



15

Programme Evaluation and Review Technique

LEARNING OBJECTIVES :

After studying this chapter, you will be able to :

- Explain generic term network analysis and such related terms as Critical Path Analysis. Critical Path Method (CPM) and Programme Evaluation and Review Technique (PERT).
- Work out how network analysis helps managers to plan when to start various tasks to monitor actual progress and to know when control action is needed to prevent delay in completion of the project
- Draw diagram and chart representing inter-relationships between various elements of the project and finally establishing critical path
- Identify dummy activities
- Identify earliest event times and latest even times for each event
- Identify total float, free float and independent float
- Crash times, costs and consumption of resources
- Help in decision making about project times and resource allocation
- Calculate the probability of completing the network in a given time when multiple time estimates of activity duration are available

15.1 INTRODUCTION

Hitherto our emphasis has been on CPM which is applicable where activity durations can be estimated from experience, past historical records and work study techniques fairly precisely. *CPM is incapable of handling uncertainty in timing* which is a rule rather than an exception for innovational projects such as introducing a new product or oil exploration project. PERT (Program Evaluation and Review Technique) is more relevant for handling such projects which have a great deal of uncertainty associated with the activity durations.



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To take these uncertainty into account, three kinds of times estimates are generally obtained. These are:

(a) *The Optimistic Times Estimate* : This is the estimate of the shortest possible time in which an activity can be completed under ideal conditions. For this estimate, no provision for delays or setbacks are made. We shall denote this estimate by t_o .

(b) *The Pessimistic Time Estimate* : This is the maximum possible time which an activity could take to accomplish the job. If everything went wrong and abnormal situations prevailed, this would be the time estimate. It is denoted by t_p .

(c) *The Most Likely Time Estimate* : This is a time estimate of an activity which lies between the optimistic and the pessimistic time estimates. It assumes that things go in a normal way with few setbacks. It is represents by t_m .

Beta distribution is assumed for these “guess estimates” and PERT analysts have found that beta-distribution curve happens to give fairly satisfactory results for most of the activities. For a distribution of this type, the *standard deviation* is approximately one sixth of the range, i.e.

$$S_t = \frac{t_p - t_o}{6}$$

The variance, therefore; is

$$S_{t,2} = \left(\frac{t_p - t_o}{6} \right)^2$$

Expected time : The expected time (t_e) is the average time taken for the completion of the job. By using beta-distribution, the expected time can be obtained by following formula.

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Probability estimate in PERT : Those experience and possessing expertise in the various technical aspects of the projects are required to make three times estimates from which is computed t_e the expected times and S_t the standard deviation of the activity duration. The expected time may then be used as the activity duration and the critical path may be found by the analytical method explained earlier. But to what use is S_t put? Well this provides quite a useful information at the expense of completing a project by a given date. Here is the procedure.

15.2 PROBABILITY OF ACHIEVING COMPLETION DATE

Suppose we wish to find out the probability that a project will be completed with the scheduled completion time. The time t_e as determined by b distribution after taking into account three times estimates viz. t_o , t_p and t_m only represents a fifty per cent chance that the activity will be completed with the time t_e . If the distribution curve for activity-i-j is as



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shown in the following figure, the vertical line corresponding to t_e would divide the area under the curve into two equal parts and the probability of completing the job in time t_e would be $\frac{1}{2}$. The probability of completing job within some other time Σ would be the area under curve upto the vertical line at Σ divided by the total area under the following curve, i.e. the probability of completing of completing the activity i-j within time Σ is P, then

$$P = \frac{\text{area under ACE}}{\text{area under ACB}}$$

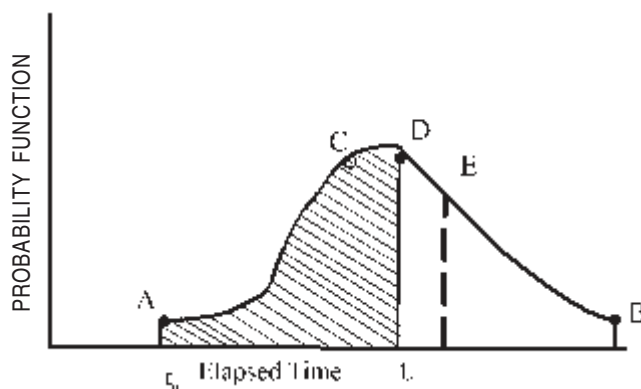


Figure 1

In general, a project consisting of several activities will have a normal distribution, that is, or the project as a whole, the distribution curve is as shown below, and the probability of completing the project in time is equal to the mean value T_e which is $\frac{1}{2}$.

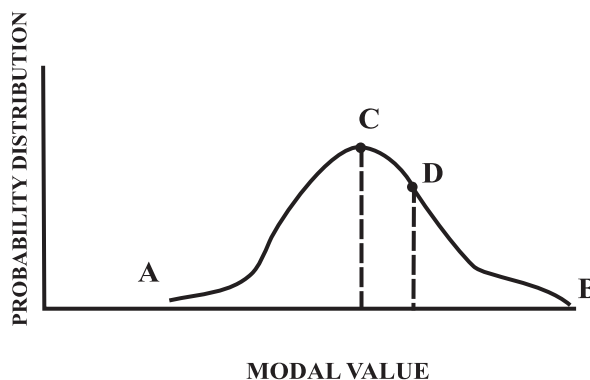


Figure 2

If the scheduled time is T_s , the probability of completing the project within time T_s is the ratio,

$$P = \frac{\text{area under ACE}}{\text{area under ACB}}$$



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In order to apply the above analysis to a network it is necessary to reduce the random curve obtained from the particular network into normal form. The advantage of this is that we can use information and data available about the normal curve in the books of Statistics.

A standardized normal curve has an area equal to unity and a standard deviation '1'. further, it is symmetrical about the mean value t_e . Hence the area under the curve AC is 50% of the total area under the curve ACB. The area under the curve ACD depends on the location of T_s along the time axis. The point t_e can be taken as the reference point and the distance t_e . T_s can expressed in terms of standard deviation. For example, if T_s is on the right t_e at a distance of 1 standard deviation then the area enclosed by ACD is 84.1%. If T_s is on the left of t_e at a distance of 1 standard deviation then area enclosed is 15.9%. These facts can be stated in a distance of -1 corresponds to 15.9% probability. By using proper statistical tables and using the probabilities as obtained above, the value of standard normal variate (Z) can be determined. But we know from statistics that

$$Z = \frac{T_1 - T_{cp}}{S.D.}$$

where T_1 denotes the duration in which we wish to complete the project and T_{cp} represent the duration on the critical path, S.D. stands for standard deviation of the earliest finish of a network.

Compute variance, $Vt (=S^2)$ of all the activity durations of the critical path. Sum these up and take square root. This yields the S.D. of the earliest finish time of a network. Let the critical path duration be designated by T_{cp} . Assuming normal distribution for the total duration, you should be in a position to find the confidence interval for T_{cp} . Note that 3SD give the limits of the total possible duration with 99% confidence. Put it in another way.

Suppose you wish that the project be completed within a duration to T_1 . Find $\frac{T_1 - T_{cp}}{SD} = Z$,

the standard normal variate.

Look to the standard normal probability distribution tables (Appendix 1) for the probability of completing the project within the given duration of T_1 .

Illustration

If the critical path of a project is 20 months alongwith a standard duration of 4 months, what is the probability that the project will be completed within :

(a) 20 months (b) 18 months (c) 24 months?

Solution

$$(a) \quad Z = \frac{20-20}{4} = 0; \text{ Probability} = 0.50$$

$$(b) \quad Z = \frac{18-20}{4} = -0.50; \text{ Probability} = 0.31$$



$$(c) \quad Z = \frac{24-20}{4} = 1; \text{ Probability} = 0.84$$

Illustration

PERT calculations yield a project length of 60 weeks with variance of 9. Within how many weeks would you expect the project to be completed with probability of 0.99? (That is the project length that you would expect to be exceeded only by 1% of the times if the project were repeated many times in an identical manner).

Solution

$$T_{cp} = 60 \text{ S.D.} = \sqrt{9} = 3.$$

$$60 + 3 \times 2.3 = 67 \text{ weeks (Answer)}$$

Illustration

A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers.

- (a) Draw the project network and identify all the paths through it.
- (b) Find the expected duration and variance for each activity. What is the expected project length ?
- (c) Calculate the variance and standard deviation of project length. What is the probability that the project will be completed :
 - (i) at least 3 weeks earlier than expected ?
 - (ii) no more than 3 weeks later than expected ?
- (d) If the project due date is 18 weeks, what is the probability of not meeting the due date ?
- (e) What due date has about 90% chance of being met?

Activity <i>i-j</i>	Estimated duration in Weeks		
	Optimistic	Most Likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15



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Solution

(a) The network is drawn in Fig. 3. The various paths are as follows.

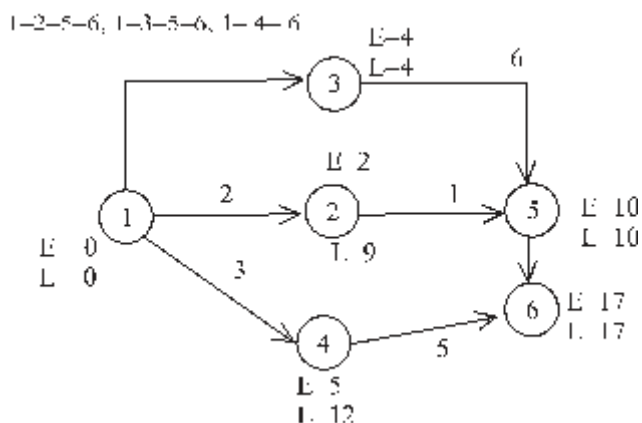


Figure 3

(b) Expected duration and variances for various activities are computed below.

	t_e	V_t
1-2;	$\frac{1+1 \times 4+7}{6} = 2$;	$\left[\frac{7-1}{6} \right] = 1$
1-3;	$\frac{1+16+7}{6} = 4$;	$\left[\frac{7-1}{6} \right] = 1$
1-4;	$\frac{2+8+8}{6} = 3$;	$\left[\frac{8-2}{6} \right] = 1$
2-5;	$\frac{1+4+1}{6} = 1$;	$\left[\frac{1-1}{6} \right] = 0$
3-5;	$\frac{2+20+14}{6} = 1$;	$\left[\frac{14-2}{6} \right] = 4$
4-6;	$\frac{2+20+8}{6} = 1$;	$\left[\frac{8-2}{6} \right] = 1$
5-6;	$\frac{3+24+15}{6} = 7$;	$\left[\frac{15-3}{6} \right] = 4$



- (c) Expected project Length = 17 weeks (Answer)
 Variance of the critical path 1-3-5-6 = 1+4+4 = 9 (Answer)
 \therefore Standard Deviation = $\sqrt{9} = 3$ (Answer)

(i) $Z = \frac{14 - 17}{3} = -1$; Probability = 0.159 (Answer)

(ii) $Z = \frac{20 - 17}{3} = 1$; Probability = 0.841 (Answer)

(d) $T_{cp} = 17$; $T_1 = 18$; $Z = \frac{18 - 17}{3} = 0.333$

Therefore, probability of meeting the due date = 0.63.

And probability of not meeting the date = 0.37 (=1-0.63)

- (e) At 90% Probability, Z = 1.3 approximately.

$\therefore 1.3 = \frac{T_2 - 17}{3}$ or $T_2 = 20.9$ weeks approximately.

Illustration

Find event variances in the network of previous illustration.

Solution

Event variance for both the T_E and T_L of each event are derived below. The computational procedure should be self evident. We shall put to use the variances of the various activities derive in part (b) of solution to example 3 above.

