



Cost Behavior

CHAPTER 3

AFTER STUDYING THIS CHAPTER, YOU SHOULD BE ABLE TO:

1. Define and describe fixed, variable, and mixed costs.
2. Explain the use of resources and activities and their relationship to cost behavior.
3. Separate mixed costs into their fixed and variable components using the high-low method, the scatterplot method, and the method of least squares.
4. Evaluate the reliability of the cost formula.
5. Explain how multiple regression can be used to assess cost behavior.
6. Define the learning curve, and discuss its impact on cost behavior.
7. Discuss the use of managerial judgment in determining cost behavior.

Costs can display variable, fixed, or mixed behavior. Knowing how costs change as activity output changes is an essential part of planning, controlling, and decision making. For example, budgeting, deciding to keep or drop a product line, and evaluating the performance of a segment all benefit from knowledge of cost behavior. In fact, not knowing and understanding cost behavior can lead to poor—and even disastrous—decisions. This chapter discusses cost behavior in depth so that a proper foundation is laid for its use in studying other cost management topics. A variable-costing system, for example, requires that all costs be classified as fixed or variable. But can all costs realistically be classified into one of these two categories? What are the assumptions and limitations associated with classifying costs in this way? Furthermore, just how good are our definitions of variable and fixed costs? Finally, what procedures can we use to break out the fixed and variable components of mixed costs? How do we assess the reliability of these procedures?

Basics of Cost Behavior

Cost behavior is the general term for describing whether a cost changes when the level of output changes. A cost that does not change as activity output changes is a *fixed cost*. A *variable cost*, on the other hand, increases in total with an increase in activity output and decreases in total with a decrease in activity output. In economics, it is usually *assumed* that fixed and variable costs are known. Management accountants must deal with the requirements to assess fixed and variable costs. Let's first review the basics of cost and output measures. Then, we will look at fixed, variable, and mixed costs. Finally, we will assess the impact of time horizon on cost behavior.

OBJECTIVE 1

Define and describe fixed, variable, and mixed costs.

Measures of Activity Output

In order to determine the behavior of a cost, we need to have a good grasp of the cost under consideration and a measure of the output associated with the cost object. The terms *fixed cost* and *variable cost* do not exist in a vacuum; they only have meaning when related to some output measure or driver. Therefore, in order to understand the behavior of costs, we must first determine the underlying activities and the associated drivers that measure the capacity of an activity and its output. For example, materials handling output may be measured by the number of moves, shipping goods output may be measured by the units sold, and laundering hospital bedding output may be measured by the pounds of laundry. The choice of driver is tailored not only to the particular firm but also to the particular activity or cost being measured.

Activity drivers explain changes in activity costs by measuring changes in activity output (usage). The two general categories of activity drivers are *unit-level drivers* and *non-unit-level drivers*. **Unit-level drivers** explain changes in cost as units produced change. Pounds of direct materials, kilowatt-hours used to run production machinery, and direct labor hours are examples of unit-based activity drivers. Notice that while none of these drivers is equal to the number of units produced, each one does vary proportionately with the number of units produced. **Non-unit-level drivers** explain changes in costs as factors other than changes in units produced. Examples of non-unit-based activity output measures include setups, work orders, engineering change orders, inspection hours, and material moves.

In a functional-based cost management system, cost behavior is assumed to be described by unit-based drivers only. In an activity-based cost management system, both unit- and non-unit-based drivers are used. Thus, the ABC system tends to produce a much richer view of cost behavior than would a functional-based system. A need exists, however, to identify cost behavior patterns for a much broader set of activities.

We now take a closer look at fixed, variable, and mixed costs. In each case, the cost is related to only one measure of output.

Fixed Costs

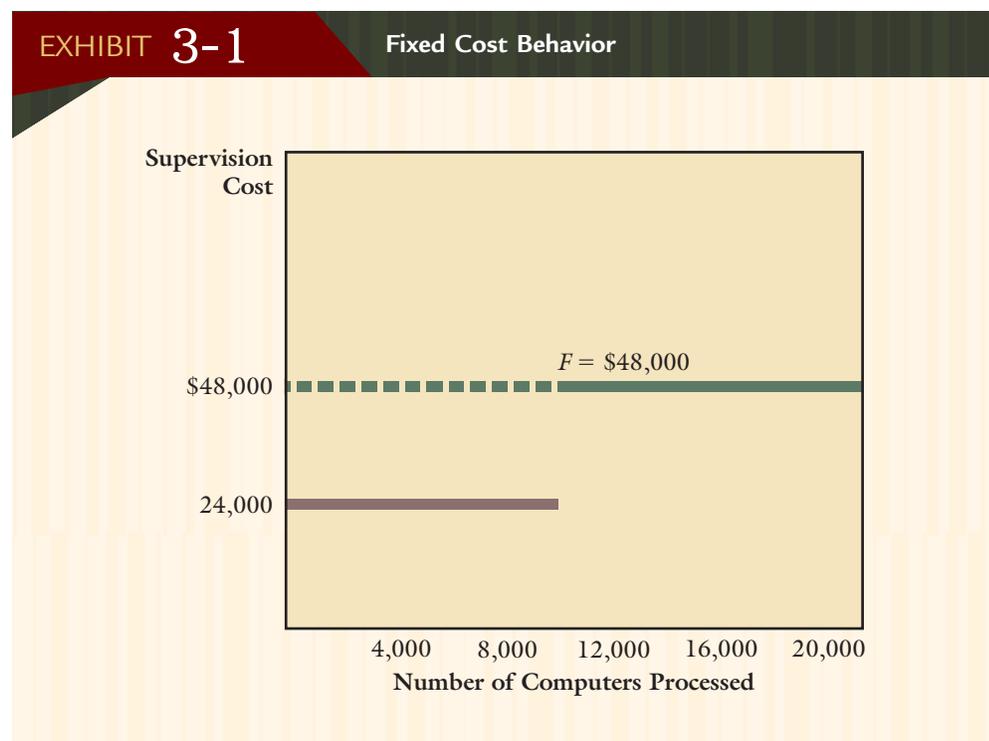
Fixed costs are costs that *in total* are constant within the relevant range as the level of the activity driver varies. To illustrate fixed cost behavior, consider a plant operated by Days Computers, Inc., that produces personal computers. One of the departments of the plant inserts a 3½-inch disk drive into each computer passing through the department. The activity is drive insertion, and the activity driver is the number of computers processed. The department operates two production lines. Each line can process up to 10,000 computers per year. The production workers of each line are supervised by a production-line manager who is paid \$24,000 per year. For production up to 10,000 units, only one manager is needed; for production between 10,001 and 20,000 units, two managers are needed. The cost of supervision for several levels of production for the plant is given as follows:

| Days Computers, Inc. | | |
|----------------------|----------------------------|------------------|
| <i>Supervision</i> | <i>Computers Processed</i> | <i>Unit Cost</i> |
| \$24,000 | 4,000 | \$6.00 |
| 24,000 | 8,000 | 3.00 |
| 24,000 | 10,000 | 2.40 |
| 48,000 | 12,000 | 4.00 |
| 48,000 | 16,000 | 3.00 |
| 48,000 | 20,000 | 2.40 |

The first step in assessing cost behavior is defining an appropriate activity driver. In this case, the activity driver is the number of computers processed. The second step is defining what is meant by **relevant range**, the range over which the assumed cost relationship is valid for the normal operations of a firm. Assume that the relevant range is 12,000 to 20,000 computers processed. Notice that the *total* cost of supervision remains constant within this range as more computers are processed. Days Computers pays \$48,000 for supervision regardless of whether it processes 12,000, 16,000, or 20,000 computers.

Pay particular attention to the words *in total* in the definition of fixed costs. While the total cost of supervision remains unchanged as more computers are processed, the unit cost does change as the level of the activity driver changes. As the example in the table shows, within the relevant range, the unit cost of supervision decreases from \$4.00 to \$2.40. Because of the behavior of per-unit fixed costs, it is easy to get the impression that fixed costs are affected by changes in the level of the activity driver, when in reality they are not. Unit fixed costs can often be misleading and may adversely affect some decisions. It is often safer to work with total fixed costs.

Let's take a look at the graph of fixed cost behavior given in Exhibit 3-1. We see that, for the relevant range, fixed cost behavior is described by a horizontal line. Notice



that at 12,000 computers processed, supervision cost is \$48,000; at 16,000 computers processed, supervision is also \$48,000. This line visually demonstrates that cost remains unchanged as the level of the activity driver varies. For the relevant range, total fixed costs can be represented by the following simple linear equation:

$$F = \text{Total fixed costs}$$

In our example for Days Computers, supervision cost amounted to \$48,000 for any level of output between 10,001 and 20,000 computers processed. Thus, supervision is a fixed cost, and the fixed cost equation in this case is $F = \$48,000$. Strictly speaking, this equation assumes that the fixed costs are \$48,000 for all levels (as if the line extends to the vertical axis as indicated by the dashed portion in Exhibit 3-1). Although this assumption is not true, it is harmless if the operating decisions are confined to the relevant range.

Can fixed costs change? Of course they can, but this does not make them variable. They are fixed at a new higher (or lower) rate. Going back to the example of Days Computers, suppose that the company gives a raise to the drive insertion line supervisors. Instead of being paid \$24,000 per year, they are paid \$26,000 per year. Now the cost of supervision is \$52,000 per year ($2 \times \$26,000$). However, supervisory costs are still fixed with respect to the number of computers produced. Can you draw in the new fixed cost line on Exhibit 3-1?¹

Variable Costs

Variable costs are defined as costs that in total vary in direct proportion to changes in an activity driver. To illustrate, let's expand the Days Computers example to include the cost of the 3½-inch disk drives. Here, the cost is the cost of direct materials—the disk drive—and the activity driver is the number of computers processed. Each computer requires one 3½-inch disk drive costing \$30. The total cost of disk drives for various levels of production is given as follows:

| Days Computers, Inc. | | |
|--------------------------------------|---|-------------------------------------|
| <i>Total Cost of Disk Drives</i> | <i>Number of Computers Produced</i> | <i>Unit Cost of Disk Drives</i> |
| \$120,000 | 4,000 | \$30 |
| 240,000 | 8,000 | 30 |
| 360,000 | 12,000 | 30 |
| 480,000 | 16,000 | 30 |
| 600,000 | 20,000 | 30 |

As more computers are produced, the total cost of disk drives increases in direct proportion. For example, as production doubles from 8,000 to 16,000 units, the *total* cost of disk drives doubles from \$240,000 to \$480,000. Notice also that the unit cost of disk drives is constant.

Variable costs can also be represented by a linear equation. Here, total variable costs depend on the level of activity driver. This relationship can be described by the following equation:

1. The new line is a horizontal line that intersects the y-axis at \$52,000. Note that it is drawn parallel to and above the original fixed cost line.

$$Y_v = VX$$

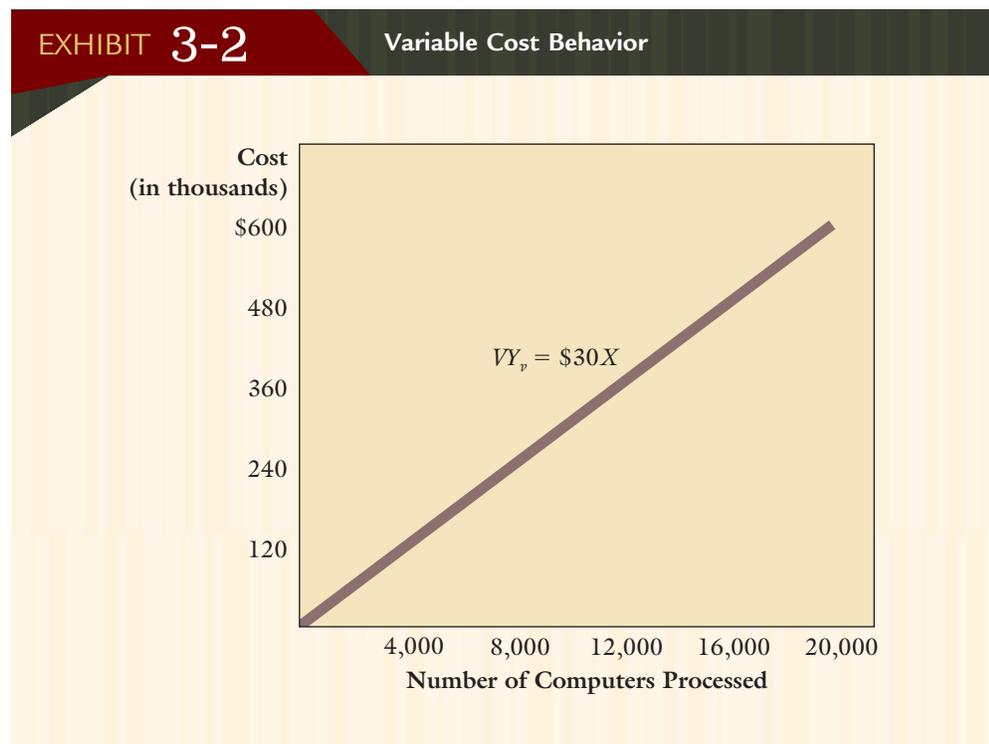
where

Y_v = Total variable costs

V = Variable cost per unit

X = Number of units of the driver

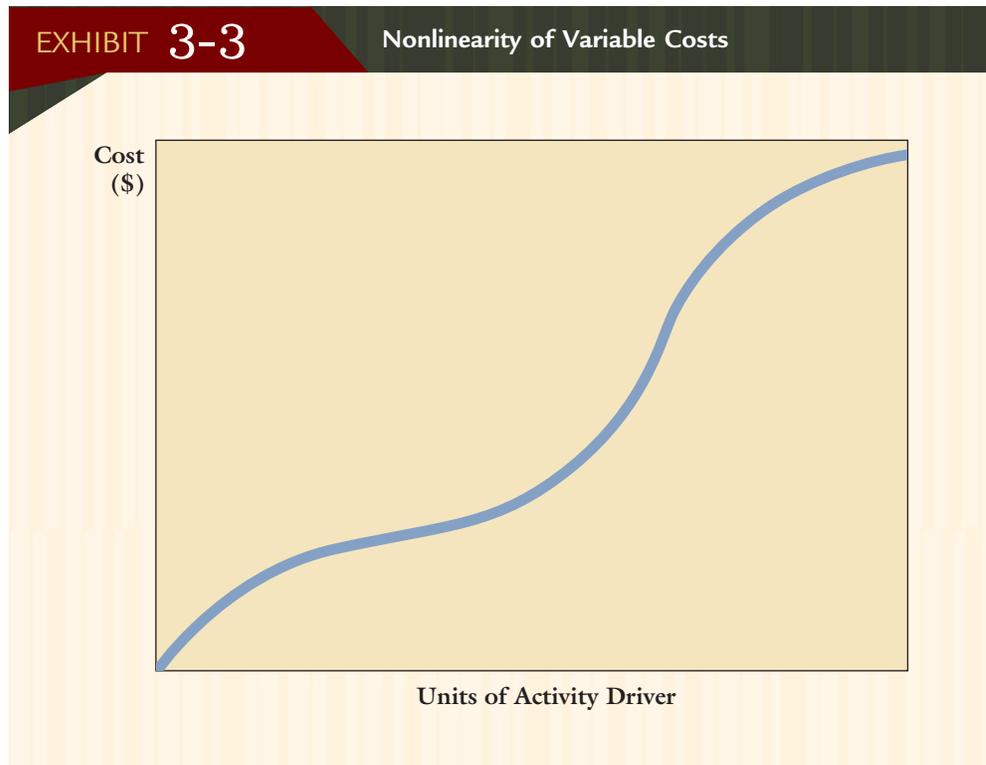
The relationship that describes the cost of disk drives is $Y_v = \$30X$, where X = the number of computers processed. Exhibit 3-2 shows graphically that variable cost behavior is represented by a straight line coming out of the origin. Notice that at zero units processed, total variable cost is zero. However, as units produced increase, the total variable cost also increases. Note that total variable cost increases in direct proportion to increases in the number of computers processed (the activity driver); the rate of increase is measured by the slope of the line. At 12,000 computers processed, the total variable cost of disk drives is \$360,000 (or $\$30 \times 12,000$ computers processed); at 16,000 computers processed, the total variable cost is \$480,000.



Linearity Assumption

The definition of variable costs just given and the graph in Exhibit 3-2 imply a linear relationship between the cost of disk drives and the number of computers processed. How reasonable is the assumption that costs are linear? Do costs really increase in direct proportion to increases in the level of the activity driver? If not, then how well does this assumed linear cost function approximate the underlying cost function?

Economists usually assume that variable costs increase at a decreasing rate up to a certain volume, at which point they increase at an increasing rate. This type of *non-linear behavior* is displayed in Exhibit 3-3. Here, variable costs increase as the number of units increases, but not in direct proportion.



What if the nonlinear view more accurately portrays reality? What do we do then? One possibility is to determine the actual cost function—but every activity could have a different cost function, and this approach could be very time consuming and expensive (if it can even be done). It is much simpler to assume a linear relationship.

If the linear relationship is assumed, then the main concern is how well this assumption approximates the underlying cost function. Exhibit 3-4 gives us some idea of the consequences of assuming a linear cost function. As with fixed costs, we can define the *relevant range* as the range of activity for which the assumed cost relationships are valid. Here, validity refers to how closely the linear cost function approximates the underlying cost function. Note that for units of the activity driver beyond X^* , the approximation appears to break down.

Mixed Costs

Mixed costs are costs that have both a fixed and a variable component. For example, sales representatives are often paid a salary plus a commission on sales. Suppose that Days Computers has 10 sales representatives, each earning a salary of \$30,000 per year plus a commission of \$50 per computer sold. The activity is selling, and the activity driver is units sold. If 10,000 computers are sold, then the total selling cost (associated with the sales representatives) is \$800,000—the sum of the fixed salary cost of \$300,000 ($10 \times \$30,000$) and the variable cost of \$500,000 ($\$50 \times 10,000$).

