16

Simulation

LEARNING OBJECTIVES :

After studying this chapter, you should be able to :

- explain the term simulation and reasons for using simulation;
- identify the steps in the simulation process;
- review the reasons why simulation may be preferable to other decision models;
- simulate a situation based on real data;
- make useful contribution in (investment decision or capital budgeting) through simulation;
- establish how simulation permits study of a system under controlled conditions.

16.1 INTRODUCTION

For various managerial problems discussed so far, we have been able to find a mathematical solution. However, in each of these cases the problem (e.g. linear programming or CPM/PERT) was simplified by certain assumptions so that the appropriate mathematical techniques could be employed. There are certain managerial situations which are so complex that mathematical solution is impossible given the current state of the art in mathematics. In many other cases, the solutions which result from simplifying assumptions are not suitable for the decision makers. In these cases, simulation offers a reasonable alternative.

16.2 WHAT IS SIMULATION?

Simulation is a quantitative procedure which describes a process by developing a model of that process and then conducting a series of organised trial and error experiments to predict the behavior of *the process over time.* Observing the experiments is much like observing the process in operation. To find how the real process would react to certain changes, we can introduce these changes in our model and simulate the reaction of the real process to them. For example, in designing an airplane, the designer can build a scale model and observe its behavior in a wind tunnel. In simulation, we build mathematical models which we cannot solve and run them on trial data to simulate the behavior of the system.

16.2.1 Steps in the simulation process : All simulations vary in complexity from situation to situation. However, in general, one would have to go through the following steps:-

1. Define the problem or system you intend to simulate.
2. Formulate the model you intend to use.
3. Test the model, compare its behavior with the behavior of the actual problem environment.
4. Identify and collect the data needed to test the model.
5. Run the simulation.
6. Analyze the results of the simulation and, if desired, change the solution you are evaluating.
7. Rerun the simulation to test the new solution.
8. Validate the simulation, that is, increase the chances that any inferences you draw about the real situation from running the simulation will be valid.

It may be noted that the fundamental principle of simulation is to make use of some device that can represent the phenomenon of a real-life system to enable us to understand the properties, behavior and functional characteristics of the system. The device used can be any convenient means-a mathematical formula or a physical model.

The aircraft simulator to train pilots on new models of aircraft represents the use of physical model as a means of experimentation. The mathematical models of real-life situations in investment analysis, scheduling or inventory control can be experimented; these are known as symbolic simulation models. These models can be either deterministic or probabilistic. The deterministic models can provide answers to 'what if' type of questions in business problems. The probabilistic simulation models deal with random phenomenon and the method of simulation applied is known as Monte Carlo simulation.

16.3 MONTE CARLO SIMULATION

The Monte Carlo method is the earliest method of simulation; the method employs random numbers and is used to solve problems that depend upon probability, where physical experimentation is impracticable and the creation of a mathematical formula impossible. It is method of Simulation by the sampling technique. That is, first of all, the probability distribution of the variable under consideration is determined; then a set of random numbers is used to generate a set of values that have the same distributional characteristics as the actual experience it is deviced to simulate. The steps involved in carrying out Monte Carlo Simulation are:

(i) Select the measure of effectiveness of the problem, that is, what element is used to measure success in improving the system modelled. This is the element one wants to maximise or minimise. For example, this might be idle time of a service facility, or inventory shortages per period etc.

(ii) Identify the variables which influence the measure of effectiveness significantly. For example, the number of service facilities in operation or the number of units in inventory and so on.

(iii) Determine the proper cumulative probability distribution of each variable selected under step (ii). Plot these, with the probability on the vertical axis and the values of variables on horizontal axis.

(iv) Get a set of random numbers.

(v) Consider each random number as a decimal value of the cumulative probability distribution. With the decimal, enter the cumulative distribution plot from the vertical axis. Project this point horizontally, until it intersects cumulative probability distribution curve. Then project the point of intersection down into the vertical axis.

(vi) Record the value (or values if several variables are being simulated) generated in step (v) into the formula derived from the chosen measure of effectiveness. Solve and record the value. This value is the measure of effectiveness for that simulated value.

(vii) Repeat steps (v) and (vi) until sample is large enough for the satisfaction of the decision maker.

In assigning a set of random numbers, we have to decide on the entire range of random numbers. If the cumulative probabilities are in two digits the range of random numbers to be assigned are 00 to 99; and if in three digits, the range is from 000 to 999, and so on.

In view of the enormous computations involved, computer is usually a necessary adjunct though below we shall deal with quite a few simple simulation problems towards its exposition.

Example : (For finding the value π experimentally by simulation):

In the figure below is shown the arc of a circle of a unit radius in the first quadrant. Also shown is a square OABC of side of one unit.

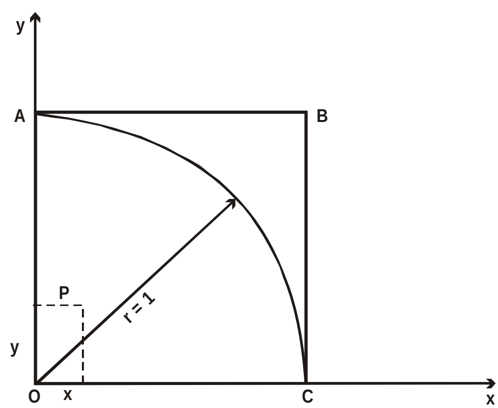

Fig. 1

The equation of the circle is given by $x^2 + y^2 = 1$. Two* random numbers, each less than unity (viz., x = 0.1906 and y = 0.3698) are picked up and the point P corresponding to these is shown plotted. Obviously, if $x^2 + y^2 < 1$, P is inside or on the circle but if $x^2 + y^2 \geq 1$, P is beyond the circle but within the square.