Earliest Time, TE (D=Duration)

Event	Longest path leading to it	*Variance
1	Null (D=0)	0
2	1-2 (D=2)	1
3	1-3 (D=3)	1
4	1-4 (D=4)	1
5	1-3, 3-5 (D = 4 + 6 = 10)	1 + 4 = 5
6	1-3, 3-5, 5-6 (D=17)	1 + 4 + 4 = 9

Suppose we wish to find the probability of reaching event 5 in 9 days. This can be computed as below:

$$Z = \frac{9 - 10}{\sqrt{5}} = \frac{-1}{\sqrt{5}} = \frac{-1}{2.236} = -0.4472$$

* (N.B. If there are two equal longest path, higher of the two variances would be picked up).



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Probability (from Appendix 1) of reaching event 5 in 9 days is equal to $0.5 - .1723 = .3275$. Likewise we can determine probabilities of reaching other events.

Longest Time T_L

Event	Longest path from it to last event 6	Variance
1	1-3, 3-5, 5-6	9
2	2-5, 5-6	4
3	3-5, 5-6	8
4	4-6	1
5	5-6	4
6	Null	0

Illustration

Shown below is a PERT network and a related set of activity times:

i-j	A	B	C	D	E	F	G	H	I	J	K	L
t_o	10	12	8	4	0	12	6	9	4	0	5	9
t_m	13	15	11	7	0	18	12	12	6	0	8	12
t_p	22	18	20	16	0	36	18	27	8	0	11	33

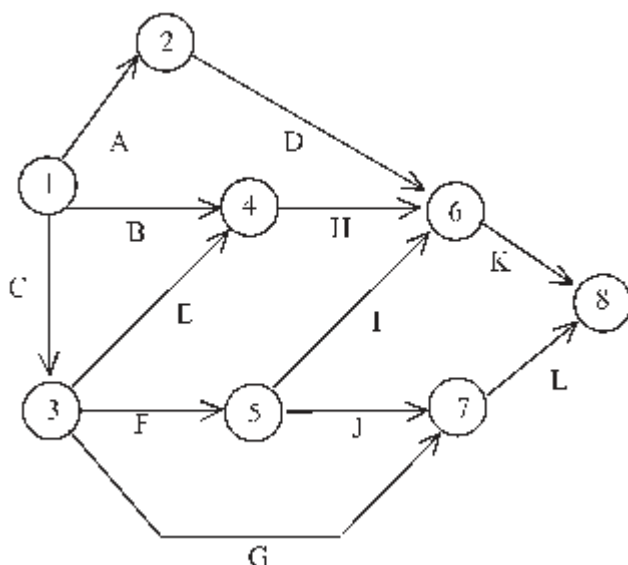


Figure 4

Required

- Determine the expected completion time of each activity.
- Determine the earliest expected completion time, the latest expected completion time and float of each activity.



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- (c) What is the total project completion time, and what are activities on the critical path?
- (d) Determine S.D. of expected completion time for only those activities on the critical path.
- (e) Determine the probability that the project will be completed within 41 weeks.

Solution

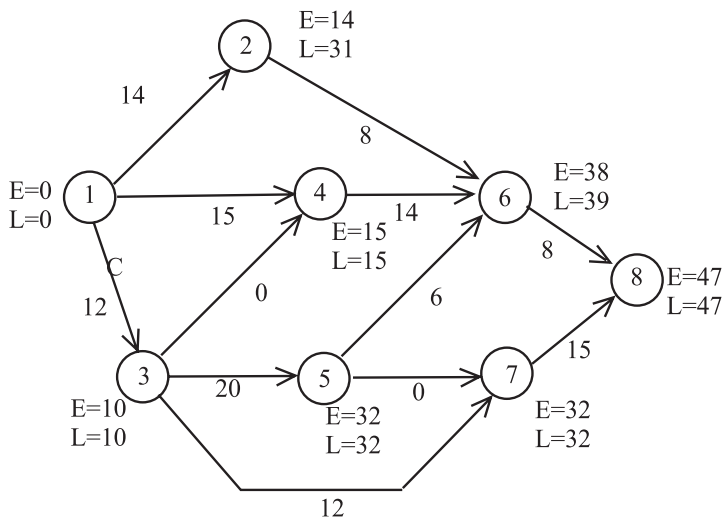


Figure 5

$$t_e = \frac{t_p + 4t_m + t_o}{6}; = \left(\frac{t_p - t_o}{6} \right)$$

i-j	Time estimates			t _e	Start		Finish		Float	V _t	S.D
	t _o	4t _m	t _p		E	L	E	L			
1-2	10	52	22	14	0	17	14	31	17		
1-3	8	44	20	12	0	0	12	12	0	4	2
1-4	12	60	18	15	0	10	15	25	10		
2-6	4	28	16	8	14	31	22	39	17		
3-4	0	0	0	0	12	25	12	25	13		
3-5	12	72	36	20	12	12	32	32	0	16	4
3-7	6	48	18	12	12	20	24	32	8		
4-6	9	48	27	14	15	25	29	39	10		
5-6	4	24	8	6	32	33	38	39	1		
5-7	0	0	0	0	32	32	32	32	0	0	0
6-8	5	32	11	8	38	39	46	47	1		
7-8	9	48	33	15	32	32	47	47	0	16	4
									Σ V _t = 36		



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Hence S.D. of the critical path = $\sqrt{36} = 6$.

Probability of completion critical path in 41 weeks is computed below:

$$z = \frac{41 - 47}{6} = -1.$$

∴ Probability = 0.159 (Answer).

Exercise

1. The following table lists the jobs of a network alongwith their time estimates.

i-j	Optimistic Duration	Most Likely Duration	Pessimistic Duration
1-2	3	6	15
1-6	2	5	14
2-3	6	12	30
2-4	2	5	8
3-5	5	11	17
4-5	3	6	15
5-8	1	4	7
6-7	3	9	26
7-8	4	19	28

- Draw the project network.
- Calculate the length and variance of critical path.
- What is the approximate probability that jobs on the critical path will be completed by the due date of 41 days?
- What is the approximate probability that jobs on the next most critical path will be completed by the due date?

Answer : (b) Length = 36 days; Variance = 25; (c) 0.84; (d) 0.84

2. Below are given the three times estimates of each activity of a project network. Construct the network. Find the variance of each activity and variances of T_E and T_L of each event.

i-j	1-2	1-3	1-4	2-5	3-5	4-6	5-6
t_o	7	15	7	9	20	14	2
t_m	10	18	8	12	24	18	3
t_p	12	20	9	17	26	20	7



Answer :

i-j	Variance	T_E		T_L	
		Event No	Variance	Event No	Variance
1-2	0.69	1	0	1	2.14
1-3	0.45	2	0.69	2	2.46
1-4	0.11	3	0.45	3	1.69
2-5	1.77	4	0.11	4	1.00
3-5	1.00	5	1.45	5	0.69
4-6	1.00	6	2.14	6	0
5-6	0.69				

15.3 A FEW COMMENTS ON ASSUMPTIONS OF PERT & CPM

1. Beta distribution may not always be applicable.
2. The formulae for the expected duration and S.D. are simplifications. Maccrinon and Ryavec reached the conclusion that in certain cases the errors, because of these assumption, may even be to the tune of 33%.
3. The errors owing to the aforesaid simplification and assumption may be compounded or may cancel each other to an extent.
4. In computing the S.D. of the critical path independence of activities is implied. Limitations of resources may invalidate the independence which exists by the very definition of an activity.
5. It may not always be possible to sort out completely identifiable activities and to state where they begin and where they end.
6. In projects fraught with in certainty it is natural that there exist alternatives with differing outcomes. For example, if a particular hardness is not obtained in a metal, an alloy might have to be used that is more expensive and also inferior on certain technical considerations. There have been theoretical developments in this regard, and it may be worthwhile to incorporate the concept of decision tree analysis depending upon the situation.
7. Time estimates have an element of subjectiveness and, to that extent, the techniques could be weak. The contractors react to this weakness shrewdly whilst bidding. If there are cost plus contracts they would deliberately “under estimate” the time for chances of being awarded with the contract. Incentive type contracts might lead to an opposite bias.
8. Cost-time trade offs, for deriving the cost curve slopes, to be discussed soon, are subjective again and call for a great deal of expertise of the technology as well as genuine effort to estimate. Often the engineers tend to be lax here; occasionally with the honest deliberation event, the guesses may be wide off the mark.



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15.4 DISTINCTION BETWEEN PERT AND CPM

The PERT and CPM models are similar in terms of their basic structure, rationale and mode of analysis. However, there are certain distinctions between PERT and CPM networks which are enumerated below: .

- (i) CPM is activity oriented i.e. CPM network is built on the basis of activities. Also results of various calculations are considered in terms of activities of the project. On the other hand, PERT is event oriented.
- (ii) CPM is a deterministic model i.e. It does not take into account the uncertainties involved in the estimation of time for execution of a job or an activity. It completely ignores the probabilistic element of the problem. PERT, however, is a probabilistic model. It uses three estimates of the activity time; optimistic, pessimistic and most likely; with a view to take into account time uncertainty. Thus, the expected duration of each activity is probabilistic and expected duration indicates that there is fifty per cent probability of getting the job done within that time.
- (iii) CPM places dual emphasis on time and cost and evaluates the trade-off between project cost and project time. By deploying, additional resources, it allows the critical path project manager to manipulate project duration within certain limits so that project duration can be shortened at an optimal cost. On the other hand, PERT is primarily concerned with time. It helps the manager to schedule and coordinate various activities so that the project can be completed on scheduled time.
- (iv) CPM is commonly used for those projects which are repetitive in nature and where one has prior experience of handling similar projects. PERT is generally used for those projects where time required to complete various activities are not known as priori. Thus, PERT is widely used for planning and scheduling research and development projects.

15.5 UPDATING THE NETWORK

The progress of various activities in a project network is measured periodically. Normally, either most of the activities are ahead or behind the schedule. It is therefore, necessary to update or redraw the network periodically to know the exact position of completion of each activity of the project. The task of updating the network may be carried out once in a month. Sometimes the updating of the network may provide useful information to such an extent that it may demand the revision of even those very activities which have not started. Even the logic may also change i.e. some of the existing activities may have to be dropped and new activities may be added up. In brief the network should be amended accordingly in the light of new developments.

It is also not unlikely that the total physical quantum of work accomplished at a point of time may exceed what was planned but the progress against the critical path alone may be slower than the scheduled pace. To understand how the task of updating is carried out, consider the following example:



Illustration

After 15 days of working the following progress is noted for the network of an erection job.

- a) Activity 1-2, 1-3 and 1-4 completed as per original schedule.
- b) Activity 2-4 is in progress and will be completed in 3 more days.
- c) Activity 3-6 is in progress and will need 18 days more for completion.
- d) Activity 6-7 appears to present some problem and its new estimated time of completion is 12 days.
- e) Activity 6-8 can be completed in 5 days instead of originally planned for 7 days.

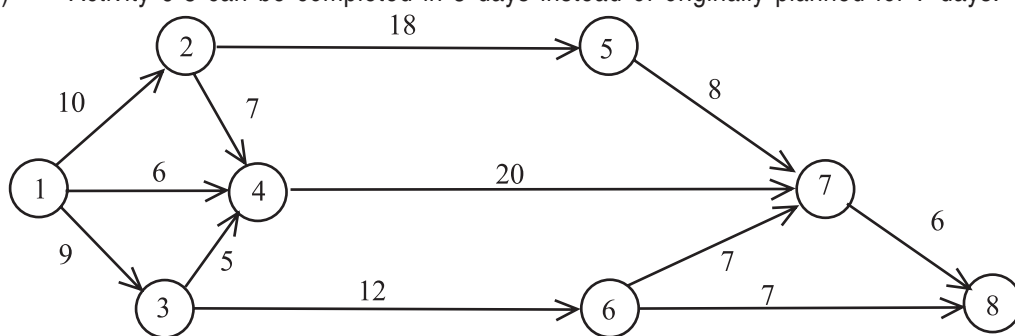


Figure 6

You are required to :

- (i) Update the above diagram after 15 days of the start of work based on the assumptions given above.
- (ii) Write down the critical path with total project duration.

