

Chapter 4

INTERTEMPORAL RISK AND CURRENCY RISK

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Abstract

Empirical work on portfolio choice and asset pricing has shown that an investor's current asset demand is affected by the possibility of uncertain changes in future investment opportunities. In addition, different countries have different prices for goods when there is a common numeraire in the international portfolio choice and asset pricing. In this survey, we present an intertemporal international asset pricing model (IAPM) that prices market hedging risk and exchange rate hedging risk in addition to market risk and exchange rate risk. This model allows us to explicitly separate hedging against changes in the investment opportunity set from hedging against exchange rate changes as well as separate exchange rate risk from intertemporal hedging risk.

Keywords: currency risk; exchange rate risk; hedging risk; inflation risk; international asset pricing; intertemporal asset pricing; intertemporal risk; intertemporal substitution; purchasing power parity; recursive preference; risk aversion

4.1. Introduction

In a dynamic economy, it is often believed that if investors anticipate information shifts, they will adjust their portfolios to hedge these shifts. To capture the dynamic hedging effect, Merton

(1973) developed a continuous-time asset pricing model which explicitly takes into account hedging demand. In contrast to the Arbitrage Pricing Theory (APT) framework, there are two factors, which are theoretically derived from Merton's model: a market factor and a hedging factor. Stulz (1981) extended the intertemporal model of Merton (1973) to develop an international asset pricing model. However, an empirical investigation is not easy to implement in the continuous-time model. In a recent paper, Campbell (1993) developed a discrete-time counterpart of Merton's model. Motivated by Campbell's results, Chang and Hung (2000) adopted a conditional two-factor asset pricing model to explain the cross-sectional pricing relationships among international stock markets. In their setup, assets are priced using their covariance with the market portfolio as well as with the hedging portfolio, both of which account for changes in the investment set. Under their proposed international two-factor asset pricing model framework, the international capital asset pricing model (CAPM) is misspecified and estimates of the CAPM model are subject to the omitted variable bias.

If purchasing power parity (PPP) is violated, investors from different countries will have different evaluations for real returns for investment in the same security. This implies that the optimal portfolio choices are different across investors

residing in different countries, and any investment in a foreign asset is exposed to currency risk. Therefore, it is reasonable to assume that investors from different countries have different estimations for real returns. This phenomenon clearly shows the existence of currency risk as well as market risk.

There are two goals in this survey. First, we want to know whether hedging demand is important to an international investor. Second, we want to separate currency hedging risk from intertemporal market hedging risk on an international asset pricing model.

The approach we describe here was first proposed by Epstein and Zin (1989, 1991). In their model, the investor is assumed to use a nonexpected utility that distinguishes the coefficient of relative risk aversion and the elasticity of intertemporal substitution. Campbell (1993) applied a log-linear approximation to the budget constraint in order to replace consumption from a standard intertemporal asset pricing model. Chang and Hung (2000) used this model to explain the intertemporal behavior in the international financial markets under no differences in consumption opportunity set.

An important challenge therefore remains – how to build a more realistic intertemporal international asset pricing model (e.g. when the consumption opportunity set is different). This essay surveys the progress that has been made on this front, drawing primarily from Chang and Hung (2000) and Chang et al. (2004).

In Section 4.2, we present a testable intertemporal capital asset pricing model proposed by Campbell. Hence, we can examine whether Campbell’s model explains the intertemporal behavior of a number of international financial markets. In Section 4.3, we separate currency hedging risk from intertemporal market hedging risk. This is accomplished by extending Campbell’s model to an international framework in which investor’s utility depends on real returns rather than on nominal returns and PPP deviation.

4.2. No Differences in Consumption Opportunity Set

This section describes the international asset pricing model we employ to estimate and test the pricing relationships among the world’s five main equity markets. The model we use is a two-factor model based on Campbell (1993). We first review the theory of nonexpected utility proposed by Weil (1989) and Epstein and Zin (1991). Then we apply a log-linear approximation to the budget constraint to derive an international asset pricing model, which is used in this chapter.

4.2.1. Asset Pricing Model

4.2.1.1. Nonexpected Utility

We consider an economy in which a single, infinitely lived representative international agent chooses consumption and portfolio composition to maximize utility and uses U.S. dollar as the numeraire and where there is one good and N assets in the economy. The international agent in this economy is assumed to be different to the timing of the resolution of uncertainty over temporal lotteries. The agent’s preferences are assumed to be represented recursively by

$$V_t = W(C_t, E_t[V_{t+1}|I_t]), \tag{4.1}$$

where $W(.,.)$ is the aggregator function, C_t is the consumption level at time t , and E_t is the mathematical expectation conditional on the information set at time t . As shown by Kreps and Porteus (1978), the agent prefers early resolution of uncertainty over temporal lotteries if $W(.,.)$ is convex in its second argument. Alternatively, if $W(.,.)$ is concave in its second argument, the agent will prefer late resolution of uncertainty over temporal lotteries.