In this manner, hundreds or thousands of pairs of random numbers are picked up and it is ascertained if the points corresponding to them fall in/on the arc or beyond the square. Suppose that n out of the total of N points fall in/on the arc.

Now the area enclosed by the arc = $\frac{\pi}{4}1^2 = \frac{\pi}{4}$ and the area enclosed by the square = $1^2=1$

$$\frac{\pi/4}{1} = \frac{n}{N} \text{ or } \pi = \frac{4n}{N}$$

The experiment value of π is thus obtained. Obviously larger the sample size N, closer shall be the true value of π.

This way, the Monte Carlo methods can be applied to solve complex and stochastic/deterministic queuing, inventory control, production scheduling, etc. problems. The methods have since recently been also applied in moon landing and studying galactic collisions, atomic behavior and in military.

Illustration

Nine villages in a certain administrative area contain 720, 130, 150, 240, 960, 100, 52, 35, 532 fields respectively. Make a random selection of six fields using the random number tables.

Solution

1 *Village No.*	2 *No. of Villages*	3 *Cumulative No. Of Villages*	4 *Random nos. fitted against the village As per column 3.*
1	720	720	0091,0684
2	130	850	0839,0814
3	150	1000	
4	240	1240	
5	960	2700	
6	100	2300	
7	52	2352	
8	36	2388	2377
9	532	2920	2471

* See the Appendix on an excerpt of random nos. at the end of the chapter. How to use it is explained by example below it. However, the nos. we are using have been taken from the other excerpt.

The first random no. picked up from the random no. tables is 2377. Since it is immediately $\leq$ 2388 in Col. 3; it is fitted against village 8 in Col. 4. The next random no. 5997 is $\geq$ 2920 in Col. 4; therefore, it is dropped. In this manner, the following random nos. are, either fitted in Col. 4 or dropped; 8269 (D= drop), 8385 (D), 6198 (D), 0091 (F_l) = fitted against village no. 1 col. 4) 4829 (D), 3322 (D), 0684 (F_l), 3267 (D), 8209 (D), 5166 (D), 0839 (F2), 0814 (F2), 7409 (D), 2471 (F9). We stop here because 6 fields have been selected, two each from village nos. I and 2 and one each from village nos. 8 and 9.

Illustration

Frontier Bakery keeps stock of a popular brand of cake. Daily demand based on past experience is as given below:-

Experience indicates

Daily demand	:	0	15	25	35	45	50
Probability	:	.01	.15	.20	.50	.12	.02

Consider the following sequence of random numbers:-

R. No. 48, 78, 09, 51, 56, 77, 15, 14, 68, 09

Using the sequence, simulate the demand for the next 10 days.

Find out the stock situation if the owner of the bakery decides to make 35 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data.

Solution

According to the given distribution of demand, the random number coding for various demand levels is shown in Table below :

Random Number Coding

Demand	*Probability*	*Cum. Prob.*	*Random nos. fitted*
0	0.01	0.0	00
15	0.15	0.16	01-15
25	0.20	0.36	16-35
35	0.50	0.86	36-85
45	0.12	0.98	86-97
50	0.02	1.00	98-99

The simulated demand for the cakes for the next 10 days is given in the Table below. Also given in the table is the stock situation for various days in accordance with the bakery decision of making 35 cakes per day.

Determination of Demand and Stock Levels

Day	*Random Number*	*Demand*	*Stock*
1	48	35	
2	78	35	–
3	09	15	20
4	51	35	20
5	56	35	20
6	77	35	20
7	15	15	40
8	14	15	60
9	68	35	60
10	09	15	80

Expected demand = 270/10 = 27 units per day.

Illustration

A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopped to 204 mopped, whose probability distribution is as given below:-

Production per day	*Probability*
196	0.05
197	0.09
198	0.12
199	0.14
200	0.20
201	0.15
202	0.11
203	0.08
204	0.06

The finished mopeds are transported in a specially designed three storeyed lorry that can accommodate only 200 mopped. Using the following 15 random numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10, simulate the process to find out:

(i) What will be the average number of mopeds waiting in the factory?

(ii) What will be the average number of empty spaces on the lorry?

Solution

The random numbers are established as in Table below:

Production Per day	*Probability*	*Cumulative Probability*	*Random Number*
196	0.05	0.05	00-04
197	0.09	0.14	05-13
198	0.12	0.26	14-25
199	0.14	0.40	26-39
200	0.20	0.60	40-59
201	0.15	0.75	60-74
202	0.11	0.86	75-85
203	0.08	0.94	86-93
204	0.06	1.00	94-99

Based on the 15 random numbers given we simulate the production per day as above in table 2 below.