Solution

The new formulation of the problem is as follows:

- (i) Activities 1-2, 1-3 and 1-4 need 9, 10 and 6 days respectively as per original programme.
- (ii) Activity 2-4 needs $15 + 3 - 9 = 9$ days instead of original programme of 7 days.
- (iii) Activity 3-6 needs $15 + 18 - 10 = 23$ days.
- (iv) Activity 3-4 needs 5 days.
- (v) Activities 2-5, 4-7 and 5-7 need 18, 20 and 8 days respectively.
- (vi) Activity 6-7 needs 12 days as no work was scheduled to be started for this activity on 15th day.
- (vii) Activities 6-8 and 7-8 need 5 and 6 days respectively.



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The new diagram based on the above listed activities will be as follows:

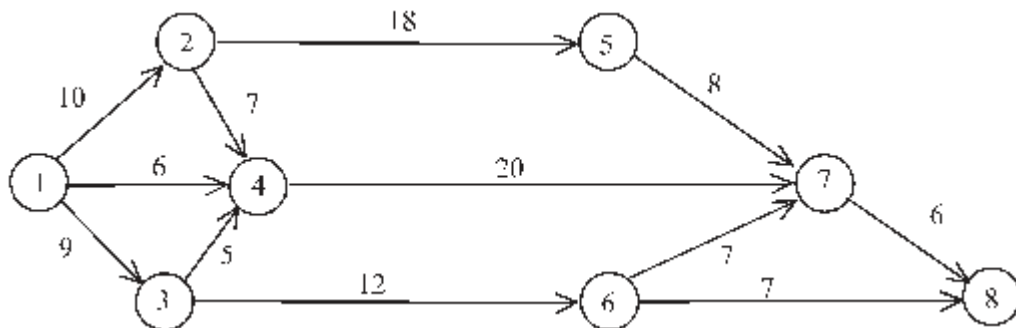


Figure 8

15.6 PROJECT CRASHING

In the discussion on PERT, we saw how the probability of completion of a project can be computed for a specified duration. There are usually compelling reasons to complete the project earlier than the originally estimated duration of the critical path computed on the basis of normal activity times, by employing extra resources. An example would be introduction of a new project. The motive in hastening the project might be to ensure that the competitors do not steal a march. In the present section we will deal with those situations which will speak of the effect of increase or decrease in the total duration for the completion of a project and are closely associated with cost considerations. In such cases when the time duration is reduced, the project cost increases, but in some exceptional cases project cost is reduced as well. The reduction in cost occurs in the case of those projects which make use of a certain type of resources, for example, a machine and whose time is more valuable than the operator's time. Before we take up an example of project cost control, it is better to understand well the following preliminaries and their definitions.

Activity Cost : It is defined as the cost of performing and completing a particular activity or task.

Crash Cost C_c : This is the direct cost that is anticipated in completing an activity within the crash time.

Crash time, C_t : This is the minimum time required to complete an activity.

Normal Cost, N_c : This is the lowest possible direct cost required to complete an activity.

Normal time, N_t : This is the minimum time required to complete an activity at normal cost.

Activity cost slope: The cost slope indicates the additional cost incurred per unit of time saved in reducing the duration of an activity. It can be understood more clearly by considering the figure 8.



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Let OA represent the normal time duration for completing a job and OC the normal cost involved to complete the job. Assume that the management wish to reduce the time of completing the job to OB from normal time OA. Therefore under such a situation the cost of the project increases and it goes up to say OD (Crash Cost). This only amounts to saving that by reducing the time period by BA the cost has increased by the amount CD. The rate of increase in the cost of activity per unit with a decrease in time is known as cost slope and is described as below.

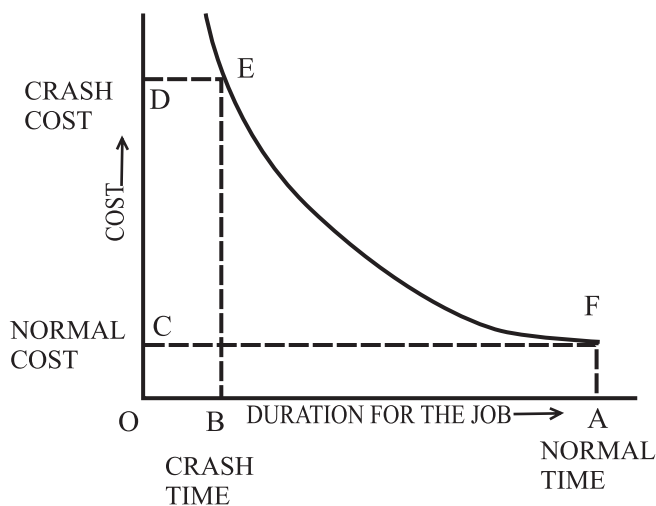


Figure 8

$$\begin{aligned} \text{Activity cost slope} &= \frac{CD}{AB} = \frac{OD - OC}{OA - OB} \\ &= \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}} \end{aligned}$$

Optimum duration : The total project cost is the sum of the direct and the indirect costs. In case the direct cost varies with the project duration time, the total project cost would have the shape indicated in the following figure:



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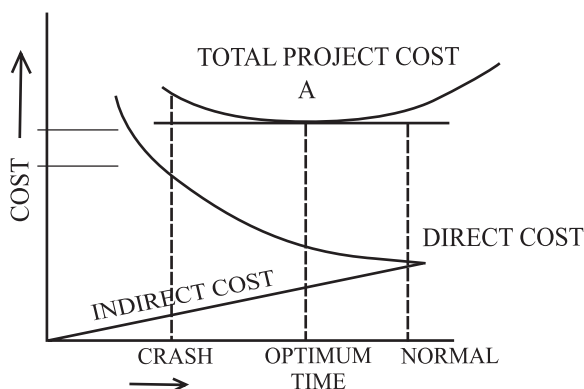


Figure 9

At the point A, the cost will be minimum. The time corresponding, to this point A is called the optimum duration and the cost as optimum cost for the project.

The example below which the student should go through carefully is intended to explain cost implications of hastening a project.

Illustration

The following data pertains to the network drawn in figure 10-A given on the next page. It is desired to compress the project to the least possible duration day by day and estimate the extra cost.

i-j	T_n	T_e	Cost slope
1-2	3	2	700
1-3	7	4	200
2-3	5	3	100
2-4	8	6	200
3-4	4	2	400

Solution

The critical path is 1-2-3-4 in figure A. It can be, seen that the critical path 1-2-3-4 is longer than either of the paths; 1-3-4 and 1-2-4 by one day. Therefore, the project can be compressed by one day along the critical path, 2-3 having the least cost slope is therefore crashed by a day.

The revised network is depicted in figure B where all activities have become critical. The following choices of compression exist now. Each set of activities is so chosen that it *reduces all the paths by a day*.

Crash each activity in one of the following sets by a day.

1-2	2-4	1-3
1-3	3-4	2-3
		2-4
Cost = 900	Cost = 600	Cost = 500

The last set of crashing 1-3, 2-3 and 2-4 is the least expensive and these activities are crashed accordingly.



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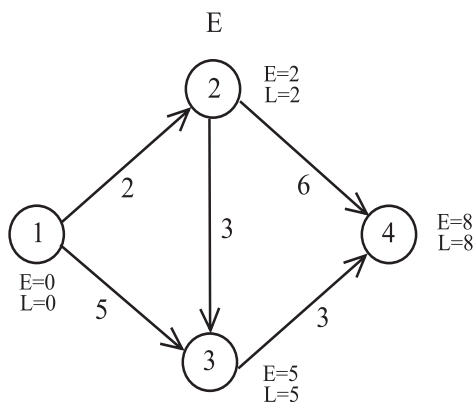
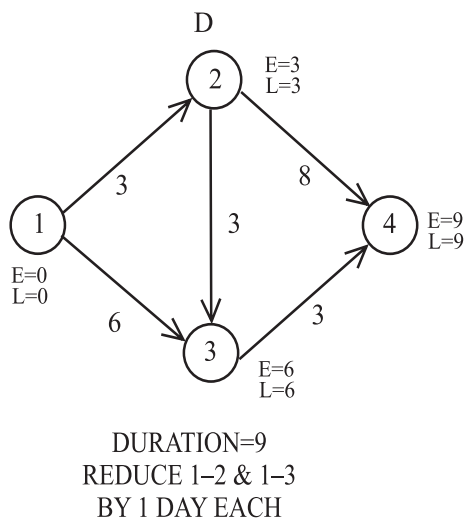
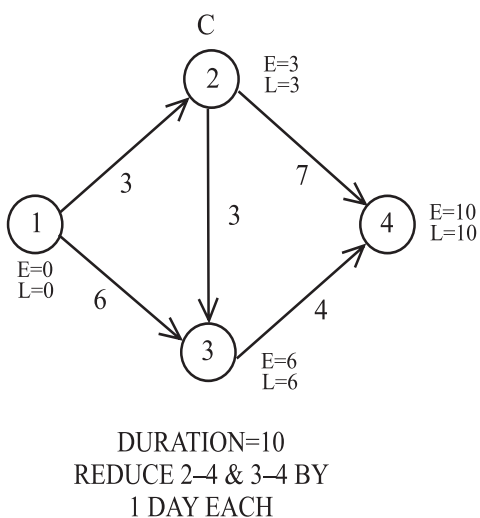
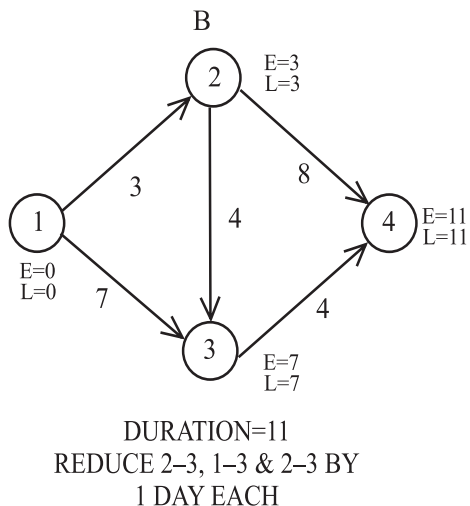
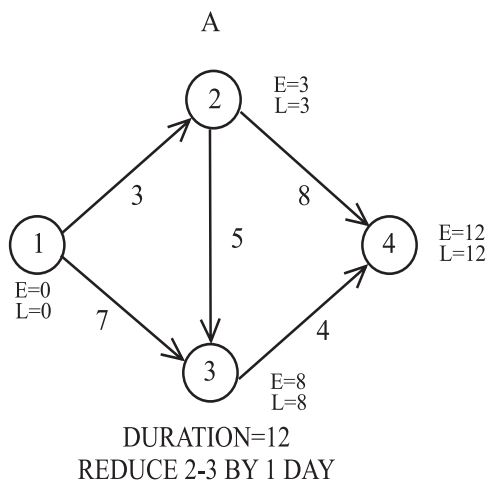


Figure 10 [(A) to (E)]



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Extra cost	=	200
		100
		200
		500
Rs.		500

This is shown in figure C. Activity 2-3 having been crashed to the limit is dropped out from further consideration. In C. the following choices of crashing each activity a day exist:

1-2	1-3	2-4
3-4	1-2	3-4
Extra cost	Rs. 1100	Rs. 900
		Rs. 600

Extra cost The last is selected **Extra Cost = Rs. 600**

The revised network is shown in figure D where 2-4 joins 2-3 in that it is also crashed to limit. The only possibility of compressing the network in figure D is to crash 1-2 and 1-3 by a day each. This is done and the final network is shown in figure E.

Extra cost = Rs. 900

Although 1-3 and 3-4 have not reached their crashing limits in figure E there is no use also to crash these since this would not compress the project which can be compressed to 8 days only.

