

Chapter 26

EXPERIMENTAL ECONOMICS AND THE THEORY OF FINANCE

HAIM LEY, *Hebrew University, Israel*

Abstract

Experimental findings and in particular Prospect Theory and Cumulative Prospect Theory contradict Expected Utility Theory, which in turn may have a direct implication to theoretical models in finance and economics. We show growing evidence against Cumulative Prospect Theory. Moreover, even if one accepts the experimental results of Cumulative Prospect Theory, we show that most theoretical models in finance are robust. In particular, the CAPM is intact even if investors make decisions based on change of wealth, employ decision weights, and are risk-seeking in the negative domain.

Keywords: decision weights; prospect theory; cumulative prospect theory; certainty effect; expected utility; stochastic dominance; prospect stochastic dominance; value function; Markowitz stochastic dominance; configural weights

26.1. Introduction

Theoretical models in finance are based on certain assumptions regarding the investors' characteristics and their investment behavior. In particular, most of these models assume rational investors who always prefer more than less consumption (money), and who maximize von Neumann–Morgenstern (1944) expected utility.

The main models in finance that we relate to in this paper are:

- (1) The Modigliani–Miller (1958) relationship between the value of the firm and its capital structure.
- (2) Black–Scholes (1973) option pricing.
- (3) Ross's (1976) Arbitrage Pricing Model (APT).
- (4) The Sharpe–Lintner (1964 and 1965, respectively) Capital Asset Pricing Model (CAPM).
- (5) Stochastic Dominance—the various investment decision rules (for a review, see Levy 1992, 1998).
- (6) Market Efficiency – though recently some empirical studies reveal (short term) autocorrelations, most academic research still assumes that the market is at least “weakly efficient,” namely one cannot employ *ex-post* rates of return to establish investment rules that provide abnormal returns. Of course, if this is the case, there is no room for “technicians” and charterists who try to predict the market based on past rates of return. (For the market efficiency hypotheses see Fama, 1965, 1991).

In this paper, we analyze the impact of recent experimental finding, and particularly the implication of Prospect Theory (PT) (see Kahneman and Tversky, 1979) (K&T), Cumulative Prospect Theory (CPT) (see Tversky and Kahneman, 1992

(T&), and Rank-Dependent Expected utility (RDEU) (see O'ggin 1982, 1993) on each of these subjects that are cornerstones in finance and in decision making under uncertainty.

The structure of this paper is as follows: In Section 26.2, we deal with the main findings of PT and their implication regarding the above mentioned topics. In Section 26.3, we cover experimental studies in finance focusing on some recent studies, which cast doubt on some of the results and claims of PT and CPT. In Section 26.4, we analyze the implication of the experimental findings to the theory of finance. Concluding remarks are given in Section 26.5.

26.2. Allias Paradox, PT, CPT, and RDEU: Claims and Implication to the Theory of Finance

26.2.1. Probability Distortions (or Decision Weights)

Most models in economics and finance assume expected utility maximization. Probably the most famous example contradicting the expected utility paradigm is provided by Allias, and is known as the Allias paradox (1953). Table 26.1 provides two

Table 26.1. Allias paradox. All outcomes are in million \$

Part I			
Outcome	A Probability	Outcome	B Probability
1	1	0	0.01
		1	0.89
		5	0.10
Part II			
Outcome	C Probability	Outcome	D Probability
0	0.89	0	0.90
1	0.11	5	0.10

choices in both part I and part II. In part I most subjects would typically choose *A*, while in part II most of them choose *D*. Such choices constitute a contradiction to the classic EU paradigm because from the choice in part I we can conclude that:

$$u(1) > 0.01u(0) + 0.89u(1) + 0.10 u(5)$$

This inequity can be rewritten as

$$0.11u(1) > 0.01 u(0) + 0.10 u(5), \tag{26.1}$$

and the choices in part II implies that

$$0.89u(0) + 0.11u(1) < 0.9u(0) + 0.10u(5)$$

The last inequality can be rewritten also as

$$0.11u(1) < 0.01u(0) + 0.10u(5) \tag{26.2}$$

As Equations. (26.1) and (26.2) contradict each other for any preference *u*, we have an inconsistency in the choices in part I and II. How can we explain this result? Does it mean that the EU paradigm is completely wrong? And if the answer is positive, do we have a better substitute to the EU paradigm?

The preference of *D* over *C* is not surprising. However, the preference of *A* over *B* in Part I seems to induce the paradox. The choice of *A* is well-known as the “certainty effect,” (see Kahneman and Tversky, 1979), i.e. the “one bird in the hand is worth more than two in the bush” effect. The explanation for the contradiction in Equations (26.1) and (26.2) is due to the “certainty effect,” or alternatively, due to probability distortion in the case where probabilities are smaller than 1. Indeed, experimental psychologists find that subjects tend to subjectively distort probabilities in their decision making. To be more specific, one makes a decision using a weight *w(p)* rather than the objective probability *p*. In our specific case, *w(0.01) > 0.01* – hence the attractiveness of *B* relative to *A* decreases, which explains the choice of *A* in this case. However, in such a case, the classical von Neuman–Morgenstern expected utility is rejected once decision weight *w(p)* is employed rather than objective probability *p*.

Probability distortions or decision weights is a subject of many experimental studies conducted mainly by psychologists. Probably the earliest experiments showing that subjects distort probabilities were conducted by Preston and Baratta (1948) and Edwards (1955, 1962). However, the publication of Prospect Theory (PT) by Kahneman and Tversky in 1979 in *Econometrica* has exposed this issue widely to economists, and hence has strongly influenced research in economics and finance. Though decision weights is an old notion, it is still currently occupying researchers (see for example, Prelec, 1998).

In their original paper, Kahneman and Tversky argue that probability p is changed to decision weight $w(p)$ in some systematic manner. However, probability distortion as suggested by Kahneman and Tversky (1979) as well as in the previous studies mentioned above may violate First degree Stochastic Dominance (FSD) or the monotonicity axiom, a property that most economists and psychologists alike are not willing to give up, because violation of FSD essentially means preferring less over more money. Before we illustrate this property, let us first define FSD.

FSD: Let F and G be the cumulative distributions of the returns on two uncertain prospects. Then F dominates G by FSD if $F(x) \leq G(x)$ for all x , and there is at least one strict inequity. Moreover,

$$F(x) \leq G(x) \text{ for all } x \Leftrightarrow E_F u(x) \geq E_G u(x)$$

for all utility function $u \in U_1$ where

U_1 is the set of all nondecreasing utility functions ($u' \geq 0$) (see Hanoch and Levy, 1969; Hadar and Russell, 1969). For a survey and more details, see Levy, 1992, 1998.

Let us illustrate with an example why the decision weights framework of PT may lead to a violation of FSD.

Example: Consider two prospects x and y . Suppose that x gets the values 3 and 4 with equal probability, and y gets the value 4 with certainty. It is obvious that y dominates x by FSD. Yet, with pos-

sible decision weights $w(1/2) = 3/4$ and $w(1) = 1$, we may find a legitimate preference showing a higher expected value for x , i.e. x is preferred to y despite the fact that y dominates x by FSD. For example, for the function $u(x) = x$ (the same is true for many other utility functions), we have

$$\begin{aligned} EU(x) &= \left(\frac{3}{4}\right)3 + \left(\frac{3}{4}\right)4 = \frac{21}{4} = 5\frac{1}{4} > EU(y) \\ &= 1 \times 4 = 4 \end{aligned}$$

Thus, the FSD inferior prospect is selected, which is an undesired result.

Fishburn (1978) shows that this distortion of probability may contradict FSD, or the monotonicity property, which is considered as a fatal flaw of such a probability distortion framework (see also Machina, 1994, p. 97). Qiggin (1982) offers a remedy to this problem. He suggests that the probability distortion should be done as a function of the cumulative distribution rather than as a function of the individual probabilities (for more studies along this line, see also Wakker et al., 1994; Yaari, 1987; Machina 1994).