Random No.		*Estimated Production Per day*	*No. of mopeds waiting*				*No. of empty spaces in the lorry*
			Opening Balance	*Current Excess production*	*Current short production*	*Total waiting*	
1	82	202	–	2	–	2	–
2	89	203	2	3	–	5	–
3	78	202	5	2	–	7	–
4	24	198	7	–	2	5	–
5	53	200	5	–	–	5	–
6	61	201	5	1	–	6	–
7	18	198	6	–	2	4	–
8	45	200	4	–	–	4	–
9	04	196	4	–	4	0	–
10	23	198	0	–	2	0	2
11	50	200	0	–	–	–	–
12	77	202	0	2	–	2	–
13	27	199	2	–	1	1	–
14	54	200	1	–	–	1	–
15	10	197	1	–	3	–	2
					Total	42	4

Average number of mopeds waiting $= \frac{42}{15} = 2.80$

Average number of empty spaces in lorry $= \frac{4}{15} = 0.266$

(Note : Some of the authors have solved this problem without adjusting excess/short production. However, we feel that above approach in better.)

Illustration

Ramu and Raju are workers on a two-station assembly line. The distribution of activity times at their stations is as follows:–

Time in Sec.	*Time frequency For Ramu*	*Time frequency for Raju*
10	4	4
20	6	5
30	10	6
40	20	7
50	40	10
60	11	8
70	5	6
80	4	4

(a) Simulate operation of the line for eight times. Use the random numbers given below:

Operation 1		*Operation 2*	
14	61	36	97
01	82	76	41
96	00	55	56
44	03	25	34

(b) Assuming Raju must wait until Ramu completes the first item before starting work, will he have to wait to process any of the other eight items? Explain your answer, based upon your simulation.

Solution

Cumulative frequency distribution for Ramu is derived below. Also fitted against it are the eight given random numbers. In parentheses are shown the serial numbers. of random numbers.

10	4	01(2)	00(7)	03(8)
20	10			
30	20	14(I)		
40	40			
50	80	44(4)	61(5)	
60	91	82(6)		
70	96	95(3)		
80	100			

Thus the eight times are: 30, 10, 70, 50, 50, 60, 10 and 10 respectively.

Likewise the eight times for Raju are derived from his cumulative distribution below:-

1	2 *Frequency*	3 *Cumulative*	4 2 × col. 3		
10	4	4	8		
20	5	9	18	13(7)	
30	6	15	30	25(4)	
40	7	22	44	36(1), 41(6),	34(8)
50	10	32	64	55(3)	
60	8	40	80	76(2)	
70	6	46	92		
80	4	50	100	97(5)	

(Note that cumulative frequency has been multiplied by 2 in Co. 4 in order that all the given random numbers are utilised).

Thus, Raju's times are 40,60,50,30,80,40,20 and 40 seconds respectively.

Ramu's and Raju's times are displayed below to observe for waiting time, if any.

1 *Ramu*	2 *Cum. time*	3 *Raju initial*	4 *Raju's cumulative time with 30 seconds included*
30	30	40	70
10	40	60	130
70	110	50	180
50	160	30	210
50	210	80	290
60	270	40	330
10	280	20	350
10	290	40	390

Since col. 4 is consistently greater than col. 2 no subsequent waiting is involved.

Illustration

Dr. STRONG is a dentist who schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time needcd to complete the work :

Category	Time required	*Probability of category*
Filling	45 minutes	0.40
Crown	60 minutes	0.15
Cleaning	15 minutes	0.15
Extraction	45 minutes	0.10
Checkup	15 minutes	0.20

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8.00 a.m. Use the following random numbers for handling the above problem:

40 82 11 34 25 66 17 79

Solution

If the numbers 00-99 are allocated in proportion to the probabilities associated with each category of work, then various kinds of dental work can be sampled, using random number table:

Type	*Probability*	*Random Numbers*
Filling	0.40	00-39
Crown	0.15	40-54
Cleaning	0.15	55-69
Extraction	0.10	70-79
Checkup	0.20	80-99

Using the given random numbers, a work sheet can now be completed as shown on next page:

Future Events

Patient	*Scheduled Arrival*	*R. No.*	*Category*	*Service Time*
1	8.00	40	Crown	60 minutes
2	8.30	82	Checkup	15 minutes
3	9.00	11	Filling	45 minutes
4	9.30	34	Filling	45 minutes
5	10.00	25	Filling	45 minutes
6	10.30	66	Cleaning	15 minutes
7	11.00	17	Filling	45 minutes
8	11.30	79	Extraction	45 minutes

Now, let us simulate the dentist's clinic for four hours starting at 8.00 A.M.

Status

Time	*Event*	*Number of the patient Being served (time to go)*	*Patients waiting*
8.00	Ist patient arrives	Ist (60)	–
8.30	2nd arrives	Ist (30)	2nd
9.00	Ist departs		
9.00	3rd arrives	2nd(15)	3rd
9.15	2nd departs	3rd(45)	
9.30	4th arrives	3rd (30)	4th
10.00	3rd depart		
	5th arrives	4th (45)	5th
10.30	6th arrives	4th (15)	5th &6th
10.45	4th departs	5th (45)	6th
11.00	7th arrives	5th (30)	6th & 7th
11.30	5th departs		
	8th arrives	6th (15)	7th & 8th
11.45	6th departs	7th (45)	8th
12.00	End	7th (30)	8th
12.30	–	8th (45)	–

The dentist was not idle during the entire simulated period:

The waiting times for the patients were as follows:

Patient	*Arrival*	*Service Starts*	*Waiting (Minutes)*
1	8.00	8.00	0
2	8.30	9.00	30
3	9.00	9.15	15
4	9.30	10.00	30
5	10.00	10.45	45
6	10.30	11.30	60
7	11.00	1l.45	45
8	11.30	12.30	60
		Total	285

The average waiting time of a patient was $= \frac{285}{8}$ = 35.625 minutes.

