

Return and Risk: The Capital Asset Pricing Model (CAPM)

OPENING CASE

In March 2010, GameStop, Cintas, and United Natural Foods, Inc., joined a host of other companies in announcing operating results. As you might expect, news such as this tends to move stock prices.

GameStop, the leading video game retailer, announced fourth-quarter earnings of \$1.29 per share, a decline compared to the \$1.34 earnings per share from the fourth quarter the previous year. Even so, the stock price rose about 6.5 percent after the announcement. Uniform supplier Cintas announced net income that was 30 percent lower than the same quarter the previous year, but did investors run away? Not exactly: The stock rose by about 1.2 percent when the news was announced. United Natural Foods announced that its sales had risen 6 percent over the previous year and net income had risen about 15 percent. Did investors cheer? Not hardly; the stock fell almost 8 percent.

GameStop and Cintas's announcements seem like bad news, yet their stock prices rose, while the news from UNFI seems good, but its stock price fell. So when is good news really good news? The answer is fundamental to understanding risk and return, and—the good news is—this chapter explores it in some detail.

11.1 INDIVIDUAL SECURITIES

In the first part of Chapter 11, we will examine the characteristics of individual securities. In particular, we will discuss:

1. *Expected Return.* This is the return that an individual expects a stock to earn over the next period. Of course, because this is only an expectation, the actual return may be either higher or lower. An individual's expectation may simply be the average return per period a security has earned in the past. Alternatively, it may be based on a detailed analysis of a firm's prospects, on some computer-based model, or on special (or inside) information.
2. *Variance and Standard Deviation.* There are many ways to assess the volatility of a security's return. One of the most common is variance, which is a measure

of the squared deviations of a security's return from its expected return. Standard deviation is the square root of the variance.

3. *Covariance and Correlation.* Returns on individual securities are related to one another. Covariance is a statistic measuring the interrelationship between two securities. Alternatively, this relationship can be restated in terms of the correlation between the two securities. Covariance and correlation are building blocks to an understanding of the beta coefficient.

11.2 EXPECTED RETURN, VARIANCE, AND COVARIANCE

Expected Return and Variance

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Suppose financial analysts believe that there are four unequally likely states of the economy next year: depression, recession, normal, and boom times. The returns on the Supertech Company, R_A , are expected to follow the economy closely, while the returns on the Slowpoke Company, R_B , are not. The return predictions are as follows:

STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	SUPERTECH RETURNS R_A	SLOWPOKE RETURNS R_B
Depression	.10	-30%	0%
Recession	.20	-10	5
Normal	.50	20	20
Boom	.20	50	-5

Variance can be calculated in four steps. An additional step is needed to calculate standard deviation. (The calculations are presented in Table 11.1.) The steps are:

1. Calculate the expected returns, $E(R_A)$ and $E(R_B)$, by multiplying each possible return by the probability that it occurs and then add them up:

Supertech

$$.10(-.30) + .20(-.10) + .50(.20) + .20(.50) = .15 = 15\% = E(R_A)$$

Slowpoke

$$.10(.00) + .20(.05) + .50(.20) + .20(-.05) = .10 = 10\% = E(R_B)$$

2. As shown in the fourth column of Table 11.1, we next calculate the deviation of each possible return from the expected returns for the two companies.
3. Next, take the deviations from the fourth column and square them as we have done in the fifth column.
4. Finally, multiply each squared deviation by its associated probability and add the products up. As shown in Table 11.1, we get a variance of .0585 for Supertech and .0110 for Slowpoke.
5. As always, to get the standard deviations, we just take the square roots of the variances:

Supertech

$$\sqrt{.0585} = .242 = 24.2\% = SD(R_A) = \sigma_A$$

Slowpoke

$$\sqrt{.0110} = .105 = 10.5\% = SD(R_B) = \sigma_B$$

TABLE 11.1

Calculating Variance and Standard Deviation

(1) STATE OF ECONOMY	(2) PROBABILITY OF STATE OF ECONOMY	(3) RATE OF RETURN	(4) DEVIATION FROM EXPECTED RETURN	(5) SQUARED VALUE OF DEVIATION	(6) PRODUCT (2) × (5)
SUPERTECH (EXPECTED RETURN = .15)					
		R_A	$R_A - E(R_A)$	$(R_A - E(R_A))^2$	
Depression	.10	-.30	-.45	.2025	.02025
Recession	.20	-.10	-.25	.0625	.01250
Normal	.50	.20	.05	.0025	.00125
Boom	.20	.50	.35	.1225	.02450
				$\text{Var}(R_A) = \sigma_A^2 =$	<u>.05850</u>
SLOWPOKE (EXPECTED RETURN = .10)					
		R_B	$R_B - E(R_B)$	$(R_B - E(R_B))^2$	
Depression	.10	.00	-.10	.0100	.00100
Recession	.20	.05	-.05	.0025	.00050
Normal	.50	.20	.10	.0100	.00500
Boom	.20	-.05	-.15	.0225	.00450
				$\text{Var}(R_B) = \sigma_B^2 =$	<u>.01100</u>

Covariance and Correlation

Variance and standard deviation measure the variability of individual stocks. We now wish to measure the relationship between the return on one stock and the return on another. Enter **covariance** and **correlation**.

Covariance and correlation measure how two random variables are related. We explain these terms by extending our Supertech and Slowpoke example presented earlier.

EXAMPLE 11.1

Calculating Covariance and Correlation

We have already determined the expected returns and standard deviations for both Supertech and Slowpoke. (The expected returns are .15 and .10 for Supertech and Slowpoke, respectively. The standard deviations are .242 and .105, respectively.) In addition, we calculated the deviation of each possible return from the expected return for each firm. Using these data, covariance can be calculated in two steps. An extra step is needed to calculate correlation.

1. For each state of the economy, multiply Supertech's deviation from its expected return and Slowpoke's deviation from its expected return together. For example, Supertech's rate of return in a depression is $-.30$, which is $-.45 (= -.30 - .15)$ from its expected return. Slowpoke's rate of return in a depression is $.00$, which is $-.10 (= .00 - .10)$ from its expected return. Multiplying the two deviations together yields $.0450 [= (-.45) \times (-.10)]$. The actual calculations are given in the last column of Table 11.2. This procedure can be written algebraically as:

$$(R_A - E(R_A)) \times (R_B - E(R_B)) \tag{11.1}$$

where R_A and R_B are the returns on Supertech and Slowpoke. $E(R_A)$ and $E(R_B)$ are the expected returns on the two securities.

(continued)

TABLE 11.2

Calculating Covariance and Correlation

(1) STATE OF ECONOMY	(2) PROBABILITY OF STATE OF ECONOMY	(3) DEVIATIONS FROM EXPECTED RETURNS		(5) PRODUCT OF DEVIATIONS	(6) PRODUCT (2) × (5)
		SUPERTECH	SLOWPOKE		
Depression	.10	−.45	−.10	.0450	.00450
Recession	.20	−.25	−.05	.0125	.00250
Normal	.50	.05	.10	.0050	.00250
Boom	.20	.35	−.15	−.0525	−.01050
$Cov(R_A, R_B) = \sigma_{A,B} =$					−.001

- Once we have the products of the deviations, we multiply each one by its associated probability and sum to get the covariance.

Note that we represent the covariance between Supertech and Slowpoke as either $Cov(R_A, R_B)$ or $\sigma_{A,B}$. Equation (11.1) illustrates the intuition of covariance. Suppose Supertech's return is generally above its average when Slowpoke's return is above its average, and Supertech's return is generally below its average when Slowpoke's return is below its average. This is indicative of a positive dependency or a positive relationship between the two returns. Note that the term in equation (11.1) will be *positive* in any state where both returns are *above* their averages. In addition, (11.1) will still be *positive* in any state where both terms are *below* their averages. Thus, a positive relationship between the two returns will give rise to a positive value for covariance.

Conversely, suppose Supertech's return is generally above its average when Slowpoke's return is below its average, and Supertech's return is generally below its average when Slowpoke's return is above its average. This is indicative of a negative dependency or a negative relationship between the two returns. Note that the term in equation (11.1) will be *negative* in any state where one return is above its average and the other return is below its average. Thus, a negative relationship between the two returns will give rise to a negative value for covariance.

Finally, suppose there is no relation between the two returns. In this case, knowing whether the return on Supertech is above or below its expected return tells us nothing about the return on Slowpoke. In the covariance formula, then, there will be no tendency for the deviations to be positive or negative together. On average, they will tend to offset each other and cancel out, making the covariance zero.

Of course, even if the two returns are unrelated to each other, the covariance formula will not equal zero exactly in any actual history. This is due to sampling error; randomness alone will make the calculation positive or negative. But for a historical sample that is long enough, if the two returns are not related to each other, we should expect the covariance to come close to zero.

Our covariance calculation seems to capture what we are looking for. If the two returns are positively related to each other, they will have a positive covariance, and if they are negatively related to each other, the covariance will be negative. Last, and very important, if they are unrelated, the covariance should be zero.

The covariance we calculated is $-.001$. A negative number like this implies that the return on one stock is likely to be above its average when the return on the other stock is below its average, and vice versa. However, the size of the number is difficult to interpret. Like the variance figure, the covariance is in squared deviation units. Until we can put it in perspective, we don't know what to make of it.

We solve the problem by computing the correlation:

- To calculate the correlation, divide the covariance by the product of the standard deviations of the two securities. For our example, we have:

$$\rho_{A,B} = \text{Corr}(R_A, R_B) = \frac{Cov(R_A, R_B)}{\sigma_A \times \sigma_B} = \frac{-.001}{.242 \times .105} = -.039 \quad [11.2]$$

(continued)

where σ_A and σ_B are the standard deviations of Supertech and Slowpoke, respectively. Note that we represent the correlation between Supertech and Slowpoke either as $\text{Corr}(R_A, R_B)$ or $\rho_{A,B}$. Note also that the ordering of the two variables is unimportant. That is, the correlation of A with B is equal to the correlation of B with A . More formally, $\text{Corr}(R_A, R_B) = \text{Corr}(R_B, R_A)$ or $\rho_{A,B} = \rho_{B,A}$. The same is true for covariance.

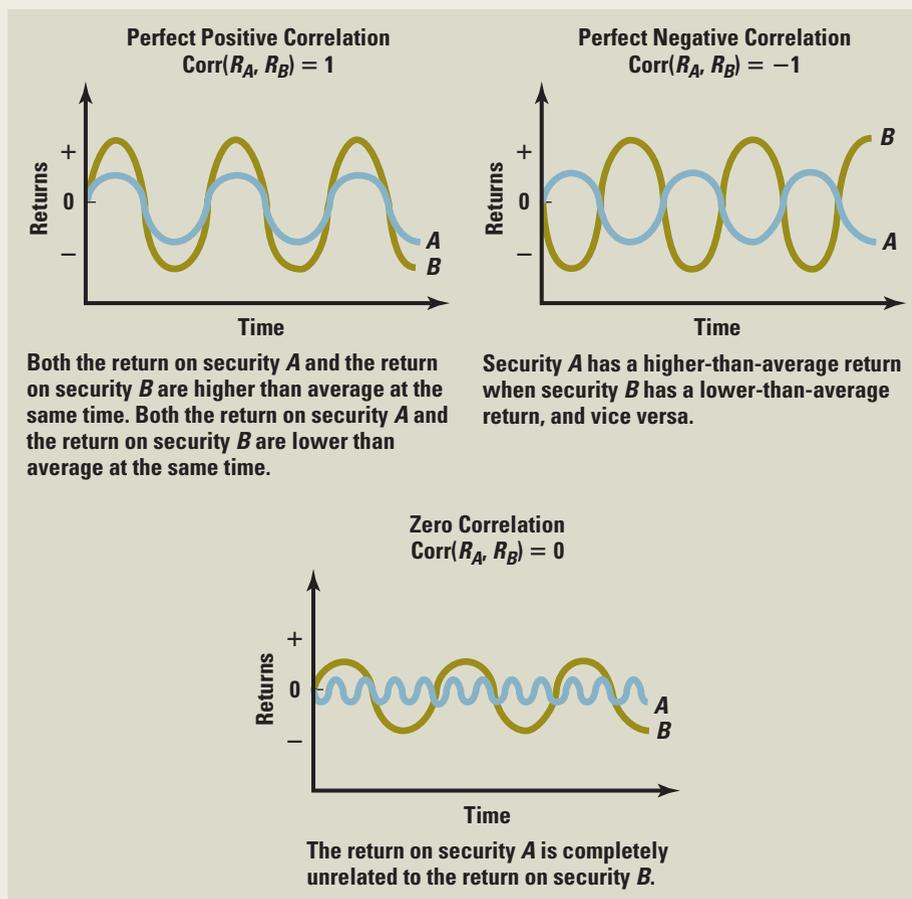
Because the standard deviation is always positive, the sign of the correlation between two variables must be the same as that of the covariance between the two variables. If the correlation is positive, we say that the variables are *positively correlated*; if it is negative, we say that they are *negatively correlated*; and if it is zero, we say that they are *uncorrelated*. Furthermore, it can be proved that the correlation is always between $+1$ and -1 . This is due to the standardizing procedure of dividing by the two standard deviations.

We can compare the correlation between different *pairs* of securities. For example, it turns out that the correlation between General Motors and Ford is much higher than the correlation between General Motors and IBM. Hence, we can state that the first pair of securities is more interrelated than the second pair.

Figure 11.1 shows the three benchmark cases for two assets, A and B . The figure shows two assets with return correlations of $+1$, -1 , and 0 . This implies perfect positive correlation, perfect negative correlation, and no correlation, respectively. The graphs in the figure plot the separate returns on the two securities through time.

FIGURE 11.1

Examples of Different Correlation Coefficients—the Graphs in the Figure Plot the Separate Returns on the Two Securities through Time



11.3 THE RETURN AND RISK FOR PORTFOLIOS

Suppose that an investor has estimates of the expected returns and standard deviations on individual securities and the correlations between securities. How then does the investor choose the best combination, or **portfolio**, of securities to hold? Obviously, the investor would like a portfolio with a high expected return and a low standard deviation of return. It is therefore worthwhile to consider:

1. The relationship between the expected return on individual securities and the expected return on a portfolio made up of these securities.
2. The relationship between the standard deviations of individual securities, the correlations between these securities, and the standard deviation of a portfolio made up of these securities.

In order to analyze the above two relationships, we will continue with our example of Supertech and Slowpoke. The relevant calculations are as follows.

The Expected Return on a Portfolio

The formula for expected return on a portfolio is very simple:

The expected return on a portfolio is simply a weighted average of the expected returns on the individual securities.

RELEVANT DATA FROM EXAMPLE OF SUPERTECH AND SLOWPOKE

ITEM	SYMBOL	VALUE
Expected return on Supertech	$E(R_{\text{Super}})$.15 = 15%
Expected return on Slowpoke	$E(R_{\text{Slow}})$.10 = 10%
Variance of Supertech	σ_{Super}^2	.0585
Variance of Slowpoke	σ_{Slow}^2	.0110
Standard deviation of Supertech	σ_{Super}	.242 = 24.2%
Standard deviation of Slowpoke	σ_{Slow}	.105 = 10.5%
Covariance between Supertech and Slowpoke	$\sigma_{\text{Super, Slow}}$	-.001
Correlation between Supertech and Slowpoke	$\rho_{\text{Super, Slow}}$	-.039

EXAMPLE 11.2

Portfolio Expected Returns

Consider Supertech and Slowpoke. From the preceding box, we find that the expected returns on these two securities are 15 percent and 10 percent, respectively.