Total extra cost = 100 + 500 + 600 + 900 = Rs. 2,100.

[If, however, just the least duration plan was required one could go about the problem in a much simpler way as follows. Draw the network with t_0 's. This is done in the figure 10F.

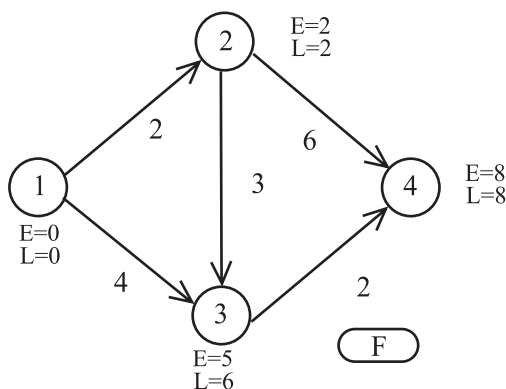


Figure 10 (F)

The network is analysed with 1-2-4 as the critical path of 8 day's duration. The other paths have to be contracted to 8 day's duration. This can be done in one of the following ways:

- (i) Increase 1-3 and 3-4 by a day each with cost reduction of Rs. 200 + 400 = 600.
- (ii) Increase 2-3 and 1-3 by a day and 2 days respectively with cost reduction of Rs. 100 + 400 = 500.



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Obviously the 1st course is to be preferred and the network, if now revised, would be identical with the one of figure E.

Often, however, management would be interested in the least total cost duration rather than the least possible duration.

Now, suppose there is an indirect cost of Rs. 800 per day. What would then be the least cost project duration for the example on hand? The various cost data are tabulated below:

Duration	Direct (Crashing) cost	Indirect cost	Total cost
12	0	9600	9600
11	100	8800	8900
10	600	8000	8600
9	1200	7200	8400
8	2100	6400	8500

Thus 9 days is the least total cost duration and rationally the management should go in for this unless high opportunity losses compel them to select a lower duration project plan.

Illustration

Consider the following network and the table for a particular project which consists of 7 activities.

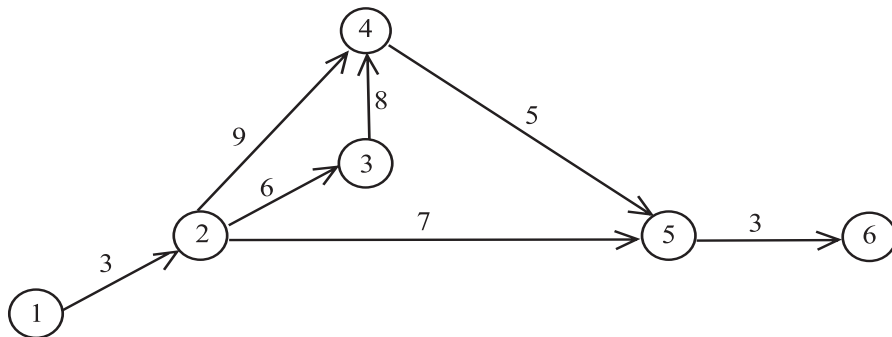


Figure 11

(The number indicated along the activity arrows are the normal duration.)

Activity	Normal		Crash		Cost slopes		
	Times (days)	Cost Rs.	Time (days)	Cost Rs.	ΔT	ΔC	$\Delta C/\Delta T$
1-2	3	360	2	400	1	40	40
2-3	6	1440	4	1620	2	180	90
2-4	9	2160	5	2380	4	220	55
2-5	7	1120	5	1600	2	480	240
3-4	8	400	4	80	4	400	100
4-5	5	1600	3	1770	2	170	85
5-6	3	480	2	760	1	280	280
		7560		9330			

The indirect cost is Rs. 160/-per day. Determine optimum duration in the above case.



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Solution

The normal duration of the project is obtained from the critical path and not by merely summing up the normal duration of all the activities. The overall normal duration in the above project is 25 days and the total direct cost is Rs. 7560/-. If due to some reason it is felt that the project is to be hastened, the question then to be answered is: "what is the minimum time required for the completion of the project?"

According to the crash time given in the table; the critical path based on these crash time estimates still appears as 1-2-3-4-5-6 and the crash duration is 15 days. Hence, the project may take 15 to 25 days depending upon the money the management is prepared to spend. If the management decides to complete the project in 15 days, then the direct cost will be Rs. 9330/-. The figure has been arrived by speeding up or crashing all the activities as mentioned in the, given table. But there are several non-critical activities in the project, which need not be crashed to reduce the project duration. Speeding up of such non critical activities involves extra amount. As any extra amount spent on these activities is not going to reduce the project duration, therefore it is better not to reduce the duration of these non-critical activities. Duration of these non-critical activities should be reduced only when they become critical activities, during the process of reducing the project duration time.

The two objectives behind the reduction of project time are:

- (1) To complete the project before a certain target date.
- (2) To reduce the overall cost of the project. This objective can only be achieved when the indirect cost per day is greater than some of the cost slopes as given in the table. In the exercise under consideration, the five activities 1-2, 2-3, 2-4, 3-4 and 4-5 have cost slopes lower than the indirect cost. In such a situation, the management would be very much interested in cutting down the project time, thereby decreasing the total indirect cost. In order to solve this example we proceed as follows:

First step is to identify those activities along the critical path whose cost slopes are less than the indirect cost. In our network such activities are 1-2,2-3,3-4 and 4-5. We take these activities in the order of increasing cost slopes. Activity 1-2 has a slope of Rs. 40/per day. This activity can be cut down by 1 day at a cost of Rs. 40/-. The next activity in the order of cheapness is 4-5. This can be cut down by 2 days at a cost of E-s. 170. Next we may take the activity 2-3 whose cost slope is Rs. 90/-. This activity can be contracted by 2 days at a cost of Rs. 180. So far three activities viz. 1-2,4-5 and 2-3 have been contracted by a total of 5 days at an overall cost of Rs, 390/-.

From the given table, we notice that the next activity in the order of priority is 3-4 with a cost slope Rs. 100/- per day. This activity can be cut down by 4 days. But a look on the network after the performance of above reduction in time reveals that activity 2-4 has float of only 3 days.

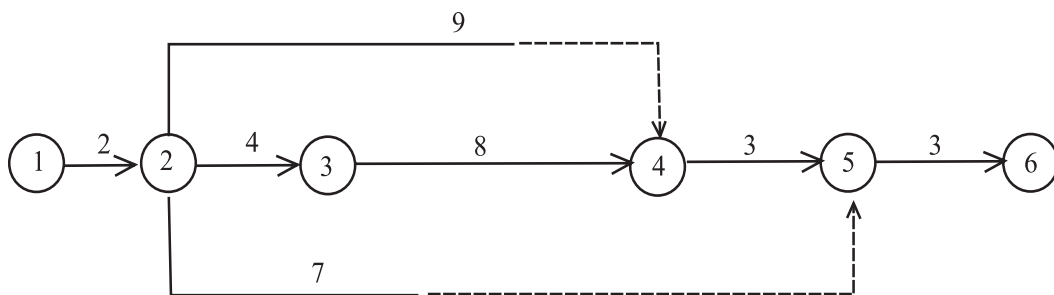


Figure 12

Hence we can not cut down the duration for the activity 3-4 by four days without affecting the activity 2-4. Therefore one should cut down the duration on activity 3-4 by 3 days at a cost of Rs. 300/- and now by this process a new sub-critical path viz. 1-2-4-5-6 may come into existence.

According to the given Table, activity 3-4 can be further contracted by 1 day. But this cannot be achieved without contracting activity 2-4. Therefore, the cost slope must include both of these activities i.e. activities 2-4 and 3-4. The combined cost slope of these : two activities is Rs. 55 + Rs. 100 = Rs. 155. The project duration has now been reduced to 116 days, as below:

Activity	Duration reduced by (Days)	Cost slope (Rs.)	Cost of contraction (Rs.)
1-2	1	40/-	40/-
4-5	2	85/-	170/-
2-3	2	90/-	180/-
3-4	3	100/-	300/-
3-4,2-4	1	155/-	155/-
	<u>9</u>		<u>845/-</u>

During the above process a total reduction of 9 days has been achieved at an extra cost of Rs. 845/-. Also the indirect cost @ Rs. 160/- per day for 9 days resulting in Rs. 1440/- has been saved.

In other words, the project duration has been reduced from 25 days to 16 days. As a result of this duration of project its direct cost has now become Rs. 8405/- and Rs. 2560/- as its indirect cost. The overall project cost with the new time schedule i.e. 16 days is Rs. 8405 + Rs. 2560 = Rs. 10965/- as against the overall cost of Rs. 11560, it thus results in a net saving of Rs. 595/-

Exercise 3. A small maintenance project consists of jobs in the table below. With each job is listed its normal time and a minimum, or crash time, in days. The cost (in Rs. per day) of each job is also given.



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<i>Job</i> <i>i-j</i>	<i>Normal day</i> <i>duration</i>	<i>Minimum (crash)</i> <i>day duration</i>	<i>Cost/day</i>
1-2	9	6	20
1-3	8	5	25
1-4	15	10	30
2-4	5	3	10
3-4	10	6	15
4-5	2	1	40

- (a) What is the normal project length and the minimum project length?
- (b) Determine the minimum crashing cost of schedules ranging from normal length down to, and including, the minimum length schedule. That is, if L = length of the schedule, find the costs of schedules which are L , $L - 1$, $L-2$ and so on days long.
- (c) Overhead costs total Rs. 60 per day. What is the optimum length schedule in terms of both crashing and overhead cost? List the scheduled duration of each job for your solution:

Answer : (a) 20 days: 12 days

(b) <i>Length (days)</i>	<i>Crashing Cost</i> <i>Rs.</i>	<i>Total Cost</i> <i>Rs.</i>
20	0	1200
19	15	1155
18	30	1110
17	45	1065
16	85	1045
15	130	1030
14	195	1035
13	260	1040
12	335	1055

(c) Optimum length = 15 days.

Illustration

The Arcot Machinery Company has been offered a contract to build and deliver nine extruding presses to the Home Bottling Company. The contract price negotiated is contingent upon meeting a specified delivery time, with a bonus offered for early delivery. The marketing department has established the following cost and time information:



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Activity	Normal time (weeks)			Normal Cost (Rs.)	Crash Time (weeks)	Crash Cost (Rs.)
	a	b	m			
1-2	1	5	3	5000	1	9000
2-3	1	7	4	8000	3	14000
2-4	1	5	3	4000	2	6000
2-5	5	11	8	5000	7	6000
3-6	2	6	4	3000	2	5000
4-6	5	7	6	2000	4	3600
5-7	4	6	5	10000	4	14000
6-7	1	5	3	7000	1	10600

Normal delivery time is 16 weeks for a contract price of Rs. 62,000.

On the basis of the calculated profitability for each delivery time specified in the following table, what delivery schedule do you recommend that the company may implement?

Contract Delivery Time (weeks)	Contract Amount (Rs.)
15	62,500
14	65,000
13	70,000
12	72,500

(Here $a = t_o$: optimistic time, $b = t_p$; pessimistic time, $m = t_m$; most likely time.)

Solution

Let us first calculate the expect duration of each activity.