According to Qiggin, a given probability p may be distorted in different ways depending on the ranking of the outcome it corresponds to. Thus, the probability $p = 1/4$ may be distorted to different values $w_i(p)$, depending on the rank of the i th outcome. For example, take the following prospect:

$$x = 1, 2, 3, 4$$

$$P(x) = 1/4 \quad 1/4 \quad 1/4 \quad 1/4$$

Then $w(1/4)$ corresponding to $x = 1$ may be larger than $1/4$ and $w(1/4)$ corresponding to $x = 2$ may be smaller than $1/4$ (the opposite relationship is also possible). Thus, the probability distortion is not only a function of the probability p_i but also on the rank of the corresponding outcome, hence the name Rank-Dependent Expected Utility (RDEU). This is in sharp contrast to the decision weights suggested by Kahneman and Tversky in 1979, because by the original PT, $w(1/4)$ is the same for all values and does not

depend on the rank of the outcome. Realizing the possible FSD violation, Qiggin (1982) suggests a modification to PT where a transformation of the cumulative distribution is employed. This idea is the basis for Cumulative Prospect Theory (CPT). By this model, the decision weight is also a function of the rank of the outcome. However, unlike Qiggin, Tversky and Kahneman distinguish between negative and positive outcomes. To be more specific, in the CPT framework, the decision weights are given as follows. Consider a prospect $(x_1, p_1; \dots; x_n, p_n)$, where p_i denote the objective probabilities and x_i denote the outcomes. Assume, without loss of generality, that $x_1 \leq \dots \leq x_k \leq 0 \leq x_{k+1} \leq \dots \leq x_n$. The decision weights, which are employed in CPT are given by

$$\begin{aligned} \pi_1 &= w^-(p_1), \pi_n = w^-(p_n), \\ \pi_i &= w^-(p_1 + \dots + p_i) - w^-(p_1 + \dots + p_{i-1}) \\ &\quad \text{for } 2 \leq i \leq k, \\ \pi_i &= w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n) \\ &\quad \text{for } k + 1 \leq i \leq n - 1, \end{aligned}$$

where w^- and w^+ are weighting functions, which Tversky and Kahneman (1992) experimentally estimate by the functions,

$$\begin{aligned} w^+(x) &= \frac{x^\gamma}{(x^\gamma + (1-x)^\gamma)^{1/\gamma}} \text{ and} \\ w^-(x) &= \frac{x^\delta}{(x^\delta + (1-x)^\delta)^{1/\delta}} \end{aligned} \tag{26.3}$$

Given these formulas, Tversky and Kahneman find the following estimates: $\hat{\gamma} = 0.61$ and $\hat{\delta} = 0.69$ (see Tversky and Kahneman, 1992, pp. 309–312). It can be easily shown that for $\gamma < 1$ and $\delta < 1$, the weighting functions have a reverse S-shape, implying the overweighting of small probabilities. The probability distortion as suggested by Kahneman and Tversky is illustrated in Figure 26.1.

Several researchers argue that in cases of equally likely outcomes, which we call here “uniform” probability distribution, probabilities are not distorted. Qiggin (1982), who was the first one to propose that cumulative probabilities are distorted

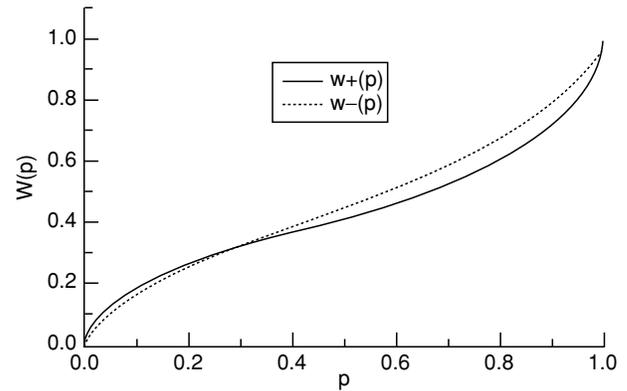


Figure 26.1. CPT decision weights

rather than the raw individual probabilities, argues that for “two equally likely” outcomes ($p = 0.50$) there will be no distortion of probabilities. This argument contradicts Equation (26.3), which suggests a distortion even in this case. Though Qiggin does not extend his argument beyond 50:50 bet (actually by his method any other uniform bet, e.g. with 3 or more equally likely outcomes, is distorted) we hypothesized that the probability of a uniform bet (with a $1/n$ probability for each of the n outcomes) should not be distorted as long as the outcomes are not extreme. This is also the result of Mcusi’s (1989) “Prospective Reference Theory” with a symmetric reference point, for which he finds experimental support. However, not all authors agree with the fact that uniform probability distributions are undistorted. Nevertheless, recall that if probabilities are distorted even with a uniform distribution, it has a devastating impact on all reported empirical studies in finance and economics (see below).

The RDEU of Qiggin transforms probabilities in the following manner. Instead of comparing the cumulative distributions F and G , the subjects compare the distributions F^* and G^* where $F^* = T(F)$ and $G^* = T(G)$, where T is the distortion function with $T' > 0$. It can be easily shown that using CPT or RDEU decision weights does not violate FSD. Namely,

$$F^* \leq G^* \Leftrightarrow T(F^*) \leq T(G^*) \tag{26.4}$$

(See Levy and Wiener, 1998. For a survey of SD rules, PT, and the impact of decision weights on choices, see Levy, 1998).

In PT and CPT frameworks, probabilities are also distorted in the uniform case. However, the advantage of PT over CPT is that with PT all probabilities with the same size, e.g. $p_i = 1/4$ are distorted in an identical way, hence the choices in a uniform bet are not affected by the probability distortion as suggested by PT. The advantage of CPT over PT is that FSD is not violated. Recalling that CPT decision weights is a technical method which was invented to avoid FSD violations, and that FSD violations do occur experimentally (see Birnbaum, 1997) leads one to question the benefit of introducing CPT decision weights.

26.2.2. *Change of Wealth Rather than Total Wealth*

Expected utility is defined on total wealth, i.e. $u(w + x)$ where w is the initial wealth and x is the change of wealth. Experimental studies reveal that subjects make decisions based on change of wealth, i.e. $u(x)$, rather than $u(w + x)$. It is interesting to note that though the change of wealth argument has been shown experimentally by Kahneman and Tversky, this idea appeared in the literature as early as 1952. Markowitz (1952b) claims that investors make decisions based on change of wealth rather than total wealth. It is easy to construct an example showing that

$$Eu(w + x) > Eu(w + y) \text{ and } Eu(x) < Eu(y)$$

when x and y are the returns on two risky projects. As we shall see later on in this paper, ignoring the initial wealth may indeed affect the choice of the “optimum” portfolio from the efficient set. However, it does not affect the division of the feasible set of portfolios to the efficient and inefficient sets.

26.2.3. *Integration of cash flows*

Expected utility maximization and portfolio selection advocate that one should select a portfolio of

assets that maximizes expected utility and one should not consider each asset in isolation. Therefore, correlations should play an important role in portfolio selection. Tversky and Kahneman experimentally find that this is not the case, hence conclude that subjects have difficulties in integrating cash flows from various sources. Let us illustrate this idea with the experiment conducted by Tversky and Kahneman in 1981 with the following two tasks.

Task I: Imagine that you face a pair of concurrent decisions. First, examine both decisions, then indicate the option you prefer.

Decision 1: Choose between A and B given below:

- (A) A sure gain of \$,400.
- (B) 25 percent chance to gain \$0,000 and 75 percent chance to gain nothing.

Decision 2: Choose between C and D given below:

- (C) A sure loss of \$,500.
- (D) 75 percent chance to lose \$0,000 and 25 percent to lose nothing.