The expected return on a portfolio of these two securities alone can be written as:

$$\text{Expected return on portfolio} = X_{\text{Super}} (15\%) + X_{\text{Slow}} (10\%) = R_p$$

where X_{Super} is the percentage of the portfolio in Supertech and X_{Slow} is the percentage of the portfolio in Slowpoke. If the investor with \$100 invests \$60 in Supertech and \$40 in Slowpoke, the expected return on the portfolio can be written as:

$$\text{Expected return on portfolio} = .6 \times 15\% + .4 \times 10\% = 13\%$$

Algebraically, we can write:

$$\text{Expected return on portfolio} = X_A E(R_A) + X_B E(R_B) = E(R_p) \quad [11.3]$$

where X_A and X_B are the proportions of the total portfolio in the assets A and B , respectively. (Because our investor can only invest in two securities, $X_A + X_B$ must equal 1 or 100 percent.) $E(R_A)$ and $E(R_B)$ are the expected returns on the two securities.

Now consider two stocks, each with an expected return of 10 percent. The expected return on a portfolio composed of these two stocks must be 10 percent, regardless of the proportions of the two stocks held. This result may seem obvious at this point, but it will become important later. The result implies that you do not reduce or *dissipate* your expected return by investing in a number of securities. Rather, the expected return on your portfolio is simply a weighted average of the expected returns on the individual assets in the portfolio.

Variance and Standard Deviation of a Portfolio

THE VARIANCE The formula for the variance of a portfolio composed of two securities, A and B , is:

The Variance of the Portfolio

$$\text{Var (portfolio)} = X_A^2 \sigma_A^2 + 2X_A X_B \sigma_{A,B} + X_B^2 \sigma_B^2 \quad [11.4]$$

Note that there are three terms on the right-hand side of the equation (in addition to X_A and X_B , the investment proportions). The first term involves the variance of A (σ_A^2), the second term involves the covariance between the two securities ($\sigma_{A,B}$), and the third term involves the variance of B (σ_B^2). (As stated earlier in this chapter, $\sigma_{A,B} = \sigma_{B,A}$. That is, the ordering of the variables is not relevant when expressing the covariance between two securities.)

The formula indicates an important point. The variance of a portfolio depends on both the variances of the individual securities and the covariance between the two securities. The variance of a security measures the variability of an individual security's return. Covariance measures the relationship between the two securities. For given variances of the individual securities, a positive relationship or covariance between the two securities increases the variance of the entire portfolio. A negative relationship or covariance between the two securities decreases the variance of the entire portfolio. This important result seems to square with common sense. If one of your securities tends to go up when the other goes down, or vice versa, your two securities are offsetting each other. You are achieving what we call a *hedge* in finance, and the risk of your entire portfolio will be low. However, if both your securities rise and fall together, you are not hedging at all. Hence, the risk of your entire portfolio will be higher.

The variance formula for our two securities, Super and Slow, is:

$$\text{Var (portfolio)} = X_{\text{Super}}^2 \sigma_{\text{Super}}^2 + 2X_{\text{Super}} X_{\text{Slow}} \sigma_{\text{Super, Slow}} + X_{\text{Slow}}^2 \sigma_{\text{Slow}}^2$$

Given our earlier assumption that an individual with \$100 invests \$60 in Supertech and \$40 in Slowpoke, $X_{\text{Super}} = .6$ and $X_{\text{Slow}} = .4$. Using this assumption and the relevant data from the previous box, the variance of the portfolio is:

$$.0223 = .36 \times .0585 + 2 \times [.6 \times .4 \times (-.001)] + .16 \times .0110$$

STANDARD DEVIATION OF A PORTFOLIO We can now determine the standard deviation of the portfolio's return. This is:

$$\begin{aligned} \sigma_p = \text{SD}(\text{portfolio}) &= \sqrt{\text{Var (portfolio)}} = \sqrt{.0223} \\ &= .1493 = 14.93\% \end{aligned} \quad [11.5]$$

The interpretation of the standard deviation of the portfolio is the same as the interpretation of the standard deviation of an individual security. The expected return on our portfolio is 13 percent. A return of -1.93 percent ($13\% - 14.93\%$) is one standard deviation below the mean and a return of 27.93 percent ($13\% + 14.93\%$) is one standard deviation above the mean. If the return on the portfolio is normally distributed, a return between -1.93 percent and $+27.93$ percent occurs about 68 percent of the time.¹

¹There are only four possible returns for Supertech and Slowpoke, so neither security possesses a normal distribution. Thus, probabilities would be somewhat different in our example.

THE DIVERSIFICATION EFFECT It is instructive to compare the standard deviation of the portfolio with the standard deviation of the individual securities. The weighted average of the standard deviations of the individual securities is:

$$\begin{aligned} \text{Weighted average of standard deviations} &= X_{\text{Super}}\sigma_{\text{Super}} + X_{\text{Slow}}\sigma_{\text{Slow}} \\ .187 &= .6 \times .242 + .4 \times .105 \end{aligned} \quad [11.6]$$

One of the most important results in this chapter concerns the difference between equations 11.5 and 11.6. In our example, the standard deviation of the portfolio is *less* than a weighted average of the standard deviations of the individual securities.

We pointed out earlier that the expected return on the portfolio is a weighted average of the expected returns on the individual securities. Thus, we get a different type of result for the standard deviation of a portfolio than we do for the expected return on a portfolio.

It is generally argued that our result for the standard deviation of a portfolio is due to diversification. For example, Supertech and Slowpoke are slightly negatively correlated ($\rho = -.039$). Supertech's return is likely to be a little below average if Slowpoke's return is above average. Similarly, Supertech's return is likely to be a little above average if Slowpoke's return is below average. Thus, the standard deviation of a portfolio composed of the two securities is less than a weighted average of the standard deviations of the two securities.

The above example has negative correlation. Clearly, there will be less benefit from diversification if the two securities exhibit positive correlation. How high must the positive correlation be before all diversification benefits vanish?

To answer this question, let us rewrite Equation 11.4 in terms of correlation rather than covariance. First, note that the covariance can be rewritten as:

$$\sigma_{\text{Super, Slow}} = \rho_{\text{Super, Slow}}\sigma_{\text{Super}}\sigma_{\text{Slow}} \quad [11.7]$$

The formula states that the covariance between any two securities is simply the correlation between the two securities multiplied by the standard deviations of each. In other words, covariance incorporates both (1) the correlation between the two assets and (2) the variability of each of the two securities as measured by standard deviation.

From our calculations earlier in this chapter, we know that the correlation between the two securities is $-.039$. Thus, the variance of our portfolio can be expressed as:

$$\begin{aligned} \text{Variance of the Portfolio's Return} \\ &= X_{\text{Super}}^2\sigma_{\text{Super}}^2 + 2X_{\text{Super}}X_{\text{Slow}}\rho_{\text{Super, Slow}}\sigma_{\text{Super}}\sigma_{\text{Slow}} + X_{\text{Slow}}^2\sigma_{\text{Slow}}^2 \\ .0223 &= .36 \times .0585 + 2 \times .6 \times .4 \times (-.039) \times .242 \times .105 + .16 \times .0110 \end{aligned} \quad [11.8]$$

The middle term on the right-hand side is now written in terms of correlation, ρ , not covariance.

Suppose $\rho_{\text{Super, Slow}} = 1$, the highest possible value for correlation. Assume all the other parameters in the example are the same. The variance of the portfolio is:

$$\begin{aligned} \text{Variance of the} &= .035 = .36 \times .0585 + 2 \times (.6 \times .4 \times 1 \times .242 \\ \text{portfolio's return} & \quad \times .105) + .16 \times .0110 \end{aligned}$$

The standard deviation is:

$$\text{Standard deviation of portfolio's return} = \sqrt{.035} = .187 = 18.7\% \quad [11.9]$$

Note that equations 11.9 and 11.6 are equal. That is, the standard deviation of a portfolio's return is equal to the weighted average of the standard deviations of the individual returns when $\rho = 1$. Inspection of Equation 11.8 indicates that the variance and hence the standard deviation of the portfolio must fall as the correlation drops below 1. This leads to:

As long as $\rho < 1$, the standard deviation of a portfolio of two securities is *less* than the weighted average of the standard deviations of the individual securities.

ASSET	STANDARD DEVIATION
S&P 500 Index	20.59%
General Electric	24.64
McDonald's	28.97
IBM	29.55
Microsoft	36.36
Harley-Davidson	36.97
Dell	38.17
Sprint Nextel	52.36
Amazon.com	92.35
Ford	115.29

TABLE 11.3

Standard Deviations for Standard & Poor's 500 Index and for Selected Stocks in the Index, 2000–2009

As long as the correlations between pairs of securities are less than 1, the standard deviation of an index is less than the weighted average of the standard deviations of the individual securities within the index.

In other words, the diversification effect applies as long as there is less than perfect correlation (as long as $\rho < 1$). Thus, our Supertech-Slowpoke example is a case of overkill. We illustrated diversification by an example with negative correlation. We could have illustrated diversification by an example with positive correlation—as long as it was not *perfect* positive correlation.

AN EXTENSION TO MANY ASSETS The preceding insight can be extended to the case of many assets. That is, as long as correlations between pairs of securities are less than 1, the standard deviation of a portfolio of many assets is less than the weighted average of the standard deviations of the individual securities.

Now consider Table 11.3, which shows the standard deviation (based on annual returns) of the Standard & Poor's 500 Index and the standard deviations of some of the individual securities listed in the index over a recent 10-year period. Note that all of the individual securities in the table have higher standard deviations than that of the index. In general, the standard deviations of most of the individual securities in an index will be above the standard deviation of the index itself, though a few of the securities could have lower standard deviations than that of the index.

11.4 THE EFFICIENT SET

The Two-Asset Case

Our results on expected returns and standard deviations are graphed in Figure 11.2. In the figure, there is a dot labeled Slowpoke and a dot labeled Supertech. Each dot represents both the expected return and the standard deviation for an individual security. As can be seen, Supertech has both a higher expected return and a higher standard deviation.

The box or “□” in the graph represents a portfolio with 60 percent invested in Supertech and 40 percent invested in Slowpoke. You will recall that we have previously calculated both the expected return and the standard deviation for this portfolio.

The choice of 60 percent in Supertech and 40 percent in Slowpoke is just one of an infinite number of portfolios that can be created. The set of portfolios is sketched by the curved line in Figure 11.3.

Consider portfolio 1. This is a portfolio composed of 90 percent Slowpoke and 10 percent Supertech. Because it is weighted so heavily toward Slowpoke, it appears close to the Slowpoke point on the graph. Portfolio 2 is higher on the curve because it is composed of

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FIGURE 11.2

Expected Returns and Standard Deviations for Supertech, Slowpoke, and a Portfolio Composed of 60 Percent in Supertech and 40 Percent in Slowpoke

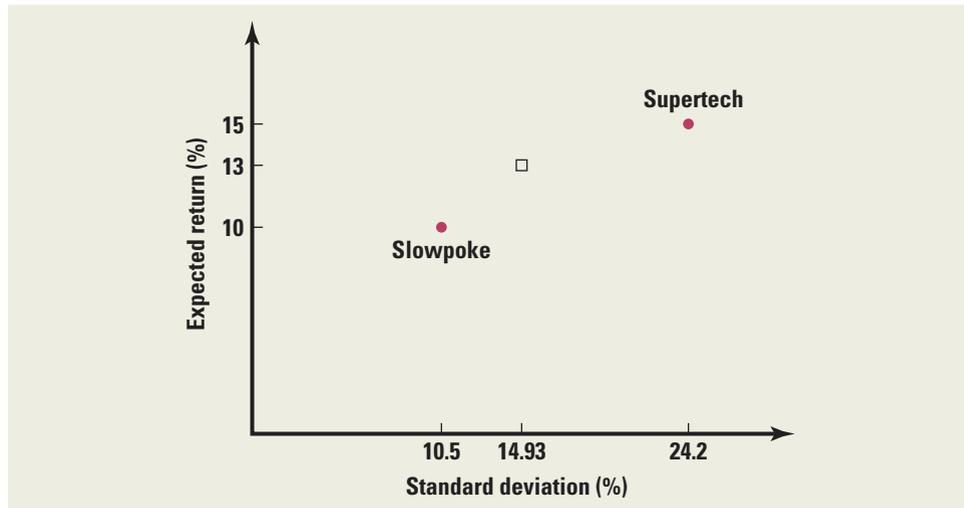
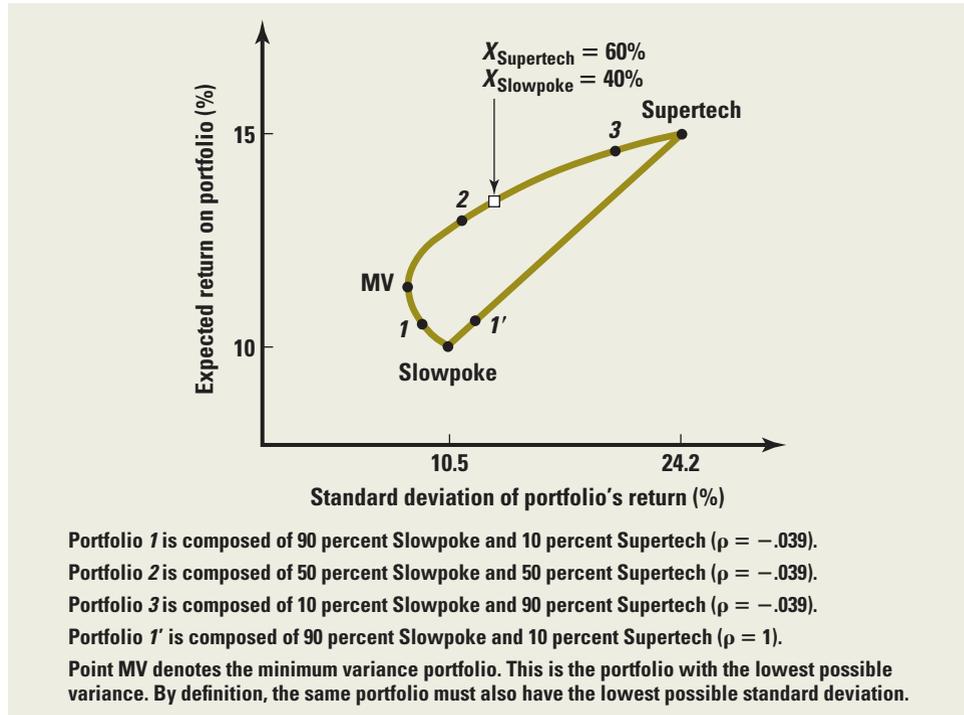


FIGURE 11.3

Set of Portfolios Composed of Holdings in Supertech and Slowpoke (correlation between the two securities is $-.039$)



50 percent Slowpoke and 50 percent Supertech. Portfolio 3 is close to the Supertech point on the graph because it is composed of 90 percent Supertech and 10 percent Slowpoke.