Activity	Normal Time (weeks)			Expected duration (weeks)	Normal Cost Rs.	Crash Time (weeks)	Crash Cost	Cost Slope (Rs.)
	a	b	m					
1-2	1	5	3	3	5000	1	9000	2000
2-3	1	7	4	4	8000	3	14000	6000
2-4	1	5	3	3	4000	2	6000	2000
2-5	5	11	8	8	5000	7	6000	1000
3-6	2	6	4	4	3000	2	5000	1000
4-6	5	7	6	6	2000	4	3600	800
5-7	4	6	5	5	10000	4	14000	4000
6-7	1	5	3	3	7000	1	10600	1800



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The network for the given problem is drawn below:

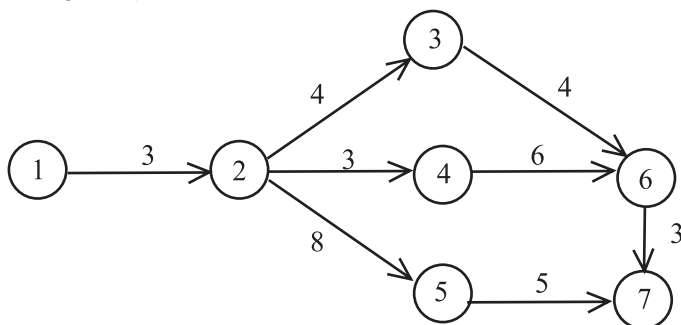


Figure 13

The critical path is 1-2-5-7 with total duration 0116 weeks. Cost of all activities is Rs.44,000. Contract price is Rs. 62,000 for normal delivery time of 16 weeks. Hence the profit is of Rs. 18000.

For calculating the most profitable delivery schedule, let us start crashing the activities on the critical path. Cost slopes for various activities are given in the above table.

Step 1: The critical activity 2-5 has the least cost slope of Rs. 1,000, so we crash this activity by 1 day. Crashing cost is Rs. 1000. The contract amount for delivery time of 15 weeks is Rs. 62,500

$$\begin{aligned}\text{Profit} &= \text{Rs. } 62,500 - (\text{Rs. } 44,000 + \text{Rs. } 1,000) \\ &= \text{Rs. } 62,500 - \text{Rs. } 45,000 = \text{Rs. } 17,500\end{aligned}$$

Step 2: Now there are two critical paths viz. 1-2-4-6-7 and 1-2-5-7 with duration of 15 weeks. So we crash activity 1-2 by 1 day at crash cost of Rs. 2000. Project duration is 14 weeks.

$$\text{Profit} = \text{Rs. } 65,000 - \text{Rs. } (45,000 + 2,000) = \text{Rs. } 18,000.$$

Step 3: We again crash activity 1-2 by 1 day. So the project duration is 13 weeks

$$\begin{aligned}\text{Profit} &= \text{Rs. } 70,000 - (\text{Rs. } 47,000 + \text{Rs. } 2,000) \\ &= \text{Rs. } 70,000 - \text{Rs. } 49,000 = \text{Rs. } 21,000\end{aligned}$$

Step 4: Now we crash activity 4-6 for path 1-2-4-6-7 at crash cost of Rs. 800 and activity 5-7 for path 1-2-5-7 at crash cost of Rs. 4000. Project duration is now 12 weeks.

$$\begin{aligned}\text{Profit} &= \text{Rs. } 72,500 - (\text{Rs. } 49,000 + \text{Rs. } 4000 + \text{Rs. } 800) \\ &= \text{Rs. } 72,500 - \text{Rs. } 53,800 = \text{Rs. } 18,700\end{aligned}$$

No further crashing is possible.

From step 3, it can be seen that the profit is maximum when the project duration is 13 weeks. Hence, the company should implement the delivery schedule of 13 weeks at a contract amount of Rs. 70,000 to gain maximum profit of Rs. 21,000



15.7 RESOURCE SMOOTHING

It is a network technique used for smoothening peak resource requirement during different periods of the project network. Under this technique the total project duration is maintained at the minimum level. For example, if the duration of a project is 15 days, then the project duration is maintained, but the resources required for completing different activities of a project are smoothened by utilising floats available on non critical activities. These non critical activities having floats are rescheduled or shifted so that a uniform demand on resources is achieved. In other words, the constraint in the case of resource smoothing operation would be on the project duration time. Resource smoothing is a useful technique or business managers to estimate the total resource requirements for various project activities.

In resources smoothing, the time-scaled diagram of various activities and their floats (if any), along with resource requirements are used. The periods of maximum demand for resources are identified and non critical activities during these periods are staggered by rescheduling them according to their floats for balancing the resource requirements.

Before explaining the process, let us first illustrate the concept of 'Time Scaled Diagram' which we will be using in the subsequent problems:

15.7.1 Time Scaled Diagrams: In the network diagrams which we have considered, it has been stressed that the length of the individual arrows has no relation to the duration of the activity which each arrow represented. It is of course possible to draw the arrows to a time scale, and this can be a very useful method of presentation for small networks.

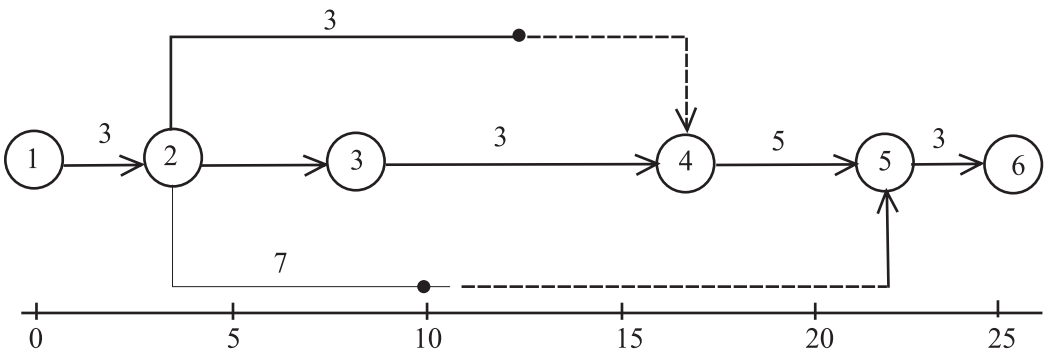


Figure 14

The figure above shows the network diagram for a problem drawn to a horizontal time scale. The critical path has been arranged as a straight line with non-critical events above or below it. Solid lines represent activities, dotted horizontal lines represent float, and dotted vertical lines represent dummies. The presentation has obvious advantages. The events are entered in the time scale version at the earliest start time. (The latest start times can also be used as an alternative).



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Illustration

Consider a project consisting of thirteen activities having duration and resource requirements shown below:

Activity	Duration (days)	Labourers required
A	2	2L
B	4	2C,2L
C	4	4C
D	3	2L
E	4	6C
F	7	2L
G	2	4C
H	5	4C,2L
J	2	2C
K	5	2C
L	2	2L
M	3	4L
N	2	4L

Here L stands for labourers and C stands for carpenters, we shall analyse this project from the point of view of resources to bring out the necessary steps involved in the analysis and smoothing of resources.

From the values given in Table above E_i 's and L_i 's are calculated and given in the network below (fig. 15)

The critical path for this network is 0–1–3–5–7–8–9–10 and is shown by dotted lines, In figure 16 is shown the time-scaled version of the same network. The critical path is shown along the horizontal line. The last two rows (also referred to as resource accumulation table) in figure 16 give the number of labourers and carpenters required each day. It can be seen that the demand on the resources is not even. On the 7th and 8th days the demand for carpenters is as high as 14, whereas on the 11th, 12th and 13th days it is two only. If the carpenters and labourers are to be hired for the entire project duration of 22 days, then during most of the days they will be idle and the company will have to hire at least 14 carpenters and 4 labourers.

We will attempt to re-schedule our activities in-such a way as to utilize the resources in a fairly uniform manner. As mentioned above the maximum demand on the resources occurs on the 6th, 7th, 8th, 9th and 10th days. The activities on these days will have to be shifted depending

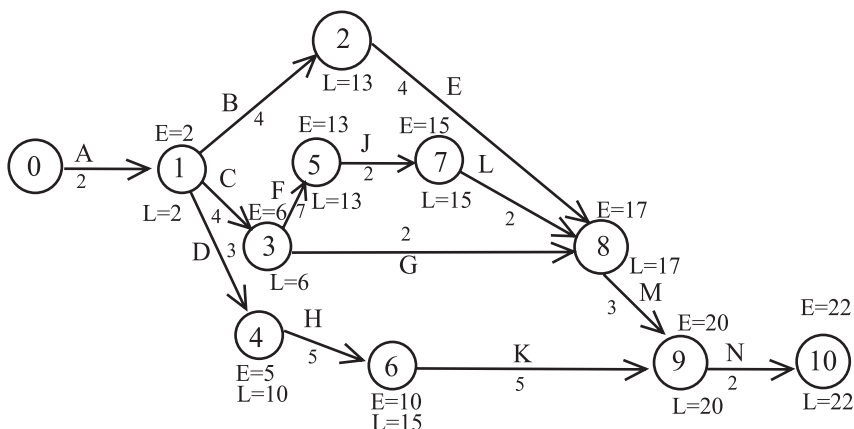


Figure 15

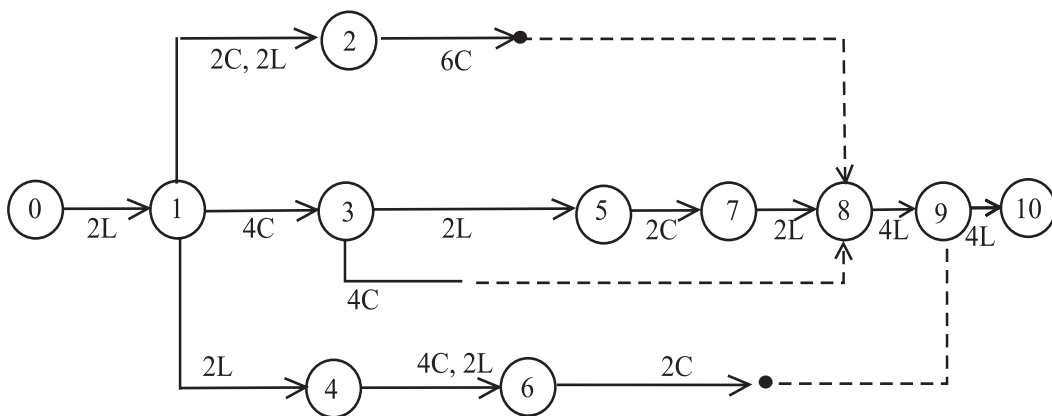
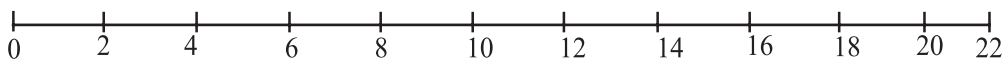


Figure 16



L	2	2	4	4	4	4	4	4	4	4	2	2	2	-	-	2	2	4	4	4	4	4
C	-	-	6	6	6	10	14	14	10	10	2	2	2	4	4	-	-	-	-	-	-	-

upon their floats such that the demand comes down. As can be seen from the above time-scaled version, activity 3-8 has maximum float, therefore we will try to shift activity 3-8 so that it starts on the 16th day instead of the 7th day. This reduces the demand on the carpenters from 14 to 10 on the 7th and 8th days so that the maximum demand for the carpenters on any day is now 10 and not 14. The modified resource accumulation table and the time-scaled version of the project is shown on next page in figure 17.



