2 Cost Functions

Purpose of cost functions

For a given firm, pricing as well as production planning require thorough knowledge of the firm's costs at different activity levels

For instance the owner of a pizzeria needs to know about costs incurred to the firm at different activity levels (number of pizzas produced) when pricing in connection with a brochure that is to be distributed door-to-door. Costs vary with activity level, as purchases of raw materials, hiring of employees, choice of oven, and maybe even size of the location depend on the chosen level of activity. Which costs vary with the activity level depend on the owner's decision-making horizon, e.g. if there are big differences between planning the daily operations and strategic considerations regarding the pizzeria's future.

With the purpose of assisting the decision maker (e.g. the owner of the pizzeria) cost functions expressing cost as a function of the activity level are devised. Depending on the functions of the decision-making situation, the following cost functions are applied:

- Total costs (TC, TVC, and TFC)
- Average costs (ATC, AVC, and AFC)
- Marginal costs (MC)

Concerning cost functions, it is important to find a suitable measurement unit for the activity level. The measurement unit varies with business sector and industry. Examples of measurement units are:

- Produced units (number of baked pizzas, printed magazines, manufactured cars)
- Turnover (expresses the activity in restaurants and clothing stores)
- Working hours (expresses the activity for architectural firms and law firms)

2.1 Cost Functions in the Short-Term

Total functions

In the following, the cost functions are explained within the frame of the firm's short-term time horizon, which is why both fixed costs and variable costs are involved. The basis of the firm's cost functions are the total-functions as described below.

- *"Total fixed costs" (TFC):* The fixed costs do not fluctuate with the activity level, within the chosen time horizon. For instance the pizzeria's rent, interest, oven installation, etc., do not vary based on the number of pizzas baked over a short-term time horizon.

- *"Total variable costs" (TVC):* The variable costs change based on the activity level within the chosen time horizon. For instance the cost of flour, meat, and workers' salaries, vary in conjunction with the number of pizzas baked, even within the short-term time horizon.

- *"Total costs" (TC):* The TC-function is found through vertical adding the TFC and TVC curves, i.e. by compiling the fixed and variable costs.

Average functions

As is the case when finding any average, the average-functions are calculated on the basis of the firm's total-functions, which are then divided by the relevant factor (in this case quantity). The average-functions are found as shown below:

- *"Average fixed costs" (AFC):* Expresses the fixed costs divided by the output, i.e. TFC/Q (Q = quantity, e.g. number of pizzas produced in a pizzeria).

- *"Average variable costs" (AVC):* Expresses the variable costs divided by firm output, i.e. TVC/Q.

- *"Average total costs" (ATC):* Expresses the total costs divided by firm output, i.e. TC/Q

Marginal functions

When the firm plans its production level, it is crucial to know how much the total costs change as related to a change in quantity. In this case marginal costs are applied.

- *"Marginal costs" (MC):* Expresses the change that appears in the total costs as a result of the firm producing one unit more. For instance, the marginal cost function for the pizzeria expresses the change in the total costs which appear when one additional pizza is produced.

The firm's marginal cost function is thus found as the change in the TVC function (or the TC function), resulting from a change in the quantity produced. For this reason the MC function is found by differentiating the TVC-function (or the TC function, as the constant is neutralized in any case):

$$MC = \frac{\partial TVC}{\partial Q} \text{ or } \frac{\partial TC}{\partial Q}$$

The cost development

Costs can develop in the following ways:

- Proportionally, i.e. if the activity increases 1% then the total costs also increase 1%.
- Digressively, i.e. if the activity increases 1% then the total costs increase less than 1%.