There are a few important points concerning this graph.

1. We argued that the diversification effect occurs whenever the correlation between the two securities is below 1. The correlation between Supertech and Slowpoke is $-.039$. The diversification effect can be illustrated by comparison with the straight line between the Supertech point and the Slowpoke point. The straight line represents points that would have been generated had the correlation coefficient between the two securities been 1. The diversification effect is

illustrated in the figure since the curved line is always to the left of the straight line. Consider point I' . This represents a portfolio composed of 90 percent Slowpoke and 10 percent Supertech if the correlation between the two were exactly 1. We argue that there is no diversification effect if $\rho = 1$. However, the diversification effect applies to the curved line, because point I has the same expected return as point I' but has a lower standard deviation. (Points $2'$ and $3'$ are omitted to reduce the clutter of Figure 11.3.)

Though the straight line and the curved line are both represented in Figure 11.3, they do not simultaneously exist in the same world. *Either* $\rho = -.039$ and the curve exists *or* $\rho = 1$ and the straight line exists. In other words, though an investor can choose between different points on the curve if $\rho = -.039$, she cannot choose between points on the curve and points on the straight line.

2. The point MV represents the minimum variance portfolio. This is the portfolio with the lowest possible variance. By definition, this portfolio must also have the lowest possible standard deviation. (The term *minimum variance portfolio* is standard in the literature, and we will use that term. Perhaps minimum standard deviation would actually be better, because standard deviation, not variance, is measured on the horizontal axis of Figure 11.3.)
3. An individual contemplating an investment in a portfolio of Slowpoke and Supertech faces an **opportunity set** or **feasible set** represented by the curved line in Figure 11.3. That is, he can achieve any point on the curve by selecting the appropriate mix between the two securities. He cannot achieve any point above the curve because he cannot increase the return on the individual securities, decrease the standard deviations of the securities, or decrease the correlation between the two securities. Neither can he achieve points below the curve because he cannot lower the returns on the individual securities, increase the standard deviations of the securities, or increase the correlation. (Of course, he would not want to achieve points below the curve, even if he were able to do so.)
Were he relatively tolerant of risk, he might choose portfolio 3. (In fact, he could even choose the end point by investing all his money in Supertech.) An investor with less tolerance for risk might choose portfolio 2. An investor wanting as little risk as possible would choose MV, the portfolio with minimum variance or minimum standard deviation.
4. Note that the curve is backward bending between the Slowpoke point and MV. This indicates that, for a portion of the feasible set, standard deviation actually decreases as one increases expected return. Students frequently ask, “How can an increase in the proportion of the risky security, Supertech, lead to a reduction in the risk of the portfolio?”

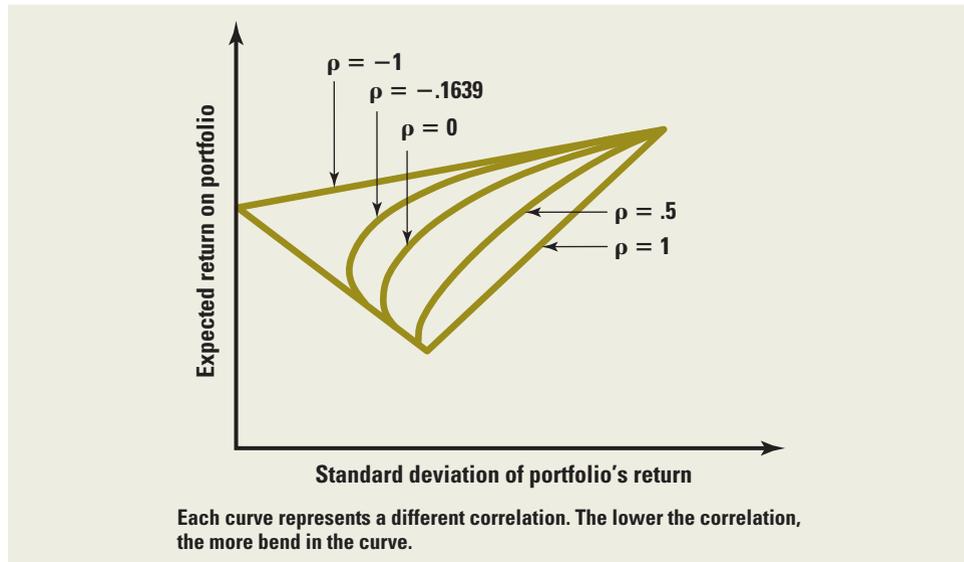
This surprising finding is due to the diversification effect. The returns on the two securities are negatively correlated with each other. One security tends to go up when the other goes down and vice versa. Thus, an addition of a small amount of Supertech acts as a hedge to a portfolio composed only of Slowpoke. The risk of the portfolio is reduced, implying backward bending. Actually, backward bending always occurs if $\rho \leq 0$. It may or may not occur when $\rho > 0$. Of course, the curve bends backward only for a portion of its length. As one continues to increase the percentage of Supertech in the portfolio, the high standard deviation of this security eventually causes the standard deviation of the entire portfolio to rise.

5. No investor would want to hold a portfolio with an expected return below that of the minimum variance portfolio. For example, no investor would choose

To find the mean variance optimal portfolio for any two stocks, go to www.wolframalpha.com and enter the ticker symbols.

FIGURE 11.4

Opportunity Sets
Composed of Holdings in
Supertech and Slowpoke



portfolio 1. This portfolio has less expected return but more standard deviation than the minimum variance portfolio has. We say that portfolios such as portfolio 1 are *dominated* by the minimum variance portfolio. Though the entire curve from Slowpoke to Supertech is called the *feasible set*, investors only consider the curve from MV to Supertech. Hence, the curve from MV to Supertech is called the **efficient set** or the **efficient frontier**.

Figure 11.3 represents the opportunity set where $\rho = -.039$. It is worthwhile to examine Figure 11.4, which shows different curves for different correlations. As can be seen, the lower the correlation, the more bend there is in the curve. This indicates that the diversification effect rises as ρ declines. The greatest bend occurs in the limiting case where $\rho = -1$. This is perfect negative correlation. While this extreme case where $\rho = -1$ seems to fascinate students, it has little practical importance. Most pairs of securities exhibit positive correlation. Strong negative correlations, let alone perfect negative correlations, are uncommon occurrences for ordinary securities such as stocks and bonds.

Note that there is only one correlation between a pair of securities. We stated earlier that the correlation between Slowpoke and Supertech is $-.039$. Thus, the curve in Figure 11.3 representing this correlation is the correct one, and the other curves in Figure 11.4 should be viewed as merely hypothetical.

The graphs we examined are not mere intellectual curiosities. Rather, efficient sets can easily be calculated in the real world. As mentioned earlier, data on returns, standard deviations, and correlations are generally taken from past observations, though subjective notions can be used to determine the values of these parameters as well. Once the parameters have been determined, any one of a whole host of software packages can be purchased to generate an efficient set. However, the choice of the preferred portfolio within the efficient set is up to you. As with other important decisions like what job to choose, what house or car to buy, and how much time to allocate to this course, there is no computer program to choose the preferred portfolio.

An efficient set can be generated where the two individual assets are portfolios themselves. For example, the two assets in Figure 11.5 are a diversified portfolio of American stocks and a diversified portfolio of foreign stocks. Expected returns, standard deviations, and the correlation coefficient were calculated over the recent past. No subjectivity entered the analysis. The U.S. stock portfolio with a standard deviation of about .17 is less risky

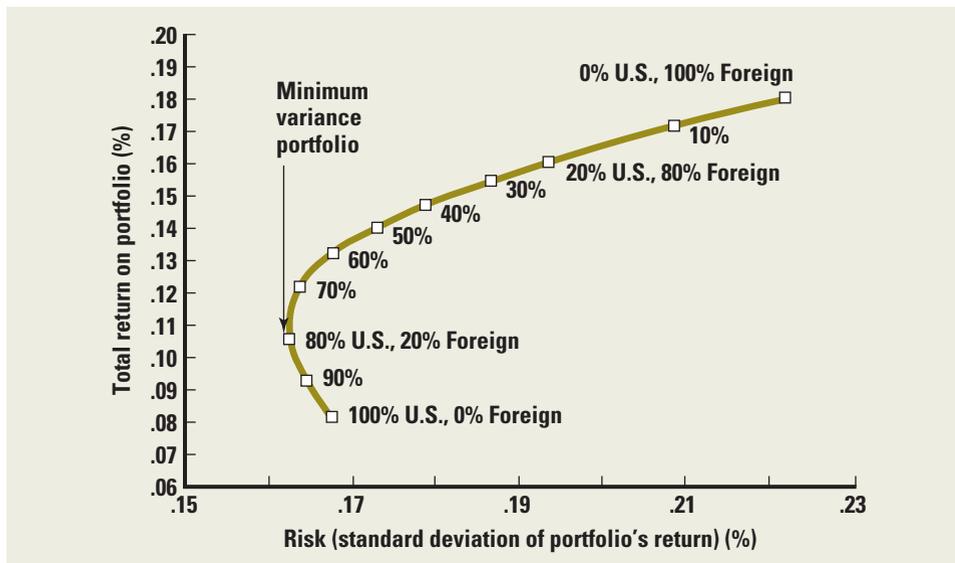


FIGURE 11.5
Return/Risk Trade-off for
World Stocks: Portfolio of
U.S. and Foreign Stocks

than the foreign stock portfolio, which has a standard deviation of about .22. However, combining a small percentage of the foreign stock portfolio with the U.S. portfolio actually reduces risk, as can be seen by the backward-bending nature of the curve. In other words, the diversification benefits from combining two different portfolios more than offset the introduction of a riskier set of stocks into one's holdings. The minimum variance portfolio occurs with about 80 percent of one's funds in American stocks and about 20 percent in foreign stocks. Addition of foreign securities beyond this point increases the risk of one's entire portfolio.

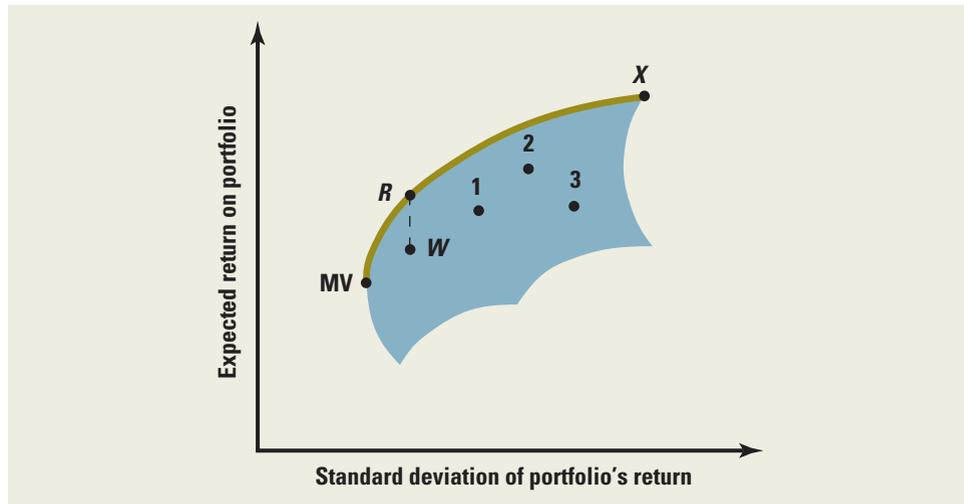
The backward-bending curve in Figure 11.5 is important information that has not bypassed American money managers. In recent years, pension fund and mutual fund managers in the United States have sought out investment opportunities overseas. Another point worth pondering concerns the potential pitfalls of using only past data to estimate future returns. The stock markets of many foreign countries have had phenomenal growth in the past 25 years. Thus, a graph like Figure 11.5 makes a large investment in these foreign markets seem attractive. However, because abnormally high returns cannot be sustained forever, some subjectivity must be used when forecasting future expected returns.

The Efficient Set for Many Securities

The previous discussion concerned two securities. We found that a simple curve sketched out all the possible portfolios. Because investors generally hold more than two securities, we should look at the same graph when more than two securities are held. The shaded area in Figure 11.6 represents the opportunity set or feasible set when many securities are considered. The shaded area represents all the possible combinations of expected return and standard deviation for a portfolio. For example, in a universe of 100 securities, point 1 might represent a portfolio of, say 40 securities. Point 2 might represent a portfolio of 80 securities. Point 3 might represent a different set of 80 securities, or the same 80 securities held in different proportions, or something else. Obviously, the combinations are virtually endless. However, note that all possible combinations fit into a confined region. No security or combination of securities can fall outside of the shaded region. That is, no one can choose a portfolio with an expected return above that given by the shaded region. Furthermore, no one can choose a portfolio with a standard deviation below that given in the shaded area. Perhaps more surprisingly, no one can choose an expected return

FIGURE 11.6

The Feasible Set of Portfolios Constructed from Many Securities



below that given in the curve. In other words, the capital markets actually prevent a self-destructive person from taking on a guaranteed loss.²

So far, Figure 11.6 is different from the earlier graphs. When only two securities are involved, all the combinations lie on a single curve. Conversely, with many securities the combinations cover an entire area. However, notice that an individual will want to be somewhere on the upper edge between MV and X. The upper edge, which we indicate in Figure 11.6 by a thick curve, is called the *efficient set*. Any point below the efficient set would receive less expected return and the same standard deviation as a point on the efficient set. For example, consider R on the efficient set and W directly below it. If W contains the risk you desire, you should choose R instead in order to receive a higher expected return.

In the final analysis, Figure 11.6 is quite similar to Figure 11.3. The efficient set in Figure 11.3 runs from MV to Supertech. It contains various combinations of the securities Supertech and Slowpoke. The efficient set in Figure 11.6 runs from MV to X. It contains various combinations of many securities. The fact that a whole shaded area appears in Figure 11.6 but not in Figure 11.3 is just not an important difference; no investor would choose any point below the efficient set in Figure 11.6 anyway.

We mentioned before that an efficient set for two securities can be traced out easily in the real world. The task becomes more difficult when additional securities are included because the number of calculations quickly becomes huge. As a result, hand calculations are impractical for more than just a few securities. A number of software packages allow the calculation of an efficient set for portfolios of moderate size. By all accounts, these packages sell quite briskly, so that our discussion above would appear to be important in practice.

11.5 RISKLESS BORROWING AND LENDING

Figure 11.6 assumes that all the securities on the efficient set are risky. Alternatively, an investor could combine a risky investment with an investment in a riskless or *risk-free* security, such as an investment in United States Treasury bills. This is illustrated in the following example.

²Of course, someone dead set on parting with his money can do so. For example, he can trade frequently without purpose, so that commissions more than offset the positive expected returns on the portfolio.