The aggregator function is further parameterized by

$$\begin{aligned} V_t &= [1 - \delta]C_t^{1-\rho} + \delta(E_t V_{t+1}^{1-\lambda})^{(1-\rho)/(1-\lambda)}]^{1/(1-\rho)} \\ &= [1 - \delta]C_t^{(1-\lambda)/\theta} + \delta(E_t V_{t+1}^{1-\lambda})^{1/\theta}]^{\theta/(1-\lambda)} \end{aligned} \tag{4.2}$$

Parameter δ is the agent's subjective time-discount factor and λ is interpreted as the Arrow–Pratt coefficient of relative risk aversion. In addition, $1/\rho$ measures the elasticity of intertemporal substitution. For instance, if the agent's coefficient of relative risk aversion (λ) is greater than the reciprocal of the agent's elasticity of intertemporal substitution (ρ), then the agent would prefer an early resolution towards uncertainty. Conversely, if the reciprocal of the agent's elasticity of intertemporal substitution is larger than the agent's coefficient of relative risk aversion, then the agent would prefer a late resolution of uncertainty. If λ is equal to ρ , the agent's utility becomes an isoelastic, von Neumann–Morgenstern utility, and the agent would be indifferent to the timing of the resolution of uncertainty.

Furthermore, θ is defined as $\theta = (1 - \lambda)/(1 - \rho)$ in accordance with Giovannini and Weil (1989). Three special cases are worth mentioning. First, $\theta \rightarrow 0$ when $\lambda \rightarrow 1$. Second, $\theta \rightarrow \infty$ when $\rho \rightarrow 1$. Third, $\theta = 1$ when $\lambda = \rho$. Under these circumstances, Equation (4.2) becomes the von Neumann–Morgenstern expected utility

$$V_t = \left[(1 - \delta) E_t \sum_{j=1}^{\infty} \delta^j \tilde{C}_{t+j}^{1-\gamma} \right]^{1/(1-\lambda)}. \quad (4.3)$$

4.2.1.2. Log-Linear Budget Constraint

We now turn to the characterization of the budget constraint of the representative investor who can invest wealth in N assets. The gross rate of return on asset i held throughout period t is given by $R_{i,t+1}$. Let

$$R_{m,t+1} \equiv \sum_{i=1}^N \alpha_{i,t} R_{i,t+1} \quad (4.4)$$

denote the rate of return on the market portfolio, and $\alpha_{i,t}$ be the fraction of the investor's total wealth held in the i th asset in period t . There are only $N - 1$ independent elements in $\alpha_{i,t}$ since the constraint

$$\sum_{i=1}^N \alpha_{i,t} = 1 \quad (4.5)$$

holds for all t . The representative agent's dynamic budget constraint can be given by

$$W_{t+1} = R_{m,t+1}(W_t - C_t), \quad (4.6)$$

where W_{t+1} is the investor's wealth at time t . The budget constraint in Equation (4.6) is nonlinear because of the interaction between subtraction and multiplication. In addition, the investor is capable of affecting future consumption flows by trading in risky assets. Campbell linearizes the budget constraint by dividing Equation (4.6) by W_t , taking log, and then using a first-order Taylor expansion around the mean log consumption/wealth ratio, $\log(C/W)$. If we define the parameter $\beta = 1 - \exp(\bar{c}_t - \bar{w}_t)$, the approximation to the intertemporal budget constraint is

$$\Delta w_{t+1} \cong r_{m,t+1} + k + \left(1 - \frac{1}{\beta}\right)(c_t - w_t), \quad (4.7)$$

where the log form of the variable is indicated by lowercase letters and k is a constant.

Combining Equation (4.7) with the following equality,

$$\Delta w_{t+1} = \Delta c_{t+1} + (c_t - w_t) - (c_{t+1} - w_{t+1}), \quad (4.8)$$

we obtain a different equation in the log consumption/wealth ratio, $c_t - w_t$. Campbell (1993) shows that if the log consumption/wealth ratio is stationary, i.e. $\lim_{j \rightarrow \infty} \beta^j (c_{t+j} - w_{t+j}) = 0$, then the approximation can be written as

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j r_{m,t+1+j} \\ &\quad - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j \Delta c_{t+1+j}. \end{aligned} \quad (4.9)$$

Equation (4.9) can be used to express the fact that an unexpected increase in consumption today is determined by an unexpected return on wealth today (the first term in the first sum on the right-hand side of the equation), or by news that future

returns will be higher (the remaining terms in the first sum), or by a downward revision in expected future consumption growth (the second sum on the right-hand side).