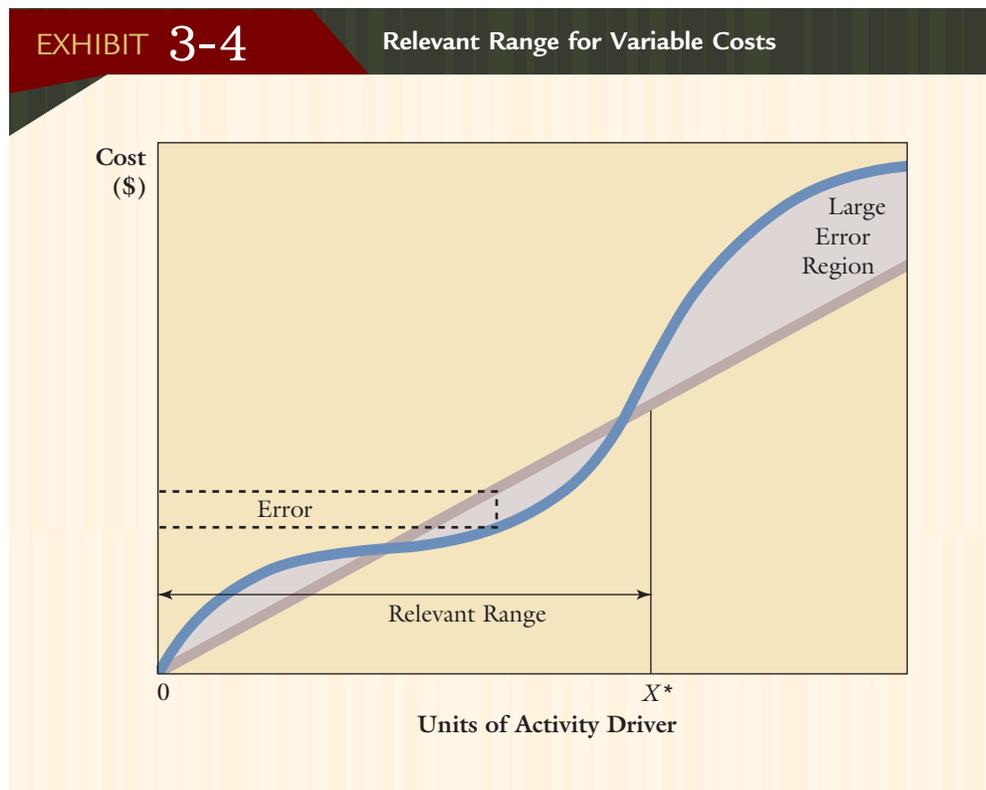
The linear equation for a mixed cost is given by:

$$Y = \text{Fixed cost} + \text{Total variable cost}$$

$$Y = F + VX$$

where

$$Y = \text{Total cost}$$



For Days Computers, the selling cost is represented by the following equation:

$$Y = \$300,000 + \$50X$$

The following table shows the selling cost for different levels of sales activity:

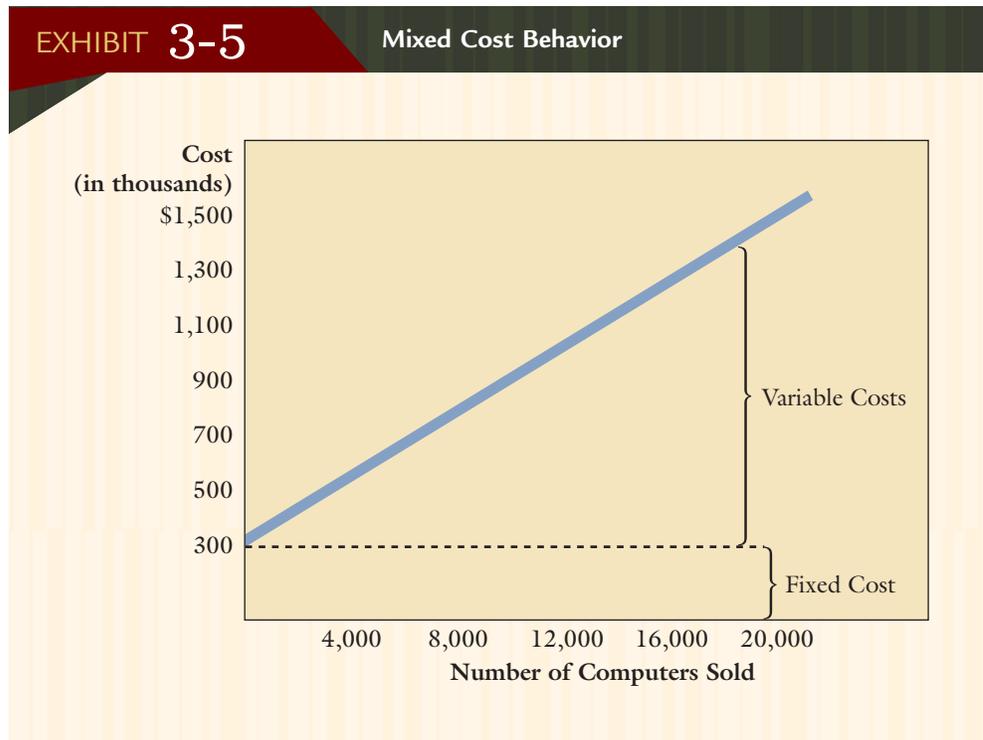
Days Computers, Inc.

| <i>Total Fixed Cost of Selling</i> | <i>Total Variable Cost of Selling</i> | <i>Total Cost</i> | <i>Computers Sold</i> | <i>Selling Cost per Unit</i> |
|--|---|-----------------------|---------------------------|----------------------------------|
| \$300,000 | \$ 200,000 | \$ 500,000 | 4,000 | \$125.00 |
| 300,000 | 400,000 | 700,000 | 8,000 | 87.50 |
| 300,000 | 600,000 | 900,000 | 12,000 | 75.00 |
| 300,000 | 800,000 | 1,100,000 | 16,000 | 68.75 |
| 300,000 | 1,000,000 | 1,300,000 | 20,000 | 65.00 |

The graph for our mixed cost example is given in Exhibit 3-5. (The graph assumes that the relevant range is 0 to 20,000 units.) Mixed costs are represented by a line that intercepts the vertical axis (at \$300,000, for this example). The intercept corresponds to the fixed cost component, and the slope of the line gives the variable cost per unit of activity driver (slope is \$50 for the example portrayed).

Time Horizon

Determining whether a cost is fixed or variable depends on the time horizon. According to economics, in the **long run**, all costs are variable; in the **short run**, at least one cost is fixed. But how long is the short run? Different costs have short runs of different



lengths. Direct materials, for example, are relatively easy to adjust. **Starbucks Coffee** may treat coffee beans (a direct material) as strictly variable, even though for the next few hours the amount already on hand is fixed. The lease of space for its coffee shop in Denver’s Cherry Creek area, however, is more difficult to adjust; it may run for one or more years. Thus, this cost is typically seen as fixed. The length of the short-run period depends to some extent on management judgment and the purpose for which cost behavior is being estimated. For example, submitting a bid on a 1-time, special order may span only a month—long enough to create a bid and produce the order. Other types of decisions, such as product discontinuance or product mix decisions, will affect a much longer period of time. In this case, the costs that must be considered are long-run variable costs, including product design, product development, market development, and market penetration. Short-run costs often do not adequately reflect all the costs necessary to design, produce, market, distribute, and support a product. Recently, there have been some new insights that help shed light on the nature of long- and short-run cost behaviors.² These insights relate to activities and the resources needed to enable an activity to be performed.

Resources, Activities, and Cost Behavior

Resources are simply economic elements that enable one to perform activities. Common resources of a manufacturing plant include direct materials, direct labor, electricity, equipment, and so on. When a company spends money on resources, it is *acquiring* the ability or capacity to perform an activity. Recall from Chapter 2, that an activity is simply a task, such as setting up equipment, purchasing materials, assembling materi-

OBJECTIVE 2

Explain the use of resources and activities and their relationship to cost behavior.

2. The concepts presented in the remainder of this section are based on Alfred M. King, “The Current Status of Activity-Based Costing: An Interview with Robin Cooper and Robert S. Kaplan,” *Management Accounting* (September 1991): 22–26; and Robin Cooper and Robert S. Kaplan, “Activity-Based Systems: Measuring the Costs of Resource Usage,” *Accounting Horizons* (September 1992): 1–13.

als, and packing completed units in boxes. When a firm acquires the resources needed to perform an activity, it is obtaining **activity capacity**. Usually, we can assume that the amount of activity capacity needed corresponds to the level where the activity is performed efficiently. This efficient level of activity performance is called **practical capacity**.

If all of the activity capacity acquired is not used, then we have **unused capacity**, which is the difference between the acquired capacity and the actual activity output. The relationship between resource spending and resource usage can be used to define variable and fixed cost behavior.

Flexible Resources

Resources can be categorized as (1) flexible and (2) committed. **Flexible resources** are supplied as used and needed; they are acquired from outside sources, where the terms of acquisition do not require any long-term commitment for any given amount of the resource. Thus, the organization is free to buy what it needs, when it needs it. As a result, the quantity of the resource supplied equals the quantity demanded. There is no unused capacity for this category of resources (resource usage = resources supplied).

Since the cost of flexible resources equals the cost of resources used, the total cost of the resource increases as demand for the resource increases. Therefore, we generally can treat the cost of flexible resources as a variable cost. For example, in a just-in-time manufacturing environment, materials are purchased when needed. Using units produced as the output measure, or driver, it is clear that as the units produced increase, the usage (and cost) of direct materials would increase proportionately. Similarly, power is a flexible resource. Using kilowatt-hours as the activity output measure (activity driver), as the demand for power increases, the cost of power increases. Note that in both examples, resource supply and usage is measured by an output measure, or driver.

Committed Resources

Committed resources are supplied in advance of usage. They are acquired by the use of either an explicit or implicit contract to obtain a given quantity of resource, regardless of whether the quantity of the resource available is fully used or not. Committed resources may exceed the demand for their usage; thus, unused capacity is possible.

Many resources are acquired before the actual demands for the resource are realized. There are two examples of this category of resource acquisition. First, organizations acquire many *multiperiod service capacities* by paying cash up front or by entering into an explicit contract that requires periodic cash payments. Buying or leasing buildings and equipment are examples of this form of advance resource acquisition. The annual expense associated with the multiperiod category is independent of actual usage of the resource. Often, these expenses are referred to as **committed fixed expenses**. They essentially correspond to committed resources—costs incurred that provide long-term activity capacity.

A second and more important example concerns organizations that acquire resources in advance through implicit contracts—usually with their employees. These implicit contracts require an ethical focus, since they imply that the organization will maintain employment levels even though there may be temporary downturns in the quantity of activity used. Companies may manage the difficulties associated with maintaining this fixed level of expense by using contingent, or temporary, workers when needed. Many companies have indicated that the key reason for the use of contingent workers is flexibility—in meeting demand fluctuations, in controlling downsizing, and in buffering core workers against job loss.³

3. “Contingent Employment on the Rise,” *Deloitte & Touche Review* (September 4, 1995): 1–2.

COST MANAGEMENT

Technology in Action

Cost behavior is important to companies. First, of course, the company must determine appropriate cost objects. This is relatively easy in a manufacturing firm; the cost object is typically the tangible product. In service firms, the logical cost object is the service. For example, hospitals may view particular services such as blood tests or radiology services as primary cost objects.

The Internet, however, has fundamentally changed the way companies do business with their suppliers and customers. Price competition is severe so firms cannot, typically, succeed using a low-price strategy. Instead, they use a customer-service strategy. Internet-based companies strive to provide a shopping experience that is user friendly, with an abundance of information tailored to customer needs, and

a secure payment system. Ideally, the company provides a seamless interface for customers, taking them from information search, through product/service choice, payment, and post-sale follow up. Software that tracks ongoing customer preferences is a large part of the enhanced customer shopping experience. (**Amazon.com** is an excellent example of this, as it welcomes new and returning customers and makes the shopping experience fun and easy.) As a result, “Internet-based firms rely much less on traditional infrastructure assets, such as buildings, and more on computers, specialized software, and intellectual capital that cater to customers in cyberspace.” This means that the customer is the appropriate cost object, and activities and drivers that are tied to customer service are important data to Internet-based firms.

Source: Taken from Lawrence A. Gordon and Martin P. Loeb, “Distinguishing Between Direct and Indirect Costs Is Crucial for Internet Companies,” *Management Accounting Quarterly*, Summer 2001, Vol. II, No. 4, pp. 12–17.

Resource spending for this category essentially corresponds to **discretionary fixed expenses**—costs incurred for the acquisition of short-term activity capacity. Hiring three sustaining engineers for \$150,000 who can supply the capacity of processing 7,500 change orders is an example of implicit contracting (change orders is the driver used to measure resource capacity and usage).⁴ Certainly, none of the three engineers would expect to be laid off if only 5,000 change orders were actually processed—unless, of course, the downturn in demand is viewed as being permanent.

Implications for Control and Decision Making

The activity-based resource usage model just described can improve both managerial control and decision making. Operational control information systems encourage managers to pay more attention to controlling resource usage and spending. A well-designed operational system would allow managers to assess the changes in resource demands that will occur from new product mix decisions. Adding new, customized products may increase the demand for various overhead activities; if sufficient unused activity capacity does not exist, then resource spending must increase.

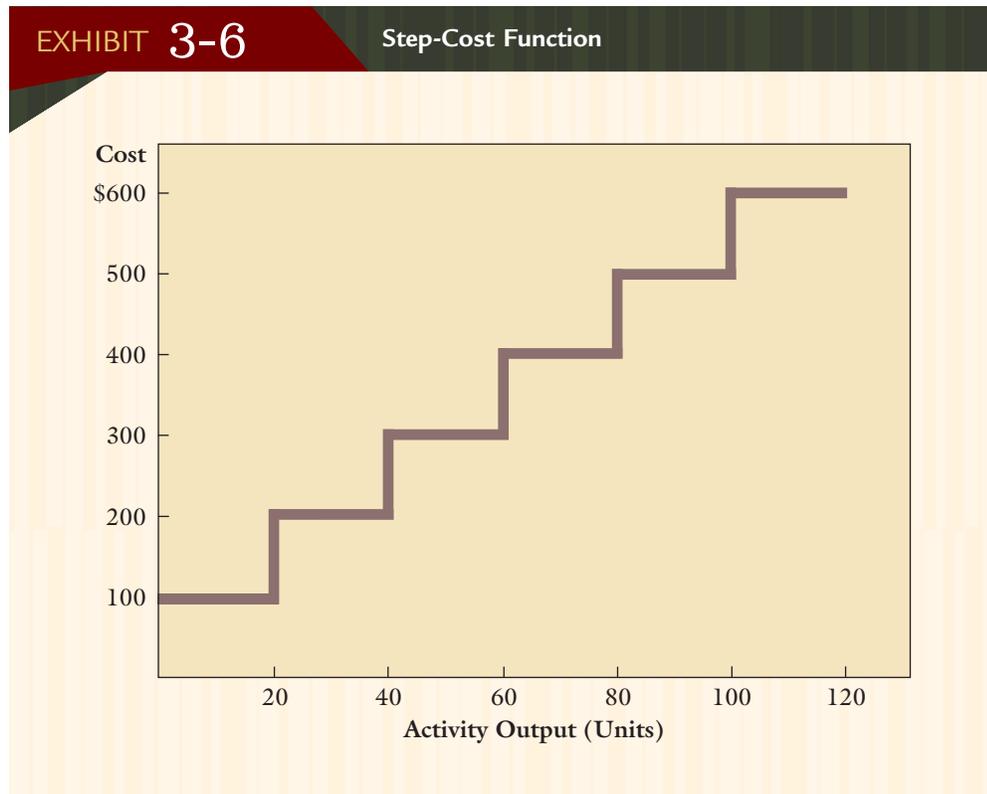
Similarly, if activity management brings about excess activity capacity (by finding ways to reduce resource usage), managers must carefully consider what to do with the excess capacity. Eliminating the excess capacity may decrease resource spending and thus improve overall profits. Alternatively, using the excess capacity to increase output could increase revenues without a corresponding increase in resource spending.

The activity-based resource usage model also allows managers to calculate the changes in resource supply and demand resulting from implementing such decisions as to make or buy, accept or reject special orders, and keep or drop product lines. Additionally, the model increases the power of a number of traditional management accounting decision-making models. The impact on decision making is explored in the decision-making chapters found in Part 4 (Chapters 17–21). Most of the decision-making models in those chapters depend heavily on knowledge of cost behavior.

4. Often, in response to customer feedback and competitive pressures, products need to be redesigned or modified. An engineering change order is the document that initiates this process.

Step-Cost Behavior

In our discussion of cost behavior, we have assumed that the cost function (either linear or nonlinear) is continuous. In reality, some cost functions may be discontinuous, as shown in Exhibit 3-6. This type of cost function is known as a *step function*. A **step-cost function** displays a constant level of cost for a range of activity output and then jumps to a higher level of cost at some point, where it remains for a similar range of activity. In Exhibit 3-6, the cost is \$100, as long as activity output is between 0 and 20 units. If the volume is between 20 and 40 units, the cost jumps to \$200. If the volume is between 40 and 60 units, the cost jumps to \$300. If the volume is between 60 and 80 units, the cost jumps to \$400. If the volume is between 80 and 100 units, the cost jumps to \$500. If the volume is between 100 and 120 units, the cost jumps to \$600.



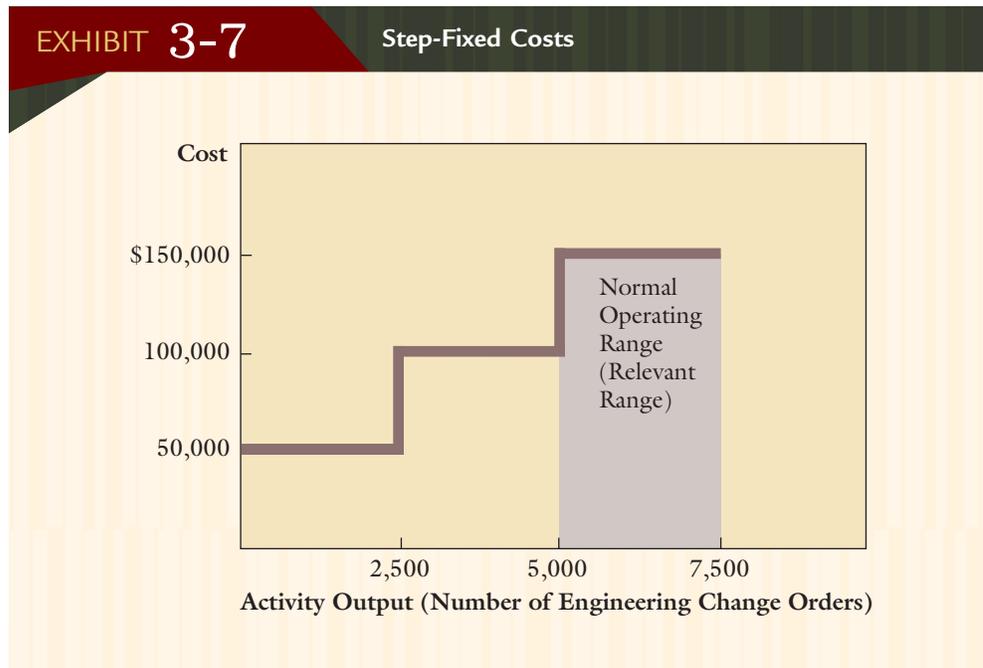
Step-Variable Costs

Items that display a step-cost behavior must be purchased in chunks. The width of the step defines the range of activity output for which a quantity of the resource must be acquired. The width of the step in Exhibit 3-6 is 20 units of activity. If the width of the step is narrow, as in Exhibit 3-6, the cost of the resource changes in response to fairly small changes in resource usage (as measured by activity output). Costs that follow a step-cost behavior with narrow steps are defined as **step-variable costs**. If the width of the step is narrow, we usually approximate step-variable costs with a strictly variable cost assumption.

Step-Fixed Costs

In reality, many so-called fixed costs probably are best described by a step-cost function. Many committed resources—particularly those that involve implicit contracting—follow a step-cost function. Suppose, for example, that a company hires three sustaining engineers—engineers who are responsible for redesigning existing products to meet customer requirements. By hiring the engineers, the company has acquired the ability to perform an activity: engineering redesign. The salaries paid to the engineers represent the cost of acquiring the engineering redesign capacity. The number of engineering

changes that can be *efficiently* processed by the three engineers is a quantitative measure of that capacity. The number of change orders processed, on the other hand, is a measure of the actual usage. Assume the engineers are each paid an annual salary of \$50,000 and that each engineer can process 2,500 engineering change orders per year. The company has acquired the capacity to process 7,500 ($3 \times 2,500$) change orders per year at a total cost of \$150,000 ($3 \times \$50,000$). The nature of the resource requires that the capacity be acquired in chunks (one engineer hired at a time). The cost function for this example is displayed in Exhibit 3-7. Notice that the width of the steps is 2,500 units—a much wider step than the cost function displayed in Exhibit 3-6. Costs that follow a step-cost behavior with wide steps are defined as **step-fixed costs**.