Riskless Lending and Portfolio Risk

Ms. Bagwell is considering investing in the common stock of Merville Enterprises. In addition, Ms. Bagwell will either borrow or lend at the risk-free rate. The relevant parameters are:

	COMMON STOCK OF MERVILLE	RISK-FREE ASSET
Expected return	14%	10%
Standard deviation	.20	0

Suppose Ms. Bagwell chooses to invest a total of \$1,000, \$350 of which is to be invested in Merville Enterprises and \$650 placed in the risk-free asset. The expected return on her total investment is simply a weighted average of the two returns:

$$\begin{aligned} \text{Expected return on portfolio composed} \\ \text{of one riskless and one risky asset} &= .114 = (.35 \times .14) + (.65 \times .10) \end{aligned} \quad [11.10]$$

Because the expected return on the portfolio is a weighted average of the expected return on the risky asset (Merville Enterprises) and the risk-free return, the calculation is analogous to the way we treated two risky assets. In other words, equation (11.3) applies here.

Using equation (11.4), the formula for the variance of the portfolio can be written as:

$$X_{\text{Merville}}^2 \sigma_{\text{Merville}}^2 + 2X_{\text{Merville}} X_{\text{Risk-free}} \sigma_{\text{Merville, Risk-free}} + X_{\text{Risk-free}}^2 \sigma_{\text{Risk-free}}^2$$

However, by definition, the risk-free asset has no variability. Thus, both $\sigma_{\text{Merville, Risk-free}}$ and $\sigma_{\text{Risk-free}}^2$ are equal to zero, reducing the above expression to:

$$\begin{aligned} \text{Variance of portfolio composed} \\ \text{of one riskless and one risky asset} &= X_{\text{Merville}}^2 \sigma_{\text{Merville}}^2 \\ &= (.35)^2 \times (.20)^2 \\ &= .0049 \end{aligned} \quad [11.11]$$

The standard deviation of the portfolio is:

$$\begin{aligned} \text{Standard deviation of portfolio composed} \\ \text{of one riskless and one risky asset} &= X_{\text{Merville}} \sigma_{\text{Merville}} \\ &= .35 \times .20 \\ &= .07 \end{aligned} \quad [11.12]$$

The relationship between risk and expected return for one risky and one riskless asset can be seen in Figure 11.7. Ms. Bagwell's split of 35–65 percent between the two assets is represented on a *straight* line between the risk-free rate and a pure investment in Merville Enterprises. Note that, unlike the case of two risky assets, the opportunity set is straight, not curved.

Suppose that, alternatively, Ms. Bagwell borrows \$200 at the risk-free rate. Combining this with her original sum of \$1,000, she invests a total of \$1,200 in Merville. Her expected return would be:

$$\begin{aligned} \text{Expected return on portfolio} \\ \text{formed by borrowing} \\ \text{to invest in risky asset} &= 14.8\% = 1.20 \times .14 + (-.2 \times .10) \end{aligned}$$

Here, she invests 120 percent of her original investment of \$1,000 by borrowing 20 percent of her original investment. Note that the return of 14.8 percent is greater than the 14 percent expected return on Merville Enterprises. This occurs because she is borrowing at 10 percent to invest in a security with an expected return greater than 10 percent.

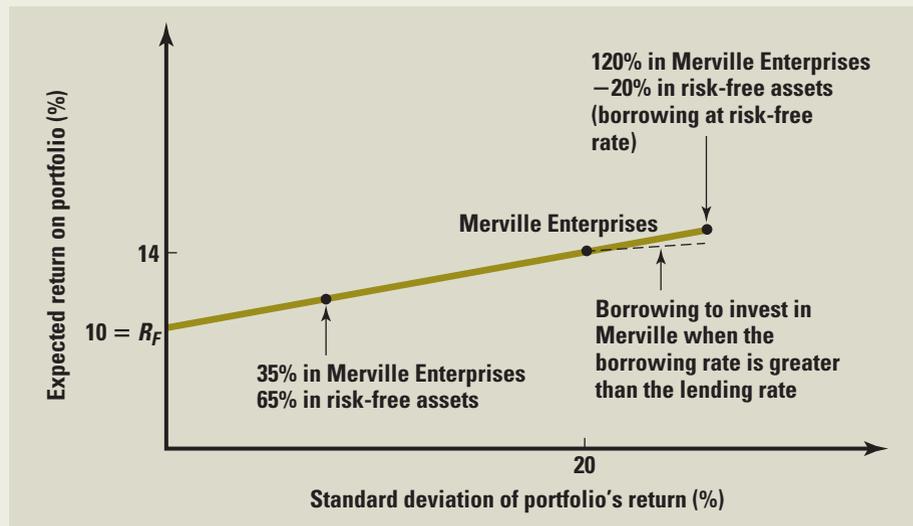
The standard deviation is:

$$\begin{aligned} \text{Standard deviation of portfolio formed} \\ \text{by borrowing to invest in risky asset} &= .24 = 1.20 \times .2 \end{aligned}$$

(continued)

FIGURE 11.7

Relationship between Expected Return and Risk for a Portfolio of One Risky Asset and One Riskless Asset



The standard deviation of .24 is greater than .20, the standard deviation of the Merville investment, because borrowing increases the variability of the investment. This investment also appears in Figure 11.7.

So far, we have assumed that Ms. Bagwell is able to borrow at the same rate at which she can lend.³ Now let us consider the case where the borrowing rate is above the lending rate. The dotted line in Figure 11.7 illustrates the opportunity set for borrowing opportunities in this case. The dotted line is below the solid line because a higher borrowing rate lowers the expected return on the investment.

The Optimal Portfolio

The previous section concerned a portfolio formed between one riskless asset and one risky asset. In reality, an investor is likely to combine an investment in the riskless asset with a *portfolio* of risky assets. This is illustrated in Figure 11.8.

Consider point Q , representing a portfolio of securities. Point Q is in the interior of the feasible set of risky securities. Let us assume the point represents a portfolio of 30 percent AT&T, 45 percent General Electric (GE), and 25 percent IBM stock. Individuals combining investments in Q with investments in the riskless asset would achieve points along the straight line from R_F to Q . We refer to this as line I . For example, point I on the line represents a portfolio of 70 percent in the riskless asset and 30 percent in stocks represented by Q . An investor with \$100 choosing point I as his portfolio would put \$70 in the risk-free asset and \$30 in Q . This can be restated as \$70 in the riskless asset, \$9 ($= .3 \times \30) in AT&T, \$13.50 ($= .45 \times \30) in GE, and \$7.50 ($= .25 \times \30) in IBM. Point 2 also represents a portfolio of the risk-free asset and Q , with more (65 percent) being invested in Q .

Point 3 is obtained by borrowing to invest in Q . For example, an investor with \$100 of his own would borrow \$40 from the bank or broker in order to invest \$140 in Q . This

³Surprisingly, this appears to be a decent approximation because a large number of investors are able to borrow from a stockbroker (called *going on margin*) when purchasing stocks. The borrowing rate here is very near the riskless rate of interest, particularly for large investors. More will be said about this in a later chapter.

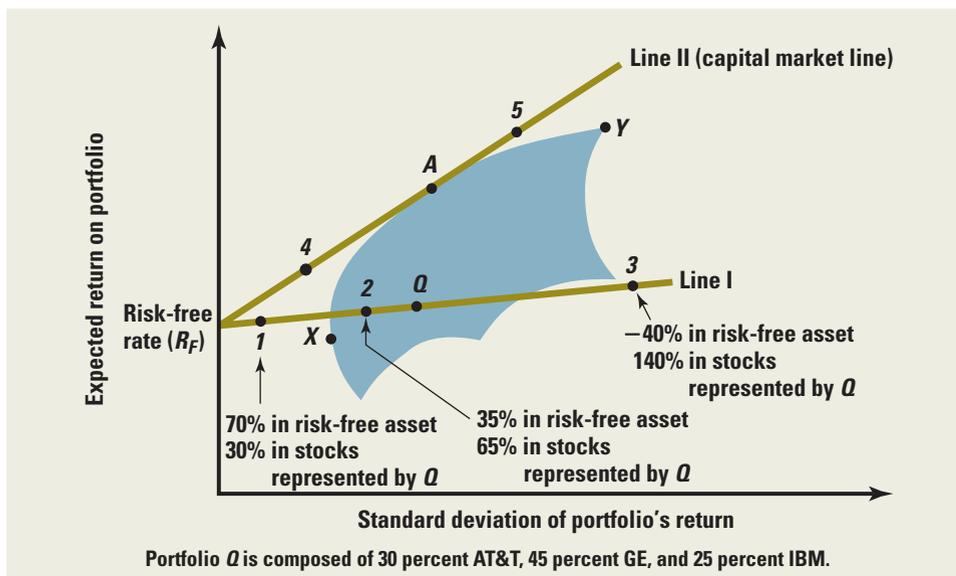


FIGURE 11.8
Relationship between Expected Return and Standard Deviation for an Investment in a Combination of Risky Securities and the Riskless Asset

can be stated as borrowing \$40 and contributing \$100 of one's own money in order to invest \$42 (= .3 × \$140) in AT&T, \$63 (= .45 × \$140) in GE, and \$35 (= .25 × \$140) in IBM.

The above investments can be summarized as:

	POINT Q	POINT 1 (LENDING \$70)	POINT 3 (BORROWING \$40)
AT&T	\$ 30	\$ 9	\$ 42
GE	45	13.50	63
IBM	25	7.50	35
Risk-free	0	70	- 40
Total investment	\$100	\$100	\$100

Though any investor can obtain any point on line *I*, no point on the line is optimal. To see this, consider line *II*, a line running from R_f through *A*. Point *A* represents a portfolio of risky securities. Line *II* represents portfolios formed by combinations of the risk-free asset and the securities in *A*. Points between R_f and *A* are portfolios in which some money is invested in the riskless asset and the rest is placed in *A*. Points past *A* are achieved by borrowing at the riskless rate to buy more of *A* than one could with one's original funds alone.

As drawn, line *II* is tangent to the efficient set of risky securities. Whatever point an individual can obtain on line *I*, he can obtain a point with the same standard deviation and a higher expected return on line *II*. In fact, because line *II* is tangent to the efficient set of risky assets, it provides the investor with the best possible opportunities. In other words, line *II* can be viewed as the efficient set of *all* assets, both risky and riskless. An investor with a fair degree of risk aversion might choose a point between R_f and *A*, perhaps point 4. An individual with less risk aversion might choose a point closer to *A* or even beyond *A*. For example, point 5 corresponds to an individual borrowing money to increase his investment in *A*.

The graph illustrates an important point. With riskless borrowing and lending, the portfolio of *risky* assets held by any investor would always be point *A*. Regardless of the

investor's tolerance for risk, he would never choose any other point on the efficient set of risky assets (represented by curve XAY) nor any point in the interior of the feasible region. Rather, he would combine the securities of A with the riskless assets if he had high aversion to risk. He would borrow the riskless asset to invest more funds in A if he had low aversion to risk.

This result establishes what financial economists call the **separation principle**. That is, the investor's investment decision consists of two separate steps:

1. After estimating (a) the expected returns and variances of individual securities, and (b) the covariances between pairs of securities, the investor calculates the efficient set of risky assets, represented by curve XAY in Figure 11.8. He then determines point A , the tangency between the risk-free rate and the efficient set of risky assets (curve XAY). Point A represents the portfolio of risky assets that the investor will hold. This point is determined solely from his estimates of returns, variances, and covariances. No personal characteristics, such as degree of risk aversion, are needed in this step.
2. The investor must now determine how he will combine point A , his portfolio of risky assets, with the riskless asset. He might invest some of his funds in the riskless asset and some in portfolio A . He would end up at a point on the line between R_f and A in this case. Alternatively, he might borrow at the risk-free rate and contribute some of his own funds as well, investing the sum in portfolio A . In this case, he would end up at a point on line II beyond A . His position in the riskless asset, that is, his choice of where on the line he wants to be, is determined by his internal characteristics, such as his ability to tolerate risk.

11.6 ANNOUNCEMENTS, SURPRISES, AND EXPECTED RETURNS

Now that we know how to construct portfolios and evaluate their returns, we begin to describe more carefully the risks and returns associated with individual securities. Thus far, we have measured volatility by looking at the difference between the actual return on an asset or portfolio, R , and the expected return, $E(R)$. We now look at why those deviations exist.

Expected and Unexpected Returns

To begin, for concreteness, we consider the return on the stock of a company called Flyers. What will determine this stock's return in, say, the coming year?

The return on any stock traded in a financial market is composed of two parts. First, the normal, or expected, return from the stock is the part of the return that shareholders in the market predict or expect. This return depends on the information shareholders have that bears on the stock, and it is based on the market's understanding today of the important factors that will influence the stock in the coming year.

The second part of the return on the stock is the uncertain, or risky, part. This is the portion that comes from unexpected information revealed within the year. A list of all possible sources of such information would be endless, but here are a few examples:

- News about Flyers research.
- Government figures released on gross domestic product (GDP).
- The results from the latest arms control talks.
- The news that Flyers's sales figures are higher than expected.
- A sudden, unexpected drop in interest rates.

Based on this discussion, one way to express the return on Flyers stock in the coming year would be:

$$\begin{aligned} \text{Total return} &= \text{Expected return} + \text{Unexpected return} \\ R &= E(R) + U \end{aligned} \quad [11.13]$$

where R stands for the actual total return in the year, $E(R)$ stands for the expected part of the return, and U stands for the unexpected part of the return. What this says is that the actual return, R , differs from the expected return, $E(R)$, because of surprises that occur during the year. In any given year, the unexpected return will be positive or negative, but, through time, the average value of U will be zero. This simply means that on average, the actual return equals the expected return.

Announcements and News

We need to be careful when we talk about the effect of news items on the return. For example, suppose Flyers's business is such that the company prospers when GDP grows at a relatively high rate and suffers when GDP is relatively stagnant. In this case, in deciding what return to expect this year from owning stock in Flyers, shareholders either implicitly or explicitly must think about what GDP is likely to be for the year.

When the government actually announces GDP figures for the year, what will happen to the value of Flyers's stock? Obviously, the answer depends on what figure is released. More to the point, however, the impact depends on how much of that figure is *new* information.

At the beginning of the year, market participants will have some idea or forecast of what the yearly GDP will be. To the extent that shareholders have predicted GDP, that prediction will already be factored into the expected part of the return on the stock, $E(R)$. On the other hand, if the announced GDP is a surprise, then the effect will be part of U , the unanticipated portion of the return. As an example, suppose shareholders in the market had forecast that the GDP increase this year would be .5 percent. If the actual announcement this year is exactly .5 percent, the same as the forecast, then the shareholders don't really learn anything, and the announcement isn't news. There will be no impact on the stock price as a result. This is like receiving confirmation of something that you suspected all along; it doesn't reveal anything new.

A common way of saying that an announcement isn't news is to say that the market has already "discounted" the announcement. The use of the word *discount* here is different from the use of the term in computing present values, but the spirit is the same. When we discount a dollar in the future, we say it is worth less to us because of the time value of money. When we discount an announcement or a news item, we say that it has less of an impact on the market because the market already knew much of it.

Going back to Flyers, suppose the government announces that the actual GDP increase during the year has been 1.5 percent. Now shareholders have learned something, namely, that the increase is one percentage point higher than they had forecast. This difference between the actual result and the forecast, one percentage point in this example, is sometimes called the *innovation* or the *surprise*.