4.2.1.3. Euler Equations

In this setup, Epstein and Zin (1989) derive the following Euler equation for each asset:

$$1 = E_t \left[\left\{ \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \right\}^\theta \left\{ \frac{1}{R_{m,t+1}} \right\}^{1-\theta} R_{i,t+1} \right] \quad (4.10)$$

Assume for the present that asset prices and consumption are jointly lognormal or apply a second-order Taylor expansion to the Euler equation. Then, the log version of the Euler equation (4.10) can be represented as

$$\begin{aligned} 0 = & \theta \log \delta - \theta \rho E_t \Delta c_{t+1} + (\theta - 1) E_t r_{m,t+1} \\ & + E_t r_{i,t+1} + \frac{1}{2} [(\theta \rho)^2 V_{cc} + (\theta - 1)^2 V_{mm} \\ & + V_{ii} - 2\theta \rho (\theta - 1) V_{cm} - 2\theta \rho V_{ci} \\ & + 2(\theta - 1) V_{im}] \end{aligned} \quad (4.11)$$

where V_{cc} denotes $\text{var}(c_{t+1})$, V_{jj} denotes $\text{var}(r_{j,t+1}) \forall j = i, m$, V_{cj} denotes $\text{cov}(c_{t+1}, r_{j,t+1}) \forall j = i, m$, and V_{im} denotes $\text{cov}(r_{i,t+1}, r_{m,t+1})$.

By replacing asset i by the market portfolio and rearranging Equation (4.11), we obtain a relationship between expected consumption growth and expected return on the market portfolio

$$\begin{aligned} E_t \Delta c_{t+1} = & \frac{1}{\rho} \log \delta + \frac{1}{2} \left[\theta \rho V_{cc} + \left(\frac{\theta}{\rho} \right) V_{mm} \right. \\ & \left. - 2\theta V_{cm} \right] + \frac{1}{\rho} E_t r_{m,t+1}. \end{aligned} \quad (4.12)$$

When we subtract Equation (4.11) for the risk-free asset from that for asset i , we obtain

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \theta(\rho V_{ic}) + (1 - \theta) V_{im} \quad (4.13)$$

where $r_{f,t+1}$ is a log riskless interest rate. Equation (4.13) expresses the expected excess log return on an asset (adjusted for Jensen's inequality effect) as a weighted sum of two terms. The first term, with a

weight θ , is the asset covariance with consumption multiplied by the intertemporal elasticity of substitution, ρ . The second term, with a weight $1 - \theta$, is the asset covariance with the return from the market portfolio.

4.2.1.4. Substituting Consumption out of the Asset Pricing Model

Now, we combine the log-linear Euler equation with the approximated log-linear budget constraint to get an intertemporal asset pricing model without consumption. Substituting Equation (4.12) into Equation (4.9), we obtain

$$\begin{aligned} c_{t+1} - E_t c_{t+1} = & r_{m,t+1} - E_t r_{m,t+1} + \left(1 - \frac{1}{\rho} \right) \\ & (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j r_{m,t+1+j} \end{aligned} \quad (4.14)$$

Equation (4.14) implies that the unexpected consumption comes from an unexpected return on invested wealth today or expected future returns.

Based on Equation (4.14), the conditional covariance of any asset return with consumption can be rewritten in terms of the covariance with the market return and revisions in expectations of future market returns which is given by

$$\text{cov}_t(r_{i,t+1}, \Delta c_{t+1}) \equiv V_{ic} = V_{im} + \left(1 - \frac{1}{\rho} \right) V_{ih} \quad (4.15)$$

where $V_{ih} = \text{cov}_t(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j r_{m,t+1+j})$.

Substituting Equation (4.15) into Equation (4.13), we obtain an international asset pricing model that is not related to consumption:

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \lambda V_{im} + (\lambda - 1) V_{ih}. \quad (4.16)$$

Equation (4.16) states that the expected excess log return in an asset, adjusted for Jensen's inequality effect, is a weighted average of two covariances—the covariance with the return from the market portfolio and the covariance with news about future returns on invested wealth.

4.2.2. Empirical Evidence

The relationship between risk and return has been the focus of recent finance research. Numerous papers have derived various versions of the international asset pricing model. For example, Solnik (1974) extends the static Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965) to an international framework. His empirical findings reveal that national factors are important in the pricing of stock markets. Furthermore, Korajczyk and Viallet (1989) propose that the international CAPM outperforms its domestic counterpart in explaining the pricing behavior of equity markets.

In a fruitful attempt to extend the conditional version of the static CAPM, Harvey (1991) employs the Generalized Method of Moments (GMM) to examine an international asset pricing model that captures some of the dynamic behavior of the country returns. De Santis and Gerard (1997) test the conditional CAPM on international stock markets, but they apply a parsimonious Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) parameterization as the specification for second moments. Their results indicate that a one-factor model cannot fully explain the dynamics of international expected returns and the price of market risk is not significant.

On the other hand, recent studies have applied the APT of Ross (1976) to an international setting. For instance, Cho et al. (1986) employ factor analysis to demonstrate that additional factors other than covariance risk are able to explain the international capital market. Ferson and Harvey (1993) investigate the predictability of national stock market returns and its relation to global economic risk. Their model includes a world market portfolio, exchange rate fluctuations, world interest rates, and international default risk. They use multifactor asset pricing models with time-varying risk premiums to examine the issue of predictability. But, one of the drawbacks of the APT approach is that the number and identity of the factors are determined either *ad hoc* or statistically from data rather than from asset pricing models directly.