Step-fixed costs are assigned to the fixed cost category. Most step-fixed costs are fixed over the normal operating range of a firm. If that range is 5,000 to 7,500 change orders (as shown in Exhibit 3-7), then the firm will spend \$150,000 on engineering resources. This is equivalent to spending \$20 per change order ($\$150,000/7,500$). The average unit cost, obtained by dividing the resource expenditure by the activity's practical capacity, is the **activity rate**. The activity rate is used to calculate the cost of resource usage and the cost of unused activity.

For example, the company may not actually process 7,500 orders during the year—that is, all of the available order-processing capacity may not be used. Resource usage is the number of change orders *actually* processed. Assume that 6,000 change orders were processed during the year. The cost of resource usage is the activity rate times the actual activity output: $\$20 \times 6,000 = \$120,000$. Further, the cost of unused activity is the activity rate times the unused activity: $\$20 \times 1,500 = \$30,000$. Note that the cost of unused capacity occurs because the resource (engineering redesign) must be acquired in lumpy (whole) amounts. Even if the company had anticipated the need for only 6,000 change orders, it would have been difficult to hire the equivalent of 2.4 engineers ($6,000/2,500$).

This example illustrates that when resources are acquired in advance, there may be a difference between the *resources supplied* and the *resources used (demanded)* to perform activities. This can only occur for activity costs that display a fixed cost behavior (resources acquired in advance of usage). Typically, the traditional cost management

system provides information only about the cost of the resources supplied. A contemporary cost management system, on the other hand, tells us how much of the activity is used and the cost of its usage. Furthermore, the relationship between resources supplied and resources used is expressed by either of the following two equations:

$$\text{Activity availability} = \text{Activity output} + \text{Unused capacity} \quad (3.1)$$

$$\text{Cost of activity supplied} = \text{Cost of activity used} + \text{Cost of unused activity} \quad (3.2)$$

Equation 3.1 expresses the relationship between supply and demand in physical units, while Equation 3.2 expresses it in financial terms.

For the engineering order example, the relationships appear as follows:

Physical units (Equation 3.1):

$$\begin{aligned} \text{Available orders} &= \text{Orders used} + \text{Orders unused} \\ 7,500 \text{ orders} &= 6,000 \text{ orders} + 1,500 \text{ orders} \end{aligned}$$

Financial terms (Equation 3.2):

$$\begin{aligned} \text{Cost of orders supplied} &= \text{Cost of orders used} + \text{Cost of unused orders} \\ 7,500(\$20) &= 6,000(\$20) + 1,500(\$20) \\ \$150,000 &= \$120,000 + \$30,000 \end{aligned}$$

Activities and Mixed Cost Behavior

It is possible for activities to have resources associated with them that are acquired in advance and resources that are acquired as needed. Thus, activity costs can display a mixed cost behavior. Assume that a plant has its own power department. The plant has acquired long-term capacity for supplying power by investing in a building and equipment (resources acquired in advance). The plant also acquires fuel to produce power as needed (resources acquired as needed). The cost of the building and equipment is independent of the kilowatt-hours produced, but the cost of fuel increases as the demand for kilowatt-hours increases. The activity of supplying power has both a fixed cost component and a variable cost component, using kilowatt-hours as the activity output measure.

What the Accounting Records Reveal

Sometimes it is easy to identify the variable and fixed components of a mixed cost, as in the example given earlier for the sales representatives. Many times, however, the only information available is the total cost of an activity and a measure of activity output (the variables Y and X). For example, the accounting system will usually record both the total cost of the maintenance activity for a given period and the number of maintenance hours provided during that period. The accounting records do not reveal the amount of the total maintenance cost representing a fixed charge and the amount representing a variable charge. (In fact, the accounting records may not even reveal the breakdown of costs in the sales representative example.) Often, the total cost is simply recorded with no attempt to segregate the fixed and variable costs.

Need for Cost Separation

Since accounting records typically reveal only the total cost and the associated activity output of a mixed cost item, it is necessary to separate the total cost into its fixed and variable components. Only through a formal effort to separate costs can they all be classified into the appropriate cost behavior categories.

If mixed costs are a very small percentage of total costs, however, formal cost separation may be more trouble than it's worth. In this case, mixed costs could be assigned to either the fixed or variable cost category without much concern for the classification error or its effect on decision making. Alternatively, the total mixed cost could

be arbitrarily divided between the two cost categories. This option is seldom available, though. Mixed costs for many firms are large enough to warrant separation. Given the need for separating costs, how is this done?

Methods for Separating Mixed Costs into Fixed and Variable Components

The three widely used methods of separating a mixed cost into its fixed and variable components are the high-low method, the scatterplot method, and the method of least squares. Each method requires us to make the simplifying assumption of a linear cost relationship. Therefore, before we examine each of these methods more closely, let's review the expression of cost as an equation for a straight line.

$$Y = F + VX$$

where

Y = Total activity cost (the dependent variable)

F = Fixed cost component (the intercept parameter)

V = Variable cost per unit of activity (the slope parameter)

X = Measure of activity output (the independent variable)

The **dependent variable** is a variable whose value depends on the value of another variable. In the preceding equation, total activity cost is the dependent variable; it is the cost we are trying to predict. The **independent variable** is a variable that measures activity output and explains changes in the activity cost. It is an activity driver. The choice of an independent variable is related to its economic plausibility. That is, the manager will attempt to find an independent variable that causes or is closely associated with the dependent variable. The **intercept parameter** corresponds to fixed activity cost. Graphically, the intercept parameter is the point at which the mixed cost line intercepts the cost (vertical) axis. The **slope parameter** corresponds to the variable cost per unit of activity. Graphically, this represents the slope of the mixed cost line.

Since the accounting records reveal only X and Y , those values must be used to estimate the parameters F and V . With estimates of F and V , the fixed and variable components can be estimated, and the behavior of the mixed cost can be predicted as activity output changes. As mentioned at the beginning of this section, three methods will be described for estimating F and V : the high-low method, the scatterplot method, and the method of least squares.

The same data will be used with each method so that comparisons among them can be made. As an example, the data have been accumulated for a materials handling activity. The plant manager believes that the number of material moves is a good activity driver for the activity. Assume that the accounting records of Anderson Company disclose the following materials handling costs and number of material moves for the past 10 months:

| <i>Month</i> | <i>Materials Handling Cost</i> | <i>Number of Moves</i> |
|--------------|--------------------------------|------------------------|
| January | \$2,000 | 100 |
| February | 3,090 | 125 |
| March | 2,780 | 175 |
| April | 1,990 | 200 |
| May | 7,500 | 500 |
| June | 5,300 | 300 |
| July | 4,300 | 250 |
| August | 6,300 | 400 |
| September | 5,600 | 475 |
| October | 6,240 | 425 |

OBJECTIVE 3

Separate mixed costs into their fixed and variable components using the high-low method, the scatterplot method, and the method of least squares.

The High-Low Method

From basic geometry, we know that two points are needed to determine a line. If we know two points on a line, then its equation can be determined. Recall that F , the fixed cost component, is the intercept of the total cost line, and V , the variable cost per unit, is the slope of the line. Given two points, the slope and the intercept can be determined. The **high-low method** preselects the two points that will be used to compute the parameters F and V . Specifically, the high-low method uses the high and low points. The *high point* is defined as the point with the *highest activity level*. The *low point* is defined as the point with the *lowest activity level*.

Letting (X_1, Y_1) be the low point and (X_2, Y_2) be the high point, the equations for determining the slope and intercept are, respectively:

$$\begin{aligned} V &= \text{Change in cost/Change in activity} \\ &= (Y_2 - Y_1)/(X_2 - X_1) \end{aligned}$$

$$\begin{aligned} F &= \text{Total mixed cost} - \text{Variable cost} \\ &= Y_2 - VX_2 \end{aligned}$$

or

$$F = Y_1 - VX_1$$

Notice that the fixed cost component is computed using the total cost at either (X_1, Y_1) or (X_2, Y_2) .

For Anderson, the high point is 500 moves with an associated cost of \$7,500, or (500, \$7,500). The low point is 100 moves with an associated cost of \$2,000, or (100, \$2,000). Once the high and low points are defined, the values of F and V can be computed.

$$\begin{aligned} V &= (Y_2 - Y_1)/(X_2 - X_1) \\ &= (\$7,500 - \$2,000)/(500 - 100) \\ &= \$5,500/400 \\ &= \$13.75 \end{aligned}$$

$$\begin{aligned} F &= Y_2 - VX_2 \\ &= \$7,500 - (\$13.75 \times 500) \\ &= \$625 \end{aligned}$$

The cost formula using the high-low method is:

$$Y = \$625 + \$13.75X$$

If the number of moves for November is expected to be 350, this cost formula will predict a total cost of \$5,437.50, with fixed costs of \$625 and variable costs of \$4,812.50.

The high-low method has two advantages. First, it is objective. That is, any two people using the high-low method on a particular data set will arrive at the same answer. Second, it is simple to calculate. The high-low method allows a manager to get a quick fix on a cost relationship using only two data points. For example, a manager may have only two months of data. Sometimes, this will be enough to get a crude approximation of the cost relationship.

The high-low method is usually not as good as the other methods. Why? First, the high and low points often can be what are known as outliers. They may represent atypical cost-activity relationships. If so, the cost formula computed using these two points will not represent what usually takes place. The **scatterplot method** can help a manager avoid this trap by selecting two points that appear to be representative of the general cost-activity pattern. Second, even if these points are not outliers, other pairs of points may clearly be more representative. Again, the scatterplot method allows the choice of the more representative points.

Scatterplot Method

The first step in applying the scatterplot method is to plot the data points so that the relationship between materials handling costs and activity output can be seen. This plot is referred to as a **scattergraph** and is shown in Exhibit 3-8, Graph A. The vertical axis is total activity cost (materials handling cost), and the horizontal axis is the driver or output measure (number of moves). Looking at Exhibit 3-8, Graph A, we see that the relationship between materials handling costs and number of moves is reasonably linear; cost goes up as the number of moves goes up, and vice versa.

Now let's examine Exhibit 3-8, Graph B, to see if the line determined by the high and low points is representative of the overall relationship. It does look relatively representative. Does that mean that the high-low line should be chosen? Not necessarily. Suppose that management believes the variable costs of materials handling will go down in the near future. In that case, the high-low line gives a somewhat higher variable cost (slope) than desired. The scatterplot line will be chosen with a shallower slope.

Thus, one purpose of a scattergraph is to assess the validity of the assumed linear relationship. Additionally, inspecting the scattergraph may reveal several points that do not seem to fit the general pattern of behavior. Upon investigation, it may be discovered that these points (the outliers) were due to some irregular occurrences. This knowledge can provide justification for their elimination and perhaps lead to a better estimate of the underlying cost function.

A scattergraph can help provide insight concerning the relationship between cost and activity output. In fact, a scattergraph allows one to visually fit a line to the points on the scattergraph. In doing so, the line chosen should appear to best fit the points. In making that choice, a manager or cost analyst is free to use past experience with the behavior of the cost item. Experience may provide a manager/analyst with a good intuitive sense of how materials handling costs behave; the scattergraph then becomes a useful tool to quantify this intuition. Fitting a line to the points in this way is how the scatterplot method works. Keep in mind that the scattergraph and the other statistical aids are tools that can help managers improve their judgment. Using the tools does not restrict the manager from using his or her own judgment to alter any of the estimates produced by formal methods.

Examine Exhibit 3-8, Graph A, carefully. Based only on the information contained in the graph, how would you fit a line to the points in it? Of course, there are an infinite number of lines that might go through the data, but let's choose one that goes through the point for January (100, \$2,000) and intersects the y-axis at \$800. Now, we have the straight line shown in Exhibit 3-8, Graph C. The fixed cost, of course, is \$800, the intercept. We can use the high-low method to determine the variable rate.

First, remember that our two points are (100, \$2,000) and (0, \$800). Next, use these two points to compute the slope:

$$\begin{aligned} V &= (Y_2 - Y_1)/(X_2 - X_1) \\ &= (\$2,000 - \$800)/(100 - 0) \\ &= \$1,200/100 \\ &= \$12 \end{aligned}$$

Thus, the variable cost per material move is \$12.

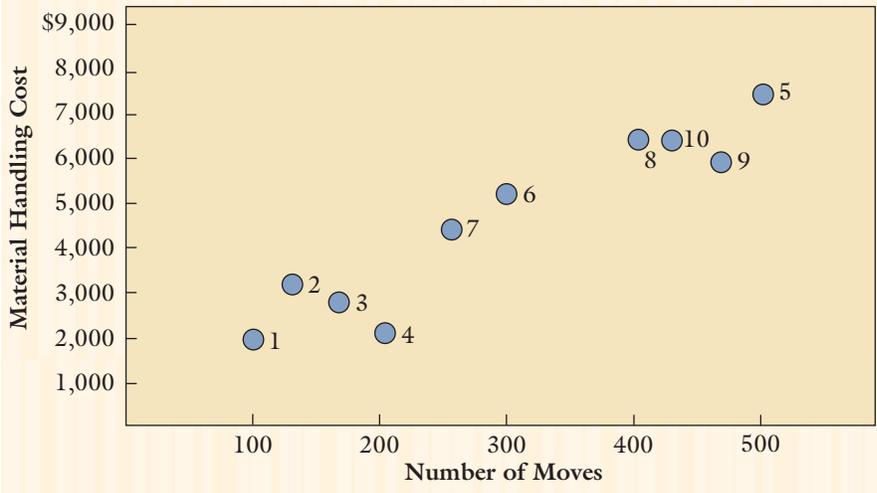
The fixed and variable components of the materials handling cost have now been identified. The cost formula for the materials handling activity can be expressed as:

$$Y = \$800 + \$12X$$

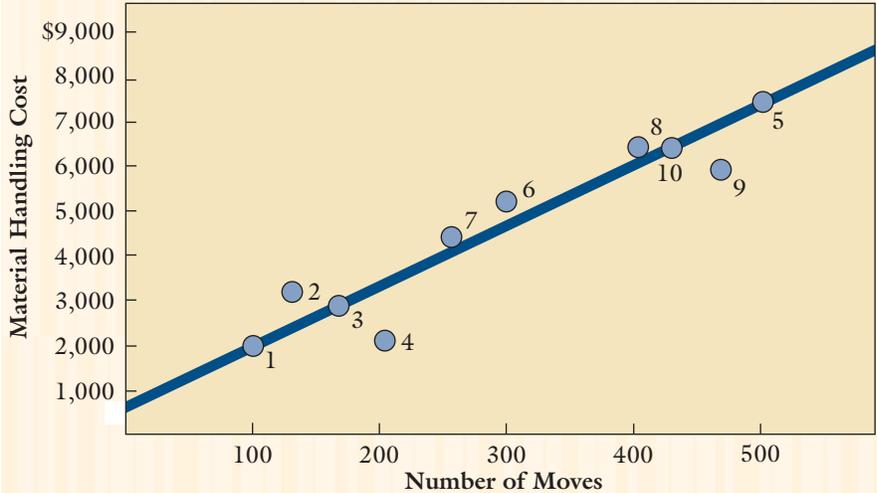
Using this formula, the total cost of materials handling for activity output between 100 and 500 can be predicted and then broken down into fixed and variable components. For example, assume that 350 moves are planned for November. Using the cost for-

EXHIBIT 3-8 Scattergraph for Anderson Company

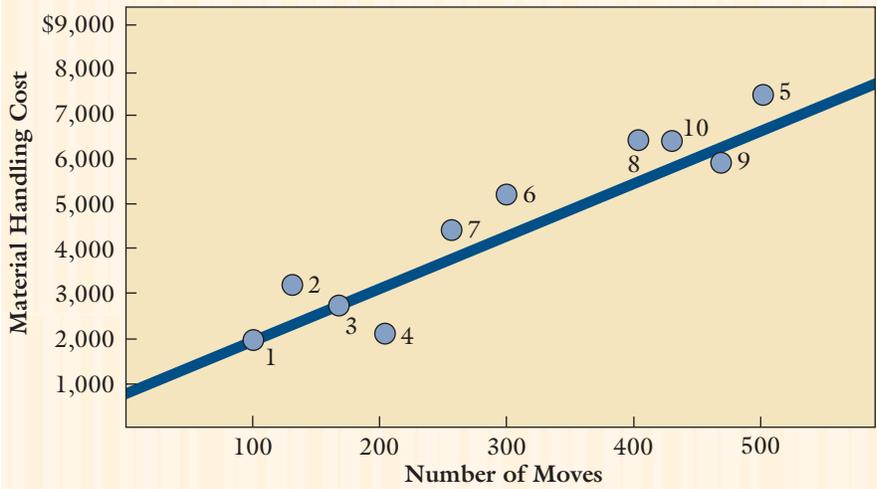
Graph A—Data Points for Anderson Company



Graph B—High-Low Line

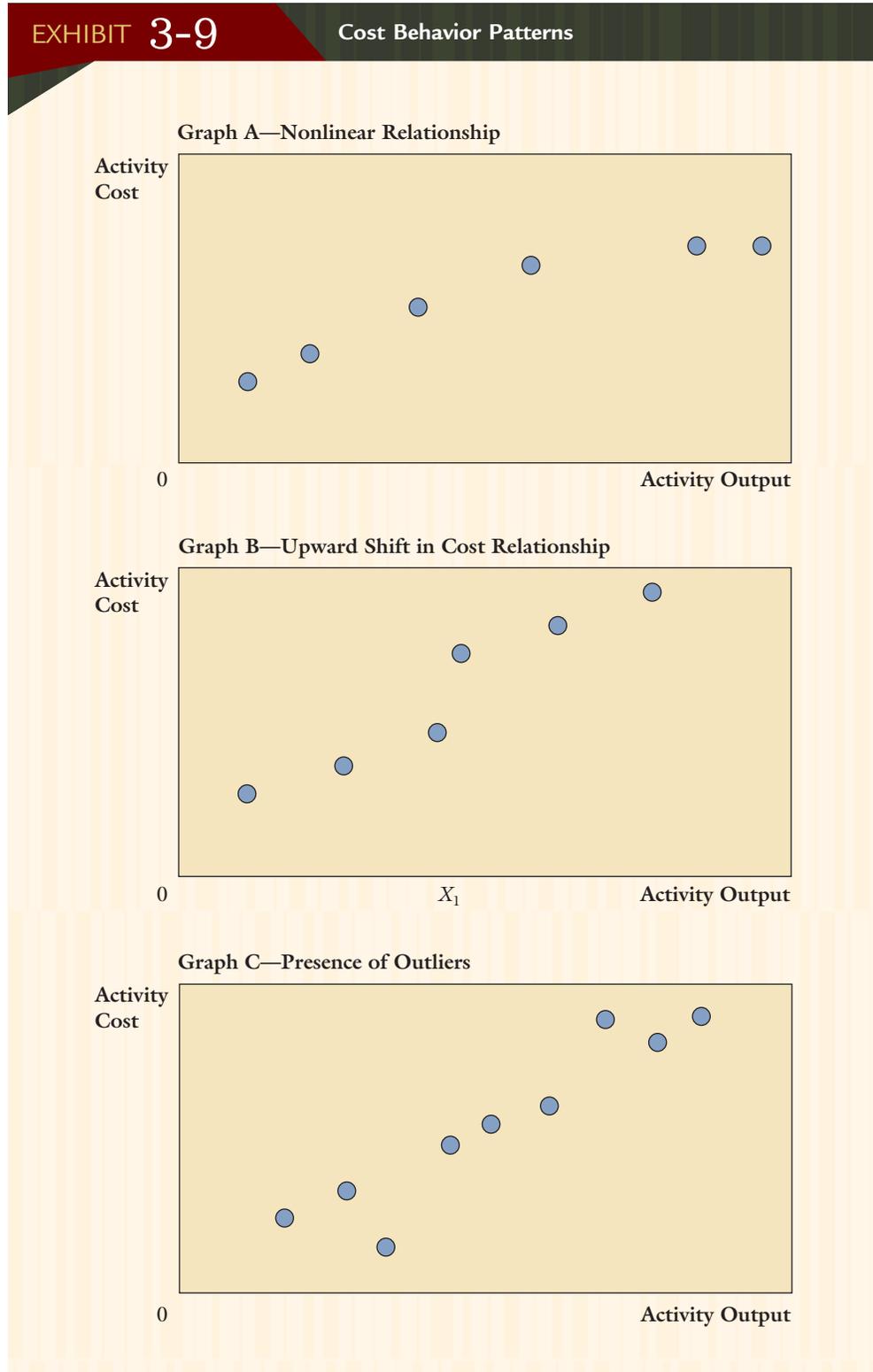


Graph C—One Possible Scattergraph Line



mula, the predicted cost is \$5,000 [$\$800 + (\$12 \times 350)$]. Of this total cost, \$800 is fixed, and \$4,200 is variable.

