://www.textchoice2.com>.

Klein-Brigham Series:

Case 2, "Peachtree Securities, Inc. (A)."

Brigham-Buzzard Series:

Case 2, "Powerline Network Corporation (Risk and Return)."