Illustration

A bakery chain delivers cakes to one of its retail stores each day. The number of cakes delivered each day is not constant but has the following distribution:-

Cakes delivered per day	*Probability*
10	0.05
11	0.10
12	0.15
13	0.35
14	0.20
15	0.10
16	0.05

The number of customers desiring cakes each day has the distribution:

No. of customers	*Probability*
5	0.10
6	0.15
7	0.20
8	0.40
9	0.10
10	0.05

Finally, the probability that a customer in need of cakes wants 1, 2, or 3 cakes is described by

Cakes to a customer	*Probability*
1	0.40
2	0.40
3	0.20

Estimate by Monte Carlo methods the average number of cakes left over per day and the average number of sales per day owing to lack of cakes. Assume that left over cakes are given away at the end of each day.

Solution

The first cumulative probability distribution is derived below:

Cumulative probability distribution of cakes delivered per day

Cakes per day	*Cumulative Probability*	*Ten random nos. fitted in*
10	0.05	.0153(9)
11	0.15	.1342(6)
12	0.30	.2291(4)
13	0.65	.5878(2),.4391(5),.5210(7),.3411(8)
14	0.85	.8136(1),.7923(10).
15	0.95	
16	1.00	.9655(3)

Thus, the cakes delivered in each of the 10 days are: 14, 13, 16, 12, 13, 1l, 13, 13, 10, 14.

Likewise the no. of customers per day are derived below from the cumulative probability distribution.

No. of customers	*Cumulative probability*	Ten random nos. fitting in
5	0.10	.0906(9)
6	0.25	.2416(4),.1934(10)
7	0.45	.3501(l),.3396(3),.2587(5),.3072(6)
8	0.85	.5054(2)
9	0.95	.8511(7), .8698(8)
10	1.00	

Thus the number of customers for the ten days are 7, 8, 7, 6, 7, 7, 9, 9, 5, 6, and the total no. of customers = 71.

Thus cakes delivered per day and the no. of customers per day are laid out below.

Days

	1	2	3	4	5	6	7	8	9	10
No. of Customers desiring cake on nth Day	7	8	7	6	7	7	9	9	5	6
No. of cakes desired by each customer	1 2 3 3 2 1 2									
No. of cakes wanted	14									
No. of cakes delivered	14	13	16	12	13	11	13	13	10	14
Lift overs	NIL									
Shortage	NIL									

This table has to be filled in by selecting no. of cakes demanded by each customer from the third cumulative distribution derived below. In the first day, there are seven customers. Thus 7 random nos. have been fitted in the cumulative probability distribution below. These, in turn, have been put under day 1 of the above table. The student may fill in the remaining columns himself, as an exercise.

Cumulative probability distribution of the no. of cakes wanted by the customers.

(7 random nos. fitted in below give the entries under day of 1 for the above table.)

No. of Cakes	*Cumulative probability*	*7 random numbers fitted in for day 1*
1	0.40	.23(1), .00(6)
2	0.80	.49(2), .61(5), .48(7)
3	1.00	.82(3), .84(4)

Thus the no. of cakes wanted by the 7 customers in the first day are 1, 2, 3, 3, 2, 1, 2. These are entered under day 1. Left-overs and shortages are also computed which may be done by student for the remaining columns.

16.4 SIMULATION AND INVENTORY CONTROL

The Monte Carlo simulation is widely used to solve inventory problems characterised by uncertainty of demand and lead time. The distribution of demand during the lead time can be obtained from an empirical analysis of past data or by computer simulation using random numbers. The cumulative probability distribution of demand during the lead time is used as a basis to determine the annual inventory costs and stock-out costs for different levels of the safety stock. The management can experiment the effect of various inventory policies by using simulation and finally select an optimum inventory policy. The purpose of simulation, as applied to inventory, is to facilitate the management in selecting an inventory policy that will result in minimum annual inventory costs-ordering, carrying and stock-out costs. The application of simulation to inventory control is explained below with the help of an example.

The distribution of demand during the lead time and the distribution of the lead time are set out in Tables 1 and 2.

Table 1 : Distribution of Demand during Lead time

Quantity demanded during lead time	*Probability*
0	0.10
1	0.45
2	0.30
3	0.15

Table 2 : Distribution of Lead time

Lead time (weeks)	*Probability*
2	0.20
3	0.65
4	0.15

Suppose, the management wants to ascertain the annual inventory costs, if they wish to reorder when the quantity on hand is 6 units and order each time a quantity of 12 units. Assume that all orders are delivered for the entire quantity. The cost of ordering is Rs. 120, and the cost of holding the inventory in stock is Rs. 5 per unit per week. Further, the management has found that when it runs out-of-stock, it costs Rs. 75 per unit.

Table 5 illustrates the manual simulation for 15 weeks. Before proceeding with simulating the demand for each week, we calculate first the cumulative probability and assign random numbers for each value of the two variables-demand and lead time. These are set out in Tables 3 and 4.

Table 3

Demand	*Cumulative Probability*	*Random No.s assigned*
0	0.10	00 to 09
1	0.55	10 to 54
2	0.85	55 to 84
3	1.00	85 to 99

Table 4

Lead Title	*Cumulative Probability*	*Random Nos. assigned*
2	0.20	00 to 19
3	0.85	20 to 84
4	1.00	85 to 99

We assume that the stock on hand at the start of the simulation process is 10 units. Further, we also assume that all orders are placed at the beginning of the week and all deliveries against orders are received at the beginning of the week.