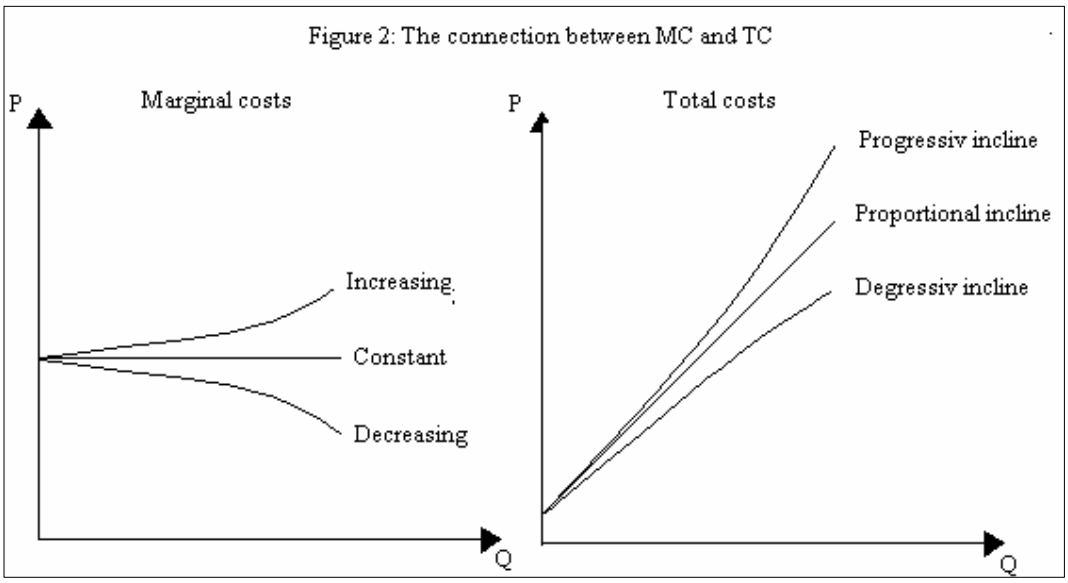
  This phenomenon is called increasing returns to scale, or economies of scale. In modern production this is the norm, as many actions support this: The more you buy the cheaper it gets. Administrative systems are cheaper for each unit, and marketing as well as production lines are utilized more efficiently.

- Progressively, i.e. if the activity increases 1% then the costs increase more than 1%

  This phenomenon is called decreasing returns to scale or diseconomies of scale.

  In modern production this is not normal. If existing it is probably found in service sectors which are heavily knowledge-based.

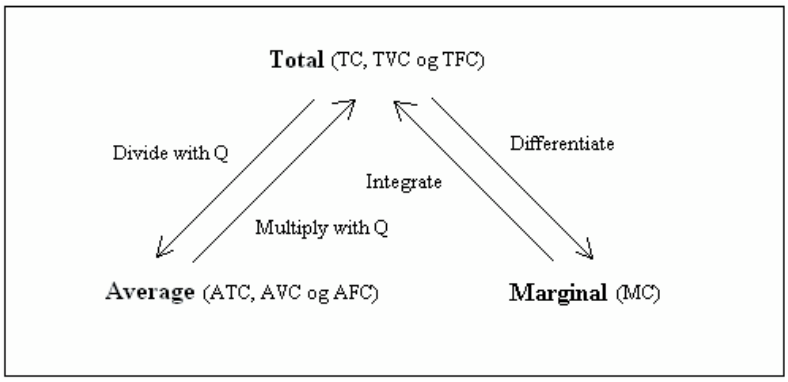
The connection is shown in figure 2:



It must be emphasized that cost development is dependent on both activity level and decision-making situation. A firm may, for example, have a digressive cost development (economies of scale) at a low level of activity because of increasing discounts on purchasing, and at the same time experience a progressive cost development because of waste, discarding, less efficient employees etc. Often these circumstances are present simultaneously.

The connection between total costs, average costs, and marginal costs.

The connection between total costs, average costs, and marginal costs is shown in model below:



The triangle model depicts: **that when you have a mathematical term for the cost function** – moving from one cost function to another is possible in the following ways:

- You move from a total function to an average function by dividing with Q.
- You move from an average function to a total function by multiplying with Q

For instance, to move from TC to ATC you divide TC with Q – and from ATC to TC by multiplying ATC with Q.

Furthermore, the triangle model shows that movement from a total function (TC or TVC) to a marginal function (MC) is possible by differentiating the total function. In this way the marginal function reflects the inclination on the total costs curve. This relationship is logical as the marginal costs show the costs of the production of one additional unit, which is exactly the increase in the total costs.

In the opposite direction, the triangle model also shows that you move from a marginal function (MC) to a total function (TVC) by integrating the marginal function. In this way the total function (TVC) reflects the area below the marginal costs curve. This relationship is equally logical as the total variable costs (TVC) are precisely the sum of all the marginal costs.