A large majority of people choose A in decision 1 and D in decision 2.

Task II: Choose between E and F given below:

- (E) 25 percent chance to win \$,400 and 75 percent chance to lose \$,600
- (F) 25 percent chance to win \$,500 and 75 percent chance to lose \$,500.

In Task II, everybody correctly preferred option F over option E . Indeed, F dominates E by FSD. Note that if you return to Task I, however, you get that the inferior option E in Task II is obtained by choosing A and D . The dominating option F in Task II is obtained by combining the two options that most people reject in Task I (i.e. $F = B + C$). Thus, a fully rational decision maker who knows to integrate cash flows from various sources should incorporate the combined decisions, and realize that the combined cash flows of $B + C$ dominate those of $A + D$ in Task I.

From this and other examples, Tversky and Kahneman conclude that investors consider decision problems one at a time instead of adopting a broader frame. Such a procedure induces a reduction in expected utility because the investors miss an opportunity to diversify, hedge, or self-insure. The “narrow framing” of investors arises from the common practice of maintaining multiple “mental accounts.” Thus, the main finding is that the subjects – at least those who participated in the study – are limited in their capability to integrate cash flows from various sources even in a relatively simple case let alone in more complicated cash flows from many sources. If these findings are relevant, not only to subjects in an experiment but also to the investors in practice, this is a severe blow to diversification theory of Markowitz (1952a, 1959, 1987) and Tobin (1958).

26.2.4. Risk Seeking Segment of Preferences

Most models in economics and finance assume risk aversion, i.e. a preference u with $u' > 0$ and $u'' < 0$ (see for example Arrow, 1965, 1971; Pratt, 1964). However, as early as 1948, Friedman and Savage, based on observed peoples' behavior, suggested a risk-seeking segment of the preference. Markowitz (1952b) modified this preference and suggested another function, which also contains a risk-seeking segment. Both of these studies rely on positive economics arguments. Kahneman and Tversky, on the other hand, base their argument on experimental findings (see also Swalm, 1966).

Figure 26.2 provides the main utility functions advocated in the literature.¹ Figure 26.2a depicts the classical utility function which is concave everywhere, in accordance with the notion of decreasing marginal utility. Such a function implies risk aversion, meaning that individuals would never accept any fair bet (let alone unfair bets). Friedman and Savage (1948) claim that the fact that investors buy insurance, lottery tickets, and both insurance and lottery tickets simultaneously, plus the fact that most lotteries have more than one big prize, imply that the utility function must have

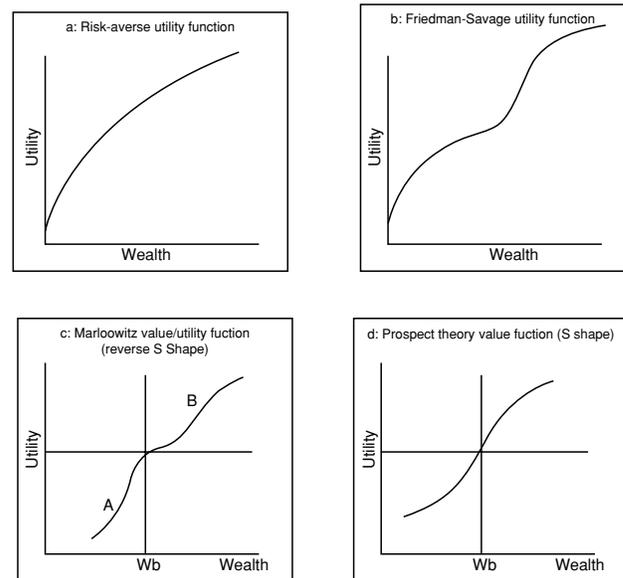


Figure 26.2. Alternative shapes of the utility/value function

two concave regions with a convex region in between, as represented in Figure 26.2b.

Markowitz (1952b) points out several severe problems with the Friedman and Savage utility function². However, he shows that the problems are solved if the first inflection point of the Friedman and Savage utility function is exactly at the individual's current wealth. Thus, Markowitz introduces the idea that decisions are based on “change” in wealth. Hence, Markowitz's utility function can be also considered as a “value function” (as later suggested by Kahneman and Tversky in 1979). By analyzing several hypothetical gambles, Markowitz suggests that individuals are risk-averse for losses and risk-seeking for gains, as long as the possible outcomes are not very extreme. For extreme outcomes, Markowitz argues that individuals become risk-averse for gains, but risk-seeking for losses. Thus, Markowitz suggests a utility function, which is characterized by three inflection points, as shown in Figure 26.2c. Notice that the central part of this function (the range between Points A and B in Figure 26.2c) has a reversed S-shape.

Based on their experimental results, with bets which are either negative or positive, Kahneman and Tversky (1979) and Tversky and Kahneman (1992) claim that the value function is concave for gains and convex for losses, yielding an S-shaped function, as shown in Figure 26.2d.

26.3. Experimental Studies In Finance

Experimental studies in finance lagged behind experimental studies in economics. Yet, the whole November/December 1999 issue of the *Financial Analyst Journal* is devoted to behavioral finance and discusses issues such as arbitrage, overconfidence, momentum strategies, market efficiency in an irrational world, and equity mispricing. In this section we discuss a few experimental studies in finance.

26.3.1. Portfolio diversification and Random Walk

In the last three decades, there has been a growing interest of economists in experimental economics. The Nobel prize committee recognized this important field by awarding the Nobel Prize in 2002 to Vernon Smith and Daniel Kahneman. On the importance of experimental research in economics, Vernon Smith asserts: "It is important to economic science for theorists to be less own-literature oriented, to take seriously the data and disciplinary function of laboratory experiments, and even to take seriously their own theories as potential generators of testable hypotheses." (See Smith, 1982, p. 924). (See also, Plott, 1979; Smith, 1976, 1982; Wilde, 1980).

While laboratory experiments are widely used in economics research, finance research is well behind in this respect. Probably, the first serious experiment in finance was done by Gordon et al. (1972), who studied portfolio choices experimentally. They indicate that to study the investors' preference, there is an advantage to the experimental method over the empirical method simply because it is difficult, if not impossible, to obtain empirically the relevant data (For a similar argument, see

Elton and Gruber, 1984). The first experimental studies in finance in diversification and portfolio choices focused on the allocation of money between the riskless asset and one risky asset as a function of various levels of wealth (see Gordon et al., 1972, Funk et al., 1979 and Rapoport, 1984).

Kroll et al. (1988a) study the choice between risky assets whose returns are normally distributed, where the riskless asset (borrowing and lending) is allowed. The subjects were undergraduate students who did not study finance or investment courses. The main findings of this experiment are:

- (1) The subjects selected a relatively high percentage of mean-variance "inefficient" portfolios.
- (2) The errors involved do not decrease with practice.
- (3) The subjects requested a lot of useless information, i.e. they asked for historical rates of returns when the parameters were known and the returns were selected randomly (information given to the subjects). Thus, the subjects presumably believed that there are some patterns in rates of return, though such patterns do not exist.

Odean (1998) in his analysis of many individual transactions reports that there is a tendency of investors to hold losing investments too long and to sell winning investments too soon (a phenomenon known as the disposition effect). This result is consistent with the results of Kroll et al. (see (3) above). He finds that when individual investors sold a stock and quickly bought another, the stock they sold outperformed on average the stock they bought by 3.4 percentage points in the first year. This costly overtrading may be explained by the fact that investors perceive patterns where none exist or do not want to admit their errors in selection of their investments. Perceiving patterns when they do not exist is exactly as reported by KLR.