The simulated demand for work 1 is 1 unit (corresponding to the random number 49 in Table 3), and the closing inventory is 9 units. The inventory-carrying cost works out to 9 × 5 = Rs. 45 (Rs. 5 per unit per week). The inventory-carrying costs are calculated for the remaining weeks in the same way. At the end of week 4, the closing inventory is 6 units (reorder point). So an order is placed at the beginning of week 5 for a quantity of 12 units (ordering quantity). The simulated lead time is 3 weeks (corresponding to random number 84 in Table 4). This order quantity is, therefore, received at the beginning of week 8. We notice from Table 5 that another order is placed at the beginning of week 12, when the stock on hand is 6 units, which is equal to the reorder point. Again, the simulated lead time is 3 weeks. Before this quantity is received At the beginning of week 15, we find that the quantity available at the beginning of week 14 is 1 unit and the simulated demand for that week is 2 units, giving rise to a stock out of 1 unit. The stock-out cost is Rs. 75.

Table 5. Simulation of Demand and Lead Time for 15 weeks.

Week	*Stock on hand beginning of week*	*Demand*		*Quantity received*	*Stock on hand end of week*	*Inventory carrying costs*	*Stock out*		*Lead Time*	
		Random No.	*Quantity demanded*				*Quantity*	*Costs*	*Random No.*	*Lead Time period*
1	10	49	1	–	9	45				
2	9	67	2	–	7	35				
3	7	06	0	–	7	35				
4	7	30	1	–	6	30				
5	6	95	3	–	3	15			84	3
6	3	01	0	–	3	15				
7	3	10	1	–	2	10				
8	2	70	2	12	12	60				
9	12	80	2	–	10	50				
10	10	66	2	–	8	40				
11	8	69	2	–	6	30				
12	6	76	2	–	4	20			79	3
13	4	86	3	–	1	5				
14	1	56	2	–	–	–	1	75		
15	–	84	2	12	10	50				

For the simulated period of 15 weeks, the total inventory costs are:-

Inventory carrying costs	=	Rs. 440
Ordering costs (2 orders × 20)	=	Rs. 240
Stock-out costs (1 stock-out)	=	Rs. 75
Total	=	Rs. 735

By simulating over a period of 2 to 3 years (i.e. 100 or 150 weeks), we can obtain a more accurate picture of the annual inventory costs.

By varying the values of the variables (the ordering quantity and the reorder point), the management can find out the effect of such a policy in terms of the annual inventory costs. In other words, simulation permits the management to evaluate the effects of alternate inventory policies. Also, if there is a change in the ordering, stock-out and inventory-carrying costs, their impact on the annual inventory costs can be determined by using simulation.

16.5 MISCELLANEOUS ILLUSTRATIONS

Illustration

A company manufactures 30 items per day. The sale of these items depends upon demand which has the following distribution:

Sales (Units)	*Probability*
27	0.10
28	0.15
29	0.20
30	0.35
31	0.15
32	0. 05

The production cost and sale price of each unit are Rs. 40 and Rs. 50 respectively. Any unsold product is to be disposed off at a loss of Rs. 15 per unit. There is a penalty of Rs. 5 per unit if the demand is not met.

Using the following random numbers estimate total profit / loss for the company for the next 10 days: 10, 99, 65, 99, 95, 01, 79, 11, 16, 20

If the company decides to produce 29 items per day, what is the advantage or disadvantage to the company?

Solution

First of all, random numbers 00-99 are allocated in proportion to the probabilities associated with the sale of the items as given below:

Sales (units)	Probability	Cumulative probability	Random numbers assigned
27	0.10	0.10	00-09
28	0.15	0.25	10-24
29	0.20	0.45	25-44
30	0.35	0.80	45-79
31	0.15	0.95	80-94
32	0.05	1.00	95–99

Let us now simulate the demand for next 10 days using the given number in order to estimate the total profit/loss for the company. Since the production cost of each item is Rs. 40 and sale price is Rs. 50, therefore the profit per unit of the sold item will be Rs. 10. There is a loss of Rs. 15 per unit associated with each unsold unit and a penalty of Rs. 5 per unit if the demand is not met. Accordingly, the profit / loss for next ten days are calculated in column (iv) of the table below if the company manufactures 30 items per day.

(i) Day	(ii) Random number	(iii) Estimated sale	(iv) Profit/Loss per day when production = 30 items per day	(v) Profit/loss per day when production = 29 items per day
1	10	28	28 × Rs. 10 – 2 × Rs. 1.5=250	28 × Rs. 10 – 1 × Rs. 15=265
2	99	32	30 × Rs. 10 – 2 × Rs.5=290	29 × Rs. 10 – 3 × Rs. 5 = 275
3	65	30	30 × Rs. 1 0=300	29 × Rs.10 – 1 × Rs.5=285
4	99	32	30 × Rs. 10 – 2 × Rs.5=290	29 × Rs. 10 – 3 × Rs.5=275
5	95	32	30 × Rs. 10 – 2 × Rs.5=290	29 × Rs. 10 – 3 × Rs.5=275
6	01	27	27 × Rs. 10 – 3 × Rs. 1 5=225	27 × Rs. 10 – 2 × Rs. 5=240
7	79	30	30 × Rs.10 = 300	29 × Rs. 10 – 1 × Rs. 5=285
8	11	28	28 × Rs. 10 – 2 × Rs. 1 5=250	28 × Rs. 1 0 -1 × Rs. 15=265
9	16	28	28 × Rs.10 – 2 × Rs.15=250	28 × Rs.10 - 1× Rs.15= 265
10	20	28	28 × Rs. 10 – 2 × Rs. 1 5=250	28 × Rs. 1 0 - 1 × Rs. 1 5=265
Total Profit =			Rs.2695	Rs.2695