Several international asset pricing models explicitly take into account currency risk, for example, see

Solnik (1974), Stulz (1981), and Adler and Dumas (1983). But investors in these models are assumed to maximize a time-additive, von Neumann–Morgenstern expected utility of lifetime consumption function. This implies that two distinct concepts of intertemporal substitution and risk aversion are characterized by the same parameter. Another approach examines consumption risk. Cumby (1990) proposes a consumption-based international asset pricing model. Difficulty occurs in the usage of aggregate consumption data, which are measured with error, and are time-aggregated. Chang and Hung (2000) show that estimations of price of market risk obtained from the De Santis and Gerard (1997) conditional CAPM model may be biased downward due to the omission of the hedging risk, which is negatively correlated to the market risk.

4.3. Differences in Consumption Opportunity Set

In this section, we consider the problem of optimal consumption and portfolio allocation in a unified world capital market with no taxes and transactions costs. Moreover, investors' preferences are assumed to be nationally heterogeneous and asset selection is the same for investors in different countries. Consider a world of $M+1$ countries and a set of S equity securities. All returns are measured in the $M+1$ st country's currency in excess of the risk-free rate and this currency is referred to as the numeraire currency. Investors are assumed to maximize Kreps–Porteus utility for their lifetime consumption function.

4.3.1. Portfolio Choice in an International Setting

4.3.1.1. Kreps–Porteus Preferences

Define C_t as the current nominal consumption level at time t , and P_t as the price level index at time t , expressed in the numeraire currency. In the setup of Kreps and Porteus (1978) nonexpected utility, the investor's value function can be represented as:

$$V_t = U \left[\frac{C_t}{P_t}, E_t V_{t+1} \right], \quad (4.17)$$

where V_t is the lifetime utility at time t , E_t is the expected value function conditional on the infor-

mation available to the investor at time t , $U[\cdot]$ is the aggregator function that aggregates current consumption with expected future value. As shown by Kreps and Porteus, the agent prefers early resolution of uncertainty over temporal lotteries if $U[\cdot]$ is convex in its second argument. On the other hand, if $U[\cdot]$ is concave in its second argument, the agent will prefer late resolution of uncertainty over temporal lotteries.

Furthermore, the aggregator function is parameterized to be homogenous of degree one in current real consumption and in the value of future state-dependent real consumption:

$$U\left[\frac{C_t}{P_t}, E_t V_{t+1}\right] = \left[(1-\delta)\left(\frac{C_t}{P}\right)^{1-\rho} + \delta(E_t V_{t+1})^{(1-\rho)/(1-\lambda)} \right]^{(1-\lambda)/(1-\rho)}, \quad (4.18)$$

where λ is the Arrow–Pratt coefficient of relative risk aversion, ρ can be interpreted as the elasticity of intertemporal substitution, and $\delta \in (0,1)$ is the subjective discount factor.

The Kreps–Porteus preference allows the separation of risk aversion from intertemporal substitution. For instance, if the agent’s coefficient of relative risk aversion, λ , is greater than the reciprocal of the agent’s elasticity of intertemporal substitution, ρ , then the agent prefers early resolution of uncertainty. Conversely, if the reciprocal of the agent’s elasticity of intertemporal substitution is larger than the agent’s coefficient of relative risk aversion, the agent prefers late resolution of uncertainty. When $\rho = \lambda$, the objective function is the time-separable power utility function with relative risk aversion λ . In addition, when both λ and ρ equal 1, we have standard time-separable log utility function. Hence, the standard time- and state-separable expected utility is a special case under Kreps–Porteus preferences.

4.3.1.2. Optimal Consumption and Portfolio Allocation

We now turn to the characterization of the budget constraint of the representative investor who can invest his wealth in $N (= M + S)$ assets that include M currencies and S equities. Currencies may be

taken as the nominal bank deposits denominated in the nonnumeraire currencies. The gross rate of nominal return on asset i held throughout period t is given by $R_{i,t+1}$. Let

$$R_{m,t+1} \equiv \sum_{i=1}^N \alpha_{i,t} R_{i,t+1} \quad (4.19)$$

denote the rate of return on the market portfolio, and $\alpha_{i,t}$ be the fraction of the investor’s total wealth held in the i th asset in period t . There are only $N - 1$ independent elements in $\alpha_{i,t}$, since the constraint

$$\sum_{i=1}^N \alpha_{i,t} = 1 \quad (4.20)$$

holds for all t . The representative agent’s dynamic budget constraint in terms of real variables can be written as:

$$\frac{W_{t+1}}{P_{t+1}} = R_{m,t+1} \frac{P_t}{P_{t+1}} \left(\frac{W_t}{P_t} - \frac{C_t}{P_t} \right) \quad (4.21)$$

where W_{t+1} is the investor’s nominal wealth at time t . The budget constraint in Equation (4.21) is nonlinear because of the interaction between subtraction and multiplication.