A significant advantage of the scatterplot method is that it allows a cost analyst to inspect the data visually. Exhibit 3-9 illustrates cost behavior situations that are not ap-



appropriate for the simple application of the high-low method. Graph A shows a non-linear relationship between cost and output. An example of this is a volume discount given on direct materials or evidence of learning by workers (e.g., as more hours are worked, the total cost increases at a decreasing rate due to the increased efficiency of the workers). Graph B shows an upward shift in cost if more than X_1 units are made—perhaps due to the costs of paying an additional supervisor or running a second shift. Graph C shows outliers that do not represent the overall cost relationship.

The cost formula for materials handling was obtained by fitting a line to two points [(0, \$800) and (100, \$2,000)] in Exhibit 3-8, Graph C. We used our judgment to select the line. Whereas one person may decide that the best-fitting line is the one passing through those two points, others, using their own judgment, may decide that the best line passes through other pairs of points.

The scatterplot method suffers from the lack of any objective criterion for choosing the best-fitting line. The quality of the cost formula depends on the quality of the subjective judgment of the analyst. The high-low method removes the subjectivity in the choice of the line. Regardless of who uses the method, the same line will result.

Looking again at Exhibit 3-8, Graphs B and C, we can compare the results of the scatterplot method with those of the high-low method. There is a difference between the fixed cost components and the variable rates. The predicted materials handling cost for 350 moves is \$5,000 according to the scatterplot method and \$5,437.50 according to the high-low method. Which is “correct”? Since the two methods can produce significantly different cost formulas, the question of which method is the best naturally arises. Ideally, a method that is objective and, at the same time, produces the best-fitting line is needed. The **method of least squares** defines *best-fitting* and is objective in the sense that using the method for a given set of data will produce the same cost formula.

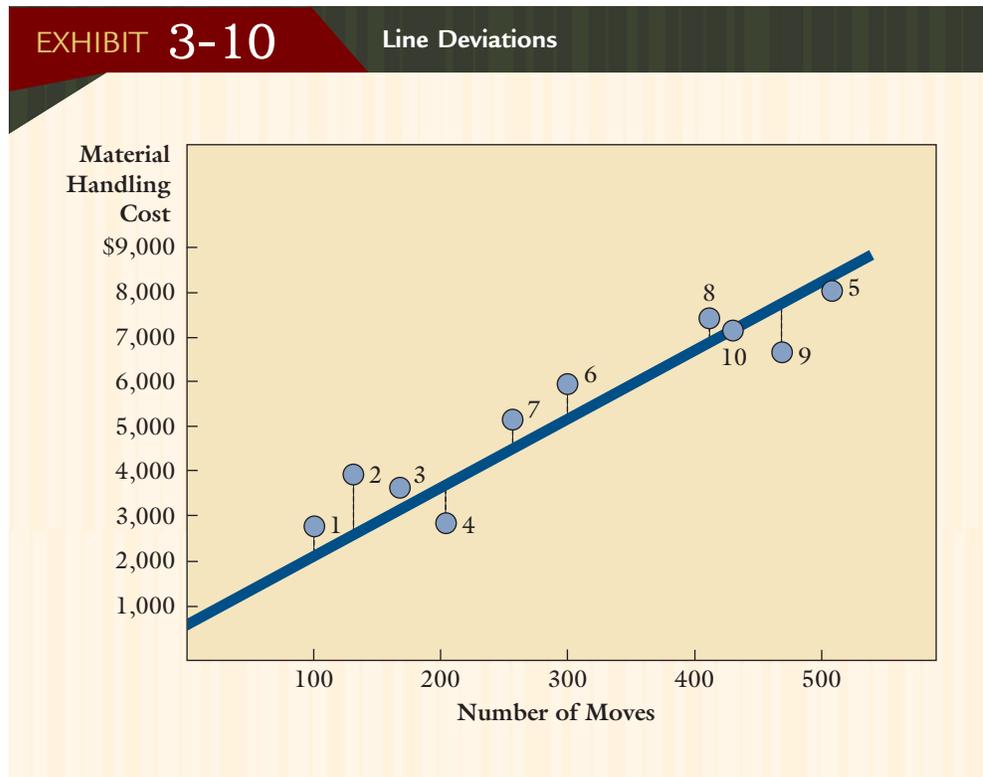
The Method of Least Squares

Up to this point, we have alluded to the concept of a line that best fits the points shown on a scattergraph. What is meant by a best-fitting line? Intuitively, it is the line to which the data points are closest. But what is meant by closest?

Consider Exhibit 3-10. Here, an arbitrary line ($Y = F + VX$) has been drawn. The closeness of each point to the line can be measured by the vertical distance of the point from the line. This vertical distance is the difference between the actual cost and the cost predicted by the line. For point 8, this is $E_8 = Y_8 - F + VX_8$, where Y_8 is the actual cost, $F + VX_8$ is the predicted cost, and the deviation is represented by E_8 . The **deviation** is the difference between the predicted and actual costs, which is shown by the distance from the point to the line.

The vertical distance measures the closeness of a single point to the line, but we really need a measure of closeness of *all* points to the line. One possibility is to add all the single measures to obtain an overall measure. However, since the single measures can have positive or negative signs, this overall measure may not be very meaningful. For example, the sum of small positive deviations could result in an overall measure greater in magnitude than the sum of large positive deviations and large negative deviations because of the cancelling effect of positive and negative numbers. To correct this problem, we could first square each single measure of closeness and then sum these squared deviations as the overall measure of closeness. Squaring the deviations avoids the cancellation problem caused by a mix of positive and negative numbers.

To illustrate this concept, a measure of closeness will be calculated for the cost formula produced by the scatterplot method.



| <i>Actual Cost</i> | <i>Predicted Cost^a</i> | <i>Deviation^b</i> | <i>Deviation Squared</i> |
|----------------------------|-----------------------------------|------------------------------|--------------------------|
| \$2,000 | \$2,000 | 0 | 0 |
| 3,090 | 2,300 | 790 | 624,100 |
| 2,780 | 2,900 | -120 | 14,400 |
| 1,990 | 3,200 | -1,210 | 1,464,100 |
| 7,500 | 6,800 | 700 | 490,000 |
| 5,300 | 4,400 | 900 | 810,000 |
| 4,300 | 3,800 | 500 | 250,000 |
| 6,300 | 5,600 | 700 | 490,000 |
| 5,600 | 6,500 | -900 | 810,000 |
| 6,240 | 5,900 | 340 | 115,600 |
| Total measure of closeness | | | <u>5,068,200</u> |

^aPredicted cost = \$800 + \$12X, where X is the actual measure of activity output associated with the actual activity cost and cost is rounded to the nearest dollar.

^bDeviation = Actual cost - Predicted cost.

Since the measure of closeness is the sum of the squared deviations of the points from the line, the smaller the measure, the better the line fits the points. For example, the scatterplot method line has a closeness measure of 5,068,200. A similar calculation produces a closeness measure of 5,402,013 for the high-low line. Thus, the scatterplot line fits the points better than the high-low line. This outcome supports the earlier claim that the use of judgment in the scatterplot method is superior to the high-low method.

In principle, comparing closeness measures can produce a ranking of all lines from best to worst. The line that fits the points better than any other line is called the *best-fitting line*. It is the line with the smallest (least) sum of squared deviations. The method

of least squares identifies the best-fitting line. We rely on statistical theory to obtain the formulas that produce the best-fitting line.

Using Regression Programs

Computing the regression formula manually is tedious, even with only a few data points. As the number of data points increases, manual computation becomes impractical. (When multiple regression is used, manual computation is virtually impossible.) Fortunately, spreadsheet packages such as Microsoft Excel⁵ have regression routines that will perform the computations. All that you need to do is input the data. The spreadsheet regression program supplies more than the estimates of the coefficients. It also provides information that can be used to see how reliable the cost equation is, a feature that is not available for the scatterplot and high-low methods.

The first step in using the computer to calculate regression coefficients is to enter the data. Exhibit 3-11 shows the computer screen you would see if you entered the Anderson Company data on materials handling cost and moves into a spreadsheet. It is a good idea to label your variables as is done in the exhibit: the months are labeled, column B is labeled for materials handling costs, and column C is labeled for the number of moves. The next step is to run the regression. In Excel and Quattro Pro, the regression routine is located under the “tools” menu (toward the top right of the screen). When you pull down the “tools” menu, you will see other menu possibilities. In Quattro Pro, choose “numeric tools” and then “regression.” In Excel, choose “add in” and then add the “data analysis tools.” When the data analysis tools have been added, “data analysis” will appear at the bottom of the “tools” menu; click on “data analysis,” and then “regression.”

EXHIBIT 3-11

Spreadsheet Data for Anderson Company

| | A | B | C | D | E | F | G | H | I |
|----|-----------|---------|---------|---|---|---|---|---|---|
| 1 | Month | Cost | # Moves | | | | | | |
| 2 | January | \$2,000 | 100 | | | | | | |
| 3 | February | 3,090 | 125 | | | | | | |
| 4 | March | 2,780 | 175 | | | | | | |
| 5 | April | 1,990 | 200 | | | | | | |
| 6 | May | 7,500 | 500 | | | | | | |
| 7 | June | 5,300 | 300 | | | | | | |
| 8 | July | 4,300 | 250 | | | | | | |
| 9 | August | 6,300 | 400 | | | | | | |
| 10 | September | 5,600 | 475 | | | | | | |
| 11 | October | 6,240 | 425 | | | | | | |
| 12 | | | | | | | | | |
| 13 | | | | | | | | | |
| 14 | | | | | | | | | |
| 15 | | | | | | | | | |
| 16 | | | | | | | | | |
| 17 | | | | | | | | | |
| 18 | | | | | | | | | |
| 19 | | | | | | | | | |
| 20 | | | | | | | | | |

5. Lotus 1-2-3 are registered trademarks of the Lotus Development Corporation. Quattro Pro is a registered trademark of Novell, Inc. Excel is a registered trademark of Microsoft Corporation. Any further reference to Lotus 1-2-3, Quattro Pro, or Excel refers to this footnote.

When the “regression” screen pops up, you can tell the program where the dependent and independent variables are located. Simply place the cursor at the beginning of the “independent” rectangle and then (again using the cursor) block the values under the independent variable column, in this case, cells c2 through c11. Then, move the cursor to the beginning of the “dependent” rectangle, and block the values in cells b2 through b11. Finally, you need to tell the computer where to place the output. Block a nice-sized rectangle—for example, cells a13 through f20—and click on “OK.” In less than the blink of an eye, the regression output is complete. The regression output is shown in Exhibit 3-12.

EXHIBIT 3-12 Regression Output for Anderson Company

| | A | R | C | D | F | F |
|----|------------------------------|---------------------|-----------------------|---------------|----------------|-----------------------|
| 1 | SUMMARY OUTPUT | | | | | |
| 2 | <i>Regression Statistics</i> | | | | | |
| 3 | Multiple R | 0.928949080 | | | | |
| 4 | R Square | 0.862946394 | | | | |
| 5 | Adjusted R | 0.845814693 | | | | |
| 6 | | | | | | |
| 7 | Square | | | | | |
| 8 | Standard Error | 770.4987038 | | | | |
| 9 | Observations | 10 | | | | |
| 10 | | | | | | |
| 11 | ANOVA | | | | | |
| 12 | | <i>df</i> | <i>SS</i> | <i>MS</i> | <i>F</i> | <i>Significance F</i> |
| 13 | Regression | 1 | 29903853.98 | 29903853.98 | 50.37132077 | 0.000102268 |
| 14 | Residual | 8 | 4749346.021 | 593668.2526 | | |
| 15 | Total | 9 | 34653200 | | | |
| 16 | | | | | | |
| 17 | | | | | | |
| 18 | | <i>Coefficients</i> | <i>Standard Error</i> | <i>t Stat</i> | <i>P-value</i> | |
| 19 | Intercept | 854.4993582 | 569.7810263 | 1.499697811 | 0.172079925 | |
| 20 | X Variable 1 | 12.3915276 | 1.745955536 | 7.097275588 | 0.000102268 | |
| 21 | | | | | | |

Now, let’s take a look at the output in Exhibit 3-12. First, locate the fixed cost and variable rate coefficients. At the bottom of the exhibit, the intercept and X Variable 1 are shown, and the next column gives their coefficients. Rounding, the fixed cost is \$854.50, and the variable rate is \$12.39. Now, we can construct the following cost formula for materials handling cost:

$$\text{Materials handling cost} = \$854.50 + (\$12.39 \times \text{Number of moves})$$

We can use this formula to predict materials handling cost for future months as we did with the formulas for the high-low and scatterplot methods.

Since the regression cost formula is the best-fitting line, it should produce better predictions of materials handling costs. For 350 moves, the estimate predicted by the least-squares line is \$5,191 [$\$854.50 + (\$12.39 \times 350)$], with a fixed component of \$854.50 plus a variable component of \$4,336.50. Using this prediction as a standard, the scatterplot line most closely approximates the least-squares line.

While the computer output in Exhibit 3-12 can give us the fixed and variable cost coefficients, its major usefulness lies in its ability to provide information about how reliable the estimated cost formula is. The scatterplot or high-low methods do not provide this feature.

OBJECTIVE 4

Evaluate the reliability of the cost formula.

Reliability of Cost Formulas

Regression routines provide information that can be used to assess how reliable the estimated cost formula is. This is a feature not provided by either the scatterplot or high-low methods. Exhibit 3-12 will serve as the point of reference for discussing three statistical assessments concerning the cost formula's reliability: *hypothesis test of cost parameters*, *goodness of fit*, and *confidence intervals*. The **hypothesis test of cost parameters** indicates whether the parameters are different from zero. For our setting, **goodness of fit** measures the degree of association between cost and activity output. This measure is important because the method of least squares identifies the best-fitting line, but it does not reveal how good the fit is. The best-fitting line may not be a good-fitting line. It may perform miserably when it comes to predicting costs. A **confidence interval** provides a range of values for the actual cost with a prespecified degree of confidence. Confidence intervals allow managers to predict a range of values instead of a single prediction. Of course, if the degree of association is perfect, then the confidence interval will consist of a single point and the actual cost will always coincide with the predicted cost. Thus, goodness of fit and confidence intervals are related, and they provide cost analysts some idea of how reliable the resulting cost equation is.

Hypothesis Test of Parameters

Refer once again to Exhibit 3-12. The fourth column of the bottom table, labeled “*t* Stat,” presents the *t* statistics for each parameter. These *t* statistics are used to test the hypothesis that the parameters are different from zero. The fifth column, labeled “*P*-value,” is the level of significance achieved. The fixed cost parameter, *F*, is significant at the 0.172 level. This is NOT significant at the 0.05 or even the 0.10 levels. The variable cost parameter is significant at the 0.0001 level. Thus, the number of moves appears to be a highly significant explanatory variable—a driver for materials handling costs. However, the presence of fixed materials handling costs is questionable. The third column presents the standard error for each parameter. This value is used to compute the *t* statistic in column 4: the coefficient in column 2 is divided by the corresponding standard error.

Goodness of Fit Measures

Initially, we assume that a single activity driver (activity output variable) explains changes (variability) in activity cost. Our experience with the Anderson Company example suggests that the number of moves can explain changes in materials handling costs. The scattergraph shown back in Exhibit 3-8 confirms this belief because it reveals that materials handling cost and activity output (as measured by the number of moves) seem to move together. It is quite likely that a significant percentage of the total variability in cost is explained by our activity output variable.

We can determine statistically just how much variability is explained by looking at the coefficient of determination. The percentage of variability in the dependent variable explained by an independent variable (in this case, a measure of activity output) is called the **coefficient of determination**. This percentage is a goodness of fit measure. The higher the percentage of cost variability explained, the better the fit. Since the coefficient is the percentage of variability explained, it always has a value between 0 and 1.00.

In Exhibit 3-12, the coefficient of determination is labeled “R Square” (R^2). The value given is 0.86, which means that 86 percent of the variability in materials handling cost is explained by the number of moves. How good are these results? There is no cut-off point for a good versus a bad coefficient of determination. Clearly, the closer R^2 is to 1.00, the better. However, is 86 percent good enough? How about 73 percent?

Or even 46 percent? The answer is that it depends. If your cost equation yields a coefficient of determination of 75 percent, you know that your independent variable explains three-fourths of the variability in cost. You also know that some other factor or combination of factors explains the remaining one-fourth. Depending on your tolerance for error, you may want to improve the equation by trying different independent variables (for example, materials handling hours worked rather than number of moves) or by trying multiple independent variables (or multiple regression, which is explained in a succeeding section of this chapter).

We note from the computer output in Exhibit 3-12, that the R^2 for materials handling cost is 0.86. In other words, material moves explain about 86 percent of the variability in the materials handling cost. This is not bad; however, something else explains the remaining 14 percent. In addition, the fixed cost coefficient is not significant. Anderson Company may want to consider other variables and perhaps use multiple regression.