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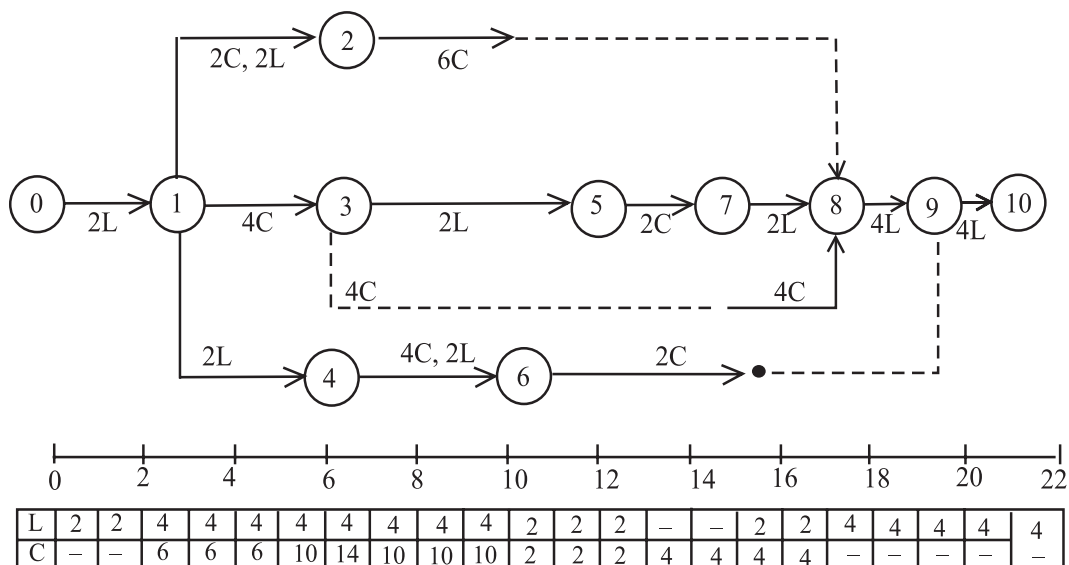


Figure 17

It is evident from the above figure that the maximum demand now is for 10 carpenters on the 6th, 7th, 8th, 9th and 10th days. We will now try to explore that possibility of further smoothing the resources which is possible because activities 1-4, 4-6 and 6-9 have a total float of 5 days. The resultant time-scaled network and the resource accumulation table are given below in fig. 18.

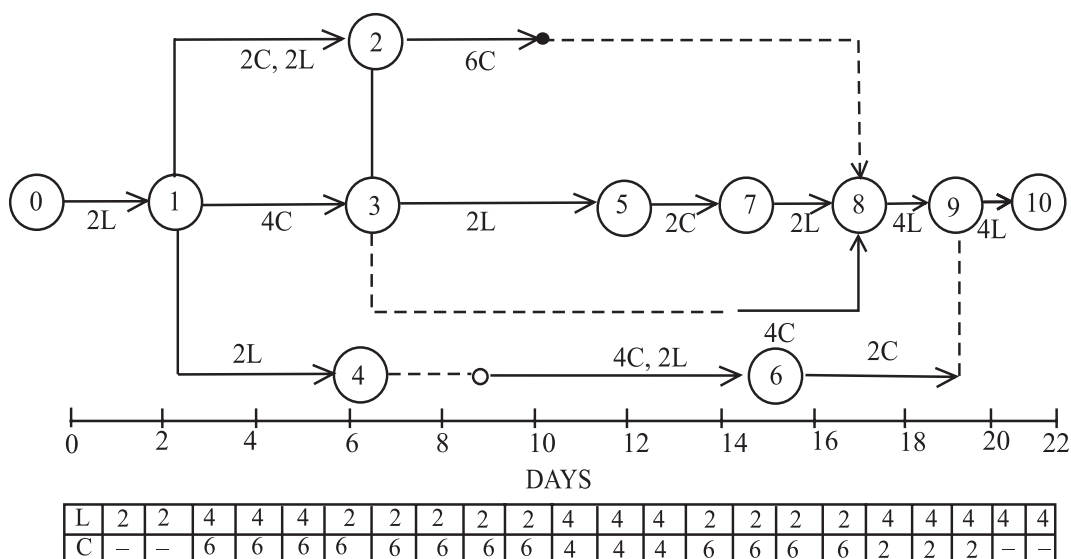


Figure 18



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As can be seen from the above figure the requirement for labourers is 4 and the requirement for carpenters reduces to 6 as against 14 carpenters originally estimated. Hence, by judiciously utilizing the float, we can smooth the demand on the resources.

15.8 RESOURCE LEVELLING

It is also a network technique which is used for reducing the requirement of a particular resource due to its paucity. The process of resource levelling utilize the large floats available on non-critical activities of the project and thus cuts down the demand on the resource. In resource levelling, the maximum demand of a resource should not exceed the available limit at any point of time. In order to achieve this, non critical activities are rescheduled by utilising their floats. *Some times, the use of resource levelling may lead to prolonging the completion time of the project.* In other words, in resource levelling, constraint is on the limit of the resource availability.

15.9 MISCELLANEOUS ILLUSTRATIONS

Illustration

A project consists of eight activities with the following relevant information:

Activity	Immediate Predecessor	Estimated Duration (Days)		
		Optimistic	Most Likely	Pessimistic
A	–	1	1	7
B	–	1	4	7
C	–	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D,E	3	6	15
H	F,G	1	2	3

- (i) Draw the PERT network and find out the expected project completion time.
- (ii) What duration will have 95% confidence for project completion?
- (iii) If the average duration for activity F increases to 14 days, what will be its effect on the expected project completion time which will have 95% confidence?



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Solution

(i) The required network is drawn below:

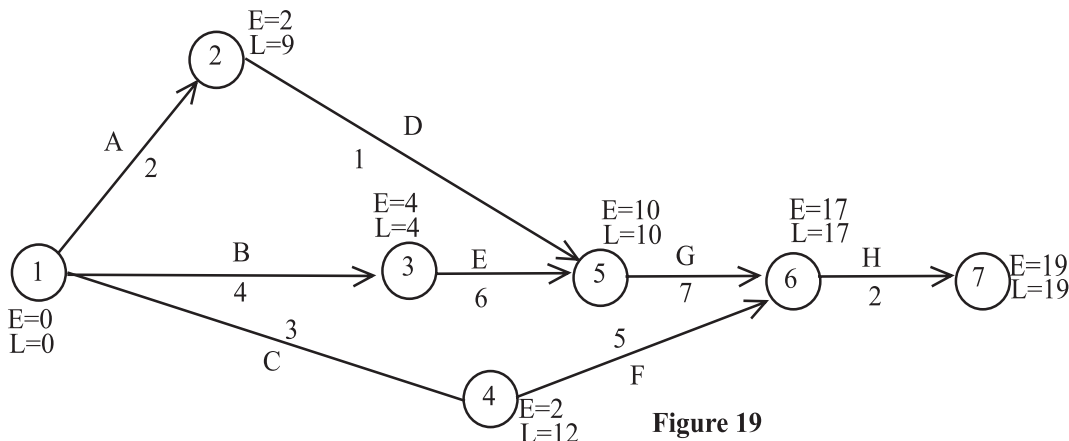


Figure 19

The expected time for each activity shown in the network above is calculated in the following table:

Activity	Estimated Duration (Days)			Expected Duration $\left(\frac{a + 4m + b}{b}\right)$	Variance $\left(\frac{b - a}{b}\right)$
	Optimistic a	Most likely m	Pessimistic b		
A 1-2	1	1	7	2	1
B 1-3	1	4	7	4	1
C 1-4	2	2	8	3	1
D 2-5	1	1	1	1	0
E 3-5	2	5	14	6	4
F 4-6	2	5	8	5	1
G 5-6	3	6	15	7	4
H 6-7	1	2	3	2	1/9

The critical path is given by 1-3-5-6-7 or B-E-G-H and the expected project completion time is 19 days. .

(ii) The variance for critical path is $1 + 4 + 4 + 1/9 = 82/9$

$$\text{Standard deviation of critical path} = \sigma_1 = \sqrt{\frac{82}{9}} = 3.02 \text{ (approx.)}$$

To calculate the project duration which will have 95% chances of its completion, we utilise the value of Z corresponding to 95% confidence which is 1.645.



Thus, $\frac{X - 19}{3.02} = 1.645,$

or $X = 1.645 \times 3.02 + 19 = 23.97 \text{ days} = 24 \text{ days}$

Hence, 24 days of project completion time will have 95% probability of its completion.

- (iii) If the average duration for activity F increases to 14 days, then the path 1-4-6-7 i.e., C-F-H will also become critical path with expected project completion time of 19 days. Now, activities C and F are also critical activities. Since we and, given only the average duration for activity F, it is assumed that the variance for this activity is zero. Further, since PERT analysis is based on the assumption that the activities are independent in terms of their variance, therefore, standard deviation of critical paths can be computed as:

$$\sigma = \sqrt{1+4+4+\frac{1}{9}+0+1} = \sqrt{10+\frac{1}{9}} = 3.18 \text{ approx}$$

We now wish to calculate the expected project completion time that will have 95% confidence level,

$P(Z < 1.645) = 0.95$

$$\frac{X - \mu}{\sigma} = \frac{X - 19}{3.18} = 1.645$$

or $X = 19 + 1.645 \times 3.18 = 24.23 \text{ days}.$

Hence the project duration of 24.23 days will have 95% confidence of completion.

Illustration

The following information is given below:

Activity	(1-2)	(2-3)	(2-4)	(3-5)	(4-6)	(5-6)	(5-7)	(6-7)
<i>Pessimistic time</i> (in weeks)	3	9	6	8	8	0	5	8
<i>Most likely time</i> (in weeks)	3	6	4	6	6	0	4	5
<i>Optimistic time</i> (in weeks)	3	3	2	4	4	0	3	2

Draw the network diagram for the above. Calculate :

- (i) Variance to each activity.
- (ii) Critical path and expected project length.
- (iii) The probability that the project will be completed in 23 weeks is drawn below:



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Solution

The required network is drawn below:

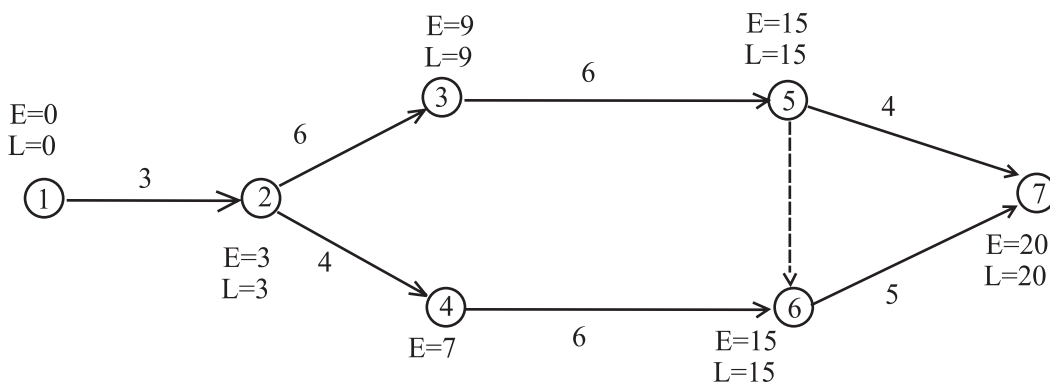


Figure 20

The expected time marked in the above network diagram for various activities is calculated in the table below:

Activities	Times (in weeks)			Expected time (weeks) $t_e = \frac{t_o + 4t_m + t_p}{6}$	Variance $\left(\frac{t_p - t_o}{6}\right)^2$
	Optimistic (t_o)	Most like (t_m)	Pessimistic (t_p)		
1-2	3	3	3	3	0
2-3	3	6	9	6	1
2-5	2	4	6	4	4/9
3-5	4	6	8	6	4/9
4-6	4	6	8	6	4/9
5-6	0	0	0	0	0
5-7	3	4	5	4	1/9
6-7	2	5	8	5	1

- Variance of each activities has been calculated in the last column of the above table.
- Critical path is given by 1-2-3-5-6-7 and the expected project length is 20 weeks.
- Variance of the critical path = $\sigma^2 = 0 + 4/9 + 0 + 1 = 22/9 = 2.444$

Mean = $x = 20$ weeks

To calculate the probability of completing the project in 23 weeks, we will first calculate the normal Z as below:

$$Z = \frac{D - x}{\sigma} = \frac{23 - 20}{\sqrt{2.444}} = 1.92$$

$P(x \leq 23) = P(z \leq 1.92) = 0.9726$ (from the Normal table)

Thus, the probability that the project will be completed in 23 weeks is 97.26%.