Define I_t as the information set available to the representative agent at time t . Denoting by $V(W/P, I)$ the maximum value of Equation (4.17) subject to Equation (4.20), the standard Bellman equations can then be written as:

$$V\left(\frac{W_t}{P_t}, I_t\right) = \max_{C_t, \{\alpha_{i,t}\}_{i=1}^N} \left\{ (1-\delta)\left(\frac{C_t}{P_t}\right)^{1-\rho} + \delta \left[E_t V\left(\frac{W_{t+1}}{P_{t+1}}, I_{t+1}\right) \right]^{(1-\rho)/(1-\lambda)} \right\}^{(1-\lambda)/(1-\rho)}. \quad (4.22)$$

Due to the homogeneity of the recursive structure of preferences, the value function can be written in the following functional form:

$$V\left(\frac{W_t}{P_t}, I_t\right) = \Phi(I_t) \left(\frac{W_t}{P_t}\right)^{1-\lambda} \equiv \Phi_t \left(\frac{W_t}{P_t}\right)^{1-\lambda}, \quad (4.23)$$

where $\Phi(\cdot)$ is an unknown function. The homogeneity of degree zero of the recursive utility function

implies that $V(W/P, I)$ satisfying Equation (4.22) must be homogeneous of degree zero in W and P .

Let the derivatives with respect to the decision variables C_t equal zero, we then obtain:

$$C_t^{-\rho} = \frac{\delta}{1-\delta} \psi(W_t - C_t)^{-\rho}, \quad (4.24)$$

$$\text{where } \psi_t = E_t \left[\Phi_{t+1} \left(R_{m,t+1} \frac{P_t}{P_{t+1}} \right)^{1-\lambda} \right]^{(1-\rho)/(1-\lambda)}.$$

Given the structure of the problem, the nominal consumption function is linear in nominal wealth. Hence, we can rewrite Equation (4.24) as:

$$\mu_t^{-\rho} = \frac{\delta}{1-\delta} \psi(1 - \mu_t)^{-\rho} \quad (4.25)$$

where $C(W_t, I_t) = \mu(I_t)W_t \equiv \mu_t W_t$. Combining the Envelope condition with respect to W_t with the first-order condition in Equation (4.25), we obtain the following functional form:

$$\Phi_t = (1-\delta)^{(1-\lambda)/(1-\rho)} \left[\left(\frac{C_t}{W_t} \right)^{-\rho} \right]^{(1-\lambda)/(1-\rho)} \quad (4.26)$$

Substituting this expression into Equation (4.25), we obtain the following Euler equation for optimal consumption decision:

$$E_t \left\{ \left[\delta \left(\frac{C_{t+1}/P_{t+1}}{C_t/P_t} \right)^{-\rho} \right]^{(1-\lambda)/(1-\rho)} \left(R_{m,t+1} \frac{P_t}{P_{t+1}} \right)^{(1-\lambda)/(1-\rho)} \right\} = 1, \quad i = 1, \dots, N \quad (4.27)$$

The maximization with respect to the decision variable $\alpha_i (i = 2, \dots, N)$, given $\alpha_1 = 1 - \sum_{i=2}^N \alpha_i$, on the right hand side of Equation (4.22), is equivalent to the following problem:

$$\begin{aligned} \max_{\{\alpha_{i,t}\}_{i=2}^N} E_t & \left[\Phi_{t+1} \left(\sum_{i=1}^N \alpha_{i,t} R_{i,t+1} \frac{P_t}{P_{t+1}} \right)^{1-\lambda} \right] \\ \text{s.t. } & \sum_{i=1}^N \alpha_{i,t} = 1 \end{aligned} \quad (4.28)$$

Using this optimal problem along with Equation (4.26), it is straightforward to show that the necessary conditions can be derived as:

$$E_t \left\{ \left[\delta \left(\frac{C_{t+1}/P_{t+1}}{C_t/P_t} \right)^{-\rho} \right]^{1-\lambda/1-\rho} \left(R_{m,t+1} \frac{P_t}{P_{t+1}} \right)^{[1-\lambda]/(1-\rho)-1} \left(R_{i,t+1} - R_{1,t+1} \right) \frac{P_t}{P_{t+1}} \right\} = 0, \quad i = 1, \dots, N \quad (4.29)$$

Taking Equations (4.27) and (4.29) together to represent the Euler equations of the optimal problem defined in Equation (4.22), we obtain a set of N equations that provide a more direct comparison with the traditional expected utility Euler equations. Multiplying Equation (4.29) by $\alpha_{i,t}$, summing up by i , and substituting from Equation (4.27), we obtain:

$$E_t \left\{ \left[\delta \left(\frac{C_{t+1}/P_{t+1}}{C_t/P_t} \right)^{-\rho} \right]^\theta \left(R_{m,t+1} \frac{P_t}{P_{t+1}} \right)^{\theta-1} R_{i,t+1} \frac{P_t}{P_{t+1}} \right\} = 1, \quad i = 1, \dots, N \quad (4.30)$$

where $\theta = (1-\lambda)/(1-\rho)$. These are the real form Euler equations which are similar to the nominal form Euler equations seen in Epstein and Zin (1989).