Coefficient of Correlation

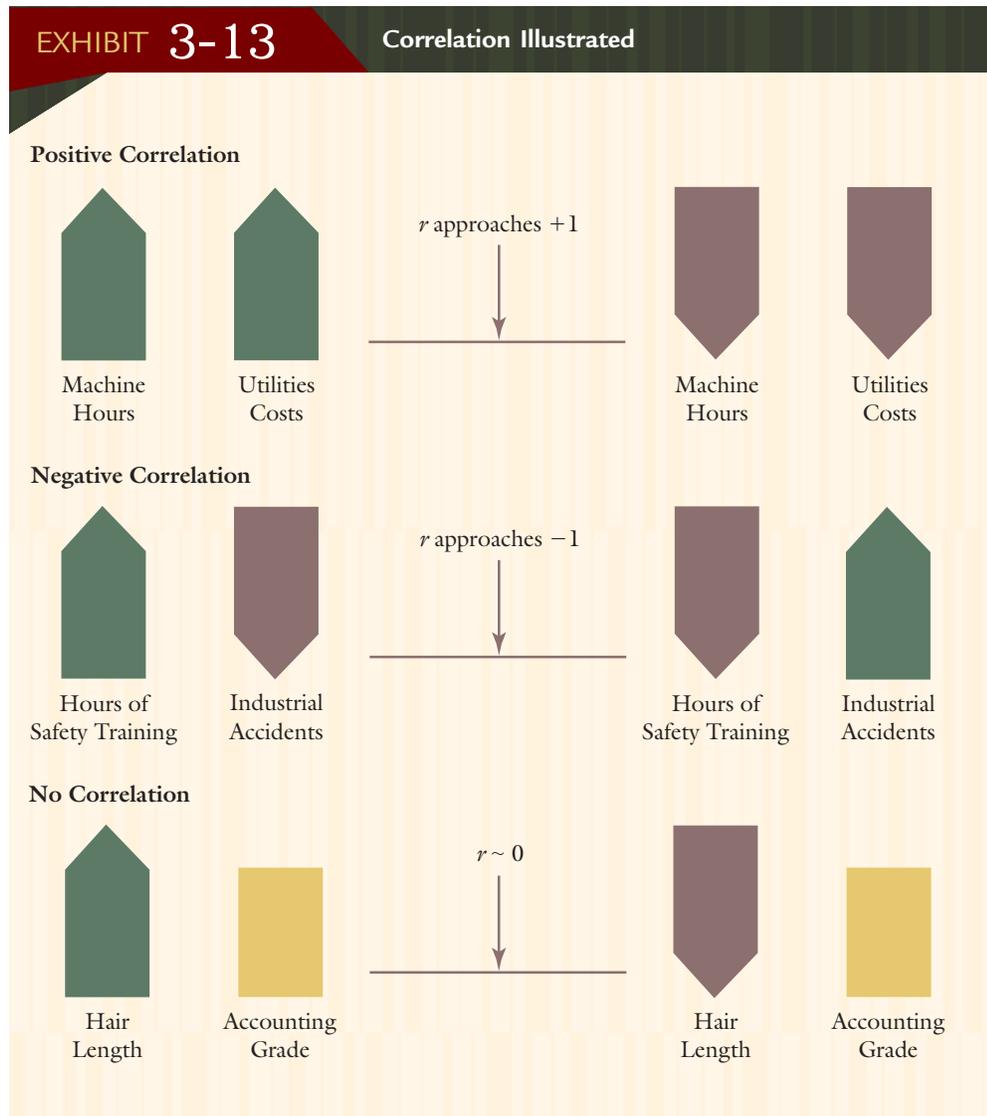
An alternative measure of goodness of fit is the **coefficient of correlation**, which is the square root of the coefficient of determination. Since square roots can be negative, the value of the coefficient of correlation can range between -1 and $+1$. If the coefficient of correlation is positive, then the two variables (in this example, cost and activity) move together in the same direction and positive correlation exists. Perfect positive correlation would yield a value of 1.00 for the coefficient of correlation. If, on the other hand, the coefficient of correlation is negative, then the two variables move in a predictable fashion but in opposite directions. Perfect negative correlation would yield a coefficient of correlation of -1.00 . A coefficient of correlation value close to zero indicates no correlation. That is, knowledge of the movement of one variable gives us no clue as to the movement of the other variable. Exhibit 3-13 illustrates the concept of correlation.

For the Anderson Company example, the coefficient of correlation (r) is 0.929 . Notice that r is the positive square root of R^2 , computed previously. The square root is positive because the correlation between X and Y is positive. In other words, as the number of moves increases, the materials handling cost increases. This positive correlation is reflected by a positive value for V , the variable rate. If cost decreases as activity output increases, then the coefficient of correlation (and the value of V) is negative. The sign of V reveals the sign of the coefficient of correlation. The very high positive correlation between materials handling cost and the number of moves indicates that the number of moves represents a good choice for an activity driver.

Confidence Intervals

The least-squares cost equation can be used to predict cost for different levels of activity output. For example, if the number of moves is 200, then the materials handling cost predicted by the least-squares equation is $\$3,332.50$ [$\$854.50 + \$12.39(200)$]. Usually, we can expect the predicted value to be different from the actual cost, and there are two reasons. First, in building the cost equation, only one activity driver (independent variable) has been used. It is possible that the cost equation has omitted other important factors, such as outlier activity output that affects cost (the dependent variable). These omitted factors are assumed to randomly affect the cost variable. The consequence of omission is to produce a distribution of cost values for every value of X (the measure of activity output appearing in the cost equation). This distribution is assumed to be normal. Second, the cost equation is based on estimated values using a sample of observed outcomes. Errors in estimating the slope, V , and the intercept, F , of the cost equation can also cause a discrepancy between the actual cost and the predicted cost.

The dispersion caused by these two effects can be measured, and the resulting measure can be used to help build a confidence interval around a predicted cost. If the



number of data points is large enough, the measure of dispersion can be approximated by the standard error, S_e . For example, in Exhibit 3-12, the standard error is \$770.50.⁶

Given S_e , a confidence interval for the predicted value of Y can be constructed by using a t statistic for the desired level of confidence:

$$Y_f \pm t S_e$$

where

Y_f = The predicted cost for a given level of activity

By adding and subtracting a multiple of the standard error to the predicted cost, a range of possible values is created. Using the t statistic, a degree of confidence can be specified. The degree of confidence is a measure of the likelihood that the prediction interval will contain the actual cost. Thus, a 95 percent confidence interval means that if repeated samples were taken and 100 confidence intervals were constructed, we would expect 95 of the 100 to contain the actual cost.

6. For simplicity, we will always use the standard error even when the sample size is small.

The construction of a confidence interval can be illustrated using the Anderson Company example. From Exhibit 3-12, the least-squares cost equation is $Y = \$854.50 + \$12.39X$ (the fixed cost parameter is rounded to the nearest cent). Let's construct a 90 percent confidence interval for materials handling cost given that $X = 200$ moves. To construct the interval, we need the predicted cost, the standard error, and the t statistic. The predicted cost is \$3,333 (computed earlier), the standard error is \$770 rounded (Exhibit 3-12), and the t statistic is 1.86 for 8 degrees of freedom and a 90 percent confidence level. The degrees of freedom are calculated by $n - p$, where n = the number of data points used to calculate the cost formula and p = the number of parameters in the cost equation (10 and two, respectively, for the Anderson example). A table of selected t values is provided in Exhibit 3-14. Using this information, the confidence interval is computed next:

$$\begin{aligned}
 & Y_f \pm t S_e \\
 & \$3,333 \pm 1.86(\$770) \\
 & \$3,333 \pm \$1,432 \\
 & \$1,901 \leq Y \leq \$4,765
 \end{aligned}$$

EXHIBIT 3-14

Table of Selected Values: t Distribution*

| Degrees of Freedom | 90% | 95% | 99% |
|--------------------|-------|--------|--------|
| 1 | 6.314 | 12.708 | 63.657 |
| 2 | 2.920 | 4.303 | 9.925 |
| 3 | 2.353 | 3.182 | 5.841 |
| 4 | 2.132 | 2.776 | 4.604 |
| 5 | 2.015 | 2.571 | 4.032 |
| 6 | 1.943 | 2.447 | 3.707 |
| 7 | 1.895 | 2.365 | 3.499 |
| 8 | 1.860 | 2.306 | 3.355 |
| 9 | 1.833 | 2.262 | 3.250 |
| 10 | 1.812 | 2.228 | 3.169 |
| 11 | 1.796 | 2.201 | 3.106 |
| 12 | 1.782 | 2.179 | 3.055 |
| 13 | 1.771 | 2.160 | 3.055 |
| 14 | 1.761 | 2.145 | 3.012 |
| 15 | 1.753 | 2.131 | 2.947 |
| 16 | 1.746 | 2.120 | 2.921 |
| 17 | 1.740 | 2.110 | 2.898 |
| 18 | 1.734 | 2.101 | 2.878 |
| 19 | 1.729 | 2.093 | 2.861 |
| 20 | 1.725 | 2.086 | 2.845 |
| 30 | 1.697 | 2.042 | 2.750 |
| 00 | 1.645 | 1.960 | 2.576 |

*Values are based on the assumption that two tails are important—as they would be with confidence intervals and hypothesis tests of regression coefficients. For values above 30, simply use the last row.

Thus, we can say with 90 percent confidence that the actual cost, Y , associated with 200 moves will be between \$1,901 and \$4,765. This outcome produces a very large range of possible values, revealing very quickly that the cost equation is not as useful for prediction as it might first appear based only on the coefficient of determination. The width of the interval diminishes the attractiveness of the cost equation. However, the width of this in-

terval often can be reduced by using a larger sample (more data points) to calculate the cost equation. With a larger sample, the standard error may decrease, and the t statistic will decrease. If a company has a limited history for the activity being evaluated (sample size must be small), it may have to rely more on the detection of association than cost prediction. Finding a strong statistical association between an activity cost and an activity driver, however, can provide evidence to a manager about the correctness of the driver selection—an important issue when searching for causal factors to assign costs to cost objects.

OBJECTIVE 5

Explain how multiple regression can be used to assess cost behavior.

Multiple Regression

In the Anderson Company example, 86 percent of the variability in materials handling cost was explained by changes in activity output (number of moves). As a result, the company may want to search for additional explanatory variables. For example, total distance moved might be useful—particularly if the plant layout is such that significant time is consumed moving parts and products from one location to another.

In the case of two explanatory variables (activity drivers), the linear equation is expanded to include the additional variable:

$$Y = F + V_1X_1 + V_2X_2$$

where

X_1 = Number of moves

X_2 = The total distance

With three variables (Y , X_1 , X_2), a minimum of three points is needed to compute the parameters F , V_1 , and V_2 . Seeing the points becomes difficult because they must be plotted in three dimensions. Using the scatterplot method or the high-low method is not practical.

However, the extension of the method of least squares is straightforward. It is relatively simple to develop a set of equations that provides values for F , V_1 , and V_2 that yields the best-fitting equation. Whenever least squares is used to fit an equation involving two or more explanatory variables, the method is called **multiple regression**. The computational complexity of multiple regression, which increases significantly, is facilitated by the computer. In fact, any practical application of multiple regression requires use of a computer.

Let's return to the Anderson Company example. Recall that the R^2 is just 86 percent and that the fixed cost coefficient was not significant. Perhaps there is another variable that could help to explain materials handling costs. Suppose that the controller for Anderson Company investigates and finds that in some months many more pounds of materials were moved than in other months. When the heavier materials were moved, additional equipment was used to handle the increased load.

The controller adds the variable “pounds moved” and gathers information on the 10 months.

| <i>Month</i> | <i>Materials Handling Cost</i> | <i>Number of Moves</i> | <i>Pounds Moved</i> |
|--------------|--------------------------------|------------------------|---------------------|
| January | \$2,000 | 100 | 6,000 |
| February | 3,090 | 125 | 15,000 |
| March | 2,780 | 175 | 7,800 |
| April | 1,990 | 200 | 600 |
| May | 7,500 | 500 | 29,000 |
| June | 5,300 | 300 | 23,000 |
| July | 4,300 | 250 | 17,000 |
| August | 6,300 | 400 | 25,000 |
| September | 5,600 | 475 | 12,000 |
| October | 6,240 | 425 | 22,400 |

Now let's run a multiple regression using the number of moves and the number of pounds moved as the independent variables. A computer screen for the regression is shown in Exhibit 3-15.

EXHIBIT 3-15 Multiple Regression for Anderson Company

| | A | B | C | D | E | F |
|----|------------------------------|---------------------|-----------------------|---------------|----------------|-----------------------|
| 1 | SUMMARY OUTPUT | | | | | |
| 2 | <i>Regression Statistics</i> | | | | | |
| 3 | Multiple R | 0.999420 | | | | |
| 4 | R Square | 0.998841 | | | | |
| 5 | Adjusted R | 0.998509 | | | | |
| 6 | | | | | | |
| 7 | Square | | | | | |
| 8 | Standard Error | 75.76272 | | | | |
| 9 | Observations | 10 | | | | |
| 10 | | | | | | |
| 11 | ANOVA | | | | | |
| 12 | | <i>df</i> | <i>SS</i> | <i>MS</i> | <i>F</i> | <i>Significance F</i> |
| 13 | Regression | 2 | 34613020 | 17306510 | 3015.076722 | 5.30799E-11 |
| 14 | Residual | 7 | 40179.93 | 5739.99 | | |
| 15 | Total | 9 | 34653200 | | | |
| 16 | | | | | | |
| 17 | | | | | | |
| 18 | | <i>Coefficients</i> | <i>Standard Error</i> | <i>t Stat</i> | <i>P-value</i> | |
| 19 | Intercept | 507.3097 | 57.3225 | 8.850098 | 4.7575E-05 | |
| 20 | X Variable 1 | 7.835162 | 0.234048 | 33.47672 | 5.49745E-09 | |
| 21 | X Variable 2 | 0.107181 | 0.003742 | 28.64286 | 1.62622E-08 | |
| 22 | | | | | | |

The computer screen conveys some very interesting and useful information. The cost equation is defined by the first two columns of the lowest table. The first column identifies the individual cost components. The intercept is the fixed activity cost, the first X variable is the number of moves, and the second X variable is the number of pounds moved. The column labeled “Coefficients” identifies the estimated fixed cost and the variable cost per unit for each activity driver. Thus, the cost equation can be written as follows:

$$Y = \$507 + \$7.84X_1 + \$0.11X_2$$

As with the cost equation involving a single activity driver, the preceding equation can be used to predict activity cost. Suppose that in November the company is expected to have 350 moves with 17,000 pounds of material moved. The predicted materials handling cost is as follows:

$$\begin{aligned} Y &= \$507 + \$7.84(350) + \$0.11(17,000) \\ &= \$507 + \$2,744 + \$1,870 \\ &= \$5,121 \end{aligned}$$

Notice in Exhibit 3-15 that the coefficient of determination is 99 percent—a significant improvement in explanatory power is achieved by adding the pounds moved variable. In addition, all three coefficients are highly significant.

For multiple regression, R^2 is usually referred to as the multiple coefficient of determination. Notice also that the standard error of estimate, S_e , is available in a multiple regression setting. As indicated earlier, the standard error of estimate can be used to build confidence intervals around cost predictions. To illustrate, consider the 95 percent confidence interval for the predicted materials handling cost when $X_1 = 350$ moves

and $X_2 = 17,000$ pounds moved ($t = 2.365$ for 95 percent confidence and 7 degrees of freedom):⁷

$$\begin{aligned} \$5,121 - 2.365(\$76) &\leq Y \leq \$5,121 + 2.365(\$76) \\ \$4,941 &\leq Y \leq \$5,301 \end{aligned}$$

Refer once again to Exhibit 3-15. Columns four and five of the lowest table present some statistical data concerning the three parameters. The fourth column presents t statistics for each of these parameters. These t statistics are used to test the hypothesis that the parameters are different from zero. The fifth column presents the level of significance achieved. All parameters are significant at the 0.0001 level. Thus, we can have some confidence that the two drivers are useful and that the materials handling activity has a fixed cost component. This example illustrates very clearly that multiple regression can be a useful tool for identifying the behavior of activity costs.

OBJECTIVE 6

Define the learning curve, and discuss its impact on cost behavior.

The Learning Curve and Nonlinear Cost Behavior

An important type of nonlinear cost curve is the learning curve. The **learning curve** shows how the labor hours worked per unit decrease as the volume produced increases. The basis of the learning curve is almost intuitive—as we perform an action over and over, we improve, and each additional performance takes less time than the preceding ones. We are learning how to do the task, becoming more efficient, and smoothing out the rough spots. In a manufacturing firm, learning takes place throughout the process: workers learn their tasks and managers learn to schedule production more efficiently and to arrange the flow of work. This effect was first documented in the aircraft industry.

Managers can now see that the ideas behind the learning curve can extend to the service industry as well as to manufacturing firms. Costs in marketing, distribution, and service after the sale also decrease as the number of units produced and sold increases.

The learning curve model takes two common forms: the cumulative average-time learning curve model and the incremental unit-time learning curve model.

Cumulative Average-Time Learning Curve

The **cumulative average-time learning curve model** states that the cumulative average time per unit decreases by a constant percentage, or learning rate, each time the cumulative quantity of units produced doubles. The **learning rate** is expressed as a percent, and it gives the percentage of time needed to make the next unit, based on the time it took to make the previous unit. The learning rate is determined through experience and must be between 50 and 100 percent. A 50 percent learning rate would eventually result in no labor time per unit—an absurd result. A 100 percent learning rate implies no learning (since the amount of decrease is zero). An 80 percent learning curve is often used to illustrate this model (possibly because the original learning curve work with the aircraft industry found an 80 percent learning curve). Exhibit 3-16 gives data for a cumulative average-time learning curve with an 80 percent learning rate and 100 direct labor hours for the first unit.

We see in Exhibit 3-16 that the bold rows give us the cumulative average time and cumulative total time according to the doubling formula. How do we obtain

7. Degrees of freedom is computed as $n - p$, where p is the number of parameters being estimated. For this example, there are 10 data points and three parameters. The three parameters are F , X_1 , and X_2 . The t statistics come from Exhibit 3-14.

EXHIBIT 3-16**Data for Cumulative Average-Time Learning Curve with 80 Percent Learning Rate**

| Cumulative Number of Units (1) | Cumulative Average Time per Unit in Hours (2) | Cumulative Total Time: Labor Hours (3) = (1) × (2) | Individual Unit Time for <i>n</i>th Unit: Labor Hours (4) |
|---|--|---|--|
| 1 | 100 | 100 | 100 |
| 2 | 80 (0.8 × 100) | 160 | 60 |
| 3 | 70.21 | 210.63 | 50.63 |
| 4 | 64 (0.8 × 80) | 256 | 45.37 |
| 5 | 59.57 | 297.85 | 41.85 |
| 6 | 56.17 | 337.02 | 39.17 |
| 7 | 53.45 | 374.15 | 37.13 |
| 8 | 51.20 (0.8 × 64) | 409.60 | 35.45 |
| 16 | 40.96 | 655.36 | 28.06 |
| 32 | 32.77 | 1,048.64 | |

Note: The rows in bold give the traditional doubling of output.

these amounts for units that are not doubles of the original amount? This is done by realizing that the cumulative average-time learning model takes a logarithmic relationship.

$$Y = pX^q$$

where

Y = Cumulative average time per unit

X = Cumulative number of units produced

p = Time in labor hours required to produce the first unit

q = Rate of learning

Therefore:

$$q = \ln(\text{percent learning}) / \ln 2$$

For an 80 percent learning curve:

$$q = -0.2231 / 0.6931 = -0.3219$$

So, when $X = 3$, $p = 100$, and $q = -0.3219$,

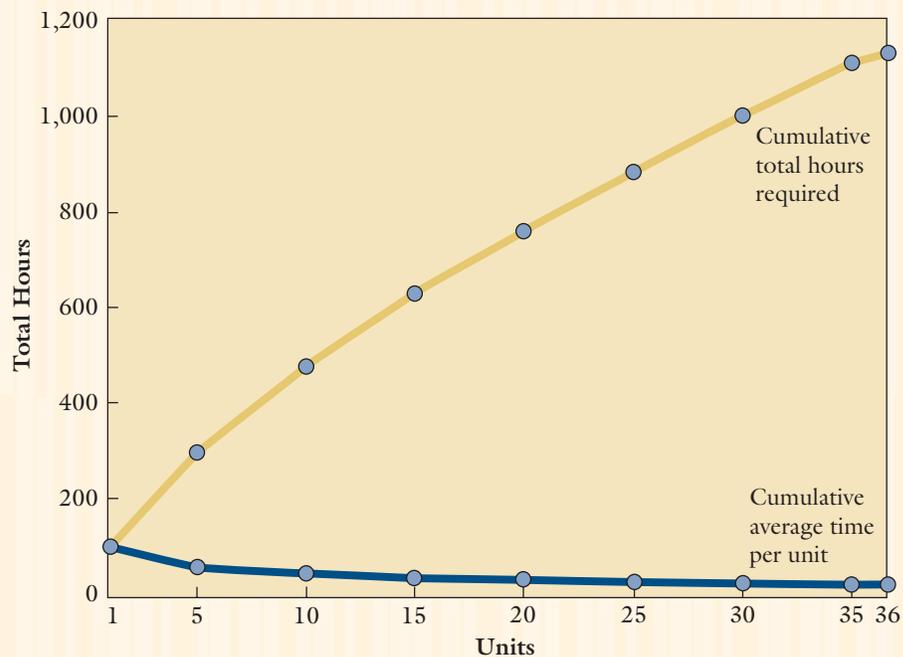
$$Y = 100 \times 3^{-0.3219} = 70.21 \text{ labor hours}$$

It is easy to see, then, that the number of hours required for the third unit is 50.63 (or $210.63 - 160.0$). Had we estimated the number of hours required for the third unit by the doubling calculations, we would have taken $256 - 160 = 96$ and then divided that result by 2 (the number of units between 2 and 4) and estimated the marginal time for the third unit as 48 hours. Notice that the more accurate result recognizes that the third unit really required 50.63 hours and the fourth unit 45.37 hours.