In a subsequent paper, Kroll et al. (1988b) experimentally test the Separation Theorem and the Capital Asset Pricing Model (CAPM). In this experiment the 42 subjects were undergraduate

students who took a course in statistics. They had to select portfolios from three available risky assets and a riskless asset. This experiment reveals some negative and some positive results. The results are summarized as follows:

- (1) As predicted by the M-Model, the subjects generally diversified between the three risky assets.
- (2) A tenfold increase in the reward to the subject significantly improved the subjects' performance. This finding casts doubt on the validity of the results of many experiments on decision making under uncertainty which involves a small amount of money.
- (3) Though the subjects were told that rates of return are drawn randomly, as before, they, again, asked for (useless) information. This finding may explain why there are "chartists" and "technical analysts" in the market even if indeed rates of returns are randomly distributed over time. Thus, academicians may continue to claim the "random walk" property of returns and practitioners will continue to find historical patterns and employ technical rules for investment based on these perceived patterns.
- (4) Changing the correlation (from -0.8 to 0.8), unlike what Markowitz's theory advocates, does not change the selected diversification investment proportions.
- (5) The introduction of the riskless asset does not change the degree of homogeneity of the investment behavior. Thus, at least with these 42 subjects the Separation Theorem (and hence the CAPM) does not hold in practice.

In the study of KL&R, the subjects were undergraduate students with no background in finance and they could not lose money. These are severe drawbacks as the subjects may not represent potential investors in the market. To overcome these drawbacks, Kroll and Levy (K&L) (1992) conducted a similar experiment with the same parameters as in KL&R but this time with second year MBA students and where financial gains and

losses were possible. The results improved dramatically in favor of Markowitz's diversification theory. Figure 26.3 shows the average portfolio with and without leverage selected in the KL&R study and in the K&L study. In the K&L study, the selected portfolios are L and U (for levered and unlevered portfolios) while in KL&R they are \hat{U} and \hat{L} . As can be seen U and L are much closer to the optimum solution (in particular, L is much closer to line rr' than \hat{L}), indicating that when real money is involved and MBA students are the subjects, much better results are achieved. Also, as predicted by portfolio theory, the subjects, unlike in KL&R study, change the investment proportions when correlation changes.

Finally, the investment proportions selected were similar to those of the optimum mean variance portfolio. Therefore, K&L conclude that the subjects behave *as if* they solve a quadratic programming problem to find the optimum portfolio even though they did not study this tool at the time the experiment was conducted.

26.3.2. The Equity Risk Premium Puzzle

The difference between the observed long-run average rate of return on equity and on bonds cannot be explained by well behaved risk averse utility function; hence the term equity risk premium puzzle. Benartzi and Thaler (1995) suggest the loss aversion preference as suggested by PT S-shape function to explain the existing equity risk premium. They show that if investors weight losses 2.5 more heavily than possible gains the observed equity risk premium can be explained. However, Levy and Levy (2002b) have shown that the same conclusion may be drawn with a reverse S-shape utility function as suggested by Markowitz, as long as the segment corresponding to $x < 0$ is steeper than the segment corresponding to $x > 0$.

Levy and Levy (2002c) (L&L) analyze the effect of PT and CPT decision weights on Arrow (1965) and Pratt (1964) risk premium. They show that a positive risk premium may be induced by decision weights $w(p)$ rather than probabilities p even in the absence of

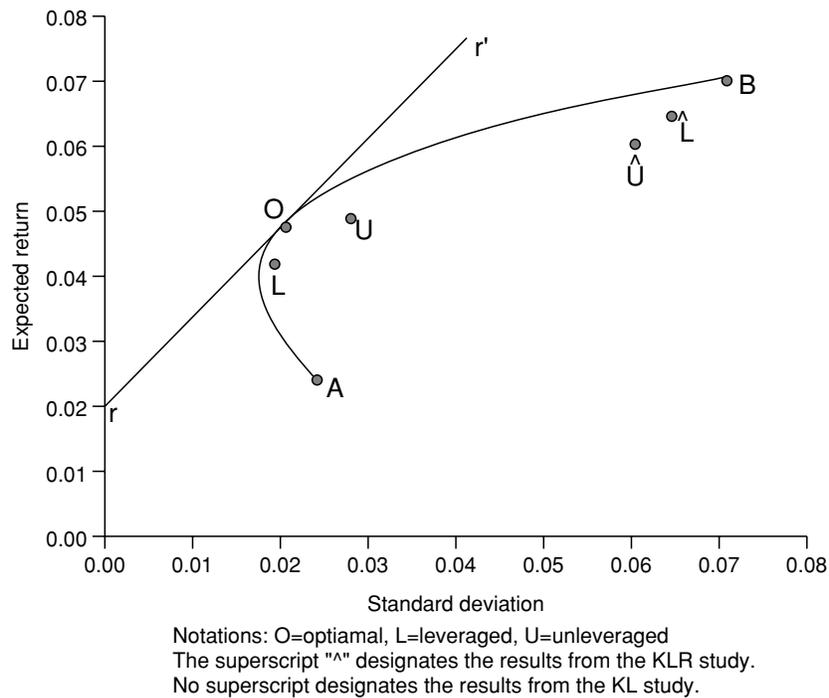


Figure 26.3. The efficient frontier and the actual portfolios selected by the subjects

risk aversion. In their experiment a large proportion of the choices contradicts risk aversion but this does not contradict the existence of a positive equity risk premium. Thus, one does not need loss aversion to explain Arrow's risk premium because it can be induced by the use of decision weights. Unlike the case of Arrow's risk premium, with Pratt's risk aversion measure or with historical data, which is composed of more than two values, the risk premium may increase or decrease due to the use of decision weights. To sum up, the equity risk premium puzzle can be explained either by loss aversion, which is consistent both with an S-shape function and a reverse S-shape function, or by decision weights, even in the absence of loss aversion.

26.3.3. The Shape of Preference

Risk aversion and a positive risk premium are two important features of most economic and finance models of assets pricing and decision making under uncertainty. Are people risk averse? As shown in Figure 26.1, Friedman and Savage, Mar-

kowitz, and ~~K&E~~ claim that this is not the case. So what can we say about preference? In a series of experiments with and without financial rewards, Levy and Levy (2002a,b) have shown that a major portion of the choices contradict risk-aversion. ~~L&E~~ conducted several experiments with 328 subjects. To test whether the subjects understood the questionnaire and did not fill it out randomly just to "get it over with," they first tested FSD which is appropriate for risk-seekers and risk averters alike. They found that 95 percent of the choices conform with FSD (i.e. with the monotonicity axiom), which validates the reliability of their results. They find that in Experiment 1 *at least 54 percent* of the choices contradict risk aversion, in Experiment 2 *at least 33 percent* of the subjects contradict risk aversion and in Experiment 3 *at least 42 percent* of the choices contradict risk aversion. It is interesting to note that the subjects in these three experiments were business school students, faculty, Ph.D students and practitioners (financial analysts and funds managers).

In these three experiments, L& also tested the effect of the subjects characteristics, the size of the outcomes as well as the framing of the bet. The results are very similar across all these factors with the exception that Ph.D. students and faculty members choose more consistently with risk aversion (71–78 percent correct second degree stochastic dominance (SSD) choices). But there may be a bias here because these subjects are more familiar with SSD rules, and it is possible that they mathematically applied it in their choices. However, even with these sophisticated subjects at least 22 percent–29 percent of them selected inconsistently with risk aversion, implying trouble for theoretical models, which rely on risk aversion.

The fact that 33 percent–54 percent of the subjects behave “as if” they are not risk averse, implies that they choose “as if” the utility function is not concave in the whole range. For example, K&, Friedman and Savage and Markowitz utility functions are consistent with L& findings. Note that L& findings do not imply risk seeking in the whole range, but rather no risk-aversion in the entire range. Therefore, their finding does not contradict the possibility that with actual equity distribution of rates of return corresponding to the US market the risk premium may even increase due to decision weights.