When $\rho = \lambda$, the Euler equations of the time additive expected utility model are also obtained in terms of real variables:

$$E_t \left[\delta \left(\frac{C_{t+1}/P_{t+1}}{C_t/P_t} \right)^{-\rho} R_{i,t+1} \frac{P_t}{P_{t+1}} \right] = 1, \quad i = 1, \dots, N \quad (4.31)$$

Another special case of this model is the logarithmic risk preferences where $\rho = \lambda = 1$. Then, the real Euler equations are equal to the nominal Euler equations, and can be written in two algebraically identical functional forms:

$$E_t \left[\delta \left(\frac{C_{t+1}}{C_t} \right)^{-1} R_{i,t+1} \right] = 1, \quad i = 1, \dots, N \quad (4.32)$$

or

$$E_t[R_{i,t+1}/R_{m,t+1}] = 1, \quad i = 1, \dots, N \quad (4.33)$$

In this case, the parameter ρ governing intertemporal substitutability cannot be identified from these equations. Hence, there is no difference between Euler equations of the nonexpected utility model with logarithmic risk preferences and those of the expected utility model with logarithmic risk preferences.

Assume that asset prices and consumption are jointly lognormal or use a second-order Taylor expansion in the Euler equation when we assume that asset prices and consumption are conditional homoskedastic, then the log-version of the real Euler equation (4.30) can be represented as:

$$\begin{aligned} 0 = & \theta \log \delta - \theta \rho E_t \Delta c_{t+1} + (\theta - 1) E_t r_{m,t+1} \\ & + E_t r_{i,t+1} + \theta(\rho - 1) E_t \Delta \pi_{t+1} \\ & + \frac{1}{2} [\theta \rho^2 V_{cc} + (\theta - 1)^2 V_{mm} + V_{ii} \\ & - 2\theta \rho(\theta - 1) V_{cm} - 2\theta \rho V_{ci} + 2(\theta - 1) V_{im}] \\ & + \frac{1}{2} [(\theta(\rho - 1))^2 V_{\pi\pi} - 2\theta^2 \rho(\rho - 1) V_{\pi c} \\ & + 2\theta(\theta - 1)(\rho - 1) V_{\pi m} + 2\theta(\rho - 1) V_{\pi i}] \end{aligned} \quad (4.34)$$

where V_{cc} denotes $\text{var}_t(c_{t+1})$, V_{jj} denotes $\text{var}_t(r_{j,t+1}) \forall j = i, m$, V_{cj} denotes $\text{cov}_t(c_{t+1}, r_{j,t+1}) \forall j = i, m$, V_{im} denotes $\text{cov}_t(r_{i,t+1}, r_{m,t+1})$, $V_{i\pi} = \text{cov}_t(r_{i,t+1}, \pi_{t+1})$, and $\pi_{t+1} = d \ln(P_{t+1}) = \frac{dP_{t+1}}{P_{t+1}}$.

Replacing asset i by market portfolio and undergoing some rearrangement, we are able to obtain a relationship between expected consumption growth and the expected return on the market portfolio:

$$E_t \Delta c_{t+1} = \mu_m + \frac{1}{\rho} E_t r_{m,t+1} + \left(1 - \frac{1}{\rho}\right) E_t \pi_{t+1} \quad (4.35)$$

$$\begin{aligned} \text{where } \mu_m = & \frac{1}{\rho} \log \delta + \frac{1}{2} \left[\theta \rho V_{cc} + \theta \frac{1}{\rho} V_{mm} \right. \\ & \left. + 2 \left(1 - \frac{1}{\rho}\right) \theta(\rho - 1) V_{\pi\pi} \right] \\ & - \frac{1}{2} \left[2\theta V_{cm} + 2\theta(\rho - 1) \right. \\ & \left. V_{\pi c} - 2\theta \left(1 - \frac{1}{\rho}\right) V_{\pi m} \right] \end{aligned}$$

When the second moments are conditional homoskedastic, Equation (4.35) indicates that the consumption growth is linearly related to the expected world market return and expected inflation. In addition, the coefficients of these two variables are summed up to 1.

When we subtract the risk free version of Equation (4.34) from the general version, we obtain:

$$\begin{aligned} E_t r_{i,t+1} - r_{f,t+1} = & -\frac{V_{ii}}{2} + \theta \rho V_{ic} \\ & + (\theta - \theta \rho) V_{i\pi} + (1 - \theta) V_{im} \end{aligned} \quad (4.36)$$

where $r_{f,t+1}$ is a log riskless real interest rate. This result is similar to that of Campbell (1993) except for the inflation term. Equation (4.36) shows that the expected excess log return on an asset is a linear combination of its own variance, which is produced by Jensen's inequality, and by a weighted average of three covariances. The weights on the consumption, inflation, and market are $\theta \rho$, $(\theta - \theta \rho)$, and $(1 - \theta)$, respectively. Moreover, the weights are summed up to 1. This is one of the most important differences between Campbell's model and our real model.

If the objective function is a time-separable power utility function, a real functional form of a log-linear version of the consumption CAPM pricing formula can thus be obtained:

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \rho V_{ic} + (1 - \rho) V_{i\pi} \quad (4.37)$$

The weights on the consumption and inflation are ρ and $(1 - \rho)$, respectively. These weights are also summed up to 1. However, when the coefficient of relative risk aversion $\lambda = 1$, then $\theta = 0$. The model is reduced to the real functional form of log-linear static CAPM, which is the same as the nominal structure of log-linear static CAPM.