Exhibit 3-17 shows the graph of both the cumulative average time per unit (the bottom line) and the cumulative total hours required (top line). We can see that the time per unit decreases as output increases, but that it decreases at a decreasing rate. We also see that the total labor hours increase as output increases, but they increase at a decreasing rate.

EXHIBIT 3-17

Graph of Cumulative Total Hours Required and the Cumulative Average Time per Unit

**Incremental Unit-Time Learning Curve**

The **incremental unit-time learning curve model** decreases by a constant percentage each time the cumulative quantity of units produced doubles. Exhibit 3-18 gives data for an incremental unit-time learning curve with an 80 percent learning rate and 100 direct labor hours for the first unit.

EXHIBIT 3-18

Data for an Incremental Unit-Time Learning Curve with an 80 Percent Learning Rate

| Cumulative Number of Units (1) | Individual Unit Time for <i>n</i> th Unit in Labor Hours (2) | Cumulative Total Time: Labor Hours (3) | Cumulative Average Time per Unit: Labor Hours (4) = (3)/(1) |
|--------------------------------|--|--|---|
| 1 | 100 | 100 | 100 |
| 2 | 80 (0.8 × 100) | 180 | 90 |
| 3 | 70.21 | 250.21 | 83.40 |
| 4 | 64 (0.8 × 80) | 314.21 | 78.55 |
| 5 | 59.57 | 373.78 | 74.76 |
| 6 | 56.17 | 429.95 | 71.66 |
| 7 | 53.45 | 483.40 | 69.06 |
| 8 | 51.20 (0.8 × 64) | 534.60 | 66.83 |
| 16 | 40.96 | 892.00 | 55.75 |

We see in Exhibit 3-18 that the bold rows give us the cumulative average time and cumulative total time according to the doubling formula. How do we obtain these amounts for units that are not doubles of the original amount? This is done by realizing that the incremental unit-time learning curve model also takes a logarithmic relationship.

$$m = pX^q$$

where

- m = Time needed to produce the last unit
- X = Cumulative number of units produced
- p = Time in labor hours required to produce the first unit
- q = Rate of learning

Therefore:

$$q = \ln(\text{percent learning}) / \ln 2$$

For an 80 percent learning curve:

$$q = -0.2231 / 0.6931 = -0.3219$$

So, when $X = 3$, $p = 100$, and $q = -0.3219$,

$$m = 100 \times 3^{-0.3219} = 70.21 \text{ labor hours}$$

The cumulative total time for three units is $100 + 80 + 70.21 = 250.21$.

Let's look more closely at Exhibits 3-16 and 3-18. Notice that the second unit produced under the cumulative average-time learning model takes 60 labor hours, while the second unit produced under the incremental unit-time learning model takes 80 hours. Why the difference? The difference is in the underlying assumptions of the two models. The cumulative average-time learning model assumes that the cumulative *average* time for every unit produced is just 80 percent of the amount for the previous output level. Thus, when we look at the time to produce two units, the average time for each of the units is assumed to be 80 percent of the time for the first unit. However, the incremental unit-time learning model assumes that only the *last* (incremental) unit experiences the decrease in time, so the second unit takes 80 hours, but the first still takes 100 hours. Thus, the total time is 180 (100 + 80) hours.

The use of the learning curve concepts permits management to be more accurate in budgeting and performance evaluation for processes in which learning occurs. While the learning curve was originally developed for manufacturing processes, it can also apply in service industries. For example, insurance companies develop new policies and new methods of selling policies. There is a learning component to each new policy as employees discover glitches that were unexpected in the development process and then learn how to fix those glitches and become more efficient.

Of course, it is important to note that the learning rate can differ for each process. Management must estimate the rate, usually on the basis of past experience.

Managerial Judgment

Managerial judgment is critically important in determining cost behavior and is by far the most widely used method in practice. Many managers simply use their experience and past observation of cost relationships to determine fixed and variable costs. This method, however, may take a number of forms. Some managers simply assign particular activity costs to the fixed category and others to the variable category. They ignore the possibility of mixed costs. Thus, a chemical firm may regard materials and utilities as strictly variable, with respect to pounds of chemical produced, and all other costs as fixed. Even labor, the textbook example of a unit-based variable cost, may be fixed for this firm. The appeal of this method is simplicity. Before opting for this course of ac-

OBJECTIVE 7

Discuss the use of managerial judgment in determining cost behavior.

tion, management would do well to make sure that each cost is predominantly fixed or variable and that the decisions being made are not highly sensitive to errors in classifying costs as fixed or variable.

To illustrate the use of judgment in assessing cost behavior, consider **Elgin Sweeper Company**, a leading manufacturer of motorized street sweepers. Using production volume as the measure of activity output, Elgin revised its chart of accounts to organize costs into fixed and variable components. Elgin's accountants used their knowledge of the company to assign expenses to either a fixed or variable category, using a decision rule that categorized an expense as fixed if it were fixed 75 percent of the time and as variable if it were variable 75 percent of the time.⁸

Management may instead identify mixed costs and divide these costs into fixed and variable components by deciding just what the fixed and variable parts are—that is, using experience to say that a certain amount of a cost is fixed and therefore that the rest must be variable. Then, the variable component can be computed using one or more cost/volume data points. This use of judgment has the advantage of accounting for mixed costs but is subject to a similar type of error as the strict fixed/variable dichotomy. That is, management may be wrong in its assessment.

Finally, management may use experience and judgment to refine statistical estimation results. Perhaps the experienced manager might “eyeball” the data and throw out several points as being highly unusual, or the manager might revise results of estimation to take into account projected changes in cost structure or technology. For example, **Tecnol Medical Products, Inc.**, radically changed its method of manufacturing medical face masks. Traditionally, face-mask production was very labor intensive, requiring hand stitching. Tecnol developed its own highly automated equipment and became the industry's low-cost supplier—besting both **Johnson & Johnson** and **3M**. Tecnol's rapid expansion into new product lines and European markets means that historical data on costs and revenues are, for the most part, irrelevant.⁹ Tecnol's management must look forward, not back, to predict the impact of changes on profit. Statistical techniques are highly accurate in depicting the past, but they cannot foresee the future, which of course is what management really wants.

The advantage of using managerial judgment to separate fixed and variable costs is its simplicity. In situations in which the manager has a deep understanding of the firm and its cost patterns, this method can give good results. However, if the manager does not have good judgment, errors will occur. Therefore, it is important to consider the experience of the manager, the potential for error, and the effect that error could have on related decisions.

8. John P. Callan, Wesley N. Tredup, and Randy S. Wissinger, “Elgin Sweeper Company's Journey Toward Cost Management,” *Management Accounting* (July 1991): 24–27.

9. Stephanie Anderson Forest, “Who's Afraid of J&J and 3M?” *Business Week* (December 5, 1994): 66, 68.

SUMMARY

Cost behavior is the way in which a cost changes in relation to changes in activity output. The time horizon is important in determining cost behavior because costs can change from fixed to variable, depending on whether the decision takes place over the short run or the long run. Variable costs are those which change in total as activity usage changes. Usually, we assume that variable costs increase in direct proportion to increases in activity output. Fixed costs are those which do not change in total as activity

output changes. Mixed costs have both a variable and a fixed component. The resource usage model adds additional understanding of cost behavior.

Resources can be classified as either flexible or committed. Flexible resources are acquired as used and needed. There is no excess capacity for these resources, and they are usually considered variable costs. Committed resources, on the other hand, are acquired in advance of usage. These resources may have excess capacity, and frequently they are fixed. Some costs—especially discretionary fixed costs—tend to follow a step-cost function. These resources are acquired in lumpy amounts. If the width of the step is sufficiently large, then the costs are viewed as fixed; otherwise, they are approximated by a variable cost function.

The three formal mathematical methods of separating mixed costs are the high-low method, the scatterplot method, and the method of least squares. In the high-low method, the two points chosen from the scattergraph are the high and the low points with respect to activity level. These two points are then used to compute the intercept and the slope of the line on which they lie. The high-low method is objective and easy. However, if either the high or low point is not representative of the true cost relationship, the relationship will be misestimated.

The scatterplot method involves inspecting a scattergraph (a plot showing total mixed cost at various activity levels) and selecting two points that seem to best represent the relationship between cost and activity. Since two points determine a line, the two selected points can be used to determine the intercept and the slope of the line on which they lie. The intercept gives an estimate of the fixed cost component, and the slope gives an estimate of the variable cost per unit of activity. The scatterplot method is a good way to identify nonlinearity, the presence of outliers, and the presence of a shift in the cost relationship. Its disadvantage is that it is subjective.

The method of least squares uses all of the data points (except outliers) on the scattergraph and produces a line that best fits all of the points. The line is best-fitting in the sense that it is closest to all the points as measured by the sum of the squared deviations of the points from the line. The method of least squares produces the line that best fits the data points and is therefore recommended over the high-low and scatterplot methods.

The least-squares method has the advantage of offering methods to assess the reliability of cost equations. The coefficient of determination allows an analyst to compute the amount of cost variability explained by a particular activity driver. The standard error of estimate can be used to build a prediction interval for cost. If the interval is too wide, it may suggest that the equation is not very useful for prediction, even if the driver explains a high percentage of the cost variability. The least-squares method can also be used to build a cost equation using more than one activity output. Equations built using multiple regression can be evaluated for their reliability as well.

The learning curve describes a nonlinear relationship between labor hours and output. The two formulations of the learning curve are the cumulative average-time curve model and the incremental unit-time learning curve model. Both show that a doubling of output requires less than a doubling of labor time.

Managerial judgment can be used alone or in conjunction with the high-low, scatterplot, or least-squares methods. Managers use their experience and knowledge of cost and activity-level relationships to identify outliers, understand structural shifts, and adjust parameters due to anticipated changing conditions.

REVIEW PROBLEMS AND SOLUTIONS

1 RESOURCE USAGE AND COST BEHAVIOR

Thompson Manufacturing Company has three salaried clerks to process purchase orders. Each clerk is paid a salary of \$28,000 and is capable of processing 5,000 purchase orders per year (working efficiently). In addition to the salaries, Thompson spends \$7,500 per year for forms, postage, etc. Thompson assumes 15,000 purchase orders will be processed. During the year, 12,500 orders were processed.

Required:

1. Calculate the activity rate for the purchase order activity. Break the activity into fixed and variable components.
2. Compute the total activity availability, and break this into activity output and unused activity.
3. Calculate the total cost of the resource supplied, and break this into the cost of activity output and the cost of unused activity.

SOLUTION

1. Activity rate = $[(3 \times \$28,000) + \$7,500] / 15,000$
 $= \$6.10/\text{order}$
 Fixed rate = $\$84,000 / 15,000$
 $= \$5.60/\text{order}$
 Variable rate = $\$7,500 / 15,000$
 $= \$0.50/\text{order}$

2. Activity availability = Activity output + Unused activity
 15,000 orders = 12,500 orders + 2,500 orders

3. Cost of activity supplied = Cost of activity output + Cost of unused activity
 $\$84,000 + (\$0.50 \times 12,500) = (\$6.10 \times 12,500) + (\$5.60 \times 2,500)$
 $\$90,250 = \$76,250 + \$14,000$

2 HIGH-LOW METHOD AND METHOD OF LEAST SQUARES

Linda Jones, an accountant for Golding, Inc., has decided to estimate the fixed and variable components associated with the company's repair activity. She has collected the following data for the past six months:

| <i>Repair Hours</i> | <i>Total Repair Costs</i> |
|---------------------|---------------------------|
| 10 | \$ 800 |
| 20 | 1,100 |
| 15 | 900 |
| 12 | 900 |
| 18 | 1,050 |
| 25 | 1,250 |

Required:

1. Estimate the fixed and variable components for the repair costs using the high-low method. Using the cost formula, predict the total cost of repair if 14 hours are used.

2. Estimate the fixed and variable components using the method of least squares. Translate your results into the form of a cost formula, and using that formula, predict the total cost of repairs if 14 hours are used.
3. Using the method of least squares, what are the coefficient of determination and the coefficient of correlation?

SOLUTION

1. The estimate of fixed and variable costs using the high-low method, where Y = total cost and X = number of hours, is as follows:

$$\begin{aligned} V &= (Y_2 - Y_1)/(X_2 - X_1) \\ &= (\$1,250 - \$800)/(25 - 10) \\ &= \$450/15 \\ &= \$30 \text{ per hour} \end{aligned}$$

$$\begin{aligned} F &= Y_2 - VX_2 \\ &= \$1,250 - \$30(25) \\ &= \$500 \end{aligned}$$

$$\begin{aligned} Y &= \$500 + \$30X \\ &= \$500 + \$30(14) \\ &= \$920 \end{aligned}$$

2. Regression is performed using Excel, with the results as follows:

Summary Output

Regression Statistics

| | |
|-------------------|----------|
| Multiple R | 0.984523 |
| R Square | 0.969285 |
| Adjusted R Square | 0.961607 |
| Standard Error | 32.19657 |
| Observations | 6 |

ANOVA

| | <i>df</i> | <i>SS</i> | <i>MS</i> | <i>F</i> | <i>Significance F</i> |
|------------|-----------|-----------|-----------|----------|-----------------------|
| Regression | 1 | 130853.5 | 130853.5 | 126.2311 | 0.000357 |
| Residual | 4 | 4146.476 | 1036.619 | | |
| Total | 5 | 135000 | | | |

| | <i>Coefficients</i> | <i>Standard Error</i> | <i>t Stat</i> | <i>P-value</i> | <i>Lower 95%</i> | <i>Upper 95%</i> | <i>Lower 95.0%</i> | <i>Upper 95.0%</i> |
|--------------|---------------------|-----------------------|---------------|----------------|------------------|------------------|--------------------|--------------------|
| Intercept | 509.9119 | 45.55789 | 11.19261 | 0.000363 | 383.4227 | 636.4011 | 383.4227 | 636.4011 |
| X Variable 1 | 29.40529 | 2.617232 | 11.23526 | 0.000357 | 22.13867 | 36.6719 | 22.13867 | 36.6719 |

The calculation using the method of least squares is as follows:

$$\begin{aligned} Y &= \$509.91 + \$29.41X \\ &= \$509.91 + \$29.41(14) \\ &= \$921.65 \end{aligned}$$

- The coefficient of determination (R^2) is 0.969, and the correlation coefficient (r) is 0.985 (the square root of 0.969).

KEY TERMS

- Activity capacity 75
- Activity rate 78
- Coefficient of correlation 90
- Coefficient of determination 89
- Committed fixed expenses 75
- Committed resources 75
- Confidence interval 89
- Cost behavior 68
- Cumulative average-time learning curve model 95
- Dependent variable 80
- Deviation 85
- Discretionary fixed expenses 76
- Fixed costs 68
- Flexible resources 75
- Goodness of fit 89
- High-low method 81
- Hypothesis test of cost parameters 89
- Incremental unit-time learning curve model 97
- Independent variable 80
- Intercept parameter 80
- Learning curve 95
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- Long run 73
- Method of least squares 85
- Mixed costs 72
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- Slope parameter 80
- Step-cost function 77
- Step-fixed costs 78
- Step-variable costs 77
- Unit-level drivers 68
- Unused capacity 75
- Variable costs 70

QUESTIONS FOR WRITING AND DISCUSSION

- Why is knowledge of cost behavior important for managerial decision making? Give an example to illustrate your answer.
- How does the length of the time horizon affect the classification of a cost as fixed or variable? What is the meaning of short run? Long run?
- Explain the difference between resource spending and resource usage.
- What is the relationship between flexible resources and cost behavior?
- What is the relationship between committed resources and cost behavior?
- Describe the difference between a variable cost and a step-variable cost. When is it reasonable to treat step-variable costs as if they were variable costs?
- Why do mixed costs pose a problem when it comes to classifying costs into fixed and variable categories?
- Why is a scattergraph a good first step in separating mixed costs into their fixed and variable components?
- What are the advantages of the scatterplot method over the high-low method? The high-low method over the scatterplot method?

10. Describe the method of least squares. Why is this method better than either the high-low method or the scatterplot method?
11. What is meant by the best-fitting line? Is the best-fitting line necessarily a good-fitting line? Explain.
12. When is multiple regression required to explain cost behavior?
13. Explain the meaning of the learning curve. How do managers determine the appropriate learning curve percentage to use?
14. Assume you are the manager responsible for implementing a new service. The time to perform the service is subject to the learning curve. Would you prefer that the new service follow the cumulative average-time learning curve model or the incremental unit-time learning curve model? Why?
15. Some firms assign mixed costs to either the fixed or variable cost categories without using any formal methodology to separate them. Explain how this practice can be defended.

EXERCISES

3-1 VARIABLE, FIXED, AND MIXED COSTS

- LO1** Classify the following costs of activity inputs as variable, fixed, or mixed. Identify the activity and the associated activity driver that allow you to define the cost behavior. For example, assume that the resource input is “cloth in a shirt.” The activity would be “sewing shirts,” the cost behavior “variable,” and the activity driver “units produced.” Prepare your answers in the following format:

| <i>Activity</i> | <i>Cost Behavior</i> | <i>Activity Driver</i> |
|-----------------|----------------------|------------------------|
| Sewing shirts | Variable | Units produced |

- a. Power to operate a drill
- b. Engine in a lawn mower
- c. Advertising
- d. Sales commissions
- e. Fuel for a forklift
- f. Depreciation on a warehouse
- g. Depreciation on a forklift used to move partially completed goods
- h. X-ray film used in the radiology department of a hospital
- i. Rental car provided for a client
- j. Amalgam used by a dentist
- k. Salaries, equipment, and materials used for setting up production equipment
- l. Forms used to file insurance claims
- m. Equipment, labor, and parts used to repair and maintain production equipment
- n. Printing and postage for advertising circulars
- o. Salaries, forms, and postage associated with purchasing

3-2 COST BEHAVIOR

- LO1** Gupta Company manufactures miniature speakers that are built into the headrests of high-end lounge chairs. Based on past experience, Gupta has found that its total annual overhead costs can be represented by the following formula: Overhead cost = \$175,000 + \$1.10X, where X = number of speakers. Last year, Gupta produced 70,000 speakers. Actual overhead costs for the year were as expected.

Required:

1. What is the driver for the overhead activity?
2. What is the total overhead cost incurred by Gupta last year?
3. What is the total fixed overhead cost incurred by Gupta last year?
4. What is the total variable overhead cost incurred by Gupta last year?
5. What is the overhead cost per unit produced?
6. What is the fixed overhead cost per unit?
7. What is the variable overhead cost per unit?
8. Recalculate Requirements 5, 6, and 7 for the following levels of production: (a) 50,000 units and (b) 100,000 units. Explain this outcome.

3-3 COST BEHAVIOR CLASSIFICATION

LO1 Mazlow Company produces specialty tubing for large-scale construction applications. Its factory has six extruding lines that form tubing of different diameters. Each line can produce up to 5,000 feet of tubing per year. Each line has one supervisor who is paid \$25,000 per year. Depreciation on equipment averages \$12,000 per year. Direct materials and power cost about \$2.50 per foot of tubing.