Rejecting risk aversion experimentally is repeated in many experiments (see for example L&, 2001, 2002a). Hence, the remainder contrasts K& S-shape function and Markowitz’s reverse S-shape function. Employing prospect stochastic dominance (PSD) and Markowitz’s stochastic dominance (MSD) L& contrast these two utility functions. Let us first present these two investment criteria:³

Prospect stochastic dominance (PSD):

Let U_s be the set of all S-shape preferences with $u' > 0$ for all $x \geq 0$ and $u'' > 0$ for $x < 0$ and $u'' < 0$ for $x > 0$. Then

$$\int_y^x [G(t) - F(t)] dt \geq 0 \text{ for all } x > 0, y < 0 \tag{26.5}$$

$$\Leftrightarrow E_F u(x) \geq E_G u(x) \text{ for all } u \in U_s.$$

Markowitz Stochastic Dominance (MSD):

Let U_M be the set of all reverse S-shape preferences with $u' > 0$ for all $x \geq 0$ and $u'' < 0$ for $x < 0$ and $u'' > 0$ for $x > 0$. Define F and G as above. Then F dominates G for all reverse S-shaped value functions, $u \in U_M$, if and only if

$$\int_{-\infty}^y [G(t) - F(t)] dt \geq 0 \text{ for all } y \leq 0 \text{ and } \int_x^{\infty} [G(t) - F(t)] dt \geq 0 \text{ for all } x \geq 0 \tag{26.6}$$

(with at least one strict inequality). And (5) holds iff $E_F u(x) > E_G u(x)$ for all $u \in U_M$. We call this dominance relation MSD–Markowitz Stochastic Dominance.

Table 26.2. The choices presented to the subjects

Suppose that you decided to invest \$0,000 either in stock F or in stock G. Which stock would you choose, F, or G, when it is given that the *dollar gain or loss* one month from now will be as follows:

TASK I:

F		G	
Gain or loss	Probability	Gain or loss	Probability
–3,000	1/2	–6,000	1/4
4,500	1/2	3,000	3/4

Please write F or G

TASK II:

Which would you prefer, F or G, if the dollar gain or loss one month from now will be as follows:

F		G	
Gain or loss	Probability	Gain or loss	Probability
–500	1/3	–500	1/2
+2,500	2/3	2,500	1/2

Please write F or G

(Continued)

Table 26.2. The choices presented to the subjects
(Continued)

TASK III:

Which would you prefer, F or G, if the dollar gain or loss one month from now will be as follows:

	F Gain or loss	G Probability	Gain or loss	Probability
	+500	3/10	-500	1/10
	+2,000	3/10	0	1/10
	+5,000	4/10	+500	1/10
			+1,000	2/10
			+2,000	1/10
			+5,000	4/10

Please write
F or G

TASK IV

Which would you prefer, F or G, if the dollar gain or loss one month from now will be as follows:

	F Gain or loss	G Probability	Gain or loss	Probability
	-500	1/4	0	1/2
	+500	1/4		
	+1,000	1/4	+1,500	1/2
	+2,000	1/4		

Please write
F or G

Source: Levy and Levy, (2002a).

Table 26.3. The results of the experiment*

Task	F	G	Indifferent	Total
I($G \phi_{PSD} F, F \phi_{MSD} G$)	71	27	2	100
II($F \phi_{FSD} G$)	96	4	0	100
III($F \phi_{FSD} G$)	82	18	0	100
IV($G \phi_{SSD} F$)	47	51	2	100

Number of subjects: 260

*Numbers in the tables are in percent, rounded to the nearest integer. The notations ϕ_{FSD} , ϕ_{SSD} , and ϕ_{MSD} indicate dominance by FSD, SSD, PSD, and MSD, respectively.

Source: Levy and Levy, (2002a).

Table 26.2 presents the four Tasks while Table 26.3 presents the results of the experimental study of Levy and Levy (2002a). Note that in Task I G dominates F by PSD, but F dominates G by MSD. As can

be seen from Table 26.3, in Task I, 71 percent of the subjects choose F despite the fact that G dominates F by PSD. Thus, at least 71 percent of the choices are in contradiction to PSD, and supporting MSD, i.e. a reverse S-shape preference as suggested by Markowitz. Note also that 82–96 percent of the choices (see Tasks II and III) are consistent with FSD. Once again, by the results of Task I we see that about 50 percent of the choices reject the assumption of risk aversion (SSD). Table 26.4 taken from Levy and Levy (2002b) reveals once again the results of another experiment showing that at least 62 percent of the choices contradict the S-shape preference of PT.

Wakker (2003) in his comment on Levy and Levy's (2002a) paper claims that the dominance by PSD (or by MSD) also depends on probability weights. Generally, his claim is valid. However, if a uniform distribution ($p_i = 1/4$ for all observations) is considered, his criticism is valid only if indeed probabilities are distorted in such a case. Are probabilities distorted in such a case? And if the answer is positive, can we blindly use the distortion formula suggested by T&K? There is evidence that in the case of uniform distributions probabilities are not distorted, or are distorted as recommended by PT but not by CPT (hence do not affect choices). Thus, Wakker's claim is invalid. Let us elaborate. In Tversky's (1989) *Prospective Reference Theory* there is also no probability distortion in the symmetric case. Also, in the original PT framework (Kahneman and Tversky, 1979), in which the probabilities are transformed directly, the choice among prospects is unaffected by subjective probability distortion in the case of equally likely outcomes. Thus, as the study reported in Table 26.4 was conducted with uniform probabilities and moderate outcomes, it is safe to ignore the effects of subjective probability distortion in this case. Wakker (2003) argues that by CPT probabilities are distorted even in the bets given in Table 26.4, hence the conclusion against the S-shape function by Levy and Levy is invalid. If one uses the distortion formula [see eq. (3)] of T&K also in the uniform case, Wakker is correct. However, recall that the formula of T&K is based on aggregate

Table 26.4a. The choices presented to the subjects

Suppose that you decided to invest \$0,000 either in stock F or in stock G. Which stock would you choose, F, or G, when it is given that the *dollar gain or loss* one month from now will be as follows:

F		G	
Gain or loss	Probability	Gain or loss	Probability
-1,600	1/4	-1,000	1/4
-200	1/4	-800	1/4
1,200	1/4	800	1/4
1,600	1/4	2,000	1/4

Please write
F or G

Table 26.4b. The results of experiment 2*

	F	G	Indifferent	Total
($F \phi_{PSD} G, G \phi_{MSD} F$)	38%	62%	0%	100%

Number of subjects: 84.

*Numbers in the tables are in percent, rounded to the nearest integer. The notations ϕ_{PSD} , and ϕ_{MSD} indicate dominance by PSD, and MSD, respectively.

Source: Levy and Levy, (2002b).

data of nonsymmetrical probability distributions and with no financial reward or penalty. So why should one think it is appropriate to apply it to the uniform probability case? Moreover, as we see in section 18, formula (3) suggests decision weights which are hard to accept if indistinguishability is employed in all cases.

Yet, even if one adheres to T&K distortion weights formula, even in the equally likely outcomes case, the S-shape preference is rejected and Wakker is wrong in his criticism. Indeed, Levy and Levy (2002b) conduct a direct confrontation of PSD and MSD where probability distortion is taken into account exactly as suggested by K&S CPT and exactly as done by Wakker (2003). Table 26.5 presents the two choices *F* and *G*, the objective probabilities as well as the decision weights as recommended by CPT [see eq. (3)]

Note that with the data of Table 26.5, *G* dominates *F* by PSD with objective as well as subjective

probabilities. Yet 50 percent of the subjects selected *F*. This implies that *at least* 50 percent (it may be much larger than 50 percent but this cannot be proven) of the subjects' choices do not conform with an S-shape preference, rejecting this important element of PT and CPT (see Table 26.5).