Illustration

A small maintenance project consists of the following twelve jobs whose precedence relations are identified with their node number:

Job (i-j)	(1,2)	(1,3)	(1,4)	(2,3)	(2,5)	(2,6)
Duration (in days)	10	4	6	5	12	9
Job (i-j)	(3,7)	(4,5)	(5,6)	(6,7)	(6,8)	(7,8)
Duration (in days)	12	15	6	5	4	7

(i) Draw an arrow diagram representing the project.

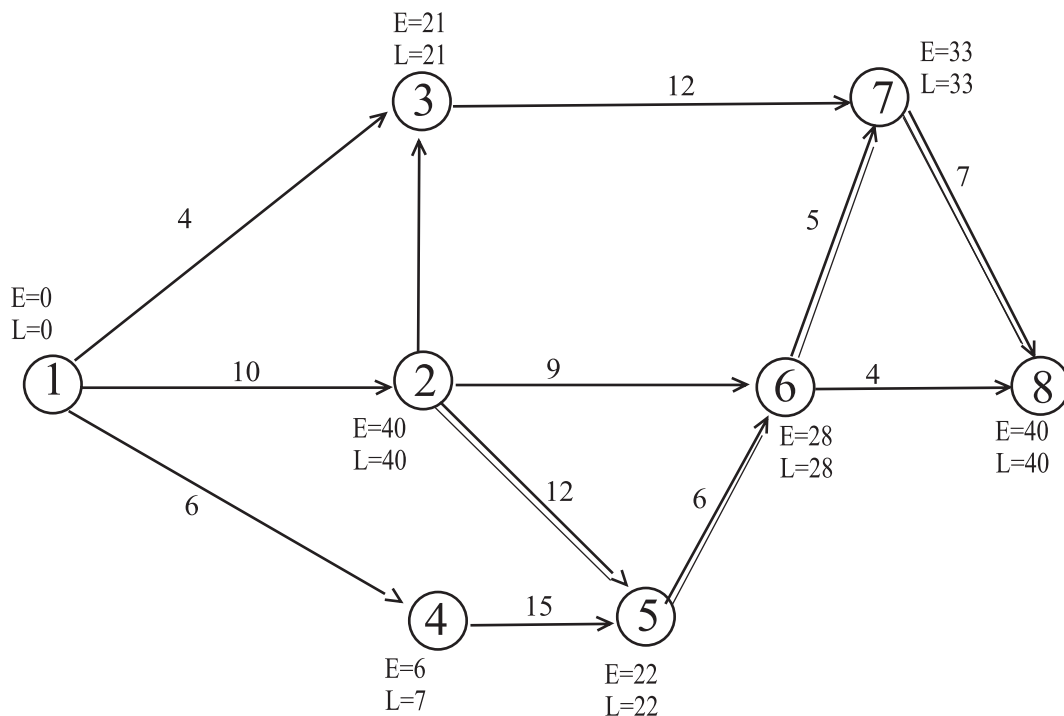


Figure 21

- (ii) Calculate earliest start, earliest finish, start and latest finish time for all jobs.
- (iii) Find the critical path and project duration.
- (iv) Tabulate total float, free float and independent float.

Solution : (i) The required arrow diagram is drawn in figure 21.



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(ii) The earliest start, earliest finish, latest start and latest finish time for all the jobs are calculated in the table given below:

Job (ij)	Dura- tion in days	Earliest time		Latest time		Slack of		Total float=TF =(TF- of head	Free float=FF Stock) of tail	Indep. float (FF-Slack)
		Start EST (E _i)	Finish EFT	Start LST (L _j)	Finish event (L _j -E _i) event	tail event (L _j -E _i) event	head =(LST- EAST)			
1-2	10	0	10	0	10	0	0	0	0	0
1-3	4	0	4	17	21	0	6	17	11	11
1-4	6	0	6	1	7	0	1	1	0	0
2-3	5	10	15	16	21	0	6	6	0	0
2-5	12	10	22	10	22	0	0	0	0	0
2-6	9	10	19	19	28	0	0	9	9	9
3-7	12	15	27	21	33	6	0	6	6	0
4-5	15	6	21	7	22	1	0	1	1	0
5-6	6	22	28	22	28	0	0	0	0	0
6-7	5	28	33	28	33	0	0	0	0	0
6-8	4	28	32	36	40	0	0	8	8	8
7-8	7	33	40	33	40	0	0	0	0	0

(iii) The critical path is 1 2 5 6 7 8 and the project duration is 40 days.

(iv) Total float, free float and independent float for various activities are calculated in the above table.

Illustration

A small project consists of seven activities for which the relevant data are given below:

Activity	Preceding Activities	Activities Duration (Days)
A	-	4
B	-	7
C	-	6
D	A,B	5
E	A,B	7
F	C,D,E	6
G	C,D,E	5

(i) Draw the network and find the project completion time.

(ii) Calculate total for each of the activities.

(iii) Draw the time scaled diagram.

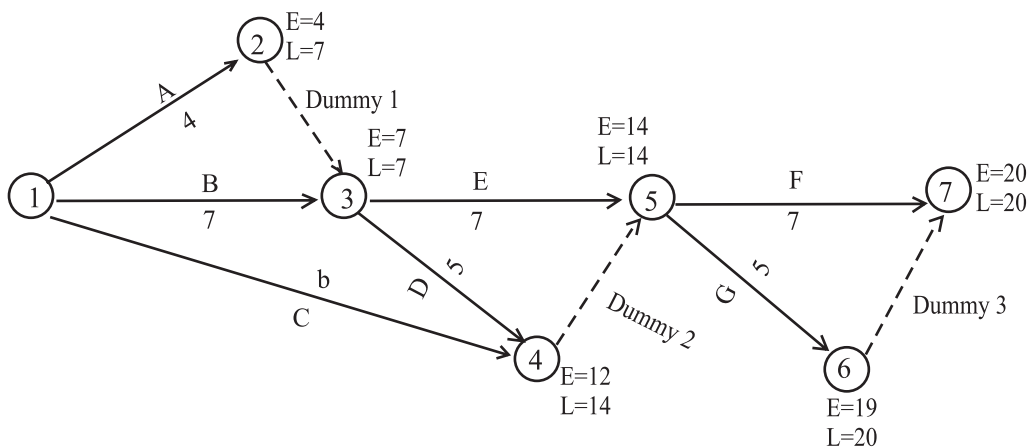


Figure 22

Solution

(i) The required network is given in figure 22.

The critical path is 1–3–5–7 with duration of 20 days. Thus, the project completion times is 20 days.

(ii) The total float for various activities is calculated as below:

Activity	Duration	Start		Finish		Total float	
		Earliest	Latest	Earliest	Latest		
A	1–2	4	0	3	4	7	3
B	1–3	7	0	0	7	7	0
C	1–4	6	0	8	6	14	8
Dummy 1	2–3	0	4	7	4	7	3
D	3–4	5	7	9	12	14	2
E	3–5	7	7	7	14	14	0
Dummy 2	4–5	0	12	17	12	14	2
F	5–7	6	14	17	20	20	0
G	5–6	5	14	15	19	20	1
Dummy 3	6–7	0	19	20	19	20	1



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(iii) The required time diagram is drawn below:

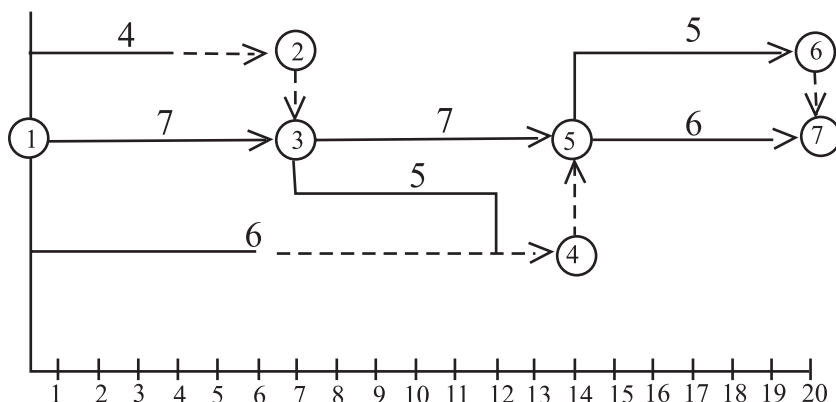


Figure 23

Illustration

A small project is having seven activities. The relevant data about these activities is given below:

Activity	Dependence	Normal duration (Days)	Crash duration (Days)	Normal cost (Rs.)	Crash cost (Rs.)
A	–	7	5	500	900
B	A	4	2	400	600
C	A	5	5	500	500
D	A	6	4	800	1,000
E	B,C	7	4	700	1,000
F	C,D	5	2	800	1,400
G	E,F	6	4	800	1,600

(i) Find out the normal duration and the minimum duration.

(ii) What is the percentage increase in cost to complete the project in 21 days ?



Solution

The network is given below:

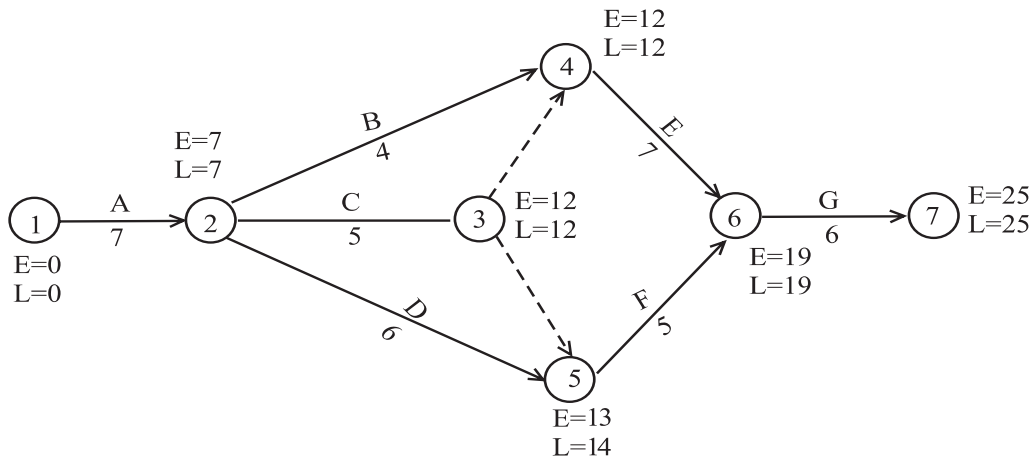


Figure 24

(i) The critical path of the project is A C E G or 1-2-3-4-6-7 with normal duration of 25 days.

The minimum duration of the project is 18 days.

(ii) The cost slope for various activities is given below:

Activity	Normal duration	Crash duration	Normal cost(Rs.)	Crash cost(Rs.)	Cost slope (Rs.)
A(1-2)	7	5	500	900	$\frac{900 - 500}{7 - 5} = 200$
B(2-4)	4	2	400	600	$\frac{600 - 400}{4 - 2} = 100$
C(2-3)	5	5	500	500	N.A
D(2-5)	6	4	800	1000	$\frac{1000 - 800}{6 - 4} = 100$
E(4-6)	7	4	700	1000	$\frac{1000 - 700}{7 - 4} = 100$
F(5-6)	5	2	800	1400	$\frac{1400 - 800}{5 - 2} = 200$
G(6-7)	6	4	800	1600	$\frac{1600 - 800}{6 - 4} = 400$
		Total	4,500		



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Step-1: Various paths of the network are given below:

1-2-3-4-6-7 with duration = 25 days

1-2-4-6-7 with duration = 24 days

1-2-3-5-6-7 with duration = 23 days

1-2-5-6-7 with duration = 24 days

In order to determine the cost of completing the project in 21 days, let us crash that activity on the critical path which has minimum cost slope. It can be seen that the minimum cost slope of Rs. 100 corresponds to activity E(4-6) and it lies on the critical path. Hence, we crash activity E (4-6) by 1 day at an additional cost of Rs. 100.