The total profit for next 10 days will be Rs. 2695 if the company manufactures 30 items per day. In case, the company decides to produce 29 items per day, then the profit of the company for next 10 days is calculated in column (v) of the above table. It is evident from this table that there is no additional profit or loss if the production is reduced to 29 items per day since the total profit remains unchanged i.e. Rs. 2695.

Illustration

An Investment Corporation wants to study the investment projects based on three factors : market demand in units, price per unit minus cost per unit, and the investment required. These factors are felt to be independent of each other. In analysing a new consumer product, the corporation estimates the following probability distributions:

Annual demand Units	Probability	(Price-Cost) per unit Rs.	Probability	Investment Rs.	Required Probability
25,000	0.05	3.00	0.10	27,50,000	0.25
30,000	0.10	5.00	0.20	30,00,000	0.50
35,000	0.20	7.00	0.40	35,00,000	0.25
40,000	0.30	9.00	0.20		
45,000	0.20	10.00	0.10		
50,000	0.10				
55,000	0.05				

Using simulation process, repeat the trial 10 times, compute the return on investment for each trial, taking these three factors into account. Approximately, what is the most likely return? Use the following random numbers for annual demand, (price-cost) and the investment required:

28, 57, 60, 17, 64, 20, 27, 58, 61, 30; 19, 07, 90, 02 57,

28, 29, 83, 58, 41, 18, 67, 16, 71, 43, 68, 47, 24, 19, 97

Solution

The yearly return can be determined by the formula:

$$\text{Return (R)} = \frac{(\text{Price} - \text{Cost}) \times \text{Number of units demanded}}{\text{Investment}}$$

The results of the simulation are shown in the table given below:

Trials	Random Number of Demand	Simulated Demand ('000)	Random Number for profit (Price-Cost) per unit	Simulated Profit	Random number for investment	Simulated investment ('000)	Simulated Return (%) Demand × Profit per Investment
1	28	35	19	5.00	18	2750	6.36
2	57	40	07	3.00	67	3000	4.00
3	60	40	90	10.00	16	2750	14.55
4	17	35	02	3.00	71	3000	3.50
5	64	40	57	7.00	43	3000	9.33
6	20	35	28	5.00	68	3000	5.83
7	27	35	29	5.00	47	3000	5.83
8	58	40	83	9.00	24	2750	13.10
9	61	40	58	7.00	19	2750	10.18
10	30	35	41	7.00	97	3500	7.00

Result: Above table shows that the highest likely return is 14.6% which is corresponding to the annual demand of 40,000 units resulting a profit of Rs. 10 per unit and the required investment will be Rs. 27,50,000.

Illustration

The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous day, the rain distribution is given by:

Event	*Probability*
No rain	0.50
1 cm. rain	0.25
2 cm. rain	0.15
3 cm. rain	0.05
4 cm. rain	0.03
5 cm. rain	0.02

If it did not rain the previous day, the rain distribution is given by:

Event	Probability
No rain	0.75
1 cm. rain	0.15
2 cm. rain	0.06
3 cm. rain	0.04

Simulate the city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random numbers:

67 63 39 55 29 78 70 06 78 76

for simulation. Assume that for the first day of the simulation it had not rained the day before.

Solution

The numbers 00-99 are allocated in proportion to the probabilities associated with each event. If it rained on the previous day, the rain distribution and the random number allocation are given below:

Event	Probability	Cumulative Probability	Random numbers assigned
No rain	0.50	0.50	00-49
1 cm. rain	0.25	0.75	50-74
2 cm. rain	0.15	0.90	75-89
3 cm. rain	0.05	0.95	90-94
4 cm. rain	0.03	0.98	95-97
5 cm. rain	0.02	1.00	98-99

Table I : Rain on previous day

Similarly, if it did not rain the previous day, the necessary distribution and the random number allocation is given below:

Event	*Probability*	*Cumulative Probability*	*Random numbers assigned*
No rain	0.75	0.75	00-74
1 cm. rain	0.15	0.90	75-89
2 cm. rain	0.06	0.96	90-95
3 cm. rain	0.04	1.00	96-99

Table 2: No rain on previous day

Let us now simulate tile rain fall for 10 days using the given random numbers. For tile first day it is assumed that it had not rained the day before:

Day	*Random Numbers*	*Event*	
1	67	No rain	(from table 2)
2	63	No rain	(from table 2)
3	39	No rain	(from table 2)
4	55	No rain	(from table 2)
5	29	No rain	(from table 2)
6	78	1 cm. rain	(from table 2)
7	70	1 cm. rain	(from table 1)
8	06	No rain	(from table 1)
9	78	1 cm. rain	(from table 2)
10	76	2 cm. rain	(from table 1)

Hence, during the simulated period, it did not rain on 6 days out of 10 days. The total rain fell during the period was 5 cm.