To sum up, with objective probabilities the S-shape preference is rejected. With PT the S-shape is also rejected by the decision weights $w(1/4)$ which is identical for all outcomes (For a proof, see Levy and Levy, 2002b). Table 26.5 reveals that the S-shape function is rejected also when decision weights are taken into account exactly as recommended by CPT's formula.

Thus, more than 50 percent of the choices contradicts risk aversion and more than 50 percent of the choices contradicts the S-shape function – the preference advocated by PT and CPT. The experiments' results yield most support Markowitz's reverse-Shape preference. From this above analysis we can conclude that investors are characterized by a variety of preferences and that there is no one dominating preference.

26.3.4. *Asset Allocation and the Investment Horizon*

Benartzi and Thaler (1999) (B&T) present subjects with a gamble reflecting a possible loss. The subjects could choose to gamble or not in an experiment which contains *N* repetitions. The subjects were reluctant to take the gamble. However, where the multi-period distributions of outcome induced by the *N* repetitions was presented to

Table 26.5. *G* dominates *F* by PSD even with CPT decision weights (task II of experiment 3 in Levy and Levy 2002b)

F			G		
Gain or loss	Probability	CPT decision weights	Gain or loss	Probability	CPT decision weights
-875	0.5	0.454	-1,000	0.4	0.392
2,025	0.5	0.421	1,800	0.6	0.474

Source: Levy and Levy, (2002b).

them, more subjects were willing to take the gamble. This shows that the subjects either have difficulties to integrate cash flows from various trials or they have “narrow framing.” These results also have strong implications to asset allocation and the investment horizon. B&E found the subjects willing to invest a substantially higher proportion of their retirement funds in stocks (risky assets) once they were shown the distributions of the long-run return relative to the investment proportion when the distribution of return is not shown to them. The results of B&E shed light on the debate between practitioners and academicians regarding the relationship between the portfolio composition and the investment horizon. While Samuelson (1994) and others correctly claim that for myopic (power) utility functions, the investment horizon should not have any effect on asset allocation, practitioners claim that the longer the horizon, the higher the proportion of assets that should be allocated to stocks. The results of B&E support the practitioners’ view provided that the subjects observe the multi-period distribution, i.e. overcoming the “narrow framing” effect. Ruling out irrationality or other possible biases, this finding means that the subjects in B&E’s experiment do not have a myopic utility function. It is interesting that Leshno and Levy (2002) have shown that as the number of period N increases, stocks “almost” dominate bonds by FSD, when “almost” means for almost all preferences, not including the myopic function. Thus, this theoretical result is consistent with B&E’s experimental results.

26.3.5. Diversification: the $1/n$ rule

Let us open this section by the following old assertion:

Man should always divide his wealth into three parts: one third in land, one third in commerce and one third retained in his own hands.

Babylonian Talmud

Two interesting conclusions can be drawn from this 1500-year-old recommendation, which is probably the first diversification recommendation. The first conclusion is consistent with what Markowitz recommended and formalized about fifty years ago: diversification pays. The second conclusion is in contrast to Markowitz’s recommendation: invest $1/3$ in each asset and ignore the optimum diversification strategy, which is a function of variances, correlations, and means.

It is interesting that Benartzi and Thaler (2001) experimentally find that this is exactly what investors do. Presented with n assets (e.g. mutual funds) the subject is inclined to invest $1/n$ in each fund. This is true regardless of the content of funds. If one fund is risky (stocks) or riskless, this does not change the $1/n$ choice, implicitly implying that the Talmud’s recommendation is intact as correlations, means and variances are ignored. From this we learn that investors believe that “a little diversification goes a long way,” but mistakenly ignore the optimal precise diversification strategy.

26.3.6. The CAPM: Experimental Study

One of the cornerstones of financial theory is asset pricing, as predicted by the CAPM. The problem with testing the CAPM empirically is that the *ex-ante* parameters may change over time. However, while the CAPM cannot be tested *empirically* with *ex-ante* parameters, it can be tested *experimentally* with *ex-ante* parameters. The subjects can provide buy-sell orders and determine collectively equilibrium prices of risky assets when the future cash flow (random variables) corresponding to the various assets are given. Levy (1997) conducted such an experiment with potential financial loss and reward to the subjects. Thus, like in Lintner’s (1969) approach for given distributions of end-of-period returns, the subjects collectively determine, exactly as in an actual market, the current market values P_{i0} . Therefore, the means μ_i , variances σ_i^2 and correlations, R_{ij} are determined simultaneously by the subjects. Having these parameters

one can test the CAPM with *ex-ante* parameters. Lintner (1969) found that subjects typically diversify in only 3–4 assets (out of the 20 available risky assets), yet the CAPM, or the $\mu - \beta$ linear relationship was as predicted by the CAPM with an R^2 of about 75 percent. Thus, Levy found a strong support to the CAPM with *ex-ante* parameters.

26.4. Implication of the Experimental findings to Finance

26.4.1. Arbitrage Models

Let us first analyze the arbitrage-based models like Modigliani and Miller (M&M) (1958), Black and Scholes (1973) model and the APT model of Ross (1976). Let us first illustrate M&M capital structure with no taxes. Denoting by V_U and V_L the value of the unlevered and levered firms, respectively, M&M claim that $V_U = V_L$ further, if $V_U \neq V_L$, one can create an arbitrage position such that the investor who holds return \tilde{y} will get after the arbitrage $\tilde{y} + a$ when $a > 0$. Thus, an FSD position is created and as the two returns are fully correlated, the FSD dominance implies an arbitrage position. Thus, arbitrage is achieved by selling short the overpriced firm's stock and holding long the underpriced firm's stock. If probability is distorted, this will not affect the results as the investor ends up with the same random variable. If preference is S-shaped, it does not affect the results as FSD position is created, which holds for all $u \in U_1$ and the S-shape functions are included in U_1 . Making decisions based on change of wealth rather than total wealth also does not affect these results (recall that FSD is not affected by initial wealth). However, the fact that investors have difficulties in integrating cash flows from various sources may affect the result. The reason is that before the arbitrage, the investor holds, say, the stock of the levered firm. If $V_L > V_U$ the investor should sell the levered firm, borrow and invest in the unlevered firm. However, the investor should be able to *integrate* the cash flows from

these two sources and realize that the levered firm's return is duplicated. According to prospect theory, "mental departments" exists and the subjects may be unable to create this cash flow integration. However, recall that to derive the condition $V_L = V_U$, it is sufficient that *one* investor will be able to integrate cash flows to guarantee this equilibrium condition and not that all investors must conduct this arbitrage transaction ("money machine" argument).

Black and Scholes (B&S) equilibrium option pricing is based on the same no-arbitrage idea. Whenever the call option deviates from B&S equilibrium price economic forces will push it back to the equilibrium price until the arbitrage opportunity disappears. This is very similar to M&M case; hence it is enough that there is one sophisticated investor in the market who can integrate cash flows.

Thus, a "money machine" is created whenever the market price of an option deviates from its equilibrium price. The same argument holds for all theoretical models which are based on an arbitrage argument, e.g, Ross's (1976) arbitrage pricing theory (APT). Thus, for arbitrage models, the integration of cash flows issue may induce a problem to some investors but luckily, in these models, *one* sophisticated investor who knows how to integrate cash flows is sufficient to guarantee the existence of an asset price as implied by these models.

Despite this argument, in a multiperiod setting, when the investment horizon is uncertain, in some cases we do not have a pure arbitrage as the gap in the price of the two assets under consideration (e.g. $V_L > V_U$) may even (irrationally) increase over time (see Thaler, 1993). Thus, while in a one-period model where all assets are liquidated at the end of the period, the above argument is valid; this is not necessarily the case in a multi-period setting.