This distinction explains why what seems to be good news can actually be bad news (and vice versa). For example to open the chapter, we compared GameStop, Cintas, and United Natural Foods. For GameStop, earnings had actually beaten analysts' estimates by a penny. Further, the company predicted future earnings growth through operational efficiencies, increasing market share, and changing to a buy-sell-trade model. In United Natural Foods's case, even though the company's earnings increased, its gross margin dropped because of lower fuel surcharge revenues and a shift in customer mix. In Cintas's case, earnings met analysts' expectations, and the news was announced on a day when investors turned positive on the future of stocks in general. (Keep this in mind as you

read the next section.) A key idea to remember about news and price changes is that news about the future is what matters.

To summarize, an announcement can be broken into two parts, the anticipated, or expected, part and the surprise, or innovation:

$$\text{Announcement} = \text{Expected part} + \text{Surprise} \quad [11.14]$$

The expected part of any announcement is the part of the information that the market uses to form the expectation, $E(R)$, of the return on the stock. The surprise is the news that influences the unanticipated return on the stock, U . Henceforth, when we speak of news, we will mean the surprise part of an announcement and not the portion that the market has expected and therefore already discounted.

11.7 RISK: SYSTEMATIC AND UNSYSTEMATIC

The unanticipated part of the return, that portion resulting from surprises, is the true risk of any investment. After all, if we always receive exactly what we expect, then the investment is perfectly predictable and, by definition, risk-free. In other words, the risk of owning an asset comes from surprises—unanticipated events.

There are important differences, though, among various sources of risk. Look back at our previous list of news stories. Some of these stories are directed specifically at Flyers, and some are more general. Which of the news items are of specific importance to Flyers?

Announcements about interest rates or GDP are clearly important for nearly all companies, whereas the news about Flyers's president, its research, or its sales is of specific interest to Flyers. We will distinguish between these two types of events, because, as we shall see, they have very different implications.

Systematic and Unsystematic Risk

The first type of surprise, the one that affects a large number of assets, we will label **systematic risk**. A systematic risk is one that influences a large number of assets, each to a greater or lesser extent. Because systematic risks have marketwide effects, they are sometimes called *market risks*.

The second type of surprise we will call **unsystematic risk**. An unsystematic risk is one that affects a single asset or a small group of assets. Because these risks are unique to individual companies or assets, they are sometimes called *unique* or *asset specific risks*. We will use these terms interchangeably.

As we have seen, uncertainties about general economic conditions, such as GDP, interest rates, or inflation, are examples of systematic risks. These conditions affect nearly all companies to some degree. An unanticipated increase, or surprise, in inflation, for example, affects wages and the costs of the supplies that companies buy; it affects the value of the assets that companies own; and it affects the prices at which companies sell their products. Forces such as these, to which all companies are susceptible, are the essence of systematic risk.

In contrast, the announcement of an oil strike by a company will primarily affect that company and, perhaps, a few others (such as primary competitors and suppliers). It is unlikely to have much of an effect on the world oil market, however, or on the affairs of companies not in the oil business, so this is an unsystematic event.

Systematic and Unsystematic Components of Return

The distinction between a systematic risk and an unsystematic risk is never really as exact as we make it out to be. Even the most narrow and peculiar bit of news about a company ripples through the economy. This is true because every enterprise, no matter how tiny, is

a part of the economy. It's like the tale of a kingdom that was lost because one horse lost a shoe. This is mostly hairsplitting, however. Some risks are clearly much more general than others. We'll see some evidence on this point in just a moment.

The distinction between the types of risk allows us to break down the surprise portion, U , of the return on the Flyers stock into two parts. Earlier, we had the actual return broken down into its expected and surprise components:

$$R = E(R) + U$$

We now recognize that the total surprise component for Flyers, U , has a systematic and an unsystematic component, so:

$$R = E(R) + \text{Systematic portion} + \text{Unsystematic portion} \quad [11.15]$$

Systematic risks are often called market risks because they affect most assets in the market to some degree.

The important thing about the way we have broken down the total surprise, U , is that the unsystematic portion is more or less unique to Flyers. For this reason, it is unrelated to the unsystematic portion of return on most other assets. To see why this is important, we need to return to the subject of portfolio risk.

11.8 DIVERSIFICATION AND PORTFOLIO RISK

We've seen earlier that portfolio risks can, in principle, be quite different from the risks of the assets that make up the portfolio. We now look more closely at the riskiness of an individual asset versus the risk of a portfolio of many different assets. We will once again examine some market history to get an idea of what happens with actual investments in U.S. capital markets.

For more on risk and diversification, visit www.investopedia.com/university.

The Effect of Diversification: Another Lesson from Market History

In our previous chapter, we saw that the standard deviation of the annual return on a portfolio of 500 large common stocks has historically been about 20 percent per year. Does this mean that the standard deviation of the annual return on a typical stock in that group of 500 is about 20 percent? As you might suspect by now, the answer is *no*. This is an extremely important observation.

To allow examination of the relationship between portfolio size and portfolio risk, Table 11.4 illustrates typical average annual standard deviations for equally weighted portfolios that contain different numbers of randomly selected NYSE securities.

In Column 2 of Table 11.4, we see that the standard deviation for a "portfolio" of one security is about 49 percent. What this means is that if you randomly selected a single NYSE stock and put all your money into it, your standard deviation of return would typically be a substantial 49 percent per year. If you were to randomly select two stocks and invest half your money in each, your standard deviation would be about 37 percent on average, and so on.

The important thing to notice in Table 11.4 is that the standard deviation declines as the number of securities is increased. By the time we have 100 randomly chosen stocks, the portfolio's standard deviation has declined by about 60 percent, from 49 percent to about 20 percent. With 500 securities, the standard deviation is 19.27 percent, similar to the 20 percent we saw in our previous chapter for the large common stock portfolio. The small difference exists because the portfolio securities and time periods examined are not identical.

The Principle of Diversification

Figure 11.9 illustrates the point we've been discussing. What we have plotted is the standard deviation of return versus the number of stocks in the portfolio. Notice in Figure 11.9 that the benefit in terms of risk reduction from adding securities drops off as we add more

TABLE 11.4

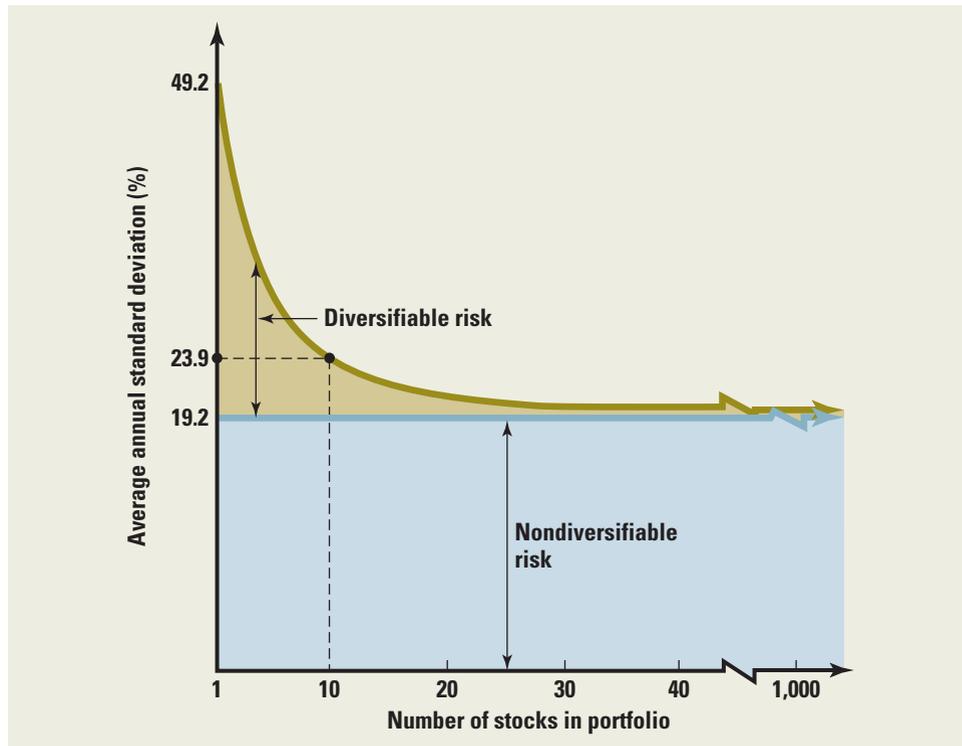
Standard Deviations of Annual Portfolio Returns

Sources: These figures are from Table 1 in M. Statman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis* 22 (September 1987), pp. 353–64. They were derived from E. J. Elton and M. J. Gruber, "Risk Reduction and Portfolio Size: An Analytic Solution," *Journal of Business* 50 (October 1977), pp. 415–37.

(1) NUMBER OF STOCKS IN PORTFOLIO	(2) AVERAGE STANDARD DEVIATION OF ANNUAL PORTFOLIO RETURNS	(3) RATIO OF PORTFOLIO STANDARD DEVIATION TO STANDARD DEVIATION OF A SINGLE STOCK
1	49.24%	1.00
2	37.36	.76
4	29.69	.60
6	26.64	.54
8	24.98	.51
10	23.93	.49
20	21.68	.44
30	20.87	.42
40	20.46	.42
50	20.20	.41
100	19.69	.40
200	19.42	.39
300	19.34	.39
400	19.29	.39
500	19.27	.39
1,000	19.21	.39

FIGURE 11.9

Portfolio Diversification



and more. By the time we have 10 securities, most of the effect is already realized, and by the time we get to 30 or so, there is very little remaining benefit.

Figure 11.9 illustrates two key points. First, some of the riskiness associated with individual assets can be eliminated by forming portfolios. The process of spreading an investment across assets (and thereby forming a portfolio) is called *diversification*. The **principle**

of diversification tells us that spreading an investment across many assets will eliminate some of the risk. The green shaded area in Figure 11.9, labeled “diversifiable risk,” is the part that can be eliminated by diversification.

The second point is equally important. There is a minimum level of risk that cannot be eliminated simply by diversifying. This minimum level is labeled “nondiversifiable risk” in Figure 11.9. Taken together, these two points are another important lesson from capital market history: Diversification reduces risk, but only up to a point. Put another way, some risk is diversifiable and some is not.

To give a recent example of the impact of diversification, the Dow Jones Industrial Average (DJIA), which is a widely followed stock market index of 30 large, well-known U.S. stocks, was up about 27 percent in 2009. As we saw in our previous chapter, this gain represents a very good year for a portfolio of large-cap stocks. The biggest individual winners for the year were American Express (up 118 percent), Microsoft (up 57 percent), and IBM (up 56 percent). But not all 30 stocks were up: The losers included ExxonMobil (down 15 percent), General Electric (down 7 percent), and Walmart (down 5 percent). Again, the lesson is clear: Diversification reduces exposure to extreme outcomes, both good and bad.

Diversification and Unsystematic Risk

From our discussion of portfolio risk, we know that some of the risk associated with individual assets can be diversified away and some cannot. We are left with an obvious question: Why is this so? It turns out that the answer hinges on the distinction we made earlier between systematic and unsystematic risk.

By definition, an unsystematic risk is one that is particular to a single asset or, at most, a small group. For example, if the asset under consideration is stock in a single company, the discovery of positive NPV projects such as successful new products and innovative cost savings will tend to increase the value of the stock. Unanticipated lawsuits, industrial accidents, strikes, and similar events will tend to decrease future cash flows and thereby reduce share values.

Here is the important observation: If we only held a single stock, then the value of our investment would fluctuate because of company-specific events. If we hold a large portfolio, on the other hand, some of the stocks in the portfolio will go up in value because of positive company-specific events and some will go down in value because of negative events. The net effect on the overall value of the portfolio will be relatively small, however, because these effects will tend to cancel each other out.

Now we see why some of the variability associated with individual assets is eliminated by diversification. When we combine assets into portfolios, the unique, or unsystematic, events—both positive and negative—tend to “wash out” once we have more than just a few assets.

This is an important point that bears repeating:

Unsystematic risk is essentially eliminated by diversification, so a portfolio with many assets has almost no unsystematic risk.

In fact, the terms *diversifiable risk* and *unsystematic risk* are often used interchangeably.

Diversification and Systematic Risk

We’ve seen that unsystematic risk can be eliminated by diversifying. What about systematic risk? Can it also be eliminated by diversification? The answer is no because, by definition, a systematic risk affects almost all assets to some degree. As a result, no matter how

many assets we put into a portfolio, the systematic risk doesn't go away. Thus, for obvious reasons, the terms *systematic risk* and *nondiversifiable risk* are used interchangeably.

Because we have introduced so many different terms, it is useful to summarize our discussion before moving on. What we have seen is that the total risk of an investment, as measured by the standard deviation of its return, can be written as:

$$\text{Total risk} = \text{Systematic risk} + \text{Unsystematic risk} \quad [11.16]$$

Systematic risk is also called *nondiversifiable risk* or *market risk*. Unsystematic risk is also called *diversifiable risk*, *unique risk*, or *asset-specific risk*. For a well-diversified portfolio, the unsystematic risk is negligible. For such a portfolio, essentially all of the risk is systematic.

11.9 MARKET EQUILIBRIUM

Definition of the Market Equilibrium Portfolio

Much of our analysis thus far concerns one investor. His estimates of the expected returns and variances for individual securities and the covariances between pairs of securities are his and his alone. Other investors would obviously have different estimates of the above variables. However, the estimates might not vary much because all investors would be forming expectations from the same data on past price movements and other publicly available information.

Financial economists often imagine a world where all investors possess the *same* estimates on expected returns, variances, and covariances. Though this can never be literally true, it can be thought of as a useful simplifying assumption in a world where investors have access to similar sources of information. This assumption is called **homogeneous expectations**.⁴

If all investors had homogeneous expectations, Figure 11.8 would be the same for all individuals. That is, all investors would sketch out the same efficient set of risky assets because they would be working with the same inputs. This efficient set of risky assets is represented by the curve *XAY*. Because the same risk-free rate would apply to everyone, all investors would view point *A* as the portfolio of risky assets to be held.

This point *A* takes on great importance because all investors would purchase the risky securities that it represents. Those investors with a high degree of risk aversion might combine *A* with an investment in the riskless asset, achieving point *4*, for example. Others with low aversion to risk might borrow to achieve, say, point *5*. Because this is a very important conclusion, we restate it:

In a world with homogeneous expectations, all investors would hold the portfolio of risky assets represented by point *A*.

If all investors choose the same portfolio of risky assets, it is possible to determine what that portfolio is. Common sense tells us that it is a market value weighted portfolio of all existing securities. It is the **market portfolio**.

In practice, financial economists use a broad-based index such as the Standard & Poor's (S&P) 500 as a proxy for the market portfolio. Of course, all investors do not hold the same portfolio. However, we know that a large number of investors hold diversified portfolios, particularly when mutual funds or pension funds are included. A broad-based index is a good proxy for the highly diversified portfolios of many investors.

⁴The assumption of homogeneous expectations states that all investors have the same beliefs concerning returns, variances, and covariances. It does not say that all investors have the same aversion to risk.

Definition of Risk When Investors Hold the Market Portfolio

The previous section states that many investors hold diversified portfolios similar to broad-based indexes. This result allows us to be more precise about the risk of a security in the context of a diversified portfolio.

Researchers have shown that the best measure of the risk of a security in a large portfolio is the *beta* of the security. We illustrate beta by an example.

