

# Chapter 34

---

## PORTFOLIO PERFORMANCE EVALUATION

---

LALITH P. SAMARAKOON, *University of St. Thomas, USA*  
TANWEER HASAN, *Roosevelt University, USA*

### Abstract

*The portfolio performance evaluation involves the determination of how a managed portfolio has performed relative to some comparison benchmark. Performance evaluation methods generally fall into two categories, namely conventional and risk-adjusted methods. The most widely used conventional methods include benchmark comparison and style comparison. The risk-adjusted methods adjust returns in order to take account of differences in risk levels between the managed portfolio and the benchmark portfolio. The major methods are the Sharpe ratio, Treynor ratio, Jensen's alpha, Modigliani and Modigliani, and Treynor Squared. The risk-adjusted methods are preferred to the conventional methods.*

**Keywords:** performance; evaluation; standard deviation; systematic risk; conventional methods; benchmark comparison; style comparison; risk-adjusted measures; Sharpe measure; Treynor measure; Jensen measure; alpha; Modigliani-Modigliani measure; Treynor squared

### 34.1. Introduction

The portfolio performance evaluation primarily refers to the determination of how a particular investment portfolio has performed relative to some comparison benchmark. The evaluation can

indicate the extent to which the portfolio has outperformed or under-performed, or whether it has performed at par with the benchmark.

The evaluation of portfolio performance is important for several reasons. First, the investor, whose funds have been invested in the portfolio, needs to know the relative performance of the portfolio. The performance review must generate and provide information that will help the investor to assess any need for rebalancing of his investments. Second, the management of the portfolio needs this information to evaluate the performance of the manager of the portfolio and to determine the manager's compensation, if that is tied to the portfolio performance. The performance evaluation methods generally fall into two categories, namely conventional and risk-adjusted methods.

### 34.2. Conventional Methods

#### 34.2.1. Benchmark Comparison

The most straightforward conventional method involves comparison of the performance of an investment portfolio against a broader market index. The most widely used market index in the United States is the S&P 500 index, which measures the price movements of 500 U.S. stocks compiled by the Standard & Poor's Corporation. If the return on the portfolio exceeds that of the benchmark

index, measured during identical time periods, then the portfolio is said to have beaten the benchmark index. While this type of comparison with a passive index is very common in the investment world, it creates a particular problem. The level of risk of the investment portfolio may not be the same as that of the benchmark index portfolio. Higher risk should lead to commensurately higher returns in the long term. This means if the investment portfolio has performed better than the benchmark portfolio, it may be due to the investment portfolio being more risky than the benchmark portfolio. Therefore, a simple comparison of the return on an investment portfolio with that of a benchmark portfolio may not produce valid results.

#### 34.2.2. *Style Comparison*

A second conventional method of performance evaluation called “style-comparison” involves comparison of return of a portfolio with that having a similar investment style. While there are many investment styles, one commonly used approach classifies investment styles as value versus growth. The “value style” portfolios invest in companies that are considered undervalued on the basis of yardsticks such as price-to-earnings and price-to-book value multiples. The “growth style” portfolios invest in companies whose revenue and earnings are expected to grow faster than those of the average company.

In order to evaluate the performance of a value-oriented portfolio, one would compare the return on such a portfolio with that of a benchmark portfolio that has value-style. Similarly, a growth-style portfolio is compared with a growth-style benchmark index. This method also suffers from the fact that while the style of the two portfolios that are compared may look similar, the risks of the two portfolios may be different. Also, the benchmarks chosen may not be truly comparable in terms of the style since there can be many important ways in which two similar style-oriented funds vary.

Reilly and Norton (2003) provide an excellent disposition of the use of benchmark portfolios and portfolios style and the issues associated with their selection. Sharpe (1992), and Christopherson (1995) have developed methods for determining this style.

### 34.3. Risk-adjusted Methods

The risk-adjusted methods make adjustments to returns in order to take account of the differences in risk levels between the managed portfolio and the benchmark portfolio. While there are many such methods, the most notable are the Sharpe ratio ( $S$ ), Treynor ratio ( $T$ ), Jensen’s alpha ( $\alpha$ ), Modigliani and Modigliani ( $M^2$ ), and Treynor Squared ( $T^2$ ). These measures, along with their applications, are discussed below.

#### 34.3.1. *Sharpe Ratio*

The Sharpe ratio (Sharpe, 1966) computes the risk premium of the investment portfolio per unit of total risk of the portfolio. The risk premium, also known as excess return, is the return of the portfolio less the risk-free rate of interest as measured by the yield of a Treasury security. The total risk is the standard deviation of returns of the portfolio. The numerator captures the reward for investing in a risky portfolio of assets in excess of the risk-free rate of interest while the denominator is the variability of returns of the portfolio. In this sense, the Sharpe measure is also called the “reward-to-variability” ratio. Equation (34.1) gives the Sharpe ratio:

$$S = \frac{r_p - r_f}{\sigma_p} \quad (34.1)$$

where  $S$  is the Sharpe ratio,  $r_p$  the return of the portfolio,  $r_f$  the risk-free rate, and  $\sigma_p$  the standard deviation of returns of the portfolio.