26.4.2. Stochastic Dominance (SD) Rules

It is easy to show that prospect F dominates prospect G in terms of total wealth ($W + x$) if only F

dominates G with change in wealth (x). Thus, shifting from *total* wealth to *change* of wealth does not affect the dominance result. The same is true with the Mean-Variance rules. SD rules deal mainly with two distinct options and not with a mix of random variables, hence generally the issues of integration of cash flows does not arise with the application of SD rules (it is relevant, however, to the Mean-Variance rule). If preference is an S-shaped or reverse S-shape, FSD is intact as it is defined for all $u \in U_1(u' > 0)$. If probabilities are distorted by a distortion function $T(\cdot)$ where $F^* = T(F)$ and $T' > 0$, the FSD relationship is also unaffected by the distortion. Let us turn now to SSD and TSD. If preference is S-shape, SSD or TSD rules which assumes $u'' < 0$ are irrelevant. However, even with risk aversion, with probability distortion $T(F)$ ($T' > 0$), the SSD and TSD dominance relationships are affected. Thus, FSD is unaffected by the experimental findings of probability distortion, but SSD and TSD are affected. With probability distortion one should first transform probabilities and only then compare F^* and G^* when $F^* = T(F)$ or $G^* = T(P)$. However, as the k th individual is characterized by probability distortion T_k , each investor has his subjective SD efficient set and therefore the classical two-step portfolio selection (i.e. determining the efficient set in its first step and selecting the optimal portfolio from the efficient set in the second step) is meaningless as there is no one efficient set for all investors. To sum up, FSD efficiency analysis is intact and SSD and TSD are not. If, however, in SSD analysis, it is assumed that $T' > 0$ and $T'' < 0$, SSD analysis also remains intact (see Levy and Wiener, 1998).

26.4.3. Mean-Variance (M-V) Rule and PT

If the utility function is S-shaped or reverse S-shaped the Mean-Variance rule does not apply and hence it is not an optimal investment decision rule even in the face of normal distributions. Probability distortion is even more devastating to the M-Efficiency analysis. The whole idea of the

M-Efficient set is that all investors face the same efficient set which depends on mean, variances and correlations. Now if the k th investor distorts F_i to $T_k(F_i)$ when F_i is the cumulative distribution of the i th asset, then we have K subjective efficient sets (composed of the individual assets) and the idea of M-Efficiency analysis breaks down. Nevertheless, as we shall see below, the M-Efficiency analysis surprisingly is intact in the presence of mutual funds, or if the probability distortion is done on portfolios but not on each individual asset.

26.4.4. Portfolios and Mutual Funds: Markowitz's M-V Rule and PT – A Consistency or a Contradiction?

It is interesting that in the same year (1952) Markowitz, published two seminal papers that seem to contradict one another. One of these papers deals with the M-V rule (Markowitz, 1952a) and the other with the reverse S-shape function (Markowitz, 1952b). The Mean-Variance rule implies implicitly or explicitly risk aversion, while the reverse S-shape function, and for that matter also the S-shape function of PT, imply that risk aversion does not hold globally. Do we have here a contradiction between these two articles of Markowitz?

To analyze this issue we must first recall that the M-V rule is intact in two alternate scenarios: (1) a quadratic utility function; and (2) risk aversion and normal distributions of return. Obviously, under S-shape or reverse S-shape preference, scenario (1) does not hold. What about scenario (2)? Generally, if one compares two assets X and Y , indeed it is possible that X dominates Y by the M-V rule, but the expected utility of Y is greater than the expected utility of X for a given S-shape function even if X and Y are normally distributed. Hence, for such a comparison of any two arbitrary prospects, the two papers of Markowitz are indeed in contradiction. However, when diversification among all available assets is allowed, Levy and Levy (2004) have shown that the two articles do

not contradict each other but conform with each other. Let us be more specific. Figure 26.4 illustrates the M-efficient frontier. It is possible that asset a dominates b by M-Vule but not for all S-shape or reverse S-shape functions. However, and that is the important point, for any interior asset like asset b , there is an asset b' on the frontier which dominates b by the M-Vule as well as by EU for all monotonically increasing utility functions, including the S-shape and reverse S-shape functions. To see this claim, recall that under scenario (2) normal distributions are assumed. Regarding assets b and b' we have $\mu(b') > \mu(b)$ and $\sigma(b') = \sigma(b)$; hence portfolio b' , dominates portfolio b by FSD, or $Eu(b') > Eu(b)$ for all utility functions, including the S-shape and reverse S-shape preferences (For FSD in the normal distribution case see Levy, 1998). Thus, Markowitz's M-Diversification analysis is intact for all $u \in U_1$ including $u \in U_M$ and $u \in U_{PT}$ as $U_M \subset U_1$ and $U_{PT} \subset U_1$. To sum up, the quadratic utility function does not conform with the experimental findings regarding preferences. Assuming normality and risk aversion is also not justified in light of the experimental findings regarding preferences. However, allowing investors to diversify (with nor-

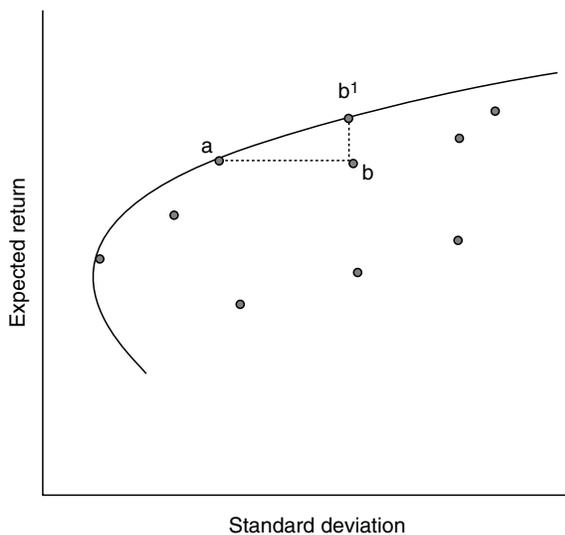


Figure 26.4. Mean–variance dominance and FSD dominance

mal distributions), the M-efficient frontier is the efficient one also in EU framework, without the need to assume risk aversion. Also, as the FSD efficient set is invariant to change in wealth instead of total wealth, looking at the change of wealth rather than the total wealth does not change our conclusion. Thus, the two papers of Markowitz are not in contradiction as long as diversification is allowed, an assumption which is well accepted. Thus, while portfolio a dominates b with risk aversion, such dominance does not exist with other preferences with a risk seeking segment. But a vertical comparison (e.g. b and b') allows us to conclude that Markowitz's M-inefficient set is inefficient for all $u \in U_1$ and not only to risk averse utility functions (see Figure 26.4).

So far, we have dealt only with the factors of preferences and change of wealth rather than total wealth. Let us now see how the other findings of PT and CPT affect the M-Analysis. First, if individual investors fail to integrate cash flows from various assets, the M-efficiency analysis and the Separation Theorem collapse. However, we have mutual funds and in particular indexed funds which carry the integration of cash flows for the investors. If such mutual funds are available – and in practice they are – the M-Analysis is intact also when one counts for the “mental departments” factor. To see this, recall that with a normal distribution mutual fund b' dominates mutual fund (or asset) b by FSD with objective distributions because $F_{b'}(X) \leq F_b(X)$ for all values x (see Figure 26.4). As FSD is not affected by CPT probability distortion, also $T(F_{b'}(X)) \leq T(F_b(X))$, hence the M-efficient set (of mutual funds) is efficient also in CPT framework. Thus, change of wealth (rather than total wealth), risk-seeking segment of preference and probability distortion (as recommended by CPT), do not affect Markowitz's M-efficiency analysis and the Separation Theorem. Hence, under the realistic assumption that mutual funds exist, the CAPM is surprisingly intact even under the many findings of experimental economics which contradict the CAPM assumptions.

26.4.5. *The Empirical Studies and Decision Weights*

Though we discuss above the probability distortion in the uniform case, we need to return to this issue here as it has a strong implication regarding empirical studies in finance.