Required:

1. Prepare a graph for each of these three costs: equipment depreciation, supervisors' wages, and direct materials and power. Use the vertical axis for cost and the horizontal axis for feet of tubing. Assume that tubing sales range from 0 to 30,000 feet of tubing.
2. Assume that the normal operating range for the company is 26,000 to 29,000 feet of tubing per year. How would you classify each of the three types of cost?

3-4 RESOURCE USAGE MODEL AND COST BEHAVIOR

LO2 For the following activities and their associated resources, identify the following: (1) a cost driver, (2) flexible resources, and (3) committed resources. Also, label each resource as one of the following with respect to the cost driver: (a) variable and (b) fixed.

| <i>Activity</i> | <i>Resource Description</i> |
|--------------------|--|
| Maintenance | Equipment, labor, and parts |
| Inspection | Test equipment, inspectors (each inspector can inspect five batches per day), and units inspected (process requires destructive sampling*) |
| Packing | Materials, labor (each packer places five units in a box), and conveyor belt |
| Payable processing | Clerks, materials, equipment, and facility |
| Assembly | Conveyor belt, supervision (one supervisor for every three assembly lines), direct labor, and materials |

*Destructive sampling occurs whenever it is necessary to destroy a unit as inspection occurs.

3-5 RESOURCE USAGE AND SUPPLY, ACTIVITY RATES, SERVICE ORGANIZATION

LO2 PhotoQuik is a film developing company. Customers mail their undeveloped rolls of film to the company and receive the completed photographs in return mail. The

PhotoQuik facility is built and staffed to handle the processing of 100,000 rolls of film per year. The lab facility cost \$330,000 to build and is expected to last 20 years. Processing equipment cost \$592,500 and has a life expectancy of five years. Both facility and equipment are depreciated on a straight-line basis. PhotoQuik has five salaried processing technicians, each of whom is paid \$15,000. In addition to the salaries, facility, and equipment, PhotoQuik expects to spend \$400,000 for chemicals, photo paper, envelopes, and other supplies (assuming 100,000 rolls of film are processed). Last year, 96,000 rolls of film were processed.

Required:

1. Classify the resources associated with the film-processing activity into one of the following types: (1) committed resources and (2) flexible resources.
2. Calculate the total activity rate for the film-processing activity. Break the activity rate into fixed and variable components.
3. Compute the total activity availability, and break this into activity output and unused activity.
4. Calculate the total cost of resources supplied, and break this into the cost of activity used and the cost of unused activity.

3-6 STEP COSTS, RELEVANT RANGE

LO2 Vargas, Inc., produces industrial machinery. Vargas has a machining department and a group of direct laborers called machinists. Each machinist is paid \$30,000 and can machine up to 500 units per year. Vargas also hires supervisors to develop machine specification plans and to oversee production within the machining department. Given the planning and supervisory work, a supervisor can oversee three machinists, at most. Vargas's accounting and production history reveal the following relationships between units produced and the costs of direct labor and supervision (measured on an annual basis):

| <i>Units Produced</i> | <i>Direct Labor</i> | <i>Supervision</i> |
|-----------------------|---------------------|--------------------|
| 0–500 | \$ 30,000 | \$ 45,000 |
| 501–1,000 | 60,000 | 45,000 |
| 1,001–1,500 | 90,000 | 45,000 |
| 1,501–2,000 | 120,000 | 90,000 |
| 2,001–2,500 | 150,000 | 90,000 |
| 2,501–3,000 | 180,000 | 90,000 |
| 3,001–3,500 | 210,000 | 135,000 |
| 3,501–4,000 | 240,000 | 135,000 |

Required:

1. Prepare two graphs: one that illustrates the relationship between direct labor cost and units produced and one that illustrates the relationship between the cost of supervision and units produced. Let cost be the vertical axis and units produced the horizontal axis.
2. How would you classify each cost? Why?
3. Suppose that the normal range of activity is between 1,300 and 1,450 units and that the exact number of machinists is currently hired to support this level of activity. Further suppose that production for the next year is expected to increase by an additional 400 units. How much will the cost of direct labor increase (and how will this increase be realized)? Cost of supervision?

3-7 SCATTERGRAPH METHOD, HIGH-LOW METHOD



Tad Jennings opened a tanning salon in a new shopping center. He had anticipated that the costs for the tanning service would be primarily fixed, but he found that tanning salon costs increased with the number of visits. Costs for this service over the past nine months are as follows:

| <i>Month</i> | <i>Tanning Visits</i> | <i>Total Cost</i> |
|--------------|-----------------------|-------------------|
| January | 700 | \$2,628 |
| February | 1,500 | 4,000 |
| March | 3,100 | 6,564 |
| April | 1,700 | 4,205 |
| May | 2,300 | 5,350 |
| June | 1,800 | 4,000 |
| July | 1,400 | 3,775 |
| August | 1,200 | 2,800 |
| September | 2,000 | 4,765 |

Required:

1. Prepare a scattergraph based on the preceding data. Use cost for the vertical axis and number of tanning visits for the horizontal axis. Based on an examination of the scattergraph, does there appear to be a linear relationship between the cost of tanning services and the number of visits?
2. Compute the cost formula for tanning services using the high-low method.
3. Calculate the predicted cost of tanning services for October for 2,200 visits using the formula found in Requirement 2.

3-8 METHOD OF LEAST SQUARES, GOODNESS OF FIT

LO3, LO4 Refer to the data in Exercise 3-7.

Required:

1. Compute the cost formula for tanning services using the method of least squares.
2. Using the formula computed in Requirement 1, what is the predicted cost of tanning services for October for 2,200 appointments?
3. What does the coefficient of determination tell you about the cost formula computed in Requirement 1? What are the t statistics for the number of appointments and the intercept term? What do these statistics tell you about the choice of number of appointments as the independent variable and the probability that there are fixed costs?

3-9 HIGH-LOW METHOD, COST FORMULAS

LO3 The controller of the Beresford plant of Gamerco, Inc., monitored activities associated with purchasing and receiving raw materials. The high and low levels of resource usage occurred in October and March for three different resources associated with purchasing and receiving. The number of purchase orders is the driver. The total costs of the three resources and the activity output, as measured by purchase orders, for the two different levels are presented as follows:

| <i>Resource</i> | <i>Purchase Orders</i> | <i>Total Cost</i> |
|-----------------------------|------------------------|-------------------|
| Supplier verification: | | |
| Low | 12,000 | \$ 43,000 |
| High | 30,000 | 43,000 |
| Receiving: | | |
| Low | 12,000 | \$ 81,600 |
| High | 30,000 | 204,000 |
| Processing purchase orders: | | |
| Low | 12,000 | \$ 43,200 |
| High | 30,000 | 97,200 |

Required:

1. Determine the cost behavior formula of each resource. Use the high-low method to assess the fixed and variable components.
2. Using your knowledge of cost behavior, predict the cost of each item for an activity output level of 25,000 purchase orders.
3. Construct a cost formula that can be used to predict the total cost of the three resources combined. Using this formula, predict the total purchasing and receiving cost if activity output is 22,000 purchase orders. In general, when can cost formulas be combined to form a single cost formula?

3-10 METHOD OF LEAST SQUARES, EVALUATION OF COST EQUATION

LO3, LO4



A company used the method of least squares to develop a cost equation to predict the cost of receiving. There were 80 data points for the regression, and the following computer output was generated:

| | |
|----------------------------|----------|
| Intercept | \$17,350 |
| Slope | 12 |
| Coefficient of correlation | 0.92 |
| Standard error | \$220 |

The activity driver used was the number of receiving orders.

Required:

1. What is the cost formula?
2. Using the cost formula, predict the cost of receiving if 10,000 orders are processed. Now prepare a 95 percent confidence interval for this prediction.
3. What percentage of the variability in receiving cost is explained by the number of receiving orders? Do you think the equation will predict well? Why or why not?

3-11 MULTIPLE REGRESSION

LO5



Kidstuff, Inc., was started 10 years ago by selling children's clothing through catalogs. Jan Switzer, Kidstuff's controller, had determined that the cost of filling and shipping orders was fairly consistently related to the number of orders. She had been using the following formula to describe monthly order filling costs:

$$\text{Order filling cost} = \$7,800 + \$7.50 \times \text{orders}$$

Lately, however, Jan noticed that order filling costs varied widely and did not seem to follow the above relationship. After a number of discussions with order pickers and

fillers, Jan determined that Kidstuff's expansion into children's toys had made order filling a more complex operation. Number of orders was still an important variable, but so were the number of categories included in an order (an order for just clothing was quicker to pick, fill, and pack than an order with both clothing and toys) and whether or not any items needed to be gift wrapped. Jan ran a multiple regression on the past 24 months of data for Kidstuff for three variables: the cost of filling orders (picking and packing the order for shipment), the number of orders, and the number of complex orders (orders with both clothing and toys) and gifts (the number of gift-wrapped items). The following printout was obtained:

| <i>Parameter</i> | <i>Estimate</i> | <i>t for H₀ Parameter = 0</i> | <i>Pr > t</i> | <i>Standard Error of Parameter</i> |
|--------------------------|-----------------|--|------------------|--|
| Intercept | 9,320 | 93.00 | 0.0001 | 479.00 |
| Number of orders | 5.14 | 3.60 | 0.0050 | 1.56 |
| Number of complex orders | 2.06 | 5.58 | 0.0050 | 2.00 |
| Number of gifts | 1.30 | 2.96 | 0.0250 | 0.75 |
| $R^2 = 0.92$ | | | | |
| $S_e = 150$ | | | | |
| Observations: 24 | | | | |

Required:

- Write out the cost equation for Kidstuff's monthly order filling cost.
- If Kidstuff expects to have 300 orders next month (65 with both clothing and toys) and expects that 100 items must be gift wrapped, what are the anticipated order filling costs?
- Calculate a 99 percent confidence interval for the prediction made in Requirement 2.
- What does R^2 mean in this equation? Overall, what is your evaluation of the cost equation that was developed for the cost of order filling? Suppose that Kidstuff charges an extra \$2.50 to gift wrap an item. How might Jan use the results of the regression equation to see whether or not the \$2.50 charge is appropriate?

3-12 MULTIPLE REGRESSION

LOS Allmen, Inc., a manufacturer of heavy machinery, is interested in improving its factory safety record. Jennifer Cybert, Allmen's controller, has investigated the past four years of industrial accidents, both at Allmen and at other similar factories. She has found that more accidents seem to happen during months with greater hours of overtime worked. In addition, she thinks that employee safety could be enhanced by a vigorous safety program. To test her hypotheses, she ran a multiple regression on 48 months of data for Allmen for three variables: the cost of industrial accidents, the number of hours of safety training, and the number of hours of overtime worked by production workers. The following printout was obtained:

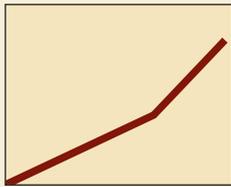
| <i>Parameter</i> | <i>Estimate</i> | <i>t for H₀ Parameter = 0</i> | <i>Pr > t</i> | <i>Standard Error of Parameter</i> |
|--------------------------|-----------------|--|------------------|--|
| Intercept | 2,150 | 70.00 | 0.0001 | 150.00 |
| Number of overtime hours | 17 | 3.60 | 0.0050 | 23.45 |
| Hours of safety training | -8.50 | -1.96 | 0.0250 | 5.13 |
| $R^2 = 0.89$ | | | | |
| $S_e = 250$ | | | | |
| Observations: 48 | | | | |

Required:

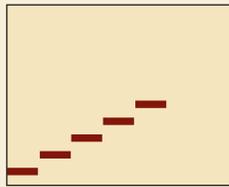
1. Write out the cost equation for Allmen's industrial accident cost.
2. If Allmen expects to have 280 overtime hours worked next month and expects to spend 200 hours on safety training, what are the anticipated accident costs?
3. Calculate a 99 percent confidence interval for the prediction made in Requirement 2.
4. Is number of overtime hours positively or negatively correlated with accident costs? Are hours of safety training positively or negatively correlated with accident costs?
5. What does R^2 mean in this equation? Overall, what is your evaluation of the cost equation that was developed for the cost of industrial accidents?

3-13 COST BEHAVIOR PATTERNS

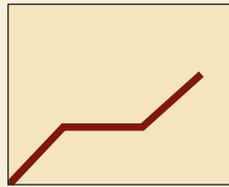
LO1, LO2 The graphs below represent cost behavior patterns that might occur in a company's cost structure. The vertical axis represents total cost, and the horizontal axis represents activity output.



a.



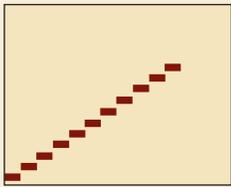
b.



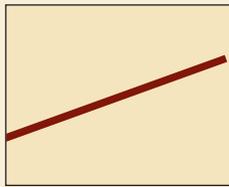
c.



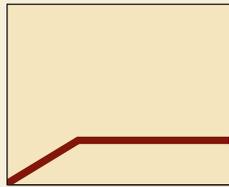
d.



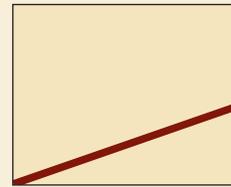
e.



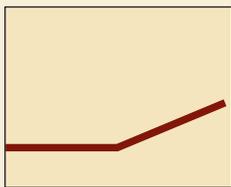
f.



g.



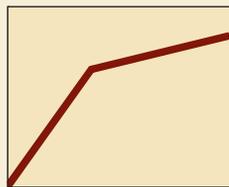
h.



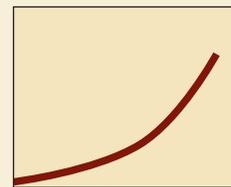
i.



j.



k.



l.

Required:

For each of the following situations, choose the graph from the group a–l that best illustrates the cost pattern involved. Also, for each situation, identify the driver that measures activity output.

1. The cost of power when a fixed fee of \$500 per month is charged plus an additional charge of \$0.12 per kilowatt-hour used.
2. Commissions paid to sales representatives. Commissions are paid at the rate of 5 percent of sales made up to total annual sales of \$500,000, and 7 percent of sales above \$500,000.
3. A part purchased from an outside supplier costs \$12 per part for the first 3,000 parts and \$10 per part for all parts purchased in excess of 3,000 units.

4. The cost of surgical gloves, which are purchased in increments of 100 units (gloves come in boxes of 100 pairs).
5. The cost of tuition at a local college that charges \$250 per credit hour up to 15 credit hours. Hours taken in excess of 15 are free.
6. The cost of tuition at another college that charges \$4,500 per semester for any course load ranging from 12 to 16 credit hours. Students taking fewer than 12 credit hours are charged \$375 per credit hour. Students taking more than 16 credit hours are charged \$4,500 plus \$300 per credit hour in excess of 16.
7. A beauty shop's purchase of soaking solution to remove artificial nails. Each jar of solution can soak off approximately 50 nails before losing its effectiveness.
8. Purchase of diagnostics equipment by a company for inspection of incoming orders.
9. Use of disposable gowns by patients in a hospital.
10. Cost of labor at a local fast-food restaurant. Three employees are always on duty during working hours; more employees can be called in during periods of heavy demand to work on an "as-needed" basis.
11. A manufacturer found that the maintenance cost of its heavy machinery was tied to the age of the equipment. Experience indicated that the maintenance cost increased at an increasing rate as the equipment aged.

PROBLEMS

3-14 COST BEHAVIOR, RESOURCE USAGE, EXCESS CAPACITY

LO1, LO2 Rolertyme Company manufactures roller skates. With the exception of the rollers, all parts of the skates are produced internally. Neeta Booth, president of Rolertyme, has decided to make the rollers instead of buying them from external suppliers. The company needs 100,000 sets per year (currently it pays \$1.90 per set of rollers).

The rollers can be produced using an available area within the plant. However, equipment for production of the rollers would need to be leased (\$30,000 per year lease payment). Additionally, it would cost \$0.50 per machine hour for power, oil, and other operating expenses. The equipment will provide 60,000 machine hours per year. Direct material costs will average \$0.75 per set, and direct labor will average \$0.25 per set. Since only one type of roller would be produced, no additional demands would be made on the setup activity. Other overhead activities (besides machining and setups), however, would be affected. The company's cost management system provides the following information about the current status of the overhead activities that would be affected. (The supply and demand figures do not include the effect of roller production on these activities.) The lumpy quantity indicates how much capacity must be purchased should any expansion of activity supply be needed. The purchase price is the cost of acquiring the capacity represented by the lumpy quantity. This price also represents the cost of current spending on existing activity supply (for each block of activity).

| <i>Activity Price</i> | <i>Cost Driver</i> | <i>Supply</i> | <i>Usage</i> | <i>Lumpy Quantity</i> | <i>Purchase</i> |
|-----------------------|--------------------|---------------|--------------|-----------------------|-----------------|
| Purchasing | Orders | 25,000 | 23,000 | 5,000 | \$25,000 |
| Inspection | Hours | 10,000 | 9,000 | 2,000 | 30,000 |
| Materials handling | Moves | 4,500 | 4,300 | 500 | 15,000 |

Production of rollers would place the following demands on the overhead activities:

| <i>Activity</i> | <i>Resource Demands</i> |
|--------------------|--|
| Machining | 50,000 machine hours |
| Purchasing | 2,000 purchase orders (associated with raw materials used to make the rollers) |
| Inspection | 750 inspection hours |
| Materials handling | 500 moves |

Producing the rollers also means that the purchase of outside rollers will cease. Thus, purchase orders associated with the outside acquisition of rollers will drop by 5,000. Similarly, the moves for the handling of incoming orders will decrease by 200. The company has not inspected the rollers purchased from outside suppliers.

Required:

1. Classify all resources associated with the production of rollers as flexible resources and committed resources. Label each committed resource as a short- or long-term commitment. How should we describe the cost behavior of these short- and long-term resource commitments? Explain.
2. Calculate the total annual resource spending (for all activities except for setups) that the company will incur after production of the rollers begins. Break this cost into fixed and variable activity costs. In calculating these figures, assume that the company will spend no more than necessary. What is the effect on resource spending caused by production of the rollers?
3. Refer to Requirement 2. For each activity, break down the cost of activity supplied into the cost of activity output and the cost of unused activity.

3-15 COST BEHAVIOR, HIGH-LOW METHOD, PRICING DECISION

LO1, LO3

St. Luke's Medical Center (SLMC) offers a number of specialized medical services, including neuroscience, cardiology, and oncology. SLMC's strong reputation for quality medical care allowed it to branch out into other services. It is now ready to expand its orthopedic services and has just added a free-standing orthopedic clinic offering a full range of outpatient, surgical, and physical therapy services. The cost of the orthopedic facility is depreciated on a straight-line basis. All equipment within the facility is leased.

Since the clinic had no experience with in-patient orthopedic services (for patients recovering for hip and knee replacements, for example), it decided to operate the orthopedic center for two months before determining how much to charge per patient day on an ongoing basis. As a temporary measure, the clinic adopted a patient-day charge of \$190, an amount equal to the fees charged by a hospital specializing in orthopedic care in a nearby city.

This initial per-day charge was quoted to patients entering the orthopedic center during the first two months with assurances that if the actual operating costs of the new center justified it, the charge could be less. In no case would the charges be more. A temporary policy of billing after 60 days was adopted so that any adjustments could be made.

