



17

Learning Curve Theory

LEARNING OBJECTIVES :

After studying this unit, you will be able to :

- Understand, visualize and explain learning curve phenomenon.
- Measure how in some industries and in some situations the incremental cost per unit of output continues to fall for each extra unit produced
- Understand how the percentage learning rate applies to the doubling of output
- Know that cumulative average time means the average time per unit for all units produced so far back to and including the first unit produced.
- Successfully use learning curve theory in such situations as pricing decisions, work scheduling and standard setting
- Describe and identify the situations where learn effect can be incorporated
- Calculate average hours/cost per unit of cumulative productions and incremental hours/cost per order using the learning curve

17.1 INTRODUCTION

Learning is the process by which an individual acquires skill, knowledge and ability. When a new product or process is started, performance of worker is not at its best and learning phenomenon takes place. As the experience is gained, the performance of worker improves, time taken per unit reduces and thus his productivity goes up. This improvement in productivity of workers is due to learning effect. Cost predictions especially those relating to direct labour must allow for the effect of learning process. This technique is a mathematical technique. It is a graphical technique used widely to predict cost. Learning curve is a geometrical progression, which reveals that there is steadily decreasing cost for the accomplishment of a given repetitive operation, as the identical operation is increasingly repeated. The amount of decrease will be less and less with each successive unit produced. The slope of the decision curve is expressed as a percentage. The other names given to learning curve are Experience curve, Improvement curve and Progress curve. It is essentially a measure of the experience gained in production of an article by an organisation. As more units are produced, people involved in production become more efficient than before. Each additional unit takes less time to produce. The amount of improvement



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or experience gained is reflected in a decrease in man-hours or cost. The application of learning curve can be extended to commercial and industrial activities as well as defence production.

The learning effect exists during a worker's start up or familiarization period on a particular job. After the limits of experimental learning are reached, productivity tends to stabilise and no further improvement is possible. The rate at which learning occurs is influenced by many factors including the relative unfamiliarity of workers with the task, the relative novelty and uniqueness of the job, the complexity of the process, the impact of incentive plans, supervision, etc.

17.2 DISTINCTIVE FEATURES OF LEARNING CURVE THEORY IN MANUFACTURING ENVIRONMENT

The Theory of learning curve was first introduced by T.P. Wright of Curtiss—Wright, Buffalo, U.S.A. engaged in production of airframes. As the production quantity of a given item doubled, the cost of that item decrease at a fixed rate. This phenomenon is the basic premise on which the theory of learning curve has been formulated. The key words, 'doubled' and 'rate' are important as the quantity produced doubles, the absolute amount of cost increase will be successively smaller but the rate of decrease will remain fixed. This is the essence of the learning curve theory and it occurs due to following distinctive features of manufacturing environment :

- (i) Better tooling methods are developed and used;
- (ii) More productive equipments are designed and used to make the product;
- (iii) Designed bugs are detected and corrected;
- (iv) Engineering changes decrease over the time. Designed engineering are prompted to achieve better design for reducing material and labour cost.
- (v) Earlier teething problems are overcome. As the product involves monthly, management is prompted to strive for better planning and better management.
- (vi) Rejections and rework tend to diminish over time.

There happen a significant influence of all these features on labour as the quantity produced increases and the cost per unit drops. The reasons for this are that each unit will entail : (a) less labour; (b) less material; (c) more units produced from the same equipments; (d) cost of fewer delays and less loss time. Every Time Study engineer or industrial engineer experienced in work measurement has seen this phenomenon happening. Learning curve applications are fast growing with time in manufacturing environment. A company should never blindly adopt another company's learning curve. The product approach for a company should be to develop knowledge of its own learning preference in its plant.