EXAMPLE 11.4

Beta

Consider the following possible returns on both the stock of Jelco, Inc., and on the market:

STATE	TYPE OF ECONOMY	RETURN ON MARKET (PERCENT)	RETURN ON JELCO, INC. (PERCENT)
I	Bull	15	25
II	Bull	15	15
III	Bear	- 5	- 5
IV	Bear	- 5	-15

Though the return on the market has only two possible outcomes (15% and -5%), the return on Jelco has four possible outcomes. It is helpful to consider the expected return on a security for a given return on the market. Assuming each state is equally likely, we have:

TYPE OF ECONOMY	RETURN ON MARKET (PERCENT)	EXPECTED RETURN ON JELCO, INC. (PERCENT)
Bull	15%	$20\% = 25\% \times .50 + 15\% \times .50$
Bear	- 5%	$-10\% = -5\% \times .50 + (-15\%) \times .50$

Jelco, Inc., responds to market movements because its expected return is greater in bullish states than in bearish states. We now calculate exactly how responsive the security is to market movements. The market's return in a bullish economy is 20 percent [= 15% - (-5%)] greater than the market's return in a bearish economy. However, the expected return on Jelco in a bullish economy is 30 percent [= 20% - (-10%)] greater than its expected return in a bearish state. Thus, Jelco, Inc., has a responsiveness coefficient of 1.5 (30%/20%).

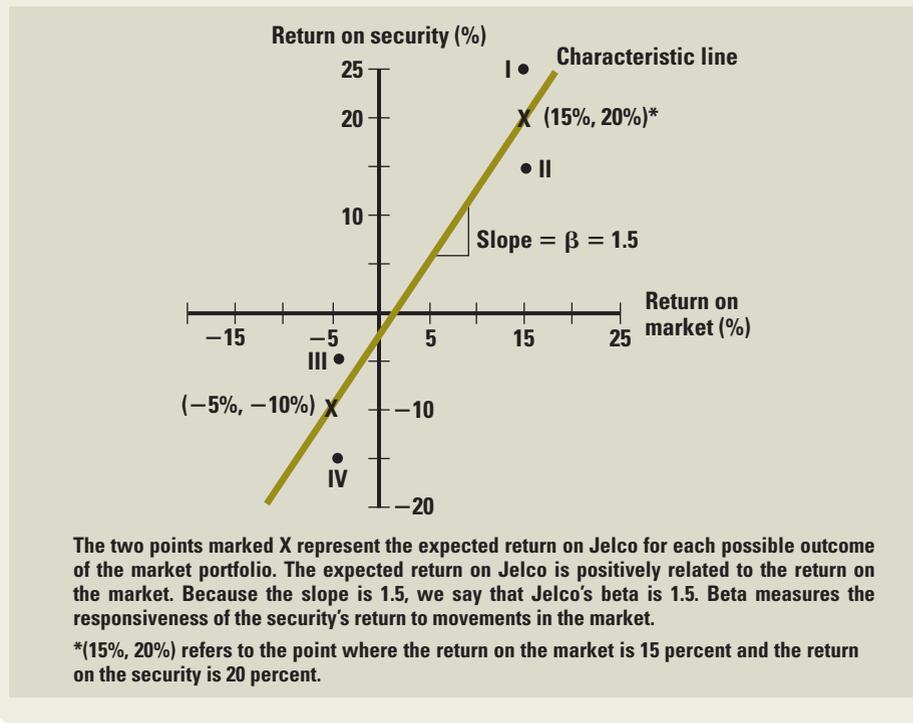
This relationship appears in Figure 11.10. The returns for both Jelco and the market in each state are plotted as four points. In addition, we plot the expected return on the security for each of the two possible returns on the market. These two points, each of which we designate by an X, are joined by a line called the **characteristic line** of the security. The slope of the line is 1.5, the number calculated in the previous paragraph. This responsiveness coefficient of 1.5 is the **beta** of Jelco.

The interpretation of beta from Figure 11.10 is intuitive. The graph tells us that the returns of Jelco are magnified 1.5 times over those of the market. When the market does well, Jelco's stock is expected to do even better. When the market does poorly, Jelco's stock is expected to do even worse. Now imagine an individual with a portfolio near that of the market who is considering the addition of Jelco to his portfolio. Because of Jelco's *magnification factor* of 1.5, he will view this stock as contributing much to the risk of the portfolio. (We will show shortly that the beta of the average security in the market is 1.) Jelco contributes more to the risk of a large, diversified portfolio than does an average security because Jelco is more responsive to movements in the market.

(continued)

FIGURE 11.10

Performance of Jelco, Inc., and the Market Portfolio



Further insight can be gleaned by examining securities with negative betas. One should view these securities as either hedges or insurance policies. The security is expected to do well when the market does poorly and vice versa. Because of this, adding a negative beta security to a large, diversified portfolio actually reduces the risk of the portfolio.⁵

Table 11.5 presents empirical estimates of betas for individual securities. As can be seen, some securities are more responsive to the market than others. For example, Tyson Foods has a beta of 1.27. This means that, for every 1 percent movement in the market, Tyson Foods is expected to move 1.27 percent in the same direction. Conversely, Disney has a beta of only 1.19. This means that, for every 1 percent movement in the market, Disney is expected to move 1.19 percent in the same direction.

TABLE 11.5
Estimates of Beta for
Selected Individual
Stocks

STOCK	BETA
McDonald's	0.55
American Electric Power	0.59
MMM	0.80
Yahoo	0.85
McGraw-Hill Co.	1.08
Amazon.com	1.15
Disney	1.19
Tyson Foods	1.27

The beta is defined as $\text{Cov}(R_i, R_M) / \text{Var}(R_M)$, where $\text{Cov}(R_i, R_M)$ is the covariance of the return on an individual stock, R_i , and the return on the market, R_M . $\text{Var}(R_M)$ is the variance of the return on the market, R_M .

⁵Unfortunately, empirical evidence shows that virtually no stocks have negative betas.

We can summarize our discussion of beta by saying:

Beta measures the responsiveness of a security to movements in the market portfolio.

Betas are easy to find on the Web. Try finance.yahoo.com and money.cnn.com.

You can find beta estimates at many sites on the Web. One of the best is finance.yahoo.com. We went there and entered the ticker symbol AMR for the AMR Corporation (American Airlines), and followed the “Key Statistics” link. Here is part of what we found:

STOCK PRICE HISTORY	
Beta	1.62
52-week change	−27.44%

INCOME STATEMENT	
Revenue (ttm)	20.82B
Revenue per share (ttm)	72.586
Qtrly revenue growth (yoy)	−20.20%
Gross profit (ttm)	4.55B
EBITDA (ttm)	423.00M
Net income avl to common (ttm)	−1.42B
Diluted EPS (ttm)	−5.086
Qtrly earnings growth (yoy)	N/A

BALANCE SHEET	
Total cash (mrq)	4.11B
Total cash per share (mrq)	12.369
Total debt (mrq)	11.59B
Total debt/equity (mrq)	N/A
Current ratio (mrq)	0.817
Book value per share (mrq)	−8.6

CASH FLOW STATEMENT	
Operating cash flow (ttm)	−438.00M
Levered free cash flow (ttm)	−1.59B

The reported beta for the AMR Corporation is 1.62 which means AMR has about 62 percent more systematic risk than the average stock. Perhaps you would expect that an airline company such as AMR would be very risky because of its dependence on discretionary consumer spending and the price of oil. Looking at the numbers, we agree. AMR’s net income and operating cash flow are negative. It has negative book value of equity, stemming from an accumulation of losses. Its quarterly revenue growth (year to year) is negative 20.2 percent. Digging a little deeper, we see that AMR has more than \$11 billion of debt and a book value per share of −\$8.6. AMR’s success going forward will clearly depend on a stronger global economy and its ability to manage its debt burden. In all, AMR seems to be a good candidate for a high beta.

The Formula for Beta

Our discussion so far has stressed the intuition behind beta. The actual definition of beta is:

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma^2(R_M)} \quad [11.17]$$



BETA, BETA, WHO'S GOT THE BETA?

Based on what we've studied so far, you can see that beta is a pretty important topic. You might wonder then, are all published betas created equal? Read on for a partial answer to this question.

We did some checking on betas and found some interesting results. The Value Line *Investment Survey* is one of the best-known sources for information on publicly traded companies. However, with the explosion of online investing, there has been a corresponding increase in the amount of investment information available online. We decided to compare the betas presented by Value Line to those reported by Yahoo! Finance (finance.yahoo.com), Google (finance.google.com), and CNN Money (money.cnn.com). What we found leads to an important note of caution.

Consider the Avis Budget group, the car rental company. Value Line reported the company's beta at 2.60, a high number. But the beta for the company reported on the Internet was 5.97, a much larger value. Avis Budget Group wasn't the only stock that showed a divergence in betas. In fact, for most of the technology companies we looked at, Value Line reported betas that were significantly lower than their online cousins. For example, the online beta for eBay was 1.71, while Value Line reported a beta of 1.15. Similarly, the online beta for 1-800-Flowers.com was 2.22 versus a Value Line beta of 1.45. Interested in something less high tech? The online beta for Martha Stewart Living was 2.21, compared to Value Line's beta of 1.35.

We also found some unusual, and even hard-to-believe, estimates for beta. Boardwalk Pipeline Partners, an oil and gas company, had a very low online beta of .05 (Value Line reported .85). The online beta for Exxon Mobil was .35, compared to Value Line's .75. Perhaps the most ridiculous numbers were the ones reported for the Etelos, Inc., and Saker Aviation; the estimated betas for those companies were 560 and -589 (notice the minus sign!), respectively. Value Line did not report a beta for these companies. How do you suppose we should interpret a beta of -589?

There are a few lessons to be learned from all of this. First, not all betas are created equal. Some are computed using weekly returns and some using daily returns. Some are computed using 60 months of stock returns; some consider more or less returns. Some betas are computed by comparing the stock to the S&P 500 Index, while others use alternative indexes. Finally, some reporting firms (including Value Line) make adjustments to raw betas to reflect information other than just the fluctuation in stock prices.

The second lesson is perhaps more subtle and comes from the betas of Etelos and Saker. We are interested in knowing what the beta of the stock will be in the future, but betas have to be estimated using historical data. Anytime we use the past to predict the future, there is the danger of a poor estimate. In our case, it is very unlikely that Etelos, Inc., has a beta anything like 560 or that Saker Aviation has a beta of -589. Instead, the estimates are almost certainly bad. The moral of the story is that, as with any financial tool, beta is not a black box that should be taken without question.

where $\text{Cov}(R_i, R_M)$ is the covariance between the return on asset i and the return on the market portfolio and $\sigma^2(R_M)$ is the variance of the market.

One useful property is that the average beta across all securities, when weighted by the proportion of each security's market value to that of the market portfolio, is 1. That is:

$$\sum_{i=1}^M X_i \beta_i = 1 \quad [11.18]$$

where X_i is the proportion of security i 's market value to that of the entire market and N is the number of securities in the market.

Equation (11.18) is intuitive, once you think about it. If you weight all securities by their market values, the resulting portfolio is the market. By definition, the beta of the market portfolio is 1. That is, for every 1 percent movement in the market, the market must move 1 percent—*by definition*.

For more on beta, visit moneycentral.msn.com.

A Test

We have put these questions on past corporate finance examinations:

1. What sort of investor rationally views the variance (or standard deviation) of an individual security's return as the security's proper measure of risk?
2. What sort of investor rationally views the beta of a security as the security's proper measure of risk?

A good answer might be something like the following:

A rational, risk-averse investor views the variance (or standard deviation) of her portfolio's return as the proper measure of the risk of her portfolio. If for some reason or another the investor can hold only one security, the variance of that security's return becomes the variance of the portfolio's return. Hence, the variance of the security's return is the security's proper measure of risk.

If an individual holds a diversified portfolio, she still views the variance (or standard deviation) of her portfolio's return as the proper measure of the risk of her portfolio. However, she is no longer interested in the variance of each individual security's return. Rather, she is interested in the contribution of an individual security to the variance of the portfolio.

Under the assumption of homogeneous expectations, all individuals hold the market portfolio. Thus, we measure risk as the contribution of an individual security to the variance of the market portfolio. This contribution, when standardized properly, is the beta of the security. While very few investors hold the market portfolio exactly, many hold reasonably diversified portfolios. These portfolios are close enough to the market portfolio so that the beta of a security is likely to be a reasonable measure of its risk.

11.10 RELATIONSHIP BETWEEN RISK AND EXPECTED RETURN (CAPM)

It is commonplace to argue that the expected return on an asset should be positively related to its risk. That is, individuals will hold a risky asset only if its expected return compensates for its risk. In this section, we first estimate the expected return on the stock market as a whole. Next, we estimate expected returns on individual securities.

Expected Return on Market

Financial economists frequently argue that the expected return on the market can be represented as:

$$E(R_M) = R_f + \text{Risk premium}$$

In words, the expected return on the market is the sum of the risk-free rate plus some compensation for the risk inherent in the market portfolio. Note that the equation refers to the *expected* return on the market, not the actual return in a particular month or year. Because stocks have risk, the actual return on the market over a particular period can, of course, be below R_f , or can even be negative.

Since investors want compensation for risk, the risk premium is presumably positive. But exactly how positive is it? It is generally argued that the place to start looking for the risk premium in the future is the average risk premium in the past. As reported in Chapter 10, the historical U.S. equity risk premium from 1900–2005 was 7.4%. Financial economists find this to be a useful estimate of the difference to occur in the future.

For example, if the risk-free rate, estimated by the current yield on a one-year Treasury bill, is 1 percent, the expected return on the market is:

$$8.4\% = 1\% + 7.4\%$$

Of course, the future equity risk premium could be higher or lower than the historical equity risk premium. This could be true if future risk is higher or lower than past risk or if individual risk aversions are higher or lower than those of the past.

Expected Return on Individual Security

Now that we have estimated the expected return on the market as a whole, what is the expected return on an individual security? We have argued that the beta of a security is the appropriate measure of risk in a large, diversified portfolio. Since most investors are diversified, the expected return on a security should be positively related to its beta. This is illustrated in Figure 11.11.

Actually, financial economists can be more precise about the relationship between expected return and beta. They posit that, under plausible conditions, the relationship between expected return and beta can be represented by the following equation:

Capital Asset Pricing Model:

$$E(R) = R_F + \beta \times (E(R_M) - R_F) \quad [11.19]$$

Expected return on a security = Risk-free rate + Beta of the security × Difference between expected return on market and risk-free rate

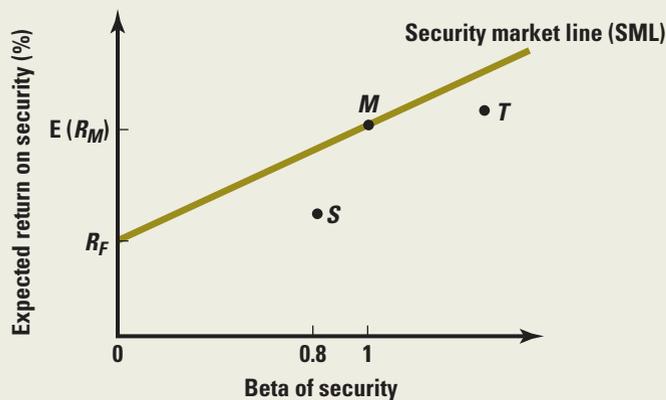
For more on CAPM and the equity risk premium, visit www.investopedia.com/terms/c/capm.asp.