The orthopedic center opened on January 1. During January, the center had 2,100 patient days of activity. During February, the activity was 2,250 patient days. Costs for these two levels of activity output are as follows:

| | <i>2,100 Patient Days</i> | <i>2,250 Patient Days</i> |
|-------------------|---------------------------|---------------------------|
| Salaries, nurses | \$ 15,000 | \$ 15,000 |
| Aides | 2,000 | 2,000 |
| Pharmacy | 217,300 | 232,300 |
| Laboratory | 55,700 | 58,700 |
| Depreciation | 12,000 | 12,000 |
| Laundry | 16,800 | 18,000 |
| Administration | 17,000 | 17,000 |
| Lease (equipment) | 30,000 | 30,000 |

Required:

1. Classify each cost as fixed, variable, or mixed, using patient days as the activity driver.
2. Use the high-low method to separate the mixed costs into fixed and variable.
3. Lynley Jackson, the administrator of the orthopedic center, has estimated that the center will average 2,000 patient days per month. If the center is to be operated as a nonprofit organization, how much will it need to charge per patient day? How much of this charge is variable? How much is fixed?
4. Suppose the orthopedic center averages 2,500 patient days per month. How much would need to be charged per patient day for the center to cover its costs? Explain why the charge per patient day decreased as the activity output increased.

3-16 HIGH-LOW METHOD, METHOD OF LEAST SQUARES, CORRELATION, CONFIDENCE INTERVAL

LO2, LO3,
LO4

PriceCut, a discount store, has gathered data on its overhead activities and associated costs for the past 10 months. Adrienne Sanjay, a member of the controller's department, believes that overhead activities and costs should be classified into groups that have the same driver. She has decided that unloading incoming goods, counting goods, and inspecting goods can be grouped together as a more general receiving activity, since these three activities are all driven by the number of purchase orders. The following 10 months of data have been gathered for the receiving activity:

| <i>Month</i> | <i>Purchase Orders</i> | <i>Receiving Cost</i> |
|--------------|------------------------|-----------------------|
| 1 | 1,000 | \$18,600 |
| 2 | 700 | 14,000 |
| 3 | 1,500 | 28,000 |
| 4 | 1,200 | 17,500 |
| 5 | 1,300 | 25,000 |
| 6 | 1,100 | 21,000 |
| 7 | 1,600 | 28,000 |
| 8 | 1,400 | 24,000 |
| 9 | 1,700 | 26,000 |
| 10 | 900 | 16,000 |

Required:

1. Prepare a scattergraph, plotting the receiving costs against the number of purchase orders. Use the vertical axis for costs and the horizontal axis for orders.
2. Select two points that make the best fit, and compute a cost formula for receiving costs.
3. Using the high-low method, prepare a cost formula for the receiving activity.

4. Using the method of least squares, prepare a cost formula for the receiving activity. What is the coefficient of determination?
5. Prepare a 95 percent confidence interval for receiving costs when 1,200 purchase orders are expected.

3-17 COST FORMULAS, SINGLE AND MULTIPLE ACTIVITY DRIVERS, COEFFICIENT OF CORRELATION

LO1, LO3,
LO4, LO5

Kimball Company has developed the following cost formulas:

$$\text{Material usage: } Y_m = \$80X; r = 0.95$$

$$\text{Labor usage (direct): } Y_l = \$20X; r = 0.96$$

$$\text{Overhead activity: } Y_o = \$350,000 + \$100X; r = 0.75$$

$$\text{Selling activity: } Y_s = \$50,000 + \$10X; r = 0.93$$

where

$$X = \text{Direct labor hours}$$

The company has a policy of producing on demand and keeps very little, if any, finished goods inventory (thus, units produced = units sold). Each unit uses one direct labor hour for production.

The president of Kimball Company has recently implemented a policy that any special orders will be accepted if they cover the costs that the orders cause. This policy was implemented because Kimball's industry is in a recession and the company is producing well below capacity (and expects to continue doing so for the coming year). The president is willing to accept orders that minimally cover their variable costs so that the company can keep its employees and avoid layoffs. Also, any orders above variable costs will increase overall profitability of the company.

Required:

1. Compute the total unit variable cost. Suppose that Kimball has an opportunity to accept an order for 20,000 units at \$220 per unit. Should Kimball accept the order? (The order would not displace any of Kimball's regular orders.)
2. Explain the significance of the coefficient of correlation measures for the cost formulas. Did these measures have a bearing on your answer in Requirement 1? Should they have a bearing? Why or why not?
3. Suppose that a multiple regression equation is developed for overhead costs: $Y = \$100,000 + \$100X_1 + \$5,000X_2 + \$300X_3$, where X_1 = direct labor hours, X_2 = number of setups, and X_3 = engineering hours. The coefficient of determination for the equation is 0.94. Assume that the order of 20,000 units requires 12 setups and 600 engineering hours. Given this new information, should the company accept the special order referred to in Requirement 1? Is there any other information about cost behavior that you would like to have? Explain.

3-18 SCATTERGRAPH, HIGH-LOW METHOD, METHOD OF LEAST SQUARES, USE OF JUDGMENT

LO3, LO4,
LO5

The management of Wheeler Company has decided to develop cost formulas for its major overhead activities. Wheeler uses a highly automated manufacturing process, and power costs are a significant manufacturing cost. Cost analysts have decided that power costs are mixed; thus, they must be broken into their fixed and variable elements so that the cost behavior of the power usage activity can be properly described. Machine hours have been selected as the activity driver for power costs. The following data for the past eight quarters have been collected:

| <i>Quarter</i> | <i>Machine Hours</i> | <i>Power Cost</i> |
|----------------|----------------------|-------------------|
| 1 | 20,000 | \$26,000 |
| 2 | 25,000 | 38,000 |
| 3 | 30,000 | 42,500 |
| 4 | 22,000 | 35,000 |
| 5 | 21,000 | 34,000 |
| 6 | 18,000 | 31,400 |
| 7 | 24,000 | 36,000 |
| 8 | 28,000 | 42,000 |

Required:

1. Prepare a scattergraph by plotting power costs against machine hours. Does the scattergraph show a linear relationship between machine hours and power cost?
2. Using the high and low points, compute a power cost formula.
3. Use the method of least squares to compute a power cost formula. Evaluate the coefficient of determination.
4. Rerun the regression and drop the point (20,000; \$26,000) as an outlier. Compare the results from this regression to those for the regression in Requirement 3. Which is better?

3-19 METHOD OF LEAST SQUARES

LO1, LO3,
LO4, LO5

DeMarco Company is developing a cost formula for its packing activity. Discussion with the workers in the packing department has revealed that packing costs are associated with the number of customer orders, the size of the orders, and the relative fragility of the items (more fragile items must be specially wrapped in bubble wrap and Styrofoam). Data for the past 20 months have been gathered:

| <i>Month</i> | <i>Packing Cost</i> | <i>Number of Orders</i> | <i>Weight of Orders</i> | <i>Number of Fragile Items</i> |
|--------------|---------------------|-------------------------|-------------------------|--------------------------------|
| 1 | \$ 45,000 | 11,200 | 24,640 | 1,120 |
| 2 | 58,000 | 14,000 | 31,220 | 1,400 |
| 3 | 39,000 | 10,500 | 18,000 | 1,000 |
| 4 | 35,600 | 9,000 | 19,350 | 850 |
| 5 | 90,000 | 21,000 | 46,200 | 4,000 |
| 6 | 126,000 | 31,000 | 64,000 | 5,500 |
| 7 | 90,600 | 20,000 | 60,000 | 1,800 |
| 8 | 63,000 | 15,000 | 40,000 | 750 |
| 9 | 79,000 | 16,000 | 59,000 | 1,500 |
| 10 | 155,000 | 40,000 | 88,000 | 2,500 |
| 11 | 450,000 | 113,500 | 249,700 | 11,800 |
| 12 | 640,000 | 150,000 | 390,000 | 14,000 |
| 13 | 41,000 | 10,000 | 23,000 | 900 |
| 14 | 54,000 | 14,000 | 29,400 | 890 |
| 15 | 58,000 | 15,000 | 30,000 | 1,500 |
| 16 | 58,090 | 14,500 | 31,900 | 1,340 |
| 17 | 80,110 | 18,000 | 50,000 | 3,000 |
| 18 | 123,000 | 30,000 | 75,000 | 2,000 |
| 19 | 108,000 | 27,000 | 63,450 | 1,900 |
| 20 | 76,000 | 18,000 | 41,400 | 1,430 |

Required:

1. Using the method of least squares, run a regression using the number of orders as the independent variable.
2. Run a multiple regression using three independent variables: the number of orders, the weight of orders, and the number of fragile items. Which regression equation is better? Why?
3. Predict the total packing cost for 25,000 orders, weighing 40,000 pounds, with 4,000 fragile items. Prepare a 99 percent confidence interval for this estimate of total packing cost.
4. How much would the cost estimated for Requirement 3 change if the 25,000 orders weighed 40,000 pounds, but only 2,000 were fragile items?

3-20 HIGH-LOW METHOD, SCATTERPLOT, REGRESSION

LO2, LO3,
LO4, LO5

Weber Valley Regional Hospital has collected data on all of its activities for the past 16 months. Data for cardiac nursing care follow:

| | <i>Y</i> <i>Cost</i> | <i>X</i> <i>Hours of Nursing Care</i> |
|----------------|-------------------------|--|
| May 2006 | \$59,600 | 1,400 |
| June 2006 | 57,150 | 1,350 |
| July 2006 | 61,110 | 1,460 |
| August 2006 | 65,800 | 1,600 |
| September 2006 | 69,500 | 1,700 |
| October 2006 | 64,250 | 1,550 |
| November 2006 | 52,000 | 1,200 |
| December 2006 | 66,000 | 1,600 |
| January 2007 | 83,000 | 1,800 |
| February 2007 | 66,550 | 1,330 |
| March 2007 | 79,500 | 1,700 |
| April 2007 | 76,000 | 1,600 |
| May 2007 | 68,500 | 1,400 |
| June 2007 | 73,150 | 1,550 |
| July 2007 | 73,175 | 1,505 |
| August 2007 | 66,150 | 1,290 |

Required:

1. Using the high-low method, calculate the variable rate per hour and the fixed cost for the nursing care activity.
2. Run a regression on the data, using hours of nursing care as the independent variable. Predict cost for the cardiac nursing care for September 2007 if 1,400 hours of nursing care are forecast. Evaluate the regression equation. How comfortable are you with the predicted cost for September 2007?
3. Upon looking into the events that happened at the end of 2006, you find that the cardiology ward bought a cardiac-monitoring machine for the nursing station. Administrators also decided to add a new supervisory position for the evening shift. Monthly depreciation on the monitor and the salary of the new supervisor together total \$10,000. Now, run two regression equations, one for the observations from 2006 and the second using only the observations for the eight months in 2007. Discuss your findings. What is your predicted cost of the cardiac nursing care activity for September 2007?

3-21 COMPARISON OF REGRESSION EQUATIONS

LO1, LO3,
LO4, LO5

Friendly Bank is attempting to determine the cost behavior of its small business lending operations. One of the major activities is the application activity. Two possible activity drivers have been mentioned: application hours (number of hours to complete the application) and number of applications. The bank controller has accumulated the following data for the setup activity:

| <i>Month</i> | <i>Application Costs</i> | <i>Application Hours</i> | <i>Number of Applications</i> |
|--------------|--------------------------|--------------------------|-------------------------------|
| February | \$ 7,700 | 2,000 | 70 |
| March | 7,650 | 2,100 | 50 |
| April | 10,052 | 3,000 | 50 |
| May | 9,400 | 2,700 | 60 |
| June | 9,584 | 3,000 | 20 |
| July | 8,480 | 2,500 | 40 |
| August | 8,550 | 2,400 | 60 |
| September | 9,735 | 2,900 | 50 |
| October | 10,500 | 3,000 | 90 |

Required:

1. Estimate a regression equation with application hours as the activity driver and the only independent variable. If the bank forecasts 2,600 application hours for the next month, what will be the budgeted application cost?
2. Estimate a regression equation with number of applications as the activity driver and the only independent variable. If the bank forecasts 80 applications for the next month, what will be the budgeted application cost?
3. Which of the two regression equations do you think does a better job of predicting application costs? Explain.
4. Run a multiple regression to determine the cost equation using both activity drivers. What are the budgeted application costs for 2,600 application hours and 80 applications?

3-22 MULTIPLE REGRESSION, CONFIDENCE INTERVALS, RELIABILITY OF COST FORMULAS

LO2, LO4,
LO5

Randy Harris, controller, has been given the charge to implement an advanced cost management system. As part of this process, he needs to identify activity drivers for the activities of the firm. During the past four months, Randy has spent considerable effort identifying activities, their associated costs, and possible drivers for the activities' costs.

Initially, Randy made his selections based on his own judgment using his experience and input from employees who perform the activities. Later, he used regression analysis to confirm his judgment. Randy prefers to use one driver per activity, provided that an R^2 of at least 80 percent can be produced. Otherwise, multiple drivers will be used, based on evidence provided by multiple regression analysis. For example, the activity of inspecting finished goods produced an R^2 of less than 80 percent for any single activity driver. Randy believes, however, that a satisfactory cost formula can be developed using two activity drivers: the number of batches and the number of inspection hours. Data collected for a 14-month period are as follows:

| <i>Inspection Costs</i> | <i>Hours of Inspection</i> | <i>Number of Batches</i> |
|-------------------------|----------------------------|--------------------------|
| \$17,689 | 100 | 10 |
| 18,350 | 120 | 20 |
| 13,125 | 60 | 15 |
| 28,000 | 320 | 30 |
| 30,560 | 240 | 25 |
| 31,755 | 200 | 40 |
| 40,750 | 280 | 35 |
| 29,500 | 230 | 22 |
| 47,570 | 350 | 50 |
| 36,740 | 270 | 45 |
| 43,500 | 350 | 38 |
| 26,780 | 200 | 18 |
| 28,500 | 140 | 28 |
| 17,000 | 160 | 14 |

Required:

1. Calculate the cost formula for inspection costs using the two drivers, inspection hours and number of batches. Are both activity drivers useful? What does the R^2 indicate about the formula?
2. Using the formula developed in Requirement 1, calculate the inspection cost when 300 inspection hours are used and 30 batches are produced. Prepare a 90 percent confidence interval for this prediction.

3-23 SIMPLE AND MULTIPLE REGRESSION, EVALUATING RELIABILITY OF AN EQUATION

LO2, LO3,
LO4

CMA

The Lockit Company manufactures door knobs for residential homes and apartments. Lockit is considering the use of simple (single-driver) and multiple regression analyses to forecast annual sales because previous forecasts have been inaccurate. The new sales forecast will be used to initiate the budgeting process and to identify more completely the underlying process that generates sales.

Larry Husky, the controller of Lockit, has considered many possible independent variables and equations to predict sales and has narrowed his choices to four equations. Husky used annual observations from 20 prior years to estimate each of the four equations.

Following are a definition of the variables used in the four equations and a statistical summary of these equations:

$$\begin{aligned}
 S_t &= \text{Forecasted sales in dollars for Lockit in period } t \\
 S_{t-1} &= \text{Actual sales in dollars for Lockit in period } t - 1 \\
 G_t &= \text{Forecasted U.S. gross domestic product in period } t \\
 G_{t-1} &= \text{Actual U.S. gross domestic product in period } t - 1 \\
 N_{t-1} &= \text{Lockit's net income in period } t - 1
 \end{aligned}$$

Required:

1. Write Equations 2 and 4 in the form $Y = a + bx$.
2. If actual sales are \$1,500,000 in 2006, what would be the forecasted sales for Lockit in 2007?
3. Explain why Larry Husky might prefer Equation 3 to Equation 2.
4. Explain the advantages and disadvantages of using Equation 4 to forecast sales.

Statistical Summary of Four Equations

| Equation | Dependent Variable | Independent Variable(s) | Intercept | Independent Variable (Rate) | Standard Error | R Square | t Value |
|----------|--------------------|-------------------------|------------|-----------------------------|----------------|----------|---------|
| 1 | S_t | S_{t-1} | \$ 500,000 | \$ 1.10 | \$500,000 | 0.94 | 5.50 |
| 2 | S_t | G_t | 1,000,000 | 0.00001 | 510,000 | 0.90 | 10.00 |
| 3 | S_t | G_{t-1} | 900,000 | 0.000012 | 520,000 | 0.81 | 5.00 |
| 4 | S_t | N_{t-1} | 600,000 | 10.00 | 490,000 | 0.96 | 4.00 |
| | | G_t | | 0.000002 | | | 1.50 |
| | | G_{t-1} | | 0.000003 | | | 3.00 |

(CMA adapted)

3-24 LEARNING CURVE

LO6 Harriman Industries manufactures engines for the aerospace industry. It has completed manufacturing the first unit of the new ZX-9 engine design. Management believes that the 1,000 labor hours required to complete this unit are reasonable and is prepared to go forward with the manufacture of additional units. An 80 percent cumulative average-time learning curve model for direct labor hours is assumed to be valid. Data on costs are as follows:

| | |
|---------------------------------|----------------------------|
| Direct materials | \$10,500 |
| Direct labor | \$30 per direct labor hour |
| Variable manufacturing overhead | \$40 per direct labor hour |

Required:

- Set up a table with columns for cumulative number of units, cumulative average time per unit in hours, cumulative total time in hours, and individual unit time for the n th unit in hours. Complete the table for 1, 2, 4, 8, 16, and 32 units.
- What are the total variable costs of producing 1, 2, 4, 8, 16, and 32 units? What is the variable cost per unit for 1, 2, 4, 8, 16, and 32 units?

3-25 LEARNING CURVE

LO6 Refer to **Problem 3-24**. Now assume that an 80 percent incremental unit-time learning curve model is applicable.

Required:

- Set up a table with columns for cumulative number of units, individual unit time for the n th unit in hours, cumulative total time in hours, and cumulative average time per unit in hours. Complete the table for 1, 2, 4, 8, 16, and 32 units.
- What are the total variable costs of producing 1, 2, 4, 8, 16, and 32 units?
- Why do the results for this problem vary from those for **Problem 3-24**?

3-26 LEARNING CURVE

LO6 Thames Assurance Company sells a variety of life and health insurance products. Recently, Thames developed a long-term care policy for sale to members of university and college alumni associations. Thames estimated that the sale and service of this type of policy would be subject to a 90 percent incremental unit-time learning curve model.

Each unit consists of 350 policies sold. The first unit is estimated to take 1,000 hours to sell and service.

Required:

1. Set up a table with columns for cumulative number of units, individual unit time for the n th unit in hours, cumulative total time in hours, and cumulative average time per unit in hours. Complete the table for 1, 2, 4, 8, 16, and 32 units.
2. Suppose that Thames revises its assumption to an 80 percent learning curve. How will this affect the amount of time needed to sell and service eight units? How do you suppose that Thames estimates the percent learning rate?

3-27 LEARNING CURVE

- LO6** Using the same data from **Problem 3-26**, apply a cumulative average-time learning curve model with a 90 percent learning rate.

Required:

1. Set up a table with columns for cumulative number of units, cumulative average time per unit in hours, cumulative total time in hours, and individual unit time for the n th unit in hours. Complete the table for 1, 2, 4, 8, 16, and 32 units.
2. Why do the results from this table differ from those in Requirement 1 of **Problem 3-26**?

3-28 COLLABORATIVE LEARNING EXERCISE

- LO1, LO2** Divide students into groups of four or five. Have each group choose a business that is familiar to them (e.g., pizza parlor) and list as many flexible and committed resources as possible. One group member, the reporter, should write down the group's responses and then share them with the rest of the class.

3-29 CYBER RESEARCH CASE

- LO6** Check the Boeing Web site at <http://www.boeing.com>, and go to commercial aircraft orders. Boeing gives the number of orders per type of plane (e.g., 767). For which type of plane would you expect Boeing to gain the most from learning effects? Why? What impact will this have on costs? Prices? Time to delivery?