17.3. THE LEARNING CURVE RATIO

In the initial stage of a new product or a new process, the learning effect pattern is so regular that the rate of decline established at the outset can be used to predict labour cost well in advance. The effect of experience on cost is summaries in the learning ratio or improvement ratio :

$$\text{Learning Curve Ratio} = \frac{\text{Average labour cost of first } 2N \text{ units}}{\text{Average labour cost of first } N \text{ units}}$$



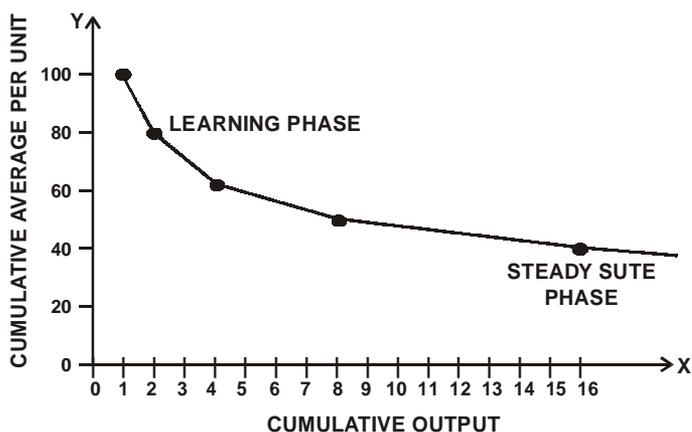
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If the average labour cost for the first 500 units of a product is Rs. 25 and the average labour cost of first 1,000 units is Rs. 20, the learning ratio will be determined as follow :

$$\text{Learning curve ratio} = \frac{\text{Rs. 20}}{\text{Rs. 25}} \times 100 \text{ or } 80\%$$

This learning curve ratio of 80% means that every time output doubles, the average cost declines to 80% of the previous amount. Since the average cost per unit of 1,000 units is Rs. 20, the average cost of first 2,000 units is likely to be 80% of this, i.e., Rs. 16 per unit. The amount of production improvement in the manufacture of an article will determine the percentage of the learning curve. The effect of learning can be presented clearly in a diagram drawn known as learning curve. The units chosen in the progression must always have a ratio of two (Unit 2 to unit 1, unit 50 to unit 25, unit 300 to unit 150, etc.). An 80% leaning curve is drawn using the following data.

<i>Incremental quantity</i>	<i>Cumulative quantity</i>		<i>Average time per unit</i>	<i>Cumulative time taken</i>
(1)	(2)	(3)	(4)	(5)
1	1		100	100
1	2	$\frac{(100 \times 80)}{100}$	80	160
2	4	$\frac{(80 \times 80)}{100}$	64	256
4	8	$\frac{(64 \times 80)}{100}$	51	408
8	16	$\frac{(51 \times 80)}{100}$	41	656





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Learning Curve

Columns (2) and (4) have been used for drawing the learning curve. Last column is not used in drawing the 80 per cent learning curve. This column shows how the cumulative time consumption will increase with decrease in cumulative average per unit. Cumulative quantity is plotted on X-axis and cumulative average time consumption per unit is plotted on Y-axis. After the learning effect phase is over, steady-state phase will start. Learning effect advantage will not be there in steady-state phase, when the product or the process gets well stabilised.

17.4 LEARNING CURVE EQUATION

Mathematicians have been able to express relationship in equations. The basic equation

$$Y_x = KX^s \quad \dots(1)$$

where,

X is the cumulative number of units or lots produced

Y is the cumulative average unit cost of those units X or lots.

K is the average cost of the first unit or lot

s is the improvement exponent or the learning coefficient or the index of learning which is calculated as follows :

$$s = \frac{\text{logarithm of learning ratio}}{\text{logarithm of 2}}$$

Learning curve equation $Y_x = KX^s$ becomes a linear equation when it is written in its logarithmic form :

$$\log Y_x = \log K + s \log X \quad \dots(2)$$

Each of the above two equations defines cumulative average cost. Either of them can be converted to a formula for the total labour cost of all units produced up to a given point. Total cost under equation 1 can be found out by the following formula :

$$\text{Total cost} = XY_x = KX^s = KX^{s+1} \quad \dots(3)$$

17.5 LEARNING CURVE APPLICATION

Knowledge of learning curve can be useful both in planning and control. Standard cost for new operations should be revised frequently to reflect the anticipated learning pattern. Its main uses are summarised below :

17.5.1 Helps to analyse CVP Relationship during familiarisation phase : Learning curve helps to analyse cost-volume-profit relationship during familiarisation phase of product or process and thus it is very useful for cost estimates. Learning curve is of immense value as a tool for forecasting.