The Sharpe ratio for an investment portfolio can be compared with the same for a benchmark portfolio such as the overall market portfolio. Suppose that a managed portfolio earned a return of

20 percent over a certain time period with a standard deviation of 32 percent. Also assume that during the same period the Treasury bill rate was 4 percent, and the overall stock market earned a return of 13 percent with a standard deviation of 20 percent. The managed portfolio's risk premium is (20 percent – 4 percent) = 16 percent, while its Sharpe ratio,  $S$ , is equal to 16 percent/32 percent = 0.50. The market portfolio's excess return is (13 percent – 4 percent) = 9 percent, while its  $S$  equals 9 percent/20 percent = 0.45. Accordingly, for each unit of standard deviation, the managed portfolio earned a risk premium of 0.50 percent, which is greater than that of the market portfolio of 0.45 percent, suggesting that the managed portfolio outperformed the market after adjusting for total risk.

### 34.3.2. Treynor Ratio

The Treynor ratio (Treynor, 1965) computes the risk premium per unit of systematic risk. The risk premium is defined as in the Sharpe measure. The difference in this method is in that it uses the systematic risk of the portfolio as the risk parameter. The systematic risk is that part of the total risk of an asset which cannot be eliminated through diversification. It is measured by the parameter known as 'beta' that represents the slope of the regression of the returns of the managed portfolio on the returns to the market portfolio. The Treynor ratio is given by the following equation:

$$T = \frac{r_p - r_f}{\beta_p} \quad (34.2)$$

where  $T$  is the Treynor ratio,  $r_p$  the return of the portfolio,  $r_f$  the risk-free rate, and  $\beta_p$  the beta of the portfolio.

Suppose that the beta of the managed portfolio in the previous example is 1.5. By definition, the beta of the market portfolio is equal to 1.0. This means the managed portfolio has one-and-half times more systematic risk than the market portfolio. We would expect the managed portfolio to earn more than the market because of its higher

risk. In fact, in the above example, the portfolio earned an excess return of 16 percent whereas the market earned only 9 percent. These two numbers alone do not tell anything about the relative performance of the portfolio since the portfolio and the market have different levels of market risk. In this instance, the Treynor ratio for the managed portfolio equals (20 percent – 4 percent)/1.5 = 10.67, while that for the market equals (13 percent – 4 percent)/1.00 = 9.00. Thus, after adjusting for systematic risk, the managed portfolio earned an excess return of 10.67 percent for each unit of beta while the market portfolio earned an excess return of 9.00 percent for each unit of beta. Thus, the managed portfolio outperformed the market portfolio after adjusting for systematic risk.

### 34.3.3. Jensen's Alpha

Jensen's alpha (Jensen, 1968) is based on the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966). The alpha represents the amount by which the average return of the portfolio deviates from the expected return given by the CAPM. The CAPM specifies the expected return in terms of the risk-free rate, systematic risk, and the market risk premium. The alpha can be greater than, less than, or equal to zero. An alpha greater than zero suggests that the portfolio earned a rate of return in excess of the expected return of the portfolio. Jensen's alpha is given by.

$$\alpha = r_p - [r_f + \beta_p(r_m - r_f)] \quad (34.3)$$

where  $\alpha$  is the Jensen's alpha,  $r_p$  the return of the portfolio,  $r_m$  the return of the market portfolio,  $r_f$  the risk-free rate, and  $\beta_p$  the beta of the portfolio.

Using the same set of numbers from the previous example, the alpha of the managed portfolio and the market portfolio can be computed as follows. The expected return of the managed portfolio is 4 percent + 1.5 (13 percent – 4 percent) = 17.5 percent. Therefore, the alpha of the managed

portfolio is equal to the actual return less the expected return, which is 20 percent – 17.5 percent = 2.5 percent. Since we are measuring the expected return as a function of the beta and the market risk premium, the alpha for the market is always zero. Thus, the managed portfolio has earned a 2.5 percent return above that must be earned given its market risk. In short, the portfolio has a positive alpha, suggesting superior performance.