4.3.2. International Asset Pricing Model Without Consumption

In order to get a pricing formula without consumption, we apply the technique of Campbell (1993). Campbell (1993) suggests to linearize the

budget constraint by dividing the nominal form of Equation (4.21) by W_t , taking log, and then using a first-order Taylor approximation around the mean log consumption/wealth ratio ($\log(C/W)$). Following his approach, approximation of the nominal budget constraint is:

$$\Delta w_{t+1} \cong r_{m,t+1} + \kappa + \left(1 - \frac{1}{\beta}\right)(c_t - w_t) \quad (4.38)$$

where the log form of the variable is indicated by lowercase letters, $\beta = 1 - \exp(c_t - w_t)$, and κ is a constant.

Combining Equation (4.38) with the following trivial equality

$$\Delta w_{t+1} = \Delta c_{t+1} + (c_t - w_t) - (c_{t+1} - w_{t+1}), \quad (4.39)$$

we obtain a difference equation in the log consumption/wealth ratio, $c_t - w_t$. When the log consumption/wealth ratio is stationary, i.e. $\lim_{j \rightarrow \infty} \beta^j (c_{t+j} - w_{t+j}) = 0$, Equation (4.38) implies that the innovation in logarithm of consumption can be represented as the innovation in the discounted present value of the world market return minus the innovation in the discounted present value of consumption growth:

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j r_{m,t+1+j} \\ &\quad - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j \Delta c_{t+1+j} \end{aligned} \quad (4.40)$$

Now we are ready to derive an international asset pricing model without consumption in terms of real variables by connecting the log-linear Euler equation to the approximation log-linear budget constraint. Substituting Equation (4.35) into Equation (4.40), we obtain:

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= r_{m,t+1} - E_t r_{m,t+1} \\ &\quad + \left(1 - \frac{1}{\rho}\right)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j r_{m,t+1+j} \\ &\quad - \left(1 - \frac{1}{\rho}\right)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j \pi_{m,t+1+j} \end{aligned} \quad (4.41)$$

Equation (4.41) implies that an unexpected consumption may come from three sources. The first one is the unexpected return on invested wealth today. The second one is the expected future nominal returns. The direction of influence depends on whether $1/\rho$ is less or greater than 1. When $1/\rho$ is less than 1, an increase (or decrease) in the expected future nominal return increases (or decreases) the unexpected consumption. Conversely, when $1/\rho$ is greater than 1, an increase (or decrease) in the expected future nominal return decreases (or increases) the unexpected consumption. The third one is the inflation in the investor's own country. The direction of influence also depends on whether $1/\rho$ is less or greater than 1. When $1/\rho$ is less than 1, an increase (or decrease) in the inflation decreases (or increases) the unexpected consumption. Conversely, when $1/\rho$ is greater than 1, an increase (or decrease) in the inflation increases (or decreases) the unexpected inflation.

Based on Equation (4.41), the conditional covariance of any asset return with consumption can be rewritten in terms of covariance with market return and revisions in expectations of future market return as:

$$\begin{aligned} \text{cov}_t(r_{i,t+1}, \Delta c_{t+1}) &\equiv V_{ic} = V_{im} + \left(1 - \frac{1}{\rho}\right) \\ &\quad V_{ih} - \left(1 - \frac{1}{\rho}\right) V_{ih\pi}, \end{aligned} \quad (4.42)$$

where

$$V_{ih} = \text{Cov}_t \left(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j r_{m,t+1+j} \right)$$

and

$$V_{ih\pi} = \text{Cov}_t \left(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j \pi_{t+1+j} \right)$$

Substituting Equation (4.42) into Equation (4.36), we thus obtain an international asset pricing model, which is not related to consumption:

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \lambda V_{im} + (\lambda - 1) V_{ih} + (1 - \lambda) V_{i\pi} + (1 - \lambda) V_{ih\pi} \quad (4.43)$$

The only preference parameter that enters Equation (4.43) is the coefficient of relative risk aversion (λ). The elasticity of intertemporal substitution ρ is not present under this international pricing model. Equation (4.43) states that the expected excess log return in an asset, adjusted for Jensen's inequality effect, is a weighted average of four covariances. These are the covariance with the market return, the covariance with news about future returns on invested wealth, the covariance with return from inflation, and the covariance with news about future inflation. This result is different from both the international model of Adler and Dumas (1983) and the intertemporal model of Campbell (1993). Adler and Dumas use von Neumann–Morgenstern utility and assume a constant investment opportunity set to derive the international model, and therefore neither V_{ih} or $V_{ih\pi}$ is included in their pricing formula. Since the intertemporal model of Campbell is a domestic model, it does not deal with the issues of inflation and currency that are emphasized in our international asset pricing model.