Illustration

The output of a production line is checked by an inspector for one or more of three different types of defects, called defects A, B and C. If defect A occurs, the item is scrapped . If defect B or C occurs, the item must be reworked. The time required to rework a B defect is 15 minutes and the time required to rework a C defect is 30 minutes. The probabilities of an A, B and C defects are .1 5, .20 and .10 respectively. For ten items coming off the assembly line, determine tile number of items without any defects, the number scrapped and the total minutes of rework time. Use the following random numbers,

RN for defect A

48 55 9 1 40 93 01 83 63 47 52

RN for defect B

47 36 57 04 79 55 10 13 57 09

RN for defect C

82 95 1 8 96 20 84 56 11 52 03

Solution

The probabilities of occurrence of A, B and C defects are 0.15, 0.20 and 0.I0 respectively. So, tile numbers 00-99 are allocated in proportion to the probabilities associated with each of the three defects

Defect A		Defect B		Defect C	
Exists	Random numbers assigned	Exists ?	Random numbers assigned	Exists ?	Random numbers assigned
Yes	00-14	Yes	00-19	Yes	00-09
No	15-99	No	20-99	No	10-99

Let us now simulate the output of the assembly line for 10 items using the given random numbers in order to determine the number of items without any defect, the number of items scrapped and the total minutes of rework time required:

Item No.	RN for defect A	RN for defect B	RN for defect C	Whether any defect exists	Rework time (in minutes)	Remarks
1	48	47	82	None	–	–
2	55	36	95	None	–	–
3	91	57	18	None	–	–
4	40	04	96	B	15	–
5	93	79	20	None	–	–
6	01	55	84	A	–	Scrap
7	83	10	56	B	15	–
8	63	13	11	B	15	–
9	47	57	52	None	–	–
10	52	09	03	B,C	15+30=45	–

During the simulated period, 5 out of the ten items had no defects, one item was scrapped and 90 minutes of total rework time was required by 3 items.

Illustration

The management of ABC company is considering the question of marketing a new product. The fixed cost required in the project is Rs. 4,000. Three factors are uncertain viz. the selling price, variable cost and the annual sales volume. The product has a life of only one year. The management has the data on these three factors as under:

Selling Price Rs.	*Probability*	*Variable Cost Rs.*	*Probability*	*Sales volume (Units)*	*Probability*
3	0.2	1	0.3	2,000	0.3
4	0.5	2	0.6	3,000	0.3
5	0.3	3	0.1	5,000	0.4

Consider the following sequence of thirty random numbers:

81 32 60 04 46 31 67 25 24 10 40 02 39 68
08 59 66 90 12 64 79 31 86 68 82 89 25 11
98 16

Using the sequence (First 3 random numbers for the first trial, etc.) simulate the average profit for the above project on the basis of 10 trials.

Solution

First of all, random numbers 00–99 are allocated in proportion to the probabilities associated with each of the three variables as given under:

Selling Price Rs.	*Probabilities*	*Cumulative Probabilities*	*Random numbers assigned*
3	0.2	0.2	00-19
4	0.5	0.7	20-69
5	0.3	1.0	70-99
Variable cost (Rs.)			
1	0.3	0.3	00-29
2	0.6	0.9	30-89
3	0.1	1.0	90-99
Sales Volumes (Units)			
2,000	0.3	0.3	00-29
3,000	0.3	0.6	30-59
5,000	0.4	1.0	60-99

Let us now simulate the output of ten trials using the given random numbers in order to find the average profit for the project:

S. No.	*Random No.*	*Selling Price (Rs.)*	*Random No.*	*Variable Cost (Rs.)*	*Random No.*	*Sales Volume ('000 units')*
1	81	5	32	2	60	5
2	04	3	46	2	31	3
3	67	4	25	1	24	2
4	10	3	40	2	02	2
5	39	4	68	2	08	2
6	59	4	66	2	90	*5*
7	12	3	64	2	79	*5*
8	31	4	86	2	68	*5*
9	82	5	89	2	25	2
10	11	3	98	3	16	2

Profit = (Selling Price – Variable cost) × Sales Volume – Fixed cost.

Simulated profit in ten trials would be as follows:

S.No.	Profit		
1	(Rs.5 – Rs.2) × 5,000 units – Rs. 4,000	=	Rs. 11,000
2	(Rs.3 – Rs. 2) × 3,000 units – Rs. 4,000	=	Rs. – 1,000
3	(Rs. 4 – Re. 1) × 2,000 units – Rs. 4,000	=	Rs. 2,000
4	(Rs.3 – Rs. 2) × 2,000 units – Rs. 4,000	=	Rs. – 2,000
5	(Rs.4 – Rs. 2) × 2,000 units – Rs. 4,000	=	0
6	(Rs.4 – Rs. 2) × 5,000 units – Rs. 4,000	=	Rs. 6,000
7	(Rs.3 – Rs. 2) × 5,000 units – Rs. 4,000	=	Rs. 1,000
8	(Rs.4 – Rs. 2) × 5,000 units – Rs. 4,000	=	Rs. 6,000
9	(Rs. 5 – Rs. 2) × 2,000 units – Rs. 4,000	=	Rs. 2,000
10	(Rs.3 – Rs. 3) × 2,000 units – Rs. 4,000	=	Rs.-4,000
		Total	Rs.2 1,000

Therefore average profit per trial = $\frac{\text{Rs. } 21{,}000}{10} = \text{Rs. } 2{,}100$

Example on the use of random number table. Suppose we want five 3-digit random nos. We can enter any where in the table e.g. the last column, first 3 digits of 5 consecutive nos give us the answers : 413, I72, 207, 511, I72. Thus we can enter the table randomly but then on proceed serially.