Moreover, it is shown that you move from an average function (ATC or AVC) to a marginal function (MC) by multiplying Q and thereafter differentiating the function; i.e. you first find the total function (TC or TVC) and thereafter find the marginal function.

2.1 The North Sealand Raspberry Plantation

North Sealand Raspberry Plantation

North Sealand Raspberry Plantation is a minor fruit plantation, that exclusively cultivates organic raspberries with organic methods. The plantation rents one hectare of land at an annual cost of 3,000 DKK Furthermore, it costs the firm 15,000 DKK to stretch out a net for protection against birds, to replant, to inspect, and cultivate the raspberries.

Harvest season is coming, which lasts about three weeks. All the raspberries are ready to be picked. The owner has to determine how many kilos are to be picked – and consequently how many pickers to hire. It is out of the question to harvest all of the raspberries as the costs of picking the last ones (the small ones or those that hang low to the ground) exceeds the value of the raspberries. The owner knows from experience that, due to internal competition the first pickers hired work faster, as succeeding pickers are hired. This positive relationship comes quickly to an end, as the pickers are in the way of each other, if too many are hired. The pickers are paid 100 DKK an hour, and with a variation each picks 8–13 kg. per hour. Naturally the best pick more and the worst pick less. Effort is put into hiring the best first.

In this short-term scenario, the costs of renting and cultivating the land are fixed. Only the costs of the raspberry pickers vary with the activity level.

The costs in a table

The costs incurred by North Sealand Raspberry Plantation for the picking of raspberries are listed in table 2.1, showing the costs as a function of the number of berries picked.

Raspberries (100 kg)	TC	TVC	TFC	ATC	AVC	MC*	AFC
0	18.000	-	18.000	-	-	-	-
10	27.800	9.800	18.000	2.780	980	810	1.800
20	35.200	17.200	18.000	1.760	860	720	900
30	43.200	25.200	18.000	1.440	840	930	600
40	54.800	36.800	18.000	1.370	920	1.440	450

** MC has been calculated by differentiating the function below at point Q (give it a try).*

Table 2.1: The costs as a function of the number of raspberries picked

Costs shown mathematically.

The costs incurred by North Sealand Raspberry Plantation in conjunction with picking raspberries, can be listed as a mathematical function, with cost in terms of the amount of raspberries picked (100 kg.):

$$\mathbf{TC = 0{,}5Q^3 - 27Q^2 + 1.200Q + 18.000}$$

The TC function can be divided into fixed and variable costs. The fixed costs do not vary in terms of production. Therefore, the TFC function and the TVC functions are found based on the TC-function:

TFC = 18.000

TVC = 0,5Q^3 – 27Q^2 + 1.200Q

Based on the total functions, the average functions can be found by dividing by Q:

ATC = TC/Q = 0,5Q^3 – 27Q^2 + 1.200Q + 18.000)/Q =
0,5Q^2 – 27Q + 1.200 + 18.000/Q

AFC = TFC/Q = **18.000/Q**

AVC = TVC/Q = (0,5Q^3 – 27Q^2 + 1.200Q)/Q = **0,5Q^2 – 27Q + 1.200**

The MC function is found by differentiating the TVC function (the TC function, as the constant unit is neutralized anyway). Below the TVC function is differentiated:

MC = (TVC)' = (0,5Q^3 – 27Q^2 + 1.200Q)' = **1,5Q^2 – 54Q + 1.200**

It is now possible to go "the other way" in the triangle model and see if the original functions result, which is a good way to check the validity of one's calculations.

The TC function is found by multiplying the ATC function with Q:

TC = (0,5Q^2 – 27Q + 1.200 + 18.000/Q) x Q =
0,5Q^3 – 27Q^2 + 1.200Q + 18.000

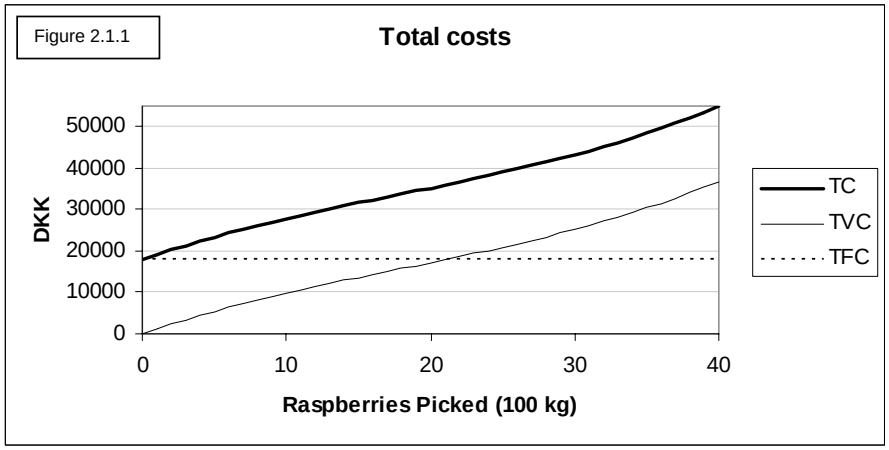
The TVC function is found by integrating the MC function :

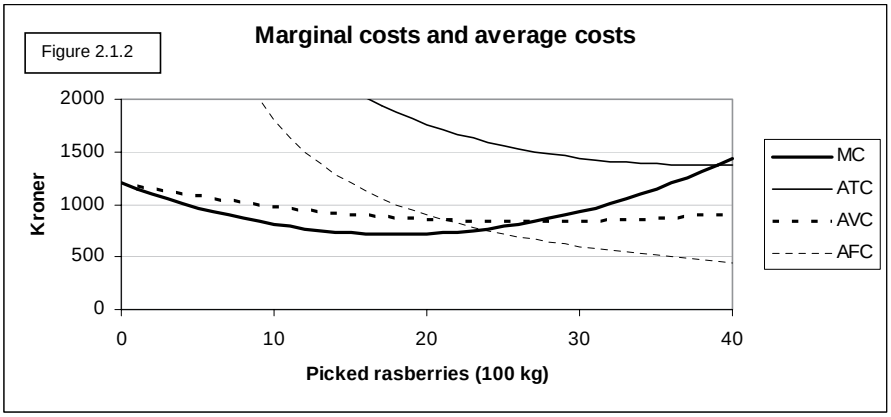
TVC = ∫MC = ∫(1,5Q^2 – 54Q + 1.200) = **0,5Q^3 – 27Q^2 + 1.200Q**

As seen from the calculations above, the original functions appear by "going the other way" in the triangle model, which indicates that the mathematical part of the cost functions are correct.

Costs shown graphically

The cost of picking the raspberries can be shown graphically, as is done in figure 2.1.1 and 2.1.2:





In figure 2.1.1 and 2.1.2 some important cost-related correlations are demonstrated:

- When MC increases and crosses AVC it always happens at a local minimum for AVC. The logic implied here is that when MC is below AVC then AVC decreases, and when MC is above AVC then AVC increase. The MC's crossing of the AVC demarcates the change between the descending and ascending sectors of AVC. This pattern corresponds to that MC, which is respectively below and above the AVC on each side of the crossing. Similar lines of reasoning are involved when MC is descending and crosses AVC.
- The reasoning explained above also applies to the relationship between MC and ATC. Notice that MC crosses ATC at a higher Q-level than when AVC was crossed. This is due to ATC being AFC+AVC, and thus ATC is at a higher level.
- In a continuous relationship, MC expresses the gradient of VC, and thus TC. When MC is low it is synonymous with TC ascending slowly.

2.2 Easymap

"Easymap" is a minor niche company producing computer mouse-pads, which are constructed in a manner that protects the mouse from collecting dirt or dust. This company's product is based on the principle that a clean mouse is fast mouse. These mouse-pads are a hit among young gamers.

Costs

Easymap has fixed costs of 400,000 DKK a year, which include rent, interests/ depreciation of machinery, management salaries, etc. Furthermore, Easymap has the following variable costs per mouse-pad produced:

- 1 DKK in materials
- 30 øre in electricity for machinery, repair and maintenance, sales commission etc.
- 3 DKK for salaries within normal working hours
- 6 DKK for salaries outside normal working hours

Variable costs within normal working hours:	**4.30 DKK per unit**
Variable costs outside normal working hours	**7.30 DKK per unit**

It is possible to produce 60,000 mouse-pads within normal working hours. Overtime allows for the production of an additional 20,000 mouse-pads.