Illustration

XYZ Co., has observed that a 90% learning curve ratio applies to all labour related costs each time a new model enters production. It is anticipated that 320 units will be manufactured during



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1999. Direct labour cost for the first lot of 10 units amounts to 1,000 hours at Rs. 8 per hour. Variable overhead cost is assigned to products at the rate of Rs. 2 per direct labour hour. You are required to determine :

- (i) Total labour and labour-related costs to manufacture 320 units of output.
- (ii) Average cost of (a) the first 40 units produced (b) the first 80 units, (c) the first 100 units.
- (iii) Incremental cost of (a) units 41–80 and (b) units 101–200.

Solution

Table showing cost projections based on 90 per cent learning curve.

<i>Incremental quantity</i>	<i>Cumulative quantity</i>	<i>Average time per unit</i>	<i>Cumulative time taken</i>	<i>Incremental hours</i>
10	10	100	1,000	—
10	20	90	1,800	800
20	40	81	3,240	1,400
40	80	72.9	5,832	2,592
80	160	65.61	10,497.6	4,665.6
160	320	59.049	18,895.68	8,398.08

Following cost information can be derived from the data given in the above table :

- (i) Total cost of 320 units = 18,895.68 × Rs. 10 = Rs. 1,88,956.80
- (ii) Average cost of first 40 units = 81 × Rs. 10 = Rs. 810 per unit.
Average cost of first 80 units = 72.9 × Rs. 10 or Rs. 729 per unit.
- (iii) Incremental cost of units 41 – 80 = 2592 × Rs. 10 or Rs. 25,920.

The basic learning curve formula must be used to derive the average cost of the first 100 units and the incremental cost of units 101 – 200.

We know that $Y_x = KX^s$

The first production lot contained 10 units. $K = 1,000$ hrs. (time taken for the first lot) and $X = 10$ (the number of lots needed to produce 100 units).

Taking the logarithm of the above relation, we get

$$\begin{aligned}
 \log Y_{10} &= \log 1000 + (-0.1518 \log 10) \\
 &= 3.0 - 0.1518 \\
 &= \underline{2.8482} \\
 Y_{10} &= \underline{705}
 \end{aligned}$$



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Y_{10} means that average processing time for the first 10 lots of output was 705 hours per lot. Each lot includes 10 units of output. Therefore, average processing time for each of the first 100 units is 70.5 hours. The labour rate is Rs. 10 per hour. Therefore, the average cost of the first 100 units was Rs. 705 per unit. The incremental cost of units 101 – 200 corresponds to the incremental cost of lots 11 – 20. This is equal to the total cost of first 20 lots minus the total cost of the first 10 lots.

$$Y_{10} = 705$$

$$Y_{10} = (1000) (20 - 0.1518) = 634.5$$

$$\text{Time for first 20 lots} = 20(634.5) = 12,690 \text{ hours}$$

$$\text{Time of first 10 lots} = 10 (705) = \underline{7,050 \text{ hours}}$$

$$\underline{5,640 \text{ hours}}$$

This means that incremental cost of lots 11 – 20 will be :

$$(5,640 \text{ hrs.} \times \text{Rs. } 10) = \text{Rs. } 56,400$$

17.5.2 Helps in budgeting and profit planning : Incorporation of the learning curve introduces new complexity and sophistication in the process of budgeting and profit planning. Budget executive should select those costs which reflect learning effect and then he should be able to incorporate this effect in process of developing budgets or in the exercises relating to project planning.

17.5.3 Helps in pricing : The use of cost data adjusted for learning effect helps in development of advantageous pricing policy. Suppose, firm A is preparing a bid to produce 220 MW turboset. Hitherto, company had produced a set of 100 MW only. There is a strong possibility of production over-run. Firm A is competing with Firm B. Firm A is not aware of the learning effect, while Firm B is aware of the learning effect. In this situation, Firm B will definitely go away with the contract.

17.5.4 Design makers : It helps design engineers in making decisions based upon expected (predictable from past experience) rates of improvement.

17.5.5 Helps in negotiations : It is very helpful to Government in negotiations about the contracts. Government receives full advantage of the decreasing unit cost in establishing the contract price.