This formula, which is called the **capital asset pricing model** (or CAPM for short), implies that the expected return on a security is linearly related to its beta. Since the average return on the market has been higher than the average risk-free rate over long periods of time, $E(R_M) - R_F$ is presumably positive. Thus, the formula implies that the expected return on a security is *positively* related to its beta. The formula can be illustrated by assuming a few special cases:

- Assume that $\beta = 0$. Here $E(R) = R_F$, that is, the expected return on the security is equal to the risk-free rate. Because a security with zero beta has no relevant risk, its expected return should equal the risk-free rate.
- Assume that $\beta = 1$. Equation 11.19 reduces to $E(R) = E(R_M)$. That is, the expected return on the security is equal to the expected return on the market. This makes sense since the beta of the market portfolio is also 1.

FIGURE 11.11

Relationship between Expected Return on an Individual Security and Beta of the Security



The security market line (SML) is the graphical depiction of the capital asset pricing model (CAPM). The expected return on a stock with a beta of 0 is equal to the risk-free rate. The expected return on a stock with a beta of 1 is equal to the expected return on the market.

Formula 11.19 can be represented graphically by the upward-sloping line in Figure 11.11. Note that the line begins at R_F and rises to $E(R_M)$ when beta is 1. This line is frequently called the **security market line (SML)**.

As with any line, the SML has both a slope and an intercept. R_F , the risk-free rate, is the intercept. Because the beta of a security is the horizontal axis, $E(R_M) - R_F$ is the slope. The line will be upward sloping as long as the expected return on the market is greater than the risk-free rate. Because the market portfolio is a risky asset, theory suggests that its expected return is above the risk-free rate. As mentioned, the empirical evidence of the previous chapter showed that the average return per year on the market portfolio (e.g., U.S. large-company stocks) from 1900 was 7.4 percent above the risk-free rate.

EXAMPLE 11.5

CAPM

The stock of Aardvark Enterprises has a beta of 1.5 and that of Zebra Enterprises has a beta of .7. The risk-free rate is assumed to be 3 percent, and the difference between the expected return on the market and the risk-free rate is assumed to be 8 percent. The expected returns on the two securities are:

Expected Return for Aardvark

$$15.0\% = 3\% + 1.5 \times 8\%$$

Expected Return for Zebra

$$8.6\% = 3\% + .7 \times 8\%$$

Three additional points concerning the CAPM should be mentioned:

1. *Linearity.* The intuition behind an upwardly sloping curve is clear. Because beta is the appropriate measure of risk, high-beta securities should have an expected return above that of low-beta securities. However, both Figure 11.11 and Equation 11.19 show something more than an upwardly sloping curve; the relationship between expected return and beta corresponds to a *straight* line.

It is easy to show that the line of Figure 11.11 is straight. To see this, consider security S with, say, a beta of .8. This security is represented by a point below the security market line in the figure. Any investor could duplicate the beta of security S by buying a portfolio with 20 percent in the risk-free asset and 80 percent in a security with a beta of 1. However, the homemade portfolio would itself lie on the SML. In other words, the portfolio dominates security S because the portfolio has a higher expected return and the same beta.

Now consider security T with, say, a beta greater than 1. This security is also below the SML in Figure 11.11. Any investor could duplicate the beta of security T by borrowing to invest in a security with a beta of 1. This portfolio must also lie on the SML, thereby dominating security T .

Because no one would hold either S or T , their stock prices would drop. This price adjustment would raise the expected returns on the two securities. The price adjustment would continue until the two securities lay on the security market line. The preceding example considered two overpriced stocks and a straight SML. Securities lying above the SML are *underpriced*. Their prices must rise until their expected returns lie on the line. If the SML is itself curved, many stocks would be mispriced. In equilibrium, all securities would be held only when prices changed so that the SML became straight. In other words, linearity would be achieved.
2. *Portfolios as well as securities.* Our discussion of the CAPM considered individual securities. Does the relationship in Figure 11.11 and Equation 11.19 hold for portfolios as well?

Yes. To see this, consider a portfolio formed by investing equally in our two securities, Aardvark and Zebra. The expected return on the portfolio is:

$$\begin{aligned} \textbf{Expected Return on Portfolio} \\ \mathbf{11.8\%} &= \mathbf{.5 \times 15.0\% + .5 \times 8.6\%} \end{aligned} \quad [11.20]$$

The beta of the portfolio is simply a weighted average of the betas of the two securities. Thus, we have:

$$\begin{aligned} \textbf{Beta of Portfolio} \\ \mathbf{1.1} &= \mathbf{.5 \times 1.5 + .5 \times .7} \end{aligned}$$

Under the CAPM, the expected return on the portfolio is:

$$\mathbf{11.8\% = 3\% + 1.1 \times 8\%}$$

Because the expected return in 11.20 is the same as the expected return in the above equation, the example shows that the CAPM holds for portfolios as well as for individual securities.

3. *A potential confusion.* Students often confuse the SML in Figure 11.11 with line *II* in Figure 11.8. Actually, the lines are quite different. Line *II* traces the efficient set of portfolios formed from both risky assets and the riskless asset. Each point on the line represents an entire portfolio. Point *A* is a portfolio composed entirely of risky assets. Every other point on the line represents a portfolio of the securities in *A* combined with the riskless asset. The axes on Figure 11.8 are the expected return on a *portfolio* and the standard deviation of a *portfolio*. Individual securities do not lie along line *II*.

The SML in Figure 11.11 relates expected return to beta. Figure 11.11 differs from Figure 11.8 in at least two ways. First, beta appears in the horizontal axis of Figure 11.11, but standard deviation appears in the horizontal axis of Figure 11.8. Second, the SML in Figure 11.11 holds both for all individual securities and for all possible portfolios, whereas line *II* in Figure 11.8 holds only for efficient portfolios.

We stated earlier that, under homogeneous expectations, point *A* in Figure 11.8 becomes the market portfolio. In this situation, line *II* is referred to as the **capital market line** (CML).

SUMMARY AND CONCLUSIONS

This chapter sets forth the fundamentals of modern portfolio theory. Our basic points are these:

1. This chapter shows us how to calculate the expected return and variance for individual securities, and the covariance and correlation for pairs of securities. Given these statistics, the expected return and variance for a portfolio of two securities *A* and *B* can be written as:

$$\text{Expected return on portfolio} = X_A E(R_A) + X_B E(R_B)$$

$$\text{Variance of portfolio return} = X_A^2 \sigma_A^2 + 2X_A X_B \sigma_{A,B} + X_B^2 \sigma_B^2$$

2. In our notation, *X* stands for the proportion of a security in one's portfolio. By varying *X*, one can trace out the efficient set of portfolios. We graphed the efficient set for the two asset case as a curve, pointing out that the degree of curvature or bend in the graph reflects the diversification effect: The lower the correlation between the two securities, the greater the bend. The same general shape of the efficient set holds in a world of many assets.
3. A diversified portfolio can eliminate only some, not all, of the risk associated with individual securities. The reason is that part of the risk with an individual asset is unsystematic, meaning essentially unique to that asset. In a well-diversified portfolio, these unsystematic risks tend to cancel out. Systematic, or market, risks are not diversifiable.

4. The efficient set of risky assets can be combined with riskless borrowing and lending. In this case, a rational investor will always choose to hold the portfolio of risky securities represented by point A in Figure 11.8. Then he can either borrow or lend at the riskless rate to achieve any desired point on line // in the figure.
5. The contribution of a security to the risk of a large, well-diversified portfolio is proportional to the covariance of the security's return with the market's return. This contribution, when standardized, is called the beta. The beta of a security can also be interpreted as the responsiveness of a security's return to that of the market.
6. The CAPM states that

$$E(R) = R_f + \beta[E(R_M) - R_f]$$
 In other words, the expected return on a security is positively (and linearly) related to the security's beta.

CONCEPT QUESTIONS

1. **Diversifiable and Nondiversifiable Risks** In broad terms, why is some risk diversifiable? Why are some risks nondiversifiable? Does it follow that an investor can control the level of unsystematic risk in a portfolio, but not the level of systematic risk?
2. **Information and Market Returns** Suppose the government announces that, based on a just-completed survey, the growth rate in the economy is likely to be 2 percent in the coming year, as compared to 5 percent for the year just completed. Will security prices increase, decrease, or stay the same following this announcement? Does it make any difference whether or not the 2 percent figure was anticipated by the market? Explain.
3. **Systematic versus Unsystematic Risk** Classify the following events as mostly systematic or mostly unsystematic. Is the distinction clear in every case?
 - a. Short-term interest rates increase unexpectedly.
 - b. The interest rate a company pays on its short-term debt borrowing is increased by its bank.
 - c. Oil prices unexpectedly decline.
 - d. An oil tanker ruptures, creating a large oil spill.
 - e. A manufacturer loses a multimillion-dollar product liability suit.
 - f. A Supreme Court decision substantially broadens producer liability for injuries suffered by product users.
4. **Systematic versus Unsystematic Risk** Indicate whether the following events might cause stocks in general to change price, and whether they might cause Big Widget Corp.'s stock to change price.
 - a. The government announces that inflation unexpectedly jumped by 2 percent last month.
 - b. Big Widget's quarterly earnings report, just issued, generally fell in line with analysts' expectations.
 - c. The government reports that economic growth last year was at 3 percent, which generally agreed with most economists' forecasts.
 - d. The directors of Big Widget die in a plane crash.
 - e. Congress approves changes to the tax code that will increase the top marginal corporate tax rate. The legislation had been debated for the previous six months.
5. **Expected Portfolio Returns** If a portfolio has a positive investment in every asset, can the expected return on the portfolio be greater than that on every asset in the portfolio? Can it

be less than that on every asset in the portfolio? If you answer yes to one or both of these questions, give an example to support your answer.

6. **Diversification** True or false: The variances of the individual assets in the portfolio are the most important characteristic in determining the expected return of a well-diversified portfolio. Explain.
7. **Portfolio Risk** If a portfolio has a positive investment in every asset, can the standard deviation on the portfolio be less than that on every asset in the portfolio? What about the portfolio beta?
8. **Beta and CAPM** Is it possible that a risky asset could have a beta of zero? Explain. Based on the CAPM, what is the expected return on such an asset? Is it possible that a risky asset could have a negative beta? What does the CAPM predict about the expected return on such an asset? Can you give an explanation for your answer?
9. **Corporate Downsizing** In recent years, it has been common for companies to experience significant stock price changes in reaction to announcements of massive layoffs. Critics charge that such events encourage companies to fire longtime employees and that Wall Street is cheering them on. Do you agree or disagree?
10. **Earnings and Stock Returns** As indicated by a number of examples in this chapter, earnings announcements by companies are closely followed by, and frequently result in, share price revisions. Two issues should come to mind. First, earnings announcements concern past periods. If the market values stocks based on expectations of the future, why are numbers summarizing past performance relevant? Second, these announcements concern accounting earnings. Going back to Chapter 2, such earnings may have little to do with cash flow, so, again, why are they relevant?
11. **Covariance** Briefly explain why the covariance of a security with the rest of a well-diversified portfolio is a more appropriate measure of the risk of the security than the security's variance.
12. **Beta** Consider the following quotation from a leading investment manager: "The shares of Southern Co. have traded close to \$12 for most of the past three years. Since Southern's stock has demonstrated very little price movement, the stock has a low beta. Texas Instruments, on the other hand, has traded as high as \$150 and as low as its current \$75. Since TI's stock has demonstrated a large amount of price movement, the stock has a very high beta." Do you agree with this analysis? Explain.
13. **Risk** A broker has advised you not to invest in oil industry stocks because they have high standard deviations. Is the broker's advice sound for a risk-averse investor like yourself? Why or why not?
14. **Security Selection** Is the following statement true or false? A risky security cannot have an expected return that is less than the risk-free rate because no risk-averse investor would be willing to hold this asset in equilibrium. Explain.

QUESTIONS AND PROBLEMS



Basic
(Questions 1–19)

1. **Determining Portfolio Weights** What are the portfolio weights for a portfolio that has 85 shares of Stock A that sell for \$38 per share and 160 shares of Stock B that sell for \$27 per share?
2. **Portfolio Expected Return** You own a portfolio that has \$3,400 invested in Stock A and \$4,100 invested in Stock B. If the expected returns on these stocks are 9.5 percent and 15.2 percent, respectively, what is the expected return on the portfolio?
3. **Portfolio Expected Return** You own a portfolio that is 45 percent invested in Stock X, 30 percent invested in Stock Y, and 25 percent invested in Stock Z. The expected returns on these three stocks are 10 percent, 13 percent, and 17 percent, respectively. What is the expected return on the portfolio?

4. Portfolio Expected Return You have \$10,000 to invest in a stock portfolio. Your choices are Stock X with an expected return of 14 percent and Stock Y with an expected return of 10.4 percent. If your goal is to create a portfolio with an expected return of 11.8 percent, how much money will you invest in Stock X? In Stock Y?



5. Calculating Expected Return Based on the following information, calculate the expected return.

STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	RATE OF RETURN IF STATE OCCURS
Recession	.20	-.06
Normal	.55	.12
Boom	.25	.25

6. Calculating Returns and Standard Deviations Based on the following information, calculate the expected return and standard deviation for the two stocks.



STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	RATE OF RETURN IF STATE OCCURS	
		STOCK A	STOCK B
Recession	.15	.03	-.20
Normal	.50	.07	.13
Boom	.35	.11	.33

7. Calculating Returns and Standard Deviations Based on the following information, calculate the expected return and standard deviation of the following stock.

STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	RATE OF RETURN IF STATE OCCURS
Depression	.05	-.245
Recession	.15	-.085
Normal	.50	.140
Boom	.30	.321

8. Calculating Expected Returns A portfolio is invested 25 percent in Stock G, 60 percent in Stock J, and 15 percent in Stock K. The expected returns on these stocks are 10 percent, 12 percent, and 19 percent, respectively. What is the portfolio's expected return? How do you interpret your answer?