When the portfolio is well diversified all three methods – Sharpe, Treynor, and Jensen – will give the same ranking of performance. In the example, the managed portfolio outperformed the market on the basis of all three ratios. When the portfolio is not well diversified or when it represents the total wealth of the investor, the appropriate measure of risk is the standard deviation of returns of the portfolio, and hence the Sharpe ratio is the most suitable. When the portfolio is well diversified, however, a part of the total risk has been diversified away and the systematic risk is the most appropriate risk metric. Both Treynor ratio and Jensen's alpha can be used to assess the performance of well-diversified portfolios of securities. These two ratios are also appropriate when the portfolio represents a sub-portfolio or only a part of the client's portfolio. Chen and Lee (1981, 1986) examined the statistical distribution of Sharpe, Treynor, and Jensen measures and show that the empirical relationship between these measures and their risk proxies is dependent on the sample size, the investment horizon and market conditions. Cumby and Glen (1990), Grinblatt and Titman (1994), Kallaberg et al. (2000), and Sharpe (1998) have provided evidence of the application of performance evaluation techniques.

#### 34.3.4. Modigliani and Modigliani Measure

The Sharpe ratio is not easy to interpret. In the example, the Sharpe ratio for the managed portfolio is 0.50, while that for the market is 0.45. We concluded that the managed portfolio outper-

formed the market. The difficulty, however, is that the differential performance of 0.05 is not an excess return. Modigliani and Modigliani (1997) measure, which is referred to as  $M^2$ , provides a risk-adjusted measure of performance that has an economically meaningful interpretation. The  $M^2$  is given by

$$M^2 = r_{p^*} - r_m \quad (34.4)$$

where  $M^2$  is the Modigliani-Modigliani measure,  $r_{p^*}$  the return on the adjusted portfolio,  $r_m$  the return on the market portfolio.

The adjusted portfolio is the managed portfolio adjusted in such a way that it has the same total risk as the market portfolio. The adjusted portfolio is constructed as a combination of the managed portfolio and risk-free asset, where weights are specified as in Equations (34.5) and (34.6).

$$w_{r_p} = \frac{\sigma_m}{\sigma_p} \quad (34.5)$$

$$w_{r_f} = 1 - w_{r_p} \quad (34.6)$$

where  $w_{r_p}$  represents the weight given to the managed portfolio, which is equal to the standard deviation of the market portfolio ( $\sigma_m$ ) divided by the standard deviation of the managed portfolio ( $\sigma_p$ ).  $w_{r_f}$  is the weight on the risk-free asset and is equal to one minus the weight on the managed portfolio. The risk of the adjusted portfolio ( $\sigma_{p^*}$ ) is the weight on the managed portfolio times the standard deviation of the managed portfolio as given in Equation (34.7). By construction, this will be equal to the risk of the market portfolio.

$$\sigma_{p^*} = w_{r_p} \times \sigma_p = \frac{\sigma_m}{\sigma_p} \times \sigma_p = \sigma_m \quad (34.7)$$

The return of the adjusted portfolio ( $r_{p^*}$ ) is computed as the weighted average of the returns of the managed portfolio and the risk-free rate, where the weights are as in Equations (34.5) and (34.6) above:

$$r_{p^*} = w_{r_f} \times r_f + w_{r_p} \times r_p \quad (34.8)$$

The return on the adjusted portfolio can be readily compared with the return on the market portfolio since both have the same degree of risk. The differential return,  $M^2$ , indicates the excess return of the managed portfolio in comparison to the benchmark portfolio after adjusting for differences in the total risk. Thus,  $M^2$  is more meaningful than the Sharpe ratio.

In the example, the standard deviation of the managed portfolio is 32 percent and the standard deviation of the market portfolio is 20 percent. Hence, the  $w_{r_p} = 20/32 = 0.625$ , and  $w_{r_f} = 1 - 0.625 = 0.375$ . The adjusted portfolio would be 62.5 percent invested in the managed portfolio and 37.5 percent invested in Treasury bills. Now the risk of the adjusted portfolio,  $\sigma_{p^*} = 0.625 \times 32$  percent = 20 percent, is the same as the risk of the market portfolio. The return on the adjusted portfolio would be  $r_{p^*} = 0.375 \times 4$  percent +  $0.625 \times 20$  percent = 14 percent. The  $M^2 = 14$  percent - 13 percent = 1 percent. Thus, on a risk-adjusted basis, the managed portfolio has performed better than the benchmark by 1 percent.

### 34.3.5. Treynor Squared

Another performance measure, called  $T^2$  analogous to  $M^2$ , can be constructed. This is a deviant of the Treynor measure, and the rationale is the same as that of  $M^2$ .  $T^2$  is defined as

$$T^2 = r_{p^*} - r_m \tag{34.9}$$

where  $T^2$  is the Treynor-squared measure,  $r_{p^*}$  the return on the adjusted portfolio, and  $r_m$  the return on the market portfolio.