17.5.6 Helps in setting standards : The learning curve is extremely useful in setting standards in learning phase.

17.6 LIMITATIONS OF LEARNING CURVE THEORY

Following limitations of learning curve must be kept in view :

1. All activities of a firm are not subject to learning effect. Following types of activities are subject to learning effect :
 - (a) Those, that have not been performed in this present operational mode.



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- (b) Those which are being performed by new workmen, new employees or others not familiar with the particular activity. In contrast, activities being performed by experienced workmen, who are thoroughly familiar with those activities will not be subject to learning effect.
- (c) Those involving utilization of material not used by firm so far.
- It is correct that learning effect does take place and average time taken is likely to reduce. But in practice it is highly unlikely that there will be a regular consistent rate of decrease, as exemplified earlier. Therefore any cost predictions based on conventional learning curves should be viewed with caution.
 - Considerable difficulty arises in obtaining valid data, that will form basis for computation of learning effect.
 - Even slight change in circumstances quickly renders the learning curve obsolete. While the regularity of conventional learning curves can be questioned, it would be wrong to ignore learning effect altogether in predicting future costs for decision purposes.

Illustration

Illustrate the use of leaning curves for calculating the expected average unit cost of making.

(a) 4 machines; (b) 8 machines

Using the data below :

Data :

Direct labour needed to make first machine 1,000 hours

Learning curve = 90%

Direct labour cost Rs. 15/per hour

Direct materials cost Rs. 1,50,000

Fixed cost for either size orders — Rs. 60,000.

Solution

Learning curve (90%) for calculating the expected average unit cost

<i>Cumulative order size</i>	<i>Cum. Avg. hours</i>	<i>Cum. Avg. labour cost @ Rs. 15 per hr.</i>	<i>Materials</i>	<i>Overheads</i>	<i>Total</i>
1	1,000	Rs. 15,000	—	—	—
2	(90% of 1,000) = 900	13,500	—	—	—
(a) 4	(90% of 900) = 810	12,150	1,50,000	60,000	2,22,150
(b) 8	(90% of 810) = 729	10,935	1,50,000	30,000	1,90,935



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Illustration

XYZ & Co. has given the following data :

80% Average – Time Curve

<i>Cumulative Units (x)</i>	<i>Average Hours</i>	<i>Total Hours</i>	<i>Marginal Hours</i>
1	100	100	100
2	80	160	60
3	?	?	?
4	64	256	?

Required : Fill in the blanks

Solution

We know that : $Y = ax^b$

where, Y = average time per unit

a = total time for the first unit

x = the cumulative number of units manufactured.

b = the learning index

The learning coefficient, b, is determined as follows :

$$b = \frac{\log(1 - \% \text{ decrease})}{\log^2} = \frac{\log(1 - 0.20)}{\log^2} = \frac{\log 0.80}{\log^2} = \frac{9.90309 - 10}{0.3010} = -0.322$$

By inserting the proper values into the equation, average time for any given number of units can be computed. For three cumulative units, we find that

$$Y = 100 (3)^{-0.322} = 100 \cdot \frac{1}{3^{.322*}}$$

$$= 100 \cdot \frac{1}{1.4244} = 100 (.7020) \text{ or } 70.2$$

$$*\text{Log } 3 = 0.47712$$

$$\underline{0.32349}$$

$$\underline{0.15363} \quad \text{Antilog} = 1.4244$$



Now it is possible to complete the table :

80% Average-Time Curve

<i>Cumulative Units</i>	<i>Average Time</i>	<i>Total Time</i>	<i>Marginal Time</i>
1	100	100	100 (100 – 0)
2	80	160 (80 × 2)	60 (160 – 100)
3	70.2	210.6 (70.2 × 3)	50.6 (210.6 – 160)
4	64	256 (64 × 4)	45.4 (256 – 210.6)

Each of the values in the table can be determined by means of the formulas :