Nowadays, most researchers probably agree that probability distortion takes place at least at the extreme case of low probabilities (probability of winning in a lottery, probability that a fire breaks out, etc.). Also probability distortion may take place when extreme gains or losses are incurred. Also the “certainty effect” is well documented. However, if one takes probability distortion too seriously and adopts it exactly as recommended by T&K (1992) [see eq. (3) above] paradoxes and absurdities emerge. For example, take the following two prospects:

x:	−\$1,000	−\$1,000	+\$1,000	+\$1,000
p(x)	1/4	1/4	1/4	1/4
y:	−\$10 ⁶	−\$1,000	+\$10 ¹²	\$10 ²⁴
p(y)	1/4	1/4	1/4	1/4

Employing the distortion of T&K [see eq. (3) above] for probability distortion implies the following decision weights:

x:	−\$1,000	−\$1,000	+\$1,000	+\$1,000
w(x)	0.29	0.16	0.13	0.29
y:	−\$10 ⁶	−\$1,000	+\$10 ¹²	+\$10 ²⁴
p(y)	0.29	0.16	0.13	0.29

Does it make sense? With x , the probability of $x = -2,000$ increases from 0.25 to 0.29, and the probability of $x = -1,000$ drops from 0.25 to 0.16. And similar is the case with y . Thus, the magnitudes of the outcomes are not important and the probabilities are distorted by the same formula regardless of whether the outcome is -10^6 or only $-2,000$. This type of probability distortion which is insensitive to the magnitude of the out-

comes, has been employed by Wakker (2003). Not everyone agrees with CPT weighting function [see eq. (3)] Birnbaum and McIntosh (1996), finds that the probability distortion depends on the configuration of the case involved, hence suggests a “configurational weighting model.” Moreover, Birnbaum experimentally shows that in some cases, FSD is violated. Note that FSD was the main reason why CPT and RDEU were suggested as substitutes to PT. Thus, if FSD is indeed violated, CPT is losing ground and PT may be a better description of investors’ behavior. Similarly, Qiggin (1982) suggests that in cases where there are two equally likely outcomes with a 50:50 chance, probability is not distorted, which contradicts eq. (3). Mcusi (1989) shows that with probability $p_i = 1/n$ with n possible outcome, $w(1/n) = 1/n$, i.e. probability is not distorted. Also, the original PT of K&K implies that in case of an equally likely outcome, the choices are not affected by the decision weights. We emphasize this issue because the issue whether probability is distorted or not in equally likely outcomes has important implications to empirical studies.

In virtually all empirical studies in finance and economics where distribution is estimated, n observations are taken and each observation is assigned an equal probability. For example, this is the case in the calculation of σ^2 , β , etc. If probabilities are also distorted in the uniform case, namely, $w(1/n) \neq 1/n$ as suggested by CPT of T&K, all the results reported in the empirical studies are questionable, including all the numerous empirical studies which have tested the CAPM. The implicit hypothesis in these studies is that in the uniform probability case, with no extreme values, probabilities are not distorted. We cannot prove this, but recall that the distortion formula of T&K is also obtained in a very limited case; an experiment with some specific lotteries when the S-shaped preference and the weighting function are tested simultaneously. Hence, the parametric assumptions concerning both functions are needed. Also, T&K report aggregate rather than individual results. Therefore, the burden of the proof that probab-

ities are distorted in the uniform case is on the advocates of PT and CPT. Finally, as mentioned above, if one employs PT (1979), decision weights where $w(1/n) = p_0$ for all observations, the decision maker who maximizes EU will not change his decision also in the uniform case (for a proof see, Levy and Levy, 2002b). However, with decision weight p_0 rather than $1/n$, the empirical results may change, despite the fact that choices are unchanged. This issue has to be further investigated.

26.5. Conclusion

Experimental research is very important as it allows us to control variables and sometimes to study issues that cannot be studied empirically, e.g. testing the CAPM with *ex-ante* parameters. Experimental findings and in particular Prospect Theory and cumulative Prospect Theory (PT and CPT, respectively) contradict expected utility theory (EUT) which, in turn, may have a direct implication to financial and economic decision making theory and to equilibrium models.

Taking the PT and CPT implication to the extreme, we can assert that virtually all models in finance and in particularly all empirical studies results and conclusions are incorrect. However, this conclusion is invalid for two reasons: (a) subjects in the experiments are not sophisticated investors who in practice risk a relatively large amount of their own money; and (b) one cannot conclude from a specific experiment (or from a few of them), conducted with no real money and with unequal probabilities that probabilities are distorted also in the uniform case, i.e. with equally likely outcomes. Therefore, PT and CPT do not have an unambiguous implication regarding the validity of the empirical studies in economics and finance which implicitly assume equally likely outcomes. Let us elaborate.

Drawing conclusions from the “average subject behavior” and assuming that also sophisticated investors behave in a similar way may be misleading. For example, for all arbitrage models it is possible that most subjects and even most investors

in practice do not know how to integrate cash flows, but it is sufficient that there are some sophisticated investors who integrate cash flows correctly to obtain the arbitrage models’ equilibrium formulas (e.g. APT, M&M and B&S models). Probability distortion, various preference shapes and change of wealth rather than the total wealth, do not affect these arbitrage models. There are exceptions however. If the gap between the prices of the two assets increases irrationally rather than decreases over time, the arbitrage profit is not guaranteed (see Thaler, 1999). To sum up, in a one-period model where the assets are liquidated at the end of the period, the arbitrage models are intact, but this is not necessarily the case in a multi-period setting with irrational asset pricing.

The capacity of investors to carry cash-flows integration is crucial in particular for M-Portfolio selection. However, in this case we may divide the group of investors into two subgroups. The first group is composed of sophisticated investors who diversify directly; hence presumably do not conduct the common mistakes done by subjects in experiments. This group includes also the financial consultants and advisers who know very well how to take correlations of returns (integration of cash flows) into account. For example, professional advisers recommend that, “You don’t want more than one company in an industry, and you don’t want companies in related industries.”⁴

This advice is quite common and one can document numerous other similar assertions made by practitioners. Therefore, it is obvious that cash-flows integration and correlations are well taken into account by this segment of investors. The other group of investors who may be exposed to all deviations from rationality are composed of the less sophisticated investors. These investors who cannot integrate cash flows from various sources may buy mutual funds managed by professional investors. Thus, if the M-efficient set contains these mutual funds the Separation Theorem and the CAPM hold even with S-shape (or reverse S-shape) utility function, with change of wealth rather than total wealth and with CPT probability

distortion. Thus, the CAPM is theoretically intact in the Rank Dependent Expected Utility (RDEU) framework. While the CAPM may not hold due to other factors (transaction costs, market segmentation, etc.), the experimental findings by themselves do not cause changes in the M-Efficiency analysis and the CAPM, as long as mutual funds exist in the market.

To sum up, though experimental findings open a new way of thinking on financial theory, most financial models, albeit not all of them, are robust even with these experimental findings. However, in some extreme cases, experimental evidence may explain phenomena, which cannot be explained by rational models. If investors follow some bounded rational behavior, booms and crashes in the stock market may be obtained (see Levy et al., 2000) even though there is no classical economic explanation for such a stock market behavior.

Acknowledgment

The author acknowledges the financial support of the Krueger Center of Finance.

NOTES

1. The analysis of the various alternate preferences of Figure 26.2 is taken from Levy and Levy, (2002).
2. For example, Markowitz argues that individuals with the Friedman and Savage utility function and wealth in the convex region would wish to take large symmetric bets, which is in contradiction to empirical observation.
3. For PSD see Levy and Weiner (1998) and Levy (1998) and Levy (1998). For MSD see Levy and Levy (2002a).
4. A quote from Mr. Lipson, a president of Horizon Financial Advisers, see *Wall Street Journal*, 10 April 1992 (an article by Ellen E. Schultz).

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