The adjusted portfolio is the managed portfolio adjusted such that it has the same degree of systematic or market risk as the market portfolio. Since the market risk or beta of the market portfolio is equal to one, the adjusted portfolio is constructed as a combination of the managed portfolio and risk-free asset such that the adjusted portfolio has a beta equal to one. The weights are specified as in equations below.

$$w_{r_p} = \frac{\beta_m}{\beta_p} \tag{34.10}$$

$$w_{r_f} = 1 - w_{r_p} \tag{34.11}$$

where  $w_{r_p}$  represents the weight given to the managed portfolio, which is equal to the beta of the market portfolio ( $\beta_m$ ) divided by the beta of the managed portfolio ( $\beta_p$ ).  $w_{r_f}$  is the weight on the risk-free asset and is equal to one minus the weight on the managed portfolio. The beta of the adjusted portfolio ( $\beta_{p^*}$ ) is the weight on the managed portfolio times the beta of the managed portfolio, and this will be equal to the risk of the market portfolio as shown in the following equation:

$$\beta_{p^*} = w_{r_p} \times \beta_p = \frac{\beta_m}{\beta_p} \times \beta_p = \beta_m \tag{34.12}$$

The return of the adjusted portfolio ( $r_{p^*}$ ) is computed as the weighted average of the returns of the managed portfolio and the risk-free rate, where the weights are as determined above in equations (34.10) and (34.11):

$$r_{p^*} = w_{r_f} \times r_f + w_{r_p} \times r_p \tag{34.13}$$

The return on the adjusted portfolio can be readily compared with the return on the market portfolio since both have the same level of market risk. The differential return,  $T^2$ , indicates the excess return of the managed portfolio in comparison to the benchmark portfolio after adjusting for differences in the market risk.

In the example, the beta of the managed portfolio is 1.5. Hence,  $w_{r_p} = 1.0/1.5 = 0.67$  and  $w_{r_f} = 1 - 0.67 = 0.33$ . The adjusted portfolio would be 67 percent invested in the managed portfolio and 33 percent invested in Treasury bills. The beta of the adjusted portfolio,  $\sigma_{p^*} = 0.67 \times 1.5 = 1.00$ , which is equal to the beta of the market portfolio. The return on the adjusted portfolio would be  $r_{p^*} = 0.33 \times 4$  percent +  $0.67 \times 20$  percent = 14.72 percent.  $T^2 = 14.72$  percent - 13.00 percent = 1.72 percent. Thus, after adjusting for market risk, the managed portfolio has performed better than the benchmark by 1.72

percent.  $T^2$  is a better measure of relative performance when the market risk of a managed portfolio is the relevant risk metric.

#### REFERENCES

- Chen, S.N. and Lee, C.F. (1981). "The sampling relationship between Sharpe's performance measure and its risk proxy: sample size, investment horizon and market conditions." *Management Science*, 27: 607–618.
- Chen, S.N. and Lee, C.F. (1986). "The effects of the sample size, the investment horizon and market conditions on the validity of composite performance measures: A generalization." *Management Science*, 32: 1410–1421.
- Christopherson, J.A. (1995). "Equity style classifications." *Journal of Portfolio Management*, 21: 32–43.
- Cumby, R.E. and Glen, J.D. (1990). "Evaluating the performance of international mutual funds." *Journal of Finance*, 45: 497–521.
- Grinblatt, M. and Titman, S. (1994). "A study of monthly mutual fund returns and performance evaluation techniques." *Journal of Financial and Quantitative Analysis*, 29: 419–444.
- Jensen, M.C. (1968). "The performance of mutual funds in the period 1945–1964." *Journal of Finance*, 23: 389–416.
- Kallaberg, J.G., Lin, C.L., and Trzcinka, C. (2000). "The value added from investment managers: an examination of funds of REITs." *Journal of Financial Quantitative Analysis*, 35: 387–408.
- Lintner, J. (1965). "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets." *Review of Economics and Statistics*, 47: 13–47.
- Modigliani, F. and Modigliani, L. (1997). "Risk-adjusted performance: how to measure it and why." *Journal of Portfolio Management*, 23: 45–54.
- Mossin, J. (1966). "Equilibrium in the capital market." *Econometrica*, 34: 768–783.
- Reilly, F.K. and Norton, E.A. (2003). *Investments*, 6th edn. Mason, Ohio: Thompson-Southwestern.
- Sharpe, W.F. (1964). "Capital asset prices: a theory of market equilibrium under conditions of risk." *Journal of Finance*, 19: 425–442.
- Sharpe, W.F. (1966). "Mutual fund performance." *Journal of Business*, 39(1): 119–138.
- Sharpe, W.F. (1992). "Asset allocation: management style and performance measurement." *Journal of Portfolio Management*, 18: 7–19.
- Sharpe, W.F. (1998). "Morningstar's risk-adjusted ratings." *Financial Analysts Journal*, 54: 21–33.
- Treynor, J.L. (1965). "How to rate management of investment funds." *Harvard Business Review*, 43: 63–75.