Units	Average time = ax^b
1	$100 (1)^{-.322} = 100 \cdot \frac{1}{1^{.322}} = 100$
2	$100 (2)^{-.322} = 100 \cdot \frac{1}{2^{.322}} = 80$
3	$100 (3)^{-.322} = 100 \cdot \frac{1}{3^{.322}} = 70.2$
4	$100 (4)^{-.322} = 100 \cdot \frac{1}{4^{.322}} = 64$

Units	Total Time = ax^{b+1}
1	$100 (1)^{-.322 + 1} = 100 (1)^{.678} = 100$
2	$100 (2)^{-.322 + 1} = 100 (2)^{.678} = 160$
3	$100 (3)^{-.322 + 1} = 800 (3)^{.678} = 210.6$
4	$100 (4)^{-.322 + 1} = 100 (4)^{.678} = 256$

Marginal Time = $a(b + 1)x^b$

Units	Formula (continuous)	(From table above)
1	$100 (-.322 + 1) (1)^{-.322} = 100 (.678) 1 \div 1^{.322} = 67.8$	100
2	$100 (-.322 + 1) (2)^{-.322} = 100 (.678) 1 \div 2^{.322} = 54.2$	60
3	$100 (-.322 + 1) (3)^{-.322} = 100 (.678) 1 \div 3^{.322} = 47.6$	50.6
4	$100 (-.322 + 1) (4)^{-.322} = 100 (.678) 1 \div 4^{.322} = 43.4$	45.4



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Illustration

A customer has asked your company to prepare a bid on supplying 800 units of a new product. Production will be in batches of 100 units. You estimate that costs for the first batch of 100 units will average Rs. 100 a unit. You also expect that a 90% learning curve will apply to the cumulative labour cost on this contract.

Required : (a) Prepare an estimate of the labour costs of fulfilling this contract.

(b) Estimate the incremental labour cost of extending the production run to produce an additional 800 units.

(c) Estimate the incremental labour cost of extending the production run from 800 units to 900 units.

Solution

(a) Average cost decreases by 10 per cent every time when the cumulative production doubles.

Therefore,

$$\text{Average cost of first 200 units} = 0.9 \times \text{Avg. cost of 100 units}$$

$$\text{Average cost of first 400 units} = 0.9 \times \text{Avg. cost of 200 units}$$

$$\text{Average cost of first 800 units} = 0.9 \times \text{Avg. cost of 400 units}$$

Combining these, we find that average cost of the first 800 units

$$= 0.9 \times 0.9 \times 0.9 \times \text{Rs.}100 = \text{Rs.} 72.90$$

$$\text{Total cost} = 800 \times \text{Rs.} 72.90 = \text{Rs.} 58,320$$

(b) Average cost of the first 1,600 units = $0.9 \times \text{Rs.} 72.9 = \text{Rs.} 65.61$

$$\therefore \text{Total cost of 1,600 units} = 1,600 \times \text{Rs.} 65.61 = \text{Rs.} 1,04,976$$

Additional cost of 2nd 800 units

$$= \text{Rs.} 1,04,976 - \text{Rs.} 58,320 = \text{Rs.} 46,656 \text{ or Rs. } 58.32 \text{ per unit}$$

(c) Because this increase will not increase cumulative production to twice of some figure we already have, formula has to be used :

$$Y = ax^b \text{ where } b = .0458 \div 0.301 = - 0.15216$$

$$\log ax \text{ cost} = \log 10,000 - 0.15216 \log 9$$

$$= 4 - 0.1452 = 3.8548$$

$$\text{Average cost} = \text{Rs.} 71.5833 \text{ per unit}$$

$$\text{Total cost} = 900 \times 71.5833 = \text{Rs.} 64,425$$

$$\text{Incremental cost} = \text{Rs.} 64,425 - \text{Rs.} 58,320 = \text{Rs.} 6,105 \text{ or Rs. } 61.50 \text{ per unit.}$$



SUMMARY

- Learning Curve Effect applies only to direct labour costs and those variable overheads, which are direct function of labour hours of input. It does not apply to material costs, non-variable costs or items which vary with output.
- Incremental hours cannot be directly determined from the learning curve graph or formula, as the results are expressed in terms of cumulative average hours.

SELF-EXAMINATION QUESTIONS

1. What is learning curve. How the learning curve ratio to calculated ?
2. Explain the application of learning curve in business situations.