Costs shown mathematically

The cost functions are defined in order to create a general view of the costs involved in mouse-pad production. All cost functions are based on the TC function;

TC = 4.30Q + 400,000	representing the interval from 0–60,000 units produced
TC = 7.30Q + 220,000	representing the interval from 60,001–80,000 units produced

Based on the TC function, the TFC function and the TVC function are determined. This process is made possible by the fact that the fixed costs do not vary with production and are therefore not affected by Q:

TFC = 400,000	regardless of the activity level

TVC = 4.30Q	from 0–60,000 units
TVC = 7.30Q – 180,000	from 60,001–80,000 units

The average functions are determined when the total functions are divided by Q:

ATC = 4.30 + 400,000/Q	from 0–60,000 units
ATC = 7.30 + 220,000/Q	from 60,001–80,000 units

AFC = 400,000/Q	regardless of the activity level

AVC = 4.30	from 0–60,000 units
AVC = 7.30 – 180,000/Q	from 60,001–80,000 units

The MC function is determined when the TVC function (or the TC function) is differentiated:

MC = 4.30	from 0–60,000 units
MC = 7.30	from 60,001–80,000 units

Costs shown graphically

The cost functions are illustrated in figure 2.2.1 and 2.2.2:

Figure 2.2.1

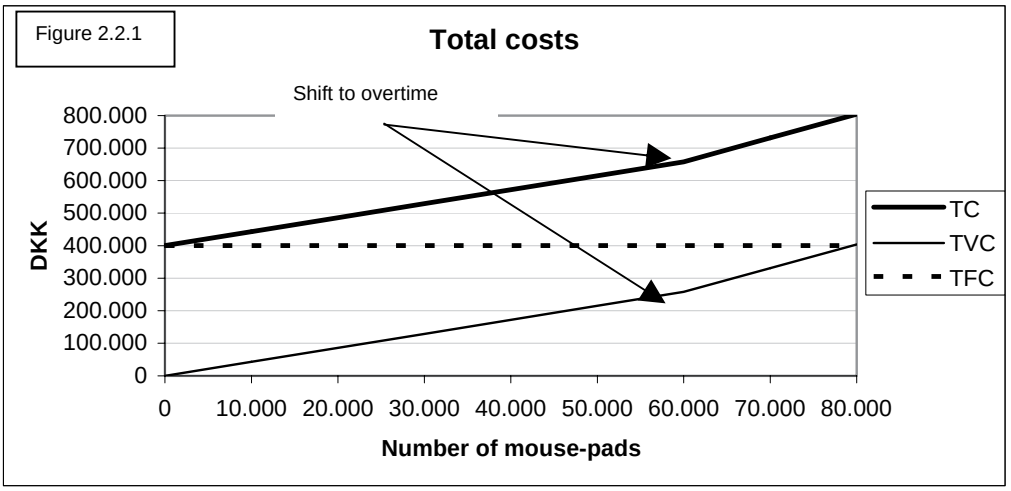
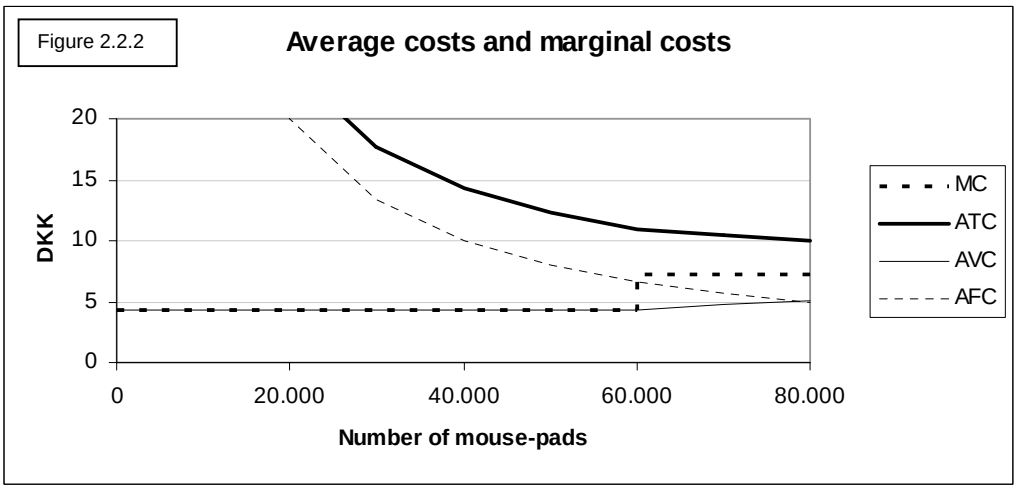