16.6 RANDOM NUMBERS TABLE

8481	5016	0080	4376	2579	8293	5950	1048	0650	4135
0744	3447	6173	3288	6378	6704	0966	9986	5202	1728
5558	7239	2976	4836	6134	5120	1541	6514	3581	2079
9371	1463	2164	2301	3142	3866	8707	9980	2011	5111
3033	1660	6365	9054	1155	8844	4085	9589	2924	1725
1053	7320	6532	7214	8972	6466	1217	0100	1458	9416
4389	3504	4086	9434	0136	5695	6876	7937	5476	3396
2158	8854	9534	1196	4941	2697	7497	1149	1952	3482
6749	3676	4943	1406	8614	2060	6433	1660	8875	3194
2878	3447	4804	6761	5309	0636	0522	2004	3207	4684
0591	6549	2206	6185	6188	2649	2389	9483	0924	1389
1025	3438	0546	2545	1089	1280	6701	9742	3453	5573
4244	9217	1628	4524	0163	9895	9586	2083	8459	0644
1331	9032	1388	5661	0472	7128	1902	0343	7724	0528
8853	3490	2589	8744	1221	4667	8396	4779	9937	7206
5059	4192	6331	5485	5922	0982	9390	8993	3621	2602
0821	4340	3194	0118	4773	8668	1891	7989	9190	2296
5262	1746	7108	6496	2570	4243	5029	8949	4989	5008
1210	1858	9365	6562	0269	9923	1796	6626	8591	1990
3642	6629	5775	3219	8801	4047	6861	0765	2379	3494
9598	5322	3747	0363	5995	5504	6804	7033	0957	9556
3894	3173	2853	9312	2498	8878	4956	8748	6247	6673
3603	3011	6762	0848	8316	3485	6388	8925	3790	0898
1121	2978	6313	5857	8457	1395	7240	8630	3895	6348
1930	4583	4227	4120	6893	7005	2264	6067	5627	7985
6309	9158	2830	3262	9809	4606	8669	1154	5841	7695
4460	3143	5383	0327	9668	1697	8335	0860	2188	1908
8371	5095	7273	1866	4193	4163	2035	2812	4996	7142
9397	5540	9298	9076	1299	0669	0088	1809	0631	3162
9304	1468	4013	7465	0861	6787	3581	7977	8409	4708
5606	2435	8546	3209	4802	6690	8527	2210	6706	1930
6693	8333	082	7546	2910	8553	8725	1237	4423	1570
0556	7715	8994	4245	1540	8150	3889	5273	6977	2703
6973	9299	4959	7146	1426	7086	8743	6982	5547	3394
4920	1223	5208	6661	4907	1102	0501	3625	8513	3192
0132	0098	8241	0858	7627	4174	1170	3142	2455	4891
4051	3101	9854	4488	6931	3266	3147	2500	8011	8848
0267	5612	5504	7917	7928	8034	9989	4353	2675	9497
0609	9469	3149	4086	8911	8547	3518	9349	1836	0548
2593	1666	5750	5105	4287	4380	7860	7792	1625	7659
8812	9491	2602	4100	4962	1037	9778	1778	4223	3193
3540	5985	0019	7155	1471	185l	8682	9957	3772	4706
9535	5375	1239	1624	5378	6803	7177	7911	4660	5669
3174	7677	8282	6669	5879	7874	9931	6581	9784	2607
8864	4760	1129	6205	4949	4205	0222	7479	6470	8194
5245	7341	0593	5656	6799	3071	1751	4339	5630	9496
5468	6038	4511	1440	2135	5777	9903	1048	6726	8602
3951	7928	6818	4161	4840	1392	1323	5014	7538	9854
7319	4064	4024	5401	2834	7518	3978	3742	1005	4619
5892	8731	6269	5189	2071	4084	9789	3620	9819	4548

SELF-EXAMINATION QUESTIONS

1. Define a Simulation model. Distinguish between deterministic and stochastic simulation model.
2. Discuss Monte Carlo simulation. Illustrate how would you use it in situations of

   (i) Queuing and (ii) Inventory Control.
3. Explain in being advantages and disadvantages of using simulation technique.
4. A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days show the following demand and supply.

Supply		*Demand*	
Availability (kg)	*No. of days*	*Demand (kg)*	*No. of days*
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at Rs. 20 per kg and sells it at Rs. 30 per kg. Any commodity remains at the end of the day, has no saleable value. Moreover, the loss (unearned profit) on any unsatisfied demand is Rs. 8 per kg. Given the following pair of random numbers, simulate 6 days sales, demand and profit.

(31,18); (63,84); (15,79); (07,32); (43,75); (81,27)

The first random number in the pair is for supply and the second random number is for demand viz. in the first pair (31,18) use 31 to simulate supply and 18 to simulate demand.

5. A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds whose probability distribution is as given below :

Production per day	Probability
196	0.05
197	0.09
198	0.12
199	0.14
200	0.20
201	0.15
202	0.11
203	0.08
204	0.06

The finished mopeds are transported in a specially designed three storeyed lorry that can accommodate only 200 mopeds. Using the following 15 random numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10 simulate the process to find out :

(i) What will be the average number of mopeds, waiting in the factory ?

(ii) What will be the average number of empty spaces on the lorry ?