4.3.3. International Asset Pricing Model When PPP Deviate

Let us now turn to the problem of aggregation across investors. It is true that different investors use different information set and different methods to forecast future world market return and inflation. To obtain the aggregation results, we first superimpose Equation (4.43) by a superscript l to indicate optimal condition for an investor l :

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \lambda^l V_{im}^l + (\lambda^l - 1) V_{ih}^l + (1 - \lambda^l) V_{i\pi}^l + (1 - \lambda^l) V_{ih\pi}^l \quad (4.43)$$

Then, Equation (4.44) can be aggregated across all investors in all countries.

The operation is to multiply Equation (4.44) by η^l , which indicates risk tolerance where $\eta^l = 1/\lambda^l$ and to take an average of all investors, where weights are their relative wealth. After aggregating all investors, we obtain:

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \frac{1}{\eta^m} V_{im}^m + \left(\frac{1}{\eta^m} - 1\right) \sum_l \omega^l V_{ih}^l + \left(1 - \frac{1}{\eta^m}\right) \sum_l \omega^l V_{i\pi}^l + \left(1 - \frac{1}{\eta^m}\right) \sum_l \omega^l V_{ih\pi}^l \quad (4.45)$$

where $\eta^m = \frac{(\sum_l W^l \eta^l)}{(\sum_l W^l)}$ and $\omega^l = \frac{(1 - \lambda^l) W^l}{\sum_l (1 - \lambda^l) W^l}$.

There are several interesting and intuitive results in this equation. First, Equation (4.45) shows that an international asset risk premium adjusted for one-half its own variance is related to its covariance with four variables. These are the world market portfolio, aggregate of the innovation in discounted expected future world market returns from different investors across countries, aggregate of the inflation from different countries, and aggregate of the innovation in discounted expected future inflation from different investors across countries. The weights are $1/\eta^m$, $1/\eta^m - 1$, $1 - 1/\eta^m$, and $1 - 1/\eta^m$, respectively. The sum of these weights is equal to 1. Moreover, it is noted that the market hedging risk is a weighted average of world market portfolio for investors from different countries. This is different from the domestic counterpart of Campbell (1993).

Second, an international asset can be priced without referring to its covariance with consumption growth. Rather, it depends on its covariance with world market return, the weighted average of news about future world market return for investors from different countries, inflation, and the weighted average of news about future inflation for investors from different countries.

Third, the coefficient of risk tolerance, η_m , is the only preference parameter that enters Equation (4.45). When consumption is substituted out in

this model, the coefficient of intertemporal substitution ρ disappears. Similar results have been documented by Kocherlakota (1990) and Svensson (1989). They show that when asset returns are independently and identically distributed over time, the coefficient of intertemporal substitution is irrelevant for asset returns.

If we are willing to make some more assumptions, we can obtain a more compact result. Namely, if investors are assumed to have the same world market portfolio and use the same method to forecast world market portfolio return, we can multiply Equation (4.44) by λ^l , and take an average of all investors, where weights are their relative wealth, to get a simple version of the international asset pricing model:

$$E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \lambda^m V_{im} + (\lambda^m - 1)V_{ih} + (1 - \lambda^m) \sum_l \omega^l V_{i\pi}^l + (1 - \lambda^m) \sum_l \omega^l V_{ih\pi}^l \quad (4.46)$$

where $\lambda^m = (\sum_l W^l \lambda^l) / (\sum_l W^l)$ and $\omega^l = \frac{(1-\lambda^l)W^l}{\sum_l (1-\lambda^l)W^l}$

Both hedging risk V_{ih} and currency risk $V_{i\pi}^l$ are related to expected return. In addition, they all depend on whether λ^m is different from 1 or not.

Furthermore, when we assume that domestic inflation is nonstochastic, the only random component in π would be the relative change in the exchange rate between the numeraire currency and the currency of the country, where the investor resides. Hence, $V_{i\pi}^l$ is a pure measure of the exposure of asset i to the currency risk of the country, where investor l resides and $V_{ih\pi}^l$ is also a measure of the exposure of asset i to hedge against the currency risk of the country, where investor l resides.

Equation (4.46) also states that the currency risk is different from the hedging risk. However, if V_{ih} and $V_{i\pi}^l$ are large enough, then whether V_{ih} and $V_{i\pi}^l$ are related to expected return depends on whether or not λ^m is different from 1: This may be the reason why Dumas and Solnik (1995) argue that exchange rate risk premium may be equivalent to intertemporal risk premium. But, their conjecture

is based on an empirical “horse race” test between international model and intertemporal model rather than a theoretical derivation.

4.4. Conclusion

The international asset pricing model without consumption developed by Chang and Hung (2004) argues that the real expected asset return is determined by market risk, market hedging risk, currency risk, and currency hedging risk. The weights are related only to relative risk aversion. Moreover, the weights are summed up to 1. Their results may be contrasted with the pioneering work of Adler and Dumas (1983), who assume a constant investment opportunity set, thus their model lacks market hedging risk and currency hedging risk.

In the Chang et al. (2004) model, the price of market hedging risk is equal to the negative price of the currency risk. This may be the reason why Dumas and Solnik (1995) argue that currency risk is equivalent to market hedging risk. But, their conjecture is based on a “horse race” test between international model and intertemporal model rather than on a theoretical derivation.

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