Figure 2.2.2



2.3 The Printing House

The Printing House

The Printing House specializes in fashion magazines. They are located in Viborg and have 3 employees. The company has annual fixed costs totalling 600,000 DKK, which include rent, interest, and installments paid on their printing machine, and insurances, etc. The cost of printing a fashion magazine is about 3 DKK, which includes materials, production worker wages, directly applicable electricity, etc. At this time, it is possible to print about 300,000 fashion magazines annually. The firm has the possibility of expanding production capacity by investing in a new and larger printing machine, which would cost 250,000 DKK. By replacing the printing machine the production capacity would be expanded to 700,000 fashion magazines annually. Simultaneously the costs of printing would decrease to 2.25 DKK per unit.

Cost functions

The Printing House would like to know what the costs are as related to different activity levels, which are especially relevant for their pricing considerations. For this reason, cost functions, which express cost as a the measurement function, are determined. The measurement unit, which defines the activity level, is the number of magazines printed.

Total costs

As mentioned earlier, the firm has annual fixed costs of 600,000 DKK. The variable costs are 3 DKK per magazine from 0–300,000 fashion magazines a year, and 2.25 DKK from 300,001–700,000 magazines a year. Total costs can therefore be described as seen below:

TC = 3Q + 600,000	from 0–300.000 fashion magazines per year
TC = 2.25Q + 1,075,000	from 300.001–700.000 fashion magazines per year

Based on the above information, the total variable costs (TVC) and the total fixed costs (TFC) are determined. TVC varies with production though TFC does not.

TVC = 3Q	from 0–300.000 fashion magazines per year
TVC = 2.25Q + 475,000	from 300.001–700.000 fashion magazines per year

TFC = 600,000 regardless of activity level.

Average costs

Based on TC, TVC, and TFC the average total costs (ATC), the average variable costs (AVC), and the average fixed costs (AFC) are found when the total cost functions are divided by Q:

ATC = 3 + 600,000/Q	from 0–300,000 fashion magazines per year
ATC = 2.25 + 1,075,000/Q	from 300,001–700,000 fashion magazines per year

AVC = 3	from 0–300,000 fashion magazines per year
AVC = 2.25 + 475,000/Q	from 300,001–700,000 fashion magazines per year

AFC = 600,000/Q regardless of activity level

Marginal costs

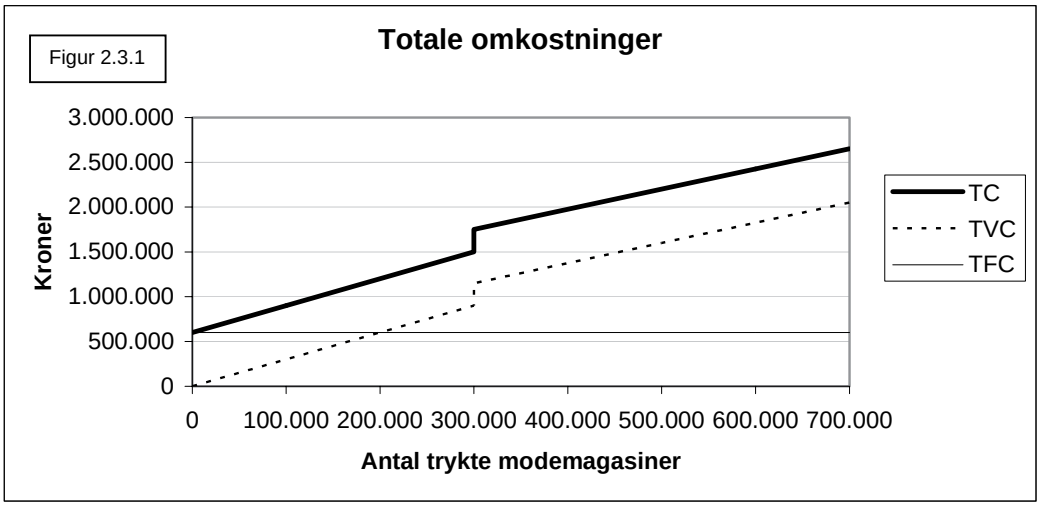
The marginal costs (MC) are deduced by differentiating the TVC function (or the TC function, as the constant will always be neutralized). The marginal costs are described below:

MC = 3	from 0–300,000 fashion magazines per year
MC = 2.25	from 300,001–700,000 fashion magazines per year

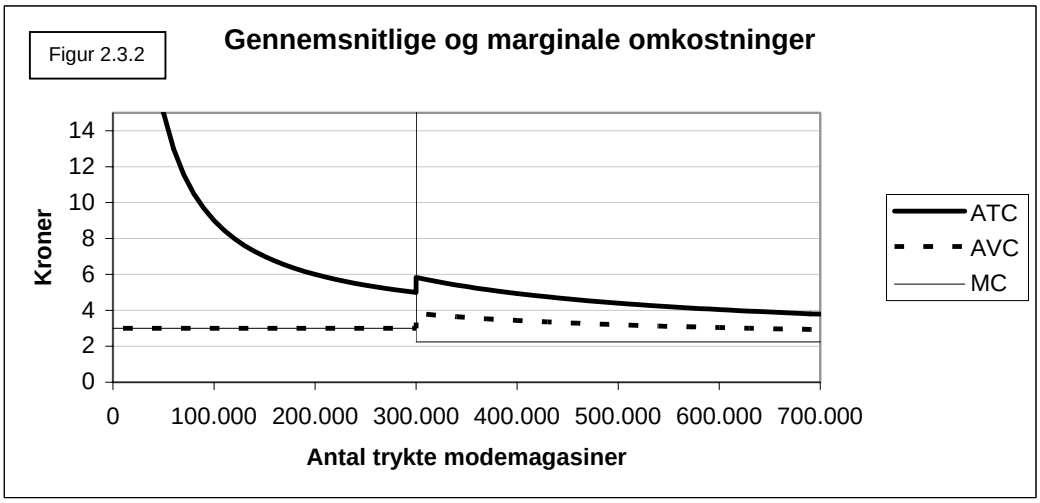
Graphic presentation of the cost functions

The costs are shown in figure 2.3.1 and 2.3.2:

Figur 2.3.1



Figur 2.3.2



Weakness of the cost functions

The weakness of the cost functions is that they do not consider the cost complexity. Understanding that a function cannot reflect the complex conditions of the firm is important. However, cost functions can, if they are drawn up in a sensible way, be a useful tool for decision makers.

The complicating factors include: Motivation, planning, production levels for other products, general market conditions, general employment conditions, etc. There are both internal and external conditions that complicate these terms.

Case 2.4: Cost complexity

When a restaurant's master chef is about to price a dinner party, several factors are considered. One of these factors is the table setting. Even though setting the table is a simple process, it is actually quite difficult to establish exactly how much it will cost before the tables are set. That the costs are difficult to establish in advance is owing to the following factors:

- **The special wishes of the party** (arrangement of the tables, table dressing, napkins, flowers, table decorations, and candles etc.) vary from party to party, which results in that the costs varying from party to party.
- **The wages used on the table setting** depend on whether the employees have a good or a bad day, the time of the table setting (when the tables are set in the morning it happens a lot faster than in the evening), who is on the job, etc.
- **The wages spent on ordering** table dressings, napkins, decorations, flowers, and candles, depend on whether the supplier that was reached first is able to supply everything, or if alternative suppliers have to be reached. It also depends on whether the supplier has the relevant information about inventory on his computer or if he has to ask about the available inventory (which results in waiting time and additional phone calls)

The master chef at Bowl'n'Fun in Viborg, Kim Bülow Mortensen, says:

"It is almost impossible to know how much it generally costs to set a table for a party of, lets say 100 people, before the table has been set. I have to know a lot of things before I can make a qualified answer. A bowling center is such a dynamic entity that plans often have to be changed; i.e. under optimal conditions I can have the tables set for about 12 DKK per person, but I run the risk that they have to be set in the last minute, with tired waiters working over time. In this situation, it may cost up to 20 DKK per person attending the party. That is why it is difficult to establish the costs in advance."