Step-2: Various paths now are

1-2-3-4-6-7 with duration = 24 days

1-2-4-6-7 with duration = 23 days

1-2-3-5-6- 7 with duration = 23 days

1-2-5-'6-7 with duration = 24 days

An examination of the above four paths clearly points out that there are two critical paths namely 1-2-3-4-6-7 and 1-2-5-6-7, each with duration = 24 days. To reduce the project duration by three days more, there are following possible combination of activities.

1. Crash activities 4 - 6 on the path 1-2-3-4-6-7 and 5 - 6 on the path 1-2-5-6-7 by one day each at an addition cost of Rs. 100 + Rs. 200 = Rs. 300
2. Crash activities 4-6 on path 1-2-3-4-6-7 and 2-5 on path 1-2-5-6-7 by one day each at an additional cost of Rs. 100 + Rs. 100 =Rs. 200
3. Crash activity 1-2 by one day at an additional cost of Rs. 200.

It can be observed that the additional cost of reducing the project duration by one day in combination 2 as well as combination 3 is Rs. 200. Hence any of these two can be selected for crashing. However, since crashing activity 1-2 by 1 day reduces the duration of all the paths by 1 day, we will crash it by 1 day. The project duration becomes = 23 days at an additional cost = Rs. 200.

Step-3: Crash activity 1-2 by 1 day further, it would reduce the project duration to 22 days at an additional cost = Rs. 200.

Step-4: Activity 1-2 can not be crashed further. So, we now select the combination 2 stated above for crashing. Crash activities 4-6 and 2-5 by one day each at an additional cost of Rs. 100 + Rs. 100 = Rs. 200.

The project duration now becomes equal to 21 days.

Hence, in order to complete the project in 21 days, an additional cost of Rs. 100 + Rs. 200+ Rs. 200 + Rs. 200 = Rs. 700 will be incurred.

The normal cost of completing the project in 25 days = Rs. 4500.



Hence, the percentage increase in cost to complete the project in

$$21 \text{ days} = \frac{\text{Rs. } 700}{\text{Rs. } 4500} \times 100 = 15.5\%$$

SUMMARY :

Three kinds of times estimates

- ✓ *The Optimistic Time Estimate*
- ✓ *The Pessimistic Time Estimate*
- ✓ *The Most Likely Time Estimate*

$$\text{Variance : } S_{t2} = \left(\frac{t_p - t_o}{6} \right)^2$$

Expected time : The expected time (t_e) is the average time taken for the completion of the job. By using beta-distribution, the expected time can be obtained by following formula.

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Activity Cost : It is defined as the cost of performing and completing a particular activity or task.

Crash Cost Cc : This is the direct cost that is anticipated in completing an activity within the crash time.

Crash time, Ct : This is the minimum time required to complete an activity.

Normal Cost, Nc : This is the lowest possible direct cost required to complete an activity.

Normal time, Nt : This is the minimum time required to complete an activity at normal cost.

Activity cost slope : The cost slope indicates the additional cost incurred per unit of time saved in reducing the duration of an activity

Resource smoothing

It is a network technique used for smoothing peak resource requirement during different periods of the project network. Under this technique the total project duration is maintained at the minimum level.

Resource levelling

It is also a network technique which is used for reducing the requirement of a particular resource due to its paucity. The process of resource levelling utilizes the large floats available on non-critical activities of the project and thus cuts down the demand on the resource. In resource levelling, the maximum demand of a resource should not exceed the available limit at any point of time. In order to achieve this, non-critical activities are rescheduled by utilizing their floats.



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SELF-EXAMINATION QUESTIONS

- Below we give a PERT network with the related time data. You are required to find the probability of completing the project in 27 days.

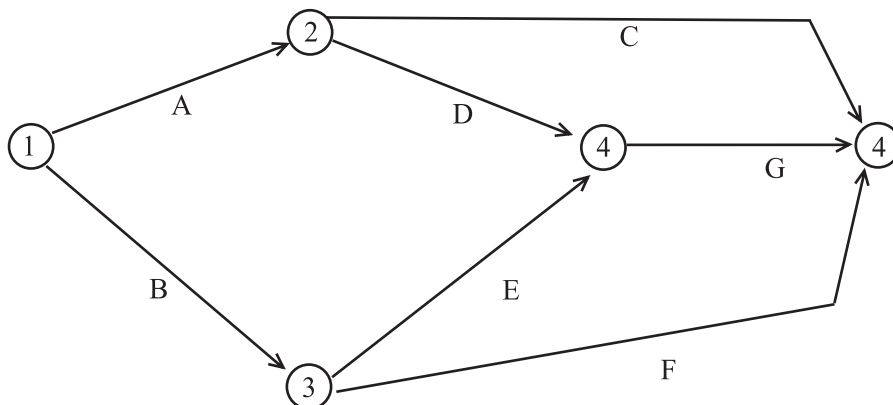


Figure 25

Activity	A	B	C	D	E	F	G
t_a	6	5	4	4	4	4	2
t_m	9	8	7	7	7	10	5
t_p	18	17	22	16	10	22	8

Answer : Critical path = 23 days, prob = 0.903.

- The following table gives the activities in a construction project and other relevant information:

Activity	Normal time Days	Crash time days	Normal cost (Rs.)	Total Crash cost (Rs.)
1-2	20	17	600	720
1-3	25	25	200	200
2-3	10	8	300	440
2-4	12	6	400	700
3-4	5	2	300	420
4-5	10	5	300	600

- Draw the network for the project.
- Find the total, free and independent floats for each activity.
- Using all information “crash” or “shorten” the project step by step until the shortest duration is reached. .



Programme Evaluation and Review Technique 15.41

3. Projects India Limited undertakes special contracts. The following table gives estimates of time and cost for activities involved in completing one contract which has been just offered to them.

Activity	Previous activities	Normal time days	Normal cost (Rs.)	Minimum time	Cost of minimum time
A	–	12	10,000	8	14,000
B	–	10	5,000	10	5,000
C	A	0	0	0	0
D	A	6	4,000	4	5,000
E	B,C	16	9,000	14	12,000
F	D	16	3,200	8	8,000
		60	31,200	44	44,000

“Previous activities” must be completed before the activity in question can be started. The minimum time represents the shortest time in which the activity can be completed given the use of special costly methods of operation. Assume that it is possible to reduce the normal time to the minimum time in small steps and the extra cost incurred will be proportional to the time saved.

- (a) Draw a network diagram and identify the critical path for normal procedures
- (b) Recommend what programme be followed if the job must be completed in 30 days and calculate the total cost for the programme.
4. Alpha construction company has an opportunity to submit a bid for the construction of a new apartment building. From specifications provided by the developer, a PERT network for the project has been developed, and is shown below. Also shown for each activity are estimates of most optimistic, most likely, and most pessimistic completion times in weeks (a, m and b, respectively).

<i>Activity</i>	<i>a</i>	<i>m</i>	<i>b</i>	<i>Activity Description</i>
A	5	8	17	Excavate basement
B	5	8	11	Building concrete elevator tower
C	3	5	7	Pour concrete foundation
D	0	0	0	
E	6	9	18	Excavate parking ramp area
F	5	8	17	Install temporary manual elevator
G	5	7	12	Erect main building
H	0	0	0	
I	0	0	0	



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J	0	0	0	
K	4	7	10	Install automatic elevator
L	7	10	31	Complete interior work
M	4	6	11	Erect parking ramp.

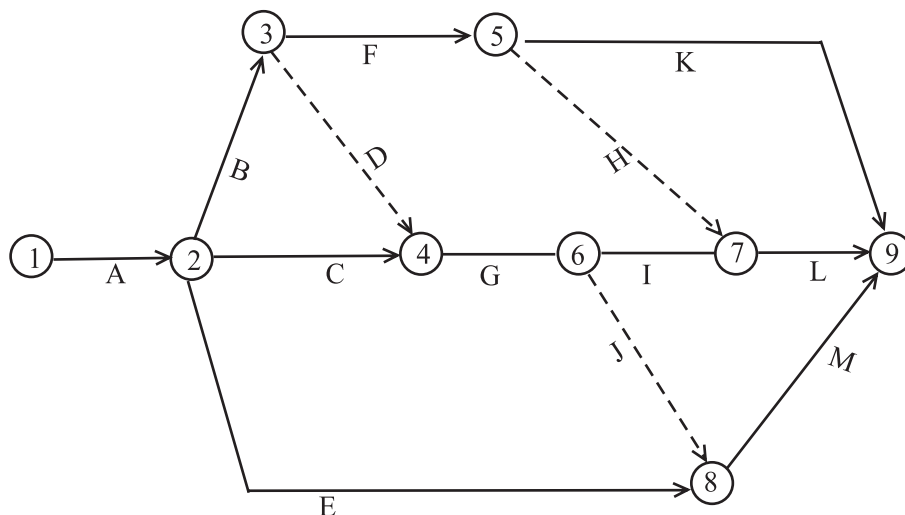


Figure 26

Required:

- Determine the critical path and its S.D.
- Alpha's management policy with respect to submitting bids is to bid minimum amount that will provide a 92% probability of at best breaking even. Materials for this project will cost Rs. 900,000 and all other costs will vary at a rate Rs. 10,000 for every week spent working on the project. What amount should be bid under this policy?
- Assume that Alpha's bid was accepted and that the project has been in progress for 20 weeks. Activities A, Band C have been completed. Activities E, F and G are in progress, with the following estimates made of time required to complete them:

Activity	a	m	b
E	1	2	3
F	3	5	10
G	3	6	9

No change has been made in the time estimates for activities K, Land M. Draw a revised PERT network representing the remainder of the project excluding completed activities. Determine the critical path for the remainder of the project and the remaining project completion time.



Programme Evaluation and Review Technique 15.43

5. Table on next page shows jobs, normal and crash time and cost for a project:
 Indirect cost for the project is Rs. 300 per day.

- (i) Draw the network of the project.
- (ii) What is the normal duration and cost of project?
- (iii) Find the optimum duration and minimum project cost.
- (iv) If all activities are crashed, what will be the project duration and corresponding cost?

Job <i>i-j</i>	Normal		Crash	
	Time Days	Cost (Rs.)	Time (Days)	Cost (Rs.)
1-2	6	1,400	4	1,900
1-3	8	2,000	5	2,800
2-3	4	1,100	2	1,500
2-4	3	800	2	14,000
3-4	Dummy	-	-	-
2-5	6	900	3	1,600
4-6	10	2,500	6	3,500
5-6	3	500	2	800

Answer. (ii) 20 days, Rs. 15,200 (iii) 12 days, Rs.13,132 (iv) 12 days. Rs. 13,500

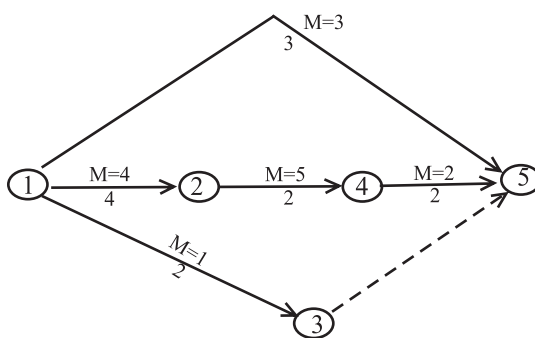


Figure 27

In the network above are indicated the duration in days, and number of men required for each activity. You are required to find the number of days in which the project can be completed with (i) unlimited resources and (ii) with a total of only 5 men available.

Answer : (i) 8 days, (ii) 9 days



15.44 Advanced Management Accounting

7. Construct a PERT network for the following project, and determine the critical path.

<i>Activity</i>	<i>Times</i>	<i>Immediate Predecessor</i>
A	4	None
B	1	None
C	1	A
D	1	B
E	6	C
F	8	E
G	2	F
H	4	F
I	1	H
J	5	G,I
K	5	D,G
L	7	K