9. Returns and Standard Deviations Consider the following information:



STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	RATE OF RETURN IF STATE OCCURS		
		STOCK A	STOCK B	STOCK C
Boom	.15	.30	.45	.33
Good	.45	.12	.10	.15
Poor	.35	.01	-.15	-.05
Bust	.05	-.06	-.30	-.09

- a. Your portfolio is invested 40 percent each in A and C, and 20 percent in B. What is the expected return of the portfolio?
 - b. What is the variance of this portfolio? The standard deviation?
- 10. Calculating Portfolio Betas** You own a stock portfolio invested 15 percent in Stock Q, 20 percent in Stock R, 30 percent in Stock S, and 35 percent in Stock T. The betas for these four stocks are .85, 1.65, 1.10, and 1.26, respectively. What is the portfolio beta?
- 11. Calculating Portfolio Betas** You own a portfolio equally invested in a risk-free asset and two stocks. If one of the stocks has a beta of 1.60 and the total portfolio is equally as risky as the market, what must the beta be for the other stock in your portfolio?
- 12. Using CAPM** A stock has a beta of 1.25, the expected return on the market is 11.5 percent, and the risk-free rate is 3.4 percent. What must the expected return on this stock be?
- 13. Using CAPM** A stock has an expected return of 11.5 percent, the risk-free rate is 3.2 percent, and the market risk premium is 7 percent. What must the beta of this stock be?
- 14. Using CAPM** A stock has an expected return of 9.8 percent, its beta is .85, and the risk-free rate is 3.6 percent. What must the expected return on the market be?
- 15. Using CAPM** A stock has an expected return of 10.1 percent, a beta of 0.82, and the expected return on the market is 11.5 percent. What must the risk-free rate be?
- 16. Using CAPM** A stock has a beta of 1.15 and an expected return of 14 percent. A risk-free asset currently earns 4.2 percent.
- a. What is the expected return on a portfolio that is equally invested in the two assets?
 - b. If a portfolio of the two assets has a beta of .75, what are the portfolio weights?
 - c. If a portfolio of the two assets has an expected return of 8 percent, what is its beta?
 - d. If a portfolio of the two assets has a beta of 2.30, what are the portfolio weights? How do you interpret the weights for the two assets in this case? Explain.
- 17. Using the SML** Asset W has an expected return of 12.9 percent and a beta of 1.30. If the risk-free rate is 4.1 percent, complete the following table for portfolios of Asset W and a risk-free asset. Illustrate the relationship between portfolio expected return and portfolio beta by plotting the expected returns against the betas. What is the slope of the line that results?



PERCENTAGE OF PORTFOLIO IN ASSET W	PORTFOLIO EXPECTED RETURN	PORTFOLIO BETA
0%		
25		
50		
75		
100		
125		
150		



- 18. Reward-to-Risk Ratios** Stock Y has a beta of 1.35 and an expected return of 14.2 percent. Stock Z has a beta of .75 and an expected return of 9.1 percent. If the risk-free rate is 4.3 percent and the market risk premium is 7 percent, are these stocks correctly priced?
- 19. Reward-to-Risk Ratios** In the previous problem, what would the risk-free rate have to be for the two stocks to be correctly priced?

- 20. Portfolio Returns** Using information from the previous chapter on capital market history, determine the return on a portfolio that is equally invested in large-company stocks and long-term government bonds. What is the return on a portfolio that is equally invested in small-company stocks and Treasury bills?
- 21. CAPM** Using the CAPM, show that the ratio of the risk premiums on two assets is equal to the ratio of their betas.
- 22. Portfolio Returns and Deviations** Consider the following information on three stocks:

Intermediate
(Questions 20–32)

STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	RATE OF RETURN IF STATE OCCURS		
		STOCK A	STOCK B	STOCK C
Boom	.25	.20	.35	.60
Normal	.60	.15	.12	.05
Bust	.15	.01	-.25	-.50

- a. If your portfolio is invested 30 percent each in A and B and 40 percent in C, what is the portfolio expected return? The variance? The standard deviation?
- b. If the expected T-bill rate is 3.80 percent, what is the expected risk premium on the portfolio?
- c. If the expected inflation rate is 3.10 percent, what are the approximate and exact expected real returns on the portfolio? What are the approximate and exact expected real risk premiums on the portfolio?
- 23 Analyzing a Portfolio** You want to create a portfolio equally as risky as the market, and you have \$1,000,000 to invest. Given this information, fill in the rest of the following table:

ASSET	INVESTMENT	BETA
Stock A	\$280,000	.80
Stock B	\$340,000	1.19
Stock C		1.30
Risk-free asset		

- 24. Analyzing a Portfolio** You have \$100,000 to invest in a portfolio containing Stock X and Stock Y. Your goal is to create a portfolio that has an expected return of 18.5 percent. If Stock X has an expected return of 17.2 percent and a beta of 1.4, and Stock Y has an expected return of 13.6 percent and a beta of .95, how much money will you invest in stock Y? How do you interpret your answer? What is the beta of your portfolio?
- 25. Covariance and Correlation** Based on the following information, calculate the expected return and standard deviation of each of the following stocks. Assume each state of the economy is equally likely to happen. What is the covariance and correlation between the returns of the two stocks?

STATE OF ECONOMY	RETURN ON STOCK A	RETURN ON STOCK B
Bear	.041	-.089
Normal	.113	-.025
Bull	.153	.416

- 26. Covariance and Correlation** Based on the following information, calculate the expected return and standard deviation for each of the following stocks. What is the covariance and correlation between the returns of the two stocks?

STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	RETURN ON STOCK J	RETURN ON STOCK K
Bear	.15	-.080	.080
Normal	.60	.130	.091
Bull	.25	.347	.062

- 27. Portfolio Standard Deviation** Security F has an expected return of 10 percent and a standard deviation of 38 percent per year. Security G has an expected return of 17 percent and a standard deviation of 53 percent per year.
- What is the expected return on a portfolio composed of 70 percent of security F and 30 percent of security G?
 - If the correlation between the returns of security F and security G is .15, what is the standard deviation of the portfolio described in part (a)?
- 28. Portfolio Standard Deviation** Suppose the expected returns and standard deviations of stocks A and B are $E(R_A) = .11$, $E(R_B) = .14$, $\sigma_A = .52$, and $\sigma_B = .65$, respectively.
- Calculate the expected return and standard deviation of a portfolio that is composed of 40 percent A and 60 percent B when the correlation between the returns on A and B is .5.
 - Calculate the standard deviation of a portfolio that is composed of 40 percent A and 60 percent B when the correlation coefficient between the returns on A and B is $-.5$.
 - How does the correlation between the returns on A and B affect the standard deviation of the portfolio?
- 29. Correlation and Beta** You have been provided the following data on the securities of three firms, the market portfolio, and the risk-free asset:

SECURITY	EXPECTED RETURN	STANDARD DEVIATION	CORRELATION*	BETA
Firm A	.11	.33	(i)	.70
Firm B	.14	(ii)	.36	1.35
Firm C	.10	.37	.40	(iii)
The market portfolio	.12	.22	(iv)	(v)
The risk-free asset	.05	(vi)	(vii)	(viii)

*With the market portfolio.

- Fill in the missing values in the table.
 - Is the stock of Firm A correctly priced according to the capital asset pricing model (CAPM)? What about the stock of Firm B? Firm C? If these securities are not correctly priced, what is your investment recommendation for someone with a well-diversified portfolio?
- 30. CML** The market portfolio has an expected return of 11 percent and a standard deviation of 21 percent. The risk-free rate is 4.5 percent.
- What is the expected return on a well-diversified portfolio with a standard deviation of 24 percent?
 - What is the standard deviation of a well-diversified portfolio with an expected return of 17 percent?

- 31. Beta and CAPM** A portfolio that combines the risk-free asset and the market portfolio has an expected return of 9 percent and a standard deviation of 11 percent. The risk-free rate is 4.2 percent, and the expected return on the market portfolio is 13 percent. Assume the capital asset pricing model holds. What expected rate of return would a security earn if it had a .55 correlation with the market portfolio and a standard deviation of 50 percent?
- 32. Beta and CAPM** Suppose the risk-free rate is 3.9 percent and the market portfolio has an expected return of 11.2 percent. The market portfolio has a variance of .0460. Portfolio Z has a correlation coefficient with the market of .55 and a variance of .3017. According to the capital asset pricing model, what is the expected return on portfolio Z?
- 33. Systematic versus Unsystematic Risk** Consider the following information on Stocks I and II:

Challenge
(Questions 33–38)

STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	RATE OF RETURN IF STATE OCCURS	
		STOCK I	STOCK II
Recession	.20	.04	−.30
Normal	.50	.21	.12
Irrational exuberance	.30	.12	.44

The market risk premium is 7.5 percent, and the risk-free rate is 4 percent. Which stock has the most systematic risk? Which one has the most unsystematic risk? Which stock is “riskier”? Explain.

- 34. SML** Suppose you observe the following situation:

SECURITY	BETA	EXPECTED RETURN
Pete Corp.	1.2	.1290
Repete Co	.8	.0985

Assume these securities are correctly priced. Based on the CAPM, what is the expected return on the market? What is the risk-free rate?

- 35. Covariance and Portfolio Standard Deviation** There are three securities in the market. The following chart shows their possible payoffs.

STATE	PROBABILITY OF OUTCOME	RETURN ON SECURITY 1	RETURN ON SECURITY 2	RETURN ON SECURITY 3
1	0.10	0.20	0.20	0.05
2	0.40	0.15	0.10	0.10
3	0.40	0.10	0.15	0.15
4	0.10	0.05	0.05	0.20

- What is the expected return and standard deviation of each security?
- What are the covariances and correlations between the pairs of securities?
- What is the expected return and standard deviation of a portfolio with half of its funds invested in security 1 and half in security 2?
- What is the expected return and standard deviation of a portfolio with half of its funds invested in security 1 and half in security 3?

- e. What is the expected return and standard deviation of a portfolio with half of its funds invested in security 2 and half in security 3?
- f. What do your answers in parts (a), (c), (d), and (e) imply about diversification?
36. **SML** Suppose you observe the following situation:

STATE OF ECONOMY	PROBABILITY OF STATE	RETURN IF STATE OCCURS	
		STOCK A	STOCK B
Bust	.20	−.08	−.05
Normal	.60	.13	.14
Boom	.20	.48	.29

- a. Calculate the expected return on each stock.
- b. Assuming the capital asset pricing model holds and stock A's beta is greater than stock B's beta by .40, what is the expected market risk premium?
37. **Standard Deviation and Beta** There are two stocks in the market: stock A and stock B. The price of stock A today is \$52. The price of stock A next year will be \$40 if the economy is in a recession, \$59 if the economy is normal, and \$68 if the economy is expanding. The probabilities of recession, normal times, and expansion are .1, .65, and .25, respectively. Stock A pays no dividends and has a correlation of .45 with the market portfolio. Stock B has a standard deviation of 51 percent, a correlation with the market portfolio of .40, and a correlation with stock A of .50. The market portfolio has a standard deviation of 20 percent. The risk-free rate is 4 percent and the market risk premium is 7.5 percent. Assume the CAPM holds.
- a. If you are a typical, risk-averse investor with a well-diversified portfolio, which stock would you prefer? Why?
- b. What are the expected return and standard deviation of a portfolio consisting of 70 percent of stock A and 30 percent of stock B?
- c. What is the beta of the portfolio in part (b)?
38. **Minimum Variance Portfolio** Assume stocks A and B have the following characteristics:

STOCK	EXPECTED RETURN (%)	STANDARD DEVIATION (%)
A	12	48
B	10	59

The covariance between the returns on the two stocks is .01.

- a. Suppose an investor holds a portfolio consisting of only stock A and stock B. Find the portfolio weights, X_A and X_B , such that the variance of his portfolio is minimized. (Hint: Remember that the sum of the two weights must equal 1.)
- b. What is the expected return on the minimum variance portfolio?
- c. If the covariance between the returns on the two stocks is $-.15$, what are the minimum variance weights?
- d. What are the variance and standard deviation of the portfolio in part (c)?

WHAT'S ON THE WEB?

- 1. Expected Return** You want to find the expected return for Honeywell using the CAPM. First you need the market risk premium. Go to money.cnn.com and find the current interest rate for three-month Treasury bills. Use the average large-company stock return in Table 10.2 to calculate the market risk premium. Next, go to finance.yahoo.com, enter the ticker symbol HON for Honeywell, and find the beta for Honeywell. What is the expected return for Honeywell using CAPM? What assumptions have you made to arrive at this number?
- 2. Portfolio Beta** You have decided to invest in an equally weighted portfolio consisting of American Express, Procter & Gamble, Home Depot, and Du Pont and need to find the beta of your portfolio. Go to finance.yahoo.com and find the beta for each of the companies. What is the beta for your portfolio?
- 3. Beta** Which companies currently have the highest and lowest betas? Go to finance.yahoo.com and find the "Stock Screener" link. Enter 0 as the maximum beta and search. How many stocks currently have a beta less than or equal to 0? What is the lowest beta? Go back to the stock screener and enter 3 as the minimum. How many stocks have a beta above 3? What stock has the highest beta?
- 4. Security Market Line** Go to finance.yahoo.com and enter the ticker symbol IP for International Paper. Follow the "Key Statistics" link to get the beta for the company. Next, find the estimated (or "target") price in 12 months according to market analysts. Using the current share price and the mean target price, compute the expected return for this stock. Don't forget to include the expected dividend payments over the next year. Now go to money.cnn.com and find the current interest rate for three-month Treasury bills. Using this information, calculate the expected return on the market using the reward-to-risk ratio. Does this number make sense? Why or why not?



A JOB AT EAST COAST YACHTS, PART 2

You are discussing your 401(k) with Dan Ervin, when he mentions that Sarah Brown, a representative from Bledsoe Financial Services, is visiting East Coast Yachts today. You decide that you should meet with Sarah, so Dan sets up an appointment for you later in the day.

When you sit down with Sarah, she discusses the various investment options available in the company's 401(k) account. You mention to Sarah that you researched East Coast Yachts before you accepted your new job. You are confident in management's ability to lead the company. Analysis of the company has led to your belief that the company is growing and will achieve a greater market share in the future. You also feel you should support your employer. Given these considerations, along with the fact that you are a conservative investor, you are leaning toward investing 100 percent of your 401(k) account in East Coast Yachts.

Assume the risk-free rate is the historical average risk-free rate (in Chapter 10). The correlation between the bond fund and the large cap stock fund is .16. (Note: The spreadsheet graphing and "Solver" functions may assist you in answering the following questions.)

1. Considering the effects of diversification, how should Sarah respond to the suggestion that you invest 100 percent of your 401(k) account in East Coast Yachts stock?

CLOSING CASE

2. After hearing Sarah's response to investing your 401(k) account entirely in East Coast Yachts stock, she has convinced you that this may not be the best alternative. Since you are a conservative investor, you tell Sarah that a 100 percent investment in the bond fund may be the best alternative. Is it?
3. Using the returns for the Bledsoe Large-Cap Stock Fund and the Bledsoe Bond Fund, graph the opportunity set of feasible portfolios.
4. After examining the opportunity set, you notice that you can invest in a portfolio consisting of the bond fund and the large-cap stock fund that will have exactly the same standard deviation as the bond fund. This portfolio will also have a greater expected return. What are the portfolio weights and expected return of this portfolio?
5. Examining the opportunity set, notice there is a portfolio that has the lowest standard deviation. This is the minimum variance portfolio. What are the portfolio weights, expected return, and standard deviation of this portfolio? Why is the minimum variance portfolio important?
6. A measure of risk-adjusted performance that is often used is the Sharpe ratio. The Sharpe ratio is calculated as the risk premium of an asset divided by its standard deviation. The portfolio with the highest possible Sharpe ratio on the opportunity set is called the Sharpe optimal portfolio. What are the portfolio weights, expected return, and standard deviation of the Sharpe optimal portfolio? How does the Sharpe ratio of this portfolio compare to the Sharpe ratio of the bond fund and the large-cap stock fund? Do you see a connection between the Sharpe optimal portfolio and the CAPM? What is the connection?