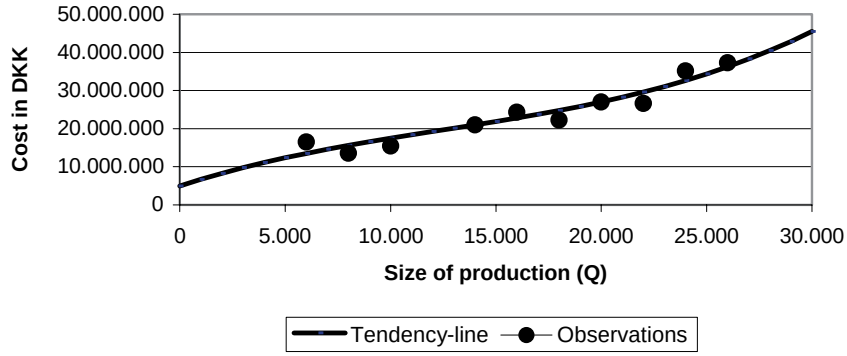
2.2 Assignments for Chapter 2

Mini case 2.1: "Guns and Thrills"

"Guns and Thrills" is an American production company with a number of quality products. The products are marketed and sold in the USA and most of Europe through national and international wholesalers, who distribute the products on to specialized stores.

"Guns and Thrills," despite quality difference, competes with some Asian paintball factories, who enjoy lower production costs. The management of the firm has been focusing specifically on cost perspectives. Management has estimated the firm's TC function for the product with the highest demand, a fully automatic AK-47 assault rifle. The TC function has been estimated based on historic cost observations, at different production levels, using Excel. Based on these observations, Excel has calculated a tendency-line, which subsequently has been applied as the management's estimate of the TC function. The observations and the resulting TC function, in this example converted into DKK, is shown in the figure below:

Figure 2.1: Estimate of the TC function



The TC function (the tendency-line) has been determined with the following equation:

$TC = 0{,}000002Q^3 - 0{,}075Q^2 + 1.800Q + 5.000.000$

Case assignment 2.1:
Determine the MC function mathematically, and if possible, show it graphically.

Case assignment 2.2:
Explain the change in the MC curve.

Case assignment 2.3:
Determine the AVC mathematically.

Case assignment 2.4:
Find the production size where the AVC function reaches its minimum.

Case assignment 2.5:
Why would other similar assault rifle manufacturers probably have different cost functions?

Case assignment 2.6:
Where can the firm go wrong in estimating the TC function?

Case assignment 2.7:
Which factors decide the development of the TC function?

Question 2.1 A company has estimated the following MC function in DKK, for production sizes of no more than 300 units:

$$MC = 0{,}003Q^2 - 0{,}8Q + 120$$

Determine the AVC function mathematically.
Draw the MC and AVC function in the same diagram and explain the connection between the two curves.

Question 2.2 Fill out the empty spaces in the table below. The company has fixed costs of DKK 1 million.

Q	0	1	2	3	4
MC	-	100.000	90.000	98.000	104.000
TVC					
TC					
AVC					

Question 2.3 Why integrate the MC function in order to get to the TVC function?

Question 2.4 A firm has an increasing MC function. Will the TVC function of this firm increase progressively, proportionally, or digressively? Explain why.

Question 2.5 Why does the AVC minimum occur at the point where the MC function crosses the AVC function from below?

Question 2.6 Why does the ATC function reach its minimum at a higher production level than the AVC function? And could you imagine a situation where ATC is minimized at a lower Q than AVC?

Question 2.7 How is MC influenced by the following actions:

- an increase in the hourly wage for production workers
- an accumulated quantity discount
- increased real estate taxes
- increased administration costs

Question 2.8 How far does the short-term time horizon stretch when determining cost functions? – include examples of "short-term" for different industries.

Question 2.9 Why is it possible to determine the MC function based on the ATC function, when the ATC function cannot be